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THE BECKER–DEGROOT–MARSCHAK MECHANISM AND GENERALIZED UTILITY THEORIES: THEORETICAL PREDICTIONS AND EMPIRICAL OBSERVATIONS

ABSTRACT. Karni and Safra [8] prove that the Becker–DeGroot–Marschak mechanism reveals a decision maker's true certainty equivalent of a lottery if and only if he satisfies the independence axiom. Segal [17] claims that this mechanism may reveal a violation of the reduction of compound lotteries axiom. This paper empirically tests these two interpretations. Our results show that the second interpretation fits better with the collected data. Moreover, we show by means of some nonexpected utility examples that these results are consistent with a wide range of functionals.

Keywords: Becker–DeGroot–Marschak mechanism, nonexpected utility models.

1. INTRODUCTION

The Becker–DeGroot–Marschak (henceforth BDM) mechanism [1] provides an economic incentive for decision makers to reveal their true (subjective) value of assets. According to this mechanism, after the decision maker states his selling price of an asset, he is presented with a random 'offer price'. If this price exceeds his selling price, he sells the asset for this offer price. Otherwise, he keeps the asset. As is explained in the next section, it is the decision maker's optimal strategy to announce his true price of the asset.

Several experiments show that when the assets are lotteries, it may happen that although a subject prefers lottery X to lottery Y, he will set a higher selling price on Y than on X.[1] It turns out, however, that the claim that the BDM mechanism necessarily reveals decision makers' true values of lotteries depends on the assumption that they are expected utility maximizers. Although many experiments have shown that people often violate expected utility, it is not clear which axioms of this theory are violated. Karni and Safra [8] claim that the independence axiom is violated, while Segal [17] offers an alternative

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interpretation of the mechanism, according to which decision makers do not obey the reduction of compound lotteries axiom. In this paper we try to check the validity of these two interpretations of the mechanism. Safra, Segal, and Spivak [15] prove several implications of the first interpretation of the mechanism, the one analyzing it as a violation of the independence axiom. To check one of these predictions, we conducted an experiment. Our results do not support this prediction. Moreover, we show that our results conform with the second interpretation, modeling the mechanism as a violation of the reduction of compound lotteries axiom.

The paper is organized as follows: in Section 2 we present the mechanism and the two interpretations. Section 3 contains the experiment. In Section 4 we show that several nonexpected utility models may agree with the theoretical implications of the second interpretation of the mechanism. Section 5 concludes the paper with a brief summary.

2. THE MECHANISM

Let L be the set of lotteries with outcomes in the $[-M, M]$ segment. The lottery $X = (x_1, p_1; \ldots; x_n, p_n) \in L$ yields x_i dollars with probability p_i, $i = 1, \ldots, n$. The cumulative distribution function of X is denoted by F_X. On L there exists a complete and transitive preference relation $\gtrsim$. Say that $X > Y$ if $X \gtrsim Y$ but not $Y \gtrsim X$, and $X \sim Y$ if $X \gtrsim Y$ and $Y \gtrsim X$. The function $V: L \to \mathbb{R}$ represents the preference relation $\gtrsim$ if $V(X) \geqslant V(Y) \Leftrightarrow X \gtrsim Y$. We assume throughout that the preference relation $\gtrsim$ is continuous (in the topology of weak convergence), and satisfies the first-order stochastic dominance axiom. That is, $[\forall x\, F_X(x) \leqslant F_Y(x)$ and there exists y such that $F_X(y) < F_Y(y)] \Rightarrow X > Y$. Under these conditions there exists a representation V of the relation $\gtrsim$.

The certainty equivalent of a lottery X, denoted $CE(X)$, is defined as the number x that makes the decision maker indifferent between X and the degenerate lottery $\delta_x = (x, 1)$ in which x is received with probability equal to 1. The existence and uniqueness of $CE(X)$ follow by the continuity and first-order stochastic dominance axioms. By the transitivity axiom it follows that the ordering of lotteries by the preference relation $\succsim$ is the same as their ordering by their certainty equivalents. In other words, $CE(X)$ is a representation function of $\succsim$. Becker, DeGroot, and Marschak [1] suggested the following mechanism to derive the decision maker's certainty equivalent of a lottery X. Let him hold a ticket for this lottery, and ask him to announce the price for which he is willing to sell this ticket. Denote this price by s_X. Next, draw at random an 'offer price' out of the $[a, b]$ interval. If the offer price exceeds the selling price s_X or is equal to it, the decision maker sells his lottery ticket for the offer price. If the offer price is less than the selling price, the decision maker keeps the ticket and plays the lottery.

