Renewable Energy, Electric Grid Integration, and Distributed Control

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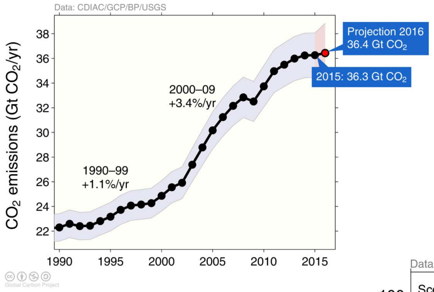
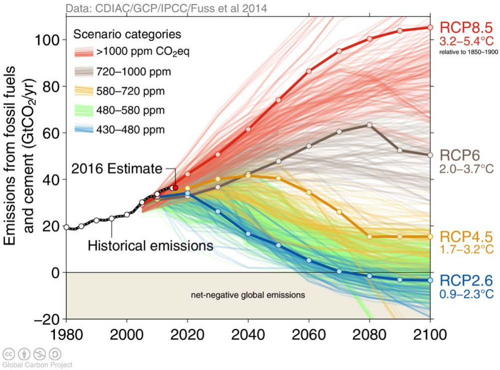
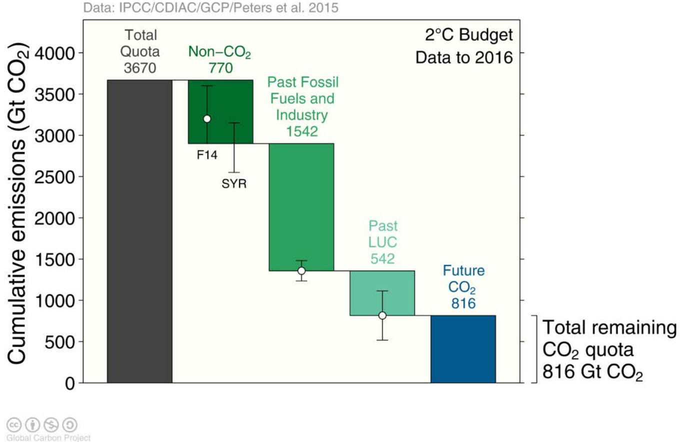
CO2 Emissions and Temperature Change Scenarios





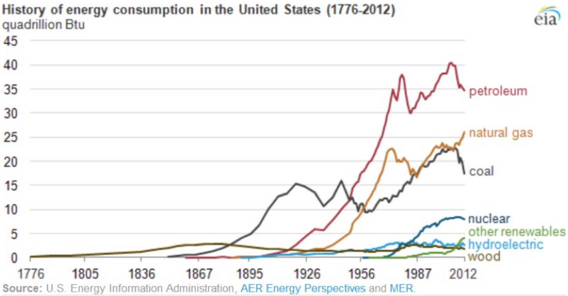
Carbon Quota

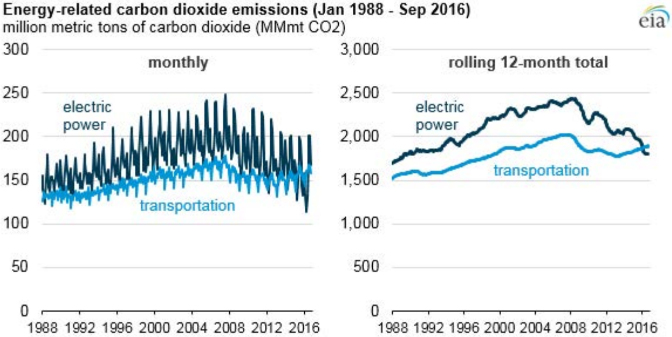


“Complete decarbonisation of the electricity sector is a necessary, but not sufficient, condition to limit average temperature increases to below 2°C ... This paper shows that even under the very optimistic assumption that other sectors reduce emissions in line with a 2°C target, no new emitting electricity infrastructure can be built after 2017 for this target to be met, unless other electricity infrastructure is retired early or retrofitted with CCS.”

