Evolution & Learning in Games
Econ 243B

Jean-Paul Carvalho

Intergenerational Cultural Transmission as an Evolutionary Game
James D. Montgomery
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Motivation

- What is the relationship between Bisin-Verdier style cultural evolution and the evolutionary dynamics we have studied?

- How can we extend the Bisin and Verdier framework to $n$ traits?
The Model

- Agents form a continuum and possess one of $n$ cultural traits, $i \in \{1, \ldots, n\}$.

- Each parent (asexually) produces one child, socializes them and then dies.

- A parent with trait $i$ will have a child with trait $j \neq i$ with probability:
  \[ P_{ij} = (1 - \tau_i)q_j \]  
  (1)
  and a child with trait $i$ with probability:
  \[ P_{ii} = \tau_i + (1 - \tau_i)q_i. \]  
  (2)
The Cultural Evolutionary Dynamic

- In discrete-time:

\[ q_i(t + 1) = \sum_j q_j(t)P_{ji}. \]  
\[ (3) \]

- Substituting (1) and (2) into (3):

\[ q_i(t + 1) = q_i(t)\left[\tau_i + (1 - \tau_i)q_i(t)\right] + \sum_{j \neq i} q_j(t)(1 - \tau_j)q_i(t) \]

\[ = q_i(t)\tau_i + (1 - \tau_i)q_i(t)^2 + q_i(t)\sum_{j \neq i} q_j(t)(1 - \tau_j) \]

\[ = q_i(t) + q_i(t)\left[\tau_i - \sum_j q_j(t)\tau_j\right]. \]
\[ (4) \]
The Cultural Evolutionary Dynamic

Taking the continuous-time limit, we have:

\[ \dot{q}_i = q_i \left[ \tau_i - \sum_j q_j \tau_j \right] \tag{5} \]

for all \( i = 1, \ldots, n \).

Clearly, when the \( \tau_s \) are exogenous, the dynamic converges to a monomorphic distribution centered on trait \( \arg \max_{i \in \{1, \ldots, n\}} \{ \tau_i \}_{i=1}^{n} \).
Endogenous Socialization

Let us proceed along the lines of Bisin and Verdier (2000) except with \(n\) traits and a quadratic socialization cost:

\[
\max_{\tau_i} \sum_j P_{ij} V_{ij} - \frac{1}{2} (\tau_i)^2,
\]

where \(V_{ij}\) is an \(i\) type’s payoff from having a child with trait \(j\).

The FOC is:

\[
\tau_i^* = (1 - q_i) V_{ii} - \sum_{j \neq i} q_j V_{ij}
\]

\[
= V_{ii} - \sum_j q_j V_{ij}
\]

\[
= \sum_j q_j [V_{ii} - V_{ij}]
\]

\[
\equiv \sum_j q_j \Delta_{ij}.
\]
Substituting into the dynamic (5), we have:

\[ \dot{q}_i = q_i \left[ \sum_j q_j \Delta_{ij} - \sum_j q_j \sum_k q_k \Delta_{jk} \right] \quad (8) \]

for all \( i = 1, \ldots n \).

Interpreting \( \Delta_{ij} \) as the payoff from playing strategy \( i \) against \( j \), this is simply the replicator dynamic operating on a particular population game: random matching to play an \( n \times n \) coordination game (CHECK).
Convergence Results

▶ Suppose that $\Delta_{ij} = \Delta_i$ for all $j \neq i$ (and $\Delta_{ii} = 0$), i.e. each group is intolerant of all other traits to an equal degree.

▶ Then this is a strictly stable game. Hence we know:
  
  ▶ There is a unique Nash equilibrium (distribution of traits), which is globally asymptotically stable.
  
  ▶ Every trajectory of the replicator dynamic in the interior of the $n$-dimensional simplex converges to this state.

▶ More generally, we can cast this as a potential game and exploit the corresponding results on such games.