This mechanism seems to force the decision maker to reveal his true certainty equivalent of the lottery X. Suppose that $s_X > CE(X)$. If the offer price is between the certainty equivalent and the selling price, the decision maker will have to play the lottery X, although he would rather have sold it. If the offer price is higher than s_X, the decision maker sells his ticket regardless of whether he said s_X or $CE(X)$, and if the offer price is less than the certainty equivalent of X, he keeps it either way. Similarly, he cannot gain by declaring a selling price below his certainty equivalent. In particular, if the offer price is between the selling price and the certainty equivalent, he will be forced to sell the ticket against his will. It thus appears to follow that the decision maker's optimal strategy is to announce his true certainty equivalent of the lottery.

Let X and Y be two lotteries such that $X \succ Y$. Since $CE(X) > CE(Y)$, a transitive decision maker will set $s_X > s_Y$. Famous experiments by Lichtenstein and Slovic [9] and Grether and Plott [6] show that people do not always conform with this. They found, among other things, that most subjects prefer the lottery $(4, \frac{35}{36}; -1, \frac{1}{36})$ to the lottery $(16, \frac{11}{36}; -1.5, \frac{25}{36})$, but many of them set a higher selling price on the second. This preference reversal phenomenon seems to indicate that people have non-transitive preferences. Grether and Plott's results are different from former experiments, because they were the first to explicitly use the BDM mechanism to obtain the selling prices.

Two recent studies suggest that the assumption that the BDM mechanism provides an incentive to state the true certainty equivalents of lotteries depends on the assumption that decision makers are expected utility maximizers. Karni and Safra [8] claim that the above interpretation of the way a person responds to the mechanism assumes the independence axiom, and Segal [17] suggests an interpretation of the response to the mechanism where the reduction of compound lotteries axiom is violated, but compound independence holds.[2] Let L_2 be the set of two-stage lotteries. The lottery $A=(X_1, q_1; \ldots; X_m, q_m) \in L_2$ yields a ticket for lottery $X_i=(x_1^i, p_1^i; \ldots; x_{n_i}^i, p_{n_i}^i)$ with probability q_i, $i=1, \ldots, m$. By the reduction of compound lotteries axiom, the compound lottery A is as attractive as the simple reduced lottery $R(A)=(x_1^1, q_1 p_1^1; \ldots; x_{n_1}^1, q_1 p_{n_1}^1; \ldots; x_1^m, q_m p_1^m; \ldots; x_{n_m}^m, q_m p_{n_m}^m)$. The compound independence axiom assumes that the two-stage lottery $(X, p; Z, 1-p)$ is preferred to the two-stage lottery $(Y, p; Z, 1-p)$ if and only if X is preferred to Y. In particular, since $X \sim \delta_{CE(X)}$, it follows that the lottery $A=(X_1, q_1; \ldots; X_m, q_m) \in L_2$ satisfies $A \sim (CE(X_1), q_1; \ldots; CE(X_m), q_m)$. Note that the right-hand side of this last equivalence is a simple lottery. For a more detailed discussion of these axioms, see [18] and [10].

Let $X=(4, \frac{35}{36}; -1, \frac{1}{36})$ and let $a=0$ and $b=9.99$. If the decision maker announces the selling price s_X, then he will participate in the following two-stage lottery. With probability $s_X/10$ the offer price is less than the selling price, and the decision maker will have to play the lottery. The offer price equals each of the numbers s_X, $s_X+0.01, \ldots, 9.99$ with probability $1/1000$, and in each of these cases he wins the offer price. The decision maker thus faces the two-stage lottery A, where

$$A=\left(\left(4, \frac{35}{36}; -1, \frac{1}{36}\right), \frac{s_X}{10}; \delta_{s_X}, \frac{1}{1000}; \delta_{s_X+0.01}, \frac{1}{1000}; \ldots; \delta_{9.99}, \frac{1}{1000}\right).$$

The decision maker has to find the optimal selling price s_X that will maximize his value of the lottery A. By the compound independence axiom,

$$A \sim \left(CE\left(4, \frac{35}{36}; -1, \frac{1}{36}\right), \frac{s_X}{10}; s_X, \frac{1}{1000}; s_X+0.01, \frac{1}{1000}; \ldots; 9.99, \frac{1}{1000}\right).$$