Pfeiffer et al, 2016, Oxford University

US Energy Consumption



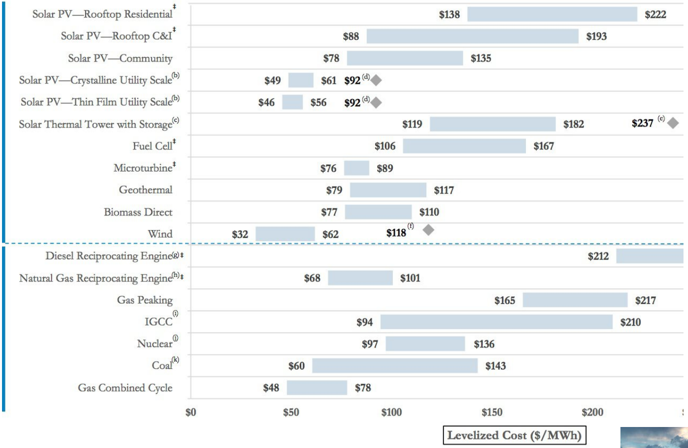


Major Energy Transitions are Slow

- Coal: 5% to 50% in 60 years starting in 1840
- Oil: 5% to 40% in 60 years starting in 1915
- Natural gas: 5% to 25% in 60 years starting in 1930
- Modern renewables $\approx$ 5%

1.2 billion people lack access to electricity
2.8 billion people rely on biomass for cooking and heating

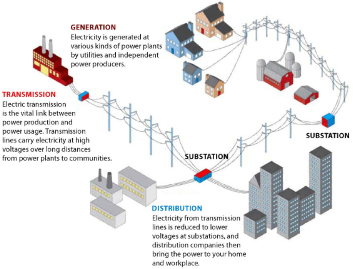
Wind and Solar are Becoming Economical

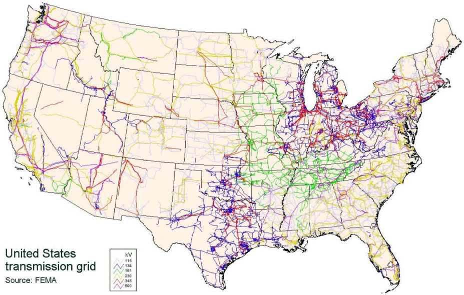


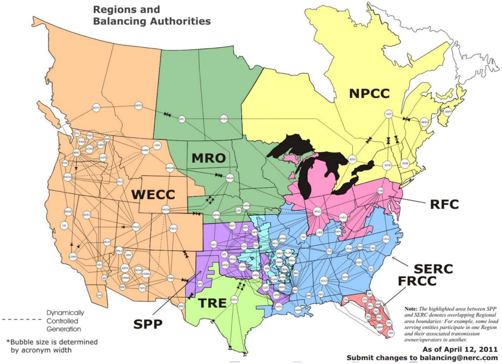
Source: Lazard

US Transmission Grid and Balancing Areas

Source: DOE







Electric Grid Overview

- Generation, transmission, distribution, consumption
- Goals: economic, reliable, and sustainable
- Key Constraint: Balancing: Supply $=$ Demand at each time instant
- Deregulation and markets
- Elaborate socio-technical control system - multiple time and spatial scales
- Robust to uncertainties in generation, transmission, distribution, load

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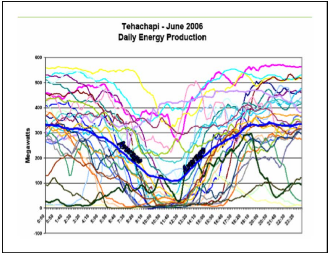
How to Power the World without Fossil Fuels

Mark Jacobson says he can run the planet solely on wind, water and solar energy. First stop: New York State

By Mark Fischetti



Could we operate an electric grid with only wind and solar as energy generation?