It follows from the first-order stochastic dominance axiom that the decision maker's optimal strategy is to set $s_X = CE(X)$. However, as pointed out by Karni and Safra [8], if the decision maker does not satisfy the compound independence axiom, there is no reason to assume that $s_X = CE(X)$ maximizes his value of the lottery

$$R(A) = \left(4, \frac{35s_X}{360}; -1, \frac{s_X}{360}; s_X, \frac{1}{1000}; \right.$$
$$\left. s_X + 0.01, \frac{1}{1000}; \ldots; 9.99, \frac{1}{1000}\right).$$

Karni and Safra actually prove that the decision maker's optimal strategy is to announce $s_X = CE(X)$ for all lotteries X if and only if he is an expected utility maximizer. Moreover, they show by means of a nonexpected utility example that the decision maker may display a preference reversal, preferring the lottery $(4, \frac{35}{36}; -1, \frac{1}{36})$ to the lottery $(16, \frac{11}{36}; -1.5, \frac{25}{36})$, but setting $s_X < s_Y$. In the sequel we refer to this interpretation of the mechanism as interpretation 1.

A different interpretation of the mechanism is suggested in Segal [17]. Let $\langle a, b\rangle$ be the uniform distribution on the $[a, b]$ interval. According to this interpretation, when he sets the selling price s_X, the decision maker perceives the mechanism as the two-stage lottery

$$\left(CE(X), \frac{s_X}{10}; \langle s_X, 9.99\rangle, 1 - \frac{s_X}{10}\right).$$

If the decision maker satisfies the compound independence axiom, but not the reduction axiom, then it is possible to find nonexpected utility preferences such that $X \succ Y$ but $s_X < s_Y$, displaying a preference reversal. We refer to this interpretation as interpretation 2.

Although the BDM mechanism does not reveal a nonexpected utility maximizer's true certainty equivalent of a lottery, it may still be useful in revealing some information about his preferences. However, this depends on the correct interpretation of the mechanism. Safra, Segal, and Spivak [14] shows that under interpretation 1, there is a strong connection between the conditions implying the Allais paradox and the preference reversal phenomenon. It turns out that both are connected to Machina's [12] hypothesis II. The same authors [15, Proposition 2] also prove that the optimal selling price and the certainty equivalent of a lottery are always on the same side of its expected value. In other words, if the decision maker is risk averse, then $s_X \leqslant E[X]$, and if he is risk loving, then $s_X \geqslant E[X]$. This is the case for all preference relations, provided interpretation 1 is valid. Different results are obtained from interpretation 2. The claim that the selling price and the certainty equivalent of a lottery must be on the same side of its expected value is not obtained under this interpretation (see Section 4 below).[3] Of course, this does not mean that they have to be on opposite sides of the expected value of the lottery. This observation enables us to discriminate between the two interpretations. Our empirical results show that about one third of our subjects set selling prices and certainty equivalents on opposite sides of the expected values of lotteries. Although this does not prove the validity of interpretation 2, it does at least indicate that interpretation 1 is not necessarily the only possible one.

3. THE EXPERIMENT

An experiment was conducted to examine the empirical validity of Proposition 2 in [15] that the certainty equivalent and the selling price of a lottery are on the same side of its expected value. Undergraduate students in introductory economics classes at the University of California, Los Angeles served as volunteer subjects during one of their tutorial sessions.

Subjects were randomly divided into two groups of 75 and 74, respectively. Each group was asked certainty equivalents and selling prices for one of two different sets of lotteries. (Two sets of lotteries were needed so no one subject had too many questions to answer.) In the first part of the session, subjects read instructions indicating they were going to be asked a series of hypothetical questions involving chances of getting different monetary amounts. They were asked to respond to the questions based on their own opinions about these monetary decisions. The instructions for the certainty equivalent elicitation were:

> The questions in this section require you to state the amount of money which makes you indifferent between a ticket to a lottery and a fixed monetary amount. You are to write in the blank in each question the dollar and cents amount which makes you like the lottery just as much as you'd like the cash amount.

In this part of the experiment, certainty equivalents were elicited for 14 lotteries, shown in Table I. All lotteries had two outcomes, with the smaller outcome always being $0. Group A got the first 7 lotteries, AA–AG; and Group B got the last 7 lotteries, BA–BG. The first lottery, AA, has a 0.65 probability of receiving $4.25, and a 0.35 probability of getting $0, etc.

Next, the BDM mechanism was explained verbally to the subjects. Then they read written instructions, and worked on sample selling price problems with feedback from the experimenter. For example, in one of the sample Becker–DeGroot–Marschak questions, the lottery was (0, 0.75; 8, 0.25) and the range $[a, b]$ was equal to [0, 10]. Finally, subjects used the BDM mechanism and wrote down their selling prices for different lotteries with different ranges for the random offer price. The 55 questions of this part are presented in Table I. These questions are constructed by taking the 14 lotteries from the first part of the experiment and linking them up with 3 to 5 different ranges for the

TABLE I

Lotteries and monetary scales used in experiment

Lottery $X = (0, 1-p; x, p)$				Upper Bound of Monetary Scale				
Label	x	p	$E[X]$ ($= px$)	$10	$20	$40	$75	$100
			Group A					
AA	4.25	0.65	2.76	√	√	√	√	√
AB	8.00	0.35	2.80	√	√	√	√	√
AC	18.00	0.15	2.70		√	√	√	√
AD	34.00	0.08	2.72			√	√	√
AE	13.00	0.90	11.70		√	√	√	√
AF	18.00	0.65	11.70		√	√	√	√
AG	33.00	0.35	11.55			√	√	√
			Group B					
BA	6.50	0.90	5.85	√	√	√	√	√
BB	9.00	0.65	5.85	√	√	√	√	√
BC	17.00	0.35	5.95		√	√	√	√
BD	39.00	0.15	5.85			√	√	√
BE	18.00	0.90	16.20		√	√	√	√
BF	25.00	0.65	16.25			√	√	√
BG	46.00	0.35	16.10			√	√	√

monetary scale. The lower bound of the range is always 0, and the upper bound can take the value \$10, \$20, \$40, \$75, and \$100. The number of ranges a lottery is linked with depends on the meaningfulness of each potential range for the lottery. For example, lottery AC has a maximum outcome of \$18. It was not linked with the monetary scale with upper bound of \$10, because even according to interpretation 1, a subject might legitimately wish to state a minimum selling price higher than the upper bound of this monetary scale. This design allows comparison of a subject's certainty equivalent for a lottery with the minimum selling price and the lottery's expected value.[4]

Let *CE*, *SP*, and *EV* be the certainty equivalent, selling price, and the expected value of a lottery, respectively. By Proposition 2 in Safra, Segal, and Spivak [15] $CE > EV \Rightarrow SP \geqslant EV$, $CV < EV \Rightarrow SP \leqslant EV$, and $CE = EV \Rightarrow SP = EV$. We therefore counted the number of times responses agreeing with these predictions occurs among all answers. Overall, as shown in Table II, only 68.5% of possible comparisons obeyed this proposition. For 31.1% of possible comparisons, both *CE* and *SP* were greater than *EV*. For 31.8% of comparisons, *CE* and *SP* were less than *EV*, and all were equal for 3.1% for the comparisons. All other cases disagree with Proposition 2 of [15]. For 30.3% of the possible comparisons, *CE* and *SP* were strictly on opposite sides of *EV*. Note that we have aggregated the data over all monetary ranges, so a single lottery is counted a few times for a subject, since it appears with 3 to 5 different monetary ranges. As the certainty equivalent and the price were not uniformly on the same side of the expected value, our results do not support Proposition 2 in [15], which resulted from interpretation 1 of the BDM mechanism. As we show in the next section, our results are consistent with interpretation 2, which does not imply such an ordering.

TABLE II

Number (and percentage) of comparisons by category

	$CE > EV$	$CE = EV$	$CE < EV$
$SP > EV$	1275 (31.1%)	46 (1.1%)	890 (21.7%)
$SP = EV$	72 (1.8%)	127 (3.1%)	28 (0.7%)
$SP < EV$	353 (8.6%)	4 (0.1%)	1302 (31.8%)

4. NUMERICAL EXAMPLES

Interpretation 1 of the BDM mechanism predicts that the selling price and the certainty equivalent of a lottery are on the same side of its expected value. This prediction is not implied by interpretation 2. In this section we will demonstrate this by examples.

According to interpretation 2, if the decision maker announces a selling price s of a lottery X, then he faces the lottery

$$Y_s = \left(CE(X), \frac{s-a}{b-a}; CE\left(\langle s, b\rangle, \frac{b-s}{b-a}\right)\right) \tag{1}$$