- Key Constraint: Balancing: Supply = Demand at time instant
- Current paradigm: Adjust supply to match random demand
- Challenge: Inherent uncertainty and uncontrollability of generation
- Cheap storage would be a game changer
- Paradigm shift: Adjust demand to match random supply
- One tool: Leverage demand flexibility

System Scenario

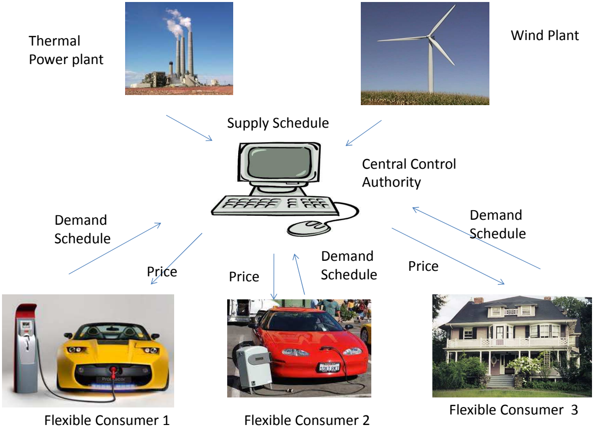


Figure: System Model

Demand Side Management

- Goal: exploit the inherent *flexibility* of electric loads
- Two approaches: incentive based and price based
- We focus on the price based approach
- Centralized control of loads — ex: direct load control
- Key Issues:
  - Consumer preferences on control and privacy
  - Large computational burden

Based on joint work with Pratyush Chakraborty

Demand Side Management

- Distributed control
  - The central authority sends the control signal, e.g., price, to the consumers.
  - The consumers optimize their consumption schedules accordingly.
- Analysis settings
  1. Consumers are price-takers
  2. Consumers are price-anticipators

Price-Anticipating Consumers

- Game theoretic modeling to capture the price anticipating behavior under distributed control
- Key Question: What is the loss of efficiency in terms of social objective by distributed control as compared with centralized control?
- Price of Anarchy (PoA) : Worst-case ratio of the objective function value of an equilibrium solution of the game to that of a centralized optimal solution.

Notation

- The time slots denoted by $t \in \mathcal{T} = (1, 2, ..., T)$
- Flexible consumers denoted by $i \in \mathcal{N} = (1, 2, ..., N)$
- $q_i(t)$: The power consumption of the i-th consumer at time t
- $\mathbf{q}_i := (q_i(t) : t \in \mathcal{T})$: the power demand vector of the i-th consumer over the time period T
- $c(t)$: The total scheduled power generation of all the thermal power plants at time t
- $w(t)$: The total predicted power supply of the renewable generators at time t
- $n(t)$: Total power consumption of uncontrolled loads/consumers at time t
- $U_i(\mathbf{q}_i)$: The utility for consuming power q_i in monetary unit.
- U_i is assumed to be a concave, strictly increasing and continuously differentiable function.

Balancing Constraints

- Supply=Demand

$$c(t) + w(t) = n(t) + \sum_{i=1}^{N} q_i(t) \quad \forall\ t\ \in\ \mathcal{T}. \tag{1}$$

- Define $v(t) := c(t) + w(t) - n(t)$.
- $\mathbf{v} := (v(t) : t \in \mathbb{T})$ denotes the net generation available for flexible demand over the time period T
- Assumption: $v(t) > 0$ for all t.
- Goal is to adjust the flexible demand to achieve

$$v(t) = \sum_{i=1}^{N} q_i(t) \quad \forall\ t\ \in\ \mathbb{T}. \tag{2}$$

Operational Constraints on Flexible Demand

Minimum, maximum, total consumption constraints can be expressed by the following linear inequalities

$$\mathbf{H}_i \mathbf{q}_i \leq \mathbf{b}_i, \;\; i \in \mathcal{N}, \tag{3}$$

where $\mathbf{H}_i \in \mathbb{R}^{M \times T}$, $\mathbf{b}_i \in \mathbb{R}^{M}$

Centralized Control

The centralized control problem is defined as follows:

$$\max_{\mathbf{q}_i} \left\{ \sum_{i \in \mathcal{N}} U_i(\mathbf{q}_i) : \mathbf{q}_i \in \mathcal{S} \right\} \tag{4}$$

where the feasible space

$$\mathcal{S} := \left\{ \mathbf{q}_i \in \mathbb{R}^T : \mathbf{v} - \sum_{i \in \mathcal{N}} \mathbf{q}_i = \mathbf{0}, \mathbf{b}_i - \mathbf{H}_i \mathbf{q}_i \geq \mathbf{0} \right\} \tag{5}$$

is assumed to be nonempty.

Distributed Control with Price Taking Consumers

- Assumption: The consumers are price takers, selfish and rational.
- Notation: $k_i(t) =$ the *monetary expenditure* for power demand of i-th consumer at time t.
- The control authority, after obtaining the values of $c(t)$, $w(t)$ and $n(t)$, calculates $v(t)$ and broadcasts its value to all the consumers.
- Each consumer then submits its bid $k_i(t)$ to the authority for all t. The authority calculates $\sum_{i=1}^{N} k_i(t)$ and sets price as

$$p(t) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)} \tag{6}$$

Distributed Control with Price Takers

- Inspired by the *proportional allocation mechanism*, the allocation of $q_i(t)$ to the i-th consumer is given by

$$q_i(t) = \frac{k_i(t)}{p(t)} \tag{7}$$

  for all i and t.
- The distributed control problem for price takers is given by

$$\max_{\mathbf{q}_i} \left\{ U_i(\mathbf{q}_i) - \mathbf{p}^\top \mathbf{q}_i : \mathbf{q}_i \in \mathcal{S}_i^{pt} \right\}, \ \ i \in \mathcal{N} \tag{8}$$

  where the set of feasible power consumptions is

$$\mathcal{S}_i^{pt} := \{\mathbf{q}_i : \mathbf{b}_i - \mathbf{H}_i \mathbf{q}_i \geq 0\}, \ \ i \in \mathcal{N}.$$

Competitive Equilibrium

Definition

The set $\{(\mathbf{q}_i^E, \mathbf{p}^E) : i \in \mathcal{N}\}$ is a *competitive equilibrium* if each consumer selects its consumption vector $\mathbf{q}_i^E$ by solving the optimization problem (8) for the price vector $\mathbf{p}^E$ and the price vector $\mathbf{p}^E$ satisfies (6)-(7).

Theorem

The set $\{(\mathbf{q}_i^E, \mathbf{p}^E) : i \in \mathcal{N}\}$ is a competitive equilibrium if and only if the set of consumptions $\{\mathbf{q}_i^E : i \in \mathcal{N}\}$ is a solution to the centralized control problem.

Algorithm I

1. The control authority computes the vector of available net generation $\mathbf{v}$.
2. Each consumer $i \in \mathcal{N}$ sends her initial bid $\mathbf{k}_i$ to the authority.
3. The control authority calculates the price vector $\mathbf{p}$ according to (6) and broadcasts it to all the consumers.
4. The consumers update their consumption maximizing their net utilities by solving (8) and communicate the new bids $\mathbf{k}_i$ obtained using (7) to the control authority.
5. Return to step 3 until convergence is attained.

Price Anticipating Users

- Price anticipating consumers will try to account for the impact of their decisions on $p(t)$ and adjust their decisions accordingly.
- Suppose they know that $p(t)$ is set by the formula $p(t) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)}$.
- We model the resulting situation as a noncooperative game as each consumer's optimization problem depends on sum of monetary values of all other consumers.

Setup

- Given a vector $\mathbf{x}$, Let $\mathbf{D}(\mathbf{x})$ denote a diagonal square matrix whose diagonal entries are elements of $\mathbf{x}$.
- Slight abuse of notation: $D^{-1}(x) := (D(x))^{-1}$
- Let $\mathbf{k}_{-i} = \{\mathbf{k}_j : j \in \mathcal{N} \backslash \{i\}\}$ denote the collection of monetary expenditure vectors of all consumers other than the consumer i.
- Note that $\mathbf{p}$ and $\mathbf{q}_i$ can be expressed as functions of $\mathbf{k}_i$ as follows:

$$\begin{aligned}
\mathbf{p}(\mathbf{k}_i; \mathbf{k}_{-i}) &= \mathbf{D}^{-1}(\mathbf{v}) \sum_{j \in \mathcal{N}} \mathbf{k}_j \\
\mathbf{q}_i(\mathbf{k}_i; \mathbf{k}_{-i}) &= \mathbf{D}^{-1}(\mathbf{p}(\mathbf{k}_i; \mathbf{k}_{-i}))\mathbf{k}_i \\
&= \mathbf{D}^{-1}(\sum_{i \in \mathcal{N}} \mathbf{k}_i)\mathbf{D}(\mathbf{v})\mathbf{k}_i
\end{aligned}$$

Let us define the search space:

$$\mathcal{S}_i^{pa}(\mathbf{k}_{-i}) := \left\{ \mathbf{k}_i : \mathbf{b}_i - \mathbf{H}_i \mathbf{D}^{-1}(\sum_{i \in \mathcal{N}} \mathbf{k}_i)\mathbf{D}(\mathbf{v})\mathbf{k}_i \geq \mathbf{0} \right\}$$

Game Formulation

The game of energy consumption is as follows:

1. Players: Set of N consumers
2. Strategy: Consumer i's strategy $\mathbf{k}_i$
3. Payoff: For each consumer i, the payoff is given by

$$\max_{k_i} \left\{ U_i(\mathbf{D}^{-1}(\sum\nolimits_{j \in \mathcal{N}} \mathbf{k}_j)\mathbf{D}(\mathbf{v})\mathbf{k}_i) - \mathbf{1}^\top \mathbf{k}_i : \mathbf{k}_i \in \mathcal{S}_i^{pa}(\mathbf{k}_{-i}) \right\} \tag{9}$$

where $\mathbf{v}$ is the available generation for flexible consumption

Nash Equilibrium

- The Nash equilibrium for the distributed control problem with price anticipators is the set of expenditures $\{\mathbf{k}_i^G : i \in \mathcal{N}\}$ such that

$$\begin{aligned} U_i(\mathbf{q}_i(\mathbf{k}_i^G, \mathbf{k}_{-i}^G)) - \mathbf{1}^\top \mathbf{k}_i^G \geq U_i(\mathbf{q}_i(\mathbf{k}_i, \mathbf{k}_{-i}^G)) - \mathbf{1}^\top \mathbf{k}_i, \\ \mathbf{k}_i \in \mathcal{S}_i^{pa}(\mathbf{k}_{-1}^G), \;\; i \in \mathcal{N}. \end{aligned} \tag{10}$$

Theorem (Existence of Nash equilibrium)

The non-cooperative game has a Nash equilibrium if the search space is nonempty.

Price of Anarchy is Less Than 25%

Theorem

Let $\{\mathbf{q}_i^C : i \in \mathcal{N}\}$ *be a solution of the centralized problem (4) and* $\{\mathbf{q}_i^G : i \in \mathcal{N}\}$ *a Nash equilibrium for the distributed problem with price anticipating consumers. Let PoA be defined by:*

$$\mathrm{PoA} := \frac{\sum_{i\in\mathcal{N}} U_i(\mathbf{q}_i^G)}{\sum_{i\in\mathcal{N}} U_i(\mathbf{q}_i^C)}.$$

then $\mathrm{PoA} \geq 0.75$*.*

Special Cases

Corollary

If all the consumers have same utility function, i.e., $U_i = U$, there is no efficiency loss at Nash equilibrium solution, i.e. PoA is 1.

Corollary

Suppose $\mathbf{q}_i = \mathbf{0}$ for all $i \in \mathcal{N}$ belongs to the set of load operational constraints, then the PoA approaches 1 as the number N of flexible consumers goes to infinity.

A Demand Response Game

- Here the price is determined by the desired energy consumption.
- Rest of the formulation is similar.
- We develop a bound on the PoA of the resulting game.

Formulation

- Price of electricity - $p(\sum_{i=1}^{N} q_i(t))$.
- The price is assumed to be a convex, continuously differentiable and monotonically increasing function.
- The constraint inequalities

$$\mathbf{H}_i \mathbf{q}_i \leq \mathbf{b}_i, \ \ i \in \mathcal{N}, \tag{11}$$

where $\mathbf{H}_i \in \mathbb{R}^{M \times T}$, $\mathbf{b}_i \in \mathbb{R}^M$

Centralized Control: Ideal Case

- The central authority aims to maximize the total utility of the consumers minus their overall cost of consumption.
- Thus, the control authority's objective is to

$$\underset{\mathbf{q}_i}{maximize} \quad V(D) = \sum_{i=1}^{N} U_i(\mathbf{q}_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} q_i(t)) \sum_{i=1}^{N} q_i(t) \qquad (12)$$

subject to (11).

Decentralized Game Formulation

- As the price is a function of power consumption of all consumers, the decentralized situation is modeled as a non-cooperative game. The game is defined as follows:
  1. Players: Set of N consumers$=\mathcal{N}$
  2. Strategy: Consumer i's strategy $\mathbf{q}_i$
  3. Payoff: For each consumer i, the payoff is to maximize

  $$L_i(\mathbf{q}_i, \mathbf{q}_{-i}) = U_i(\mathbf{q}_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} q_i(t))q_i(t) \tag{13}$$

  subject to (11),
- This game is called "demand response game" and is denoted by $\mathcal{G}$. Nash equilibrium is the strategy q_i^* such that,

  $$L_i(\mathbf{q}_i^*, \mathbf{q}_{-i}^*) \geq L_i(\mathbf{q}_i, \mathbf{q}_{-i}^*) \quad \forall i \ \in \ \mathcal{N}. \tag{14}$$

General Payoff Maximization Game

A payoff maximization game is defined as follows.

- Set of players $\mathcal{N} = \{1, 2, ..., N\}$ and is indexed by i.
- Player i's strategy vector is $\mathbf{q}_i$.
- $D = \{\mathbf{q}_i : i \in \mathcal{N}\}$ denotes the set of all players' strategies.
- The payoff of a player is $L_i(D) = L_i(\mathbf{q}_i, \mathbf{q}_{-i})$.
- The objective function is $V(D)$ where $V : 2^D \to \mathbb{R}$ is a general function defined over all subsets of D.

Valid Monotone Utility Game

1. A payoff maximization game is called a valid utility game if it satisfies the following three properties:

- V is submodular , i.e., for any $A \subset A' \subset D$ and any element $\mathbf{a} \in D \setminus A'$

$$V(A \cup \{\mathbf{a}\}) - V(A) \geq V(A' \cup \{\mathbf{a}\}) - V(A') \tag{15}$$

- The objective value of a player is at least her added value for the societal objective, i.e.,

$$L_j(D) \geq V(D) - V(D - \mathbf{q}_j) \tag{16}$$

where $\mathbf{q}_j$ is the strategy vector of a player j.

Valid Monotone Utility Game

- The total value for the players is less than or equal to the total societal value, i.e.,

$$\sum_{i=1}^{N} L_i(D) \leq V(D) \tag{17}$$

2. A payoff maximization game is called a monotone game if for all $A \subseteq A' \subseteq D$,

$$V(A) \leq V(A') \tag{18}$$

Demand Response Game and Monotone Games

- Assumption **A1:** The utility function of each consumer satisfies

$$U_j(\mathbf{q}_j) \geq \sum_{t=1}^{T} \{p(k + q_j(t))(k + q_j(t)) - p(k)k\} \tag{19}$$

where $k = \sum_{i=1, i \neq j}^{N} q_i^{max}$.

Demand Response Game is a Valid Monotone Utility Game

Theorem

Consider the demand response game defined by $\mathcal{G} = \langle \mathcal{N}, \{\mathbf{q}_i\}, \{L_i(\mathbf{q}_i, \mathbf{q}_{-i})\}\rangle$ with objective function $V(D)$ as defined by (12). If the assumption **A1** *holds then this game is a valid monotone utility game.*

PoA for Nash Equilibrium Solution

- Consider a general payoff maximization game which satisfies (17). This game is called a (λ, μ) smooth game if

$$\sum_{i=1}^{N} L_i(D^*) \geq \lambda V(D^{'}) - \mu V(D^*) \quad (20)$$

where D^* and $D^{'}$ are any two solution sets of the game. $PoA = \frac{\lambda}{1+\mu}$

- It is shown in Roughgarden [2012] that a valid monotone utility game is (1,1) smooth and the lower bound on the PoA is 1/2.

Corollary

The demand response game $\mathcal{G} = \langle \mathcal{N}, \{\mathbf{q}_i\}, \{L_i(\mathbf{q}_i, \mathbf{q}_{-i})\}\rangle$ is a (1,1) smooth game. Moreover, the lower bound of the price of anarchy of a pure Nash equilibrium is at least 1/2.

Coarse Correlated Equilibrium-a Weaker Notion of Equilibrium

Notations:

- σ_i: probability distribution over the strategy space of a player for all $i \in \mathcal{N}$
- $\sigma = \Pi_{i=1}^{N} \sigma_i$: product probability distribution

System Structure:

- A benevolent mediator draws a strategy set D from σ and privately recommends the strategy $\mathbf{q}_i$ to each player.

Definition:

- The coarse correlated equilibrium of the demand response game can be defined as probability distribution σ over strategies that satisfies

$$\mathbb{E}_{D^* \sim \sigma}(L_i(\mathbf{q}_i^*, \mathbf{q}_{-i}^*)) \geq \mathbb{E}_{D \sim \sigma}(L_i(\mathbf{q}_i, \mathbf{q}_{-i}^*)) \tag{21}$$

PoA for Coarse Correlated Equilibrium Solution

- Nash equilibrium for a game may exist, but there can be a number of reasons for which the players may not reach an equilibrium.
- The coarse correlated equilibrium for a game always exist and easy to compute.
- **Intrinsic Robustness property of the PoA**: The bound derived via smoothness argument extends with no quantitative degradation to other weaker equilibria notions (Roughgarden [2012]).

Corollary

Consider the demand response game $\mathcal{G} = \langle \mathbb{N}, \{\mathbf{q}_i\}, \{L_i(\mathbf{q}_i, \mathbf{q}_{-i})\} \rangle$ *that reaches a coarse correlated equilibrium. Then*

$$\mathbb{E}_{D \sim \sigma}(V(D)) \geq 0.5 V(D^O) \tag{22}$$

where σ *is the coarse correlated equilibrium and* D^O *is the optimal solution of the centralized control problem.*

Conclusions

- Grid integration of renewable energy offers an interesting domain for distributed control and optimization
- Game theory offers a rich set of ideas for understanding distributed control
- Energy systems present a unique mix of science, engineering, economics and social policy
- Decarbonization of the energy system remains a true grand challenge for humanity