where $\langle s, b\rangle$ is the uniform distribution on $[s, b]$. The decision maker wants to maximize, with respect to s, the value of $V(Y_s)$. Denote the optimal value of s by s^*. To prove that Proposition 2 of Safra, Segal, and Spivak [15] may not be satisfied under interpretation 2, we show that for some lottery X, $s^* < CE(X)$ and $s^* > CE(X)$ can occur for both risk averse and risk loving preferences. We demonstrate it for three types of non-expected utility functionals; anticipated utility (AU) [13], quadratic utility (Q) [4], and weighted utility (WU) [2] functionals. All of them are axiomatic extensions of expected utility theory, and all are based on weakening the independence axiom. This axiomatization feature distinguishes them from the class of Fréchet differentiable functionals, introduced by Machina [12], where the independence axiom is completely abandoned. Also, all of the above three theories are transitive, unlike, for example, regret theory [11]. Here we would like to keep expected utility as our benchmark model, therefore we adopt only guarded departures from it. Secondly, even though we just consider these three classes of functionals, they are general enough. All the other known classes of transitive utility functionals based on axiomatizations either are subclasses of them or have substantial overlap with one of them. Weighted utility, and under some trivial conditions, quadratic utility functionals, are Fréchet differentiable. However, anticipated utility is not Fréchet differentiable [3]. The three functionals' forms are given by:

4.1. *Anticipated Utility*

$$AU(X) = \int_{-M}^{M} u(x)\, \mathrm{d}f(F_X(x)) \tag{2}$$

where u is strictly increasing and continuous, and $f: [0, 1] \to [0, 1]$ is strictly increasing, onto, and continuous. This functional represents risk aversion (seeking) if and only if u and f are both concave (convex). For this, and other properties of the functional, see [3] and [16].

4.2. *Weighted Utility*

$$WU(X) = \frac{\int_{-M}^{M} w(x)v(x)\,\mathrm{d}F_X(x)}{\int_{-M}^{M} w(x)\,\mathrm{d}F_X(x)} \tag{3}$$

where $v: [-M, M] \to \mathbb{R}$ is increasing and $w: [-M, M] \to \mathbb{R}_{++}$ is non-vanishing. This functional satisfies the betweenness axiom, that is, if $F \gtrsim G$, then for every $\alpha \in [0, 1]$, $F \gtrsim \alpha F + (1 - \alpha)G \gtrsim G$ (see [2] and [5]). Also let $h(x, s) = w(x)[v(x) - v(s)]$. By [2], WU represents risk aversion (seeking) if $h(x, s)$ is concave (convex) in x for every s.

4.3. *Quadratic Utility*

$$Q(X) = \int_{-M}^{M} \int_{-M}^{M} \varphi(x, y)\,\mathrm{d}F_X(x)\,\mathrm{d}F_X(y) \tag{4}$$

where $\varphi(x, y)$ is symmetric, nondecreasing in both arguments, and for $x > y$, $\varphi(x, x) > \varphi(y, y)$. For an axiomatization of this functional see [4]. This functional represents risk aversion (seeking) if both $\partial^2\varphi/\partial x^2$ and $\partial^2\varphi/\partial y^2$ are non-positive (non-negative). Next, we show that all three families of functionals are consistent with the certainty equivalent and the selling price being on the same or on opposite sides of the expected value of a lottery. According to the second interpretation, the decision maker wants to maximize, with respect to the selling price s, the value of the lottery Y_s (see (1)). We obtain

$$AU(Y_s) = \begin{cases} u(CE(X))f(\frac{s-a}{b-a}) \\ \quad + u(CE(\langle s, b \rangle))[1 - f(\frac{s-a}{b-a})] & \text{if } CE(\langle s, b \rangle) \geqslant CE(X) \\ \\ u(CE(\langle s, b \rangle))f(\frac{s-a}{b-a}) \\ \quad + u(CE(X))[1 - f(\frac{s-a}{b-a})] & \text{if } CE(\langle s, b \rangle) < CE(X) \end{cases}$$

where for every lottery Z, $CE(Z) = u^{-1}[AU(Z)]$. It is always optimal for the decision maker to set s such that $CE(\langle s, b\rangle) \geqslant CE(X)$. Indeed, if $CE(\langle s^*, b\rangle) < CE(X)$, then

$$\left(CE(X), \frac{CE(X)-a}{b-a}; \langle CE(X), b\rangle, 1-\frac{CE(X)-a}{b-a}\right) >$$
$$(CE(X), 1) > \left(CE(X), \frac{s^*-a}{b-a}; \langle s^*, b\rangle, 1-\frac{s^*-a}{b-a}\right)$$

In that case s^* is not the optimal selling price, as $CE(X)$ is better for the decision maker.

Consider now the following four examples. In all of them the lottery X is $(4.25, 0.65; 0, 0.35)$ with the expected value \$2.76 and the range $[a, b] = [0, 10]$. The optimal selling prices were found by using the *Math*Cad software:

Case					
Risk attitude	Order of EV, CE, s^*	$u(x)$	$f(p)$	$CE(X)$	s_X^*
averse	$s^* > EV > CE$	x	$p^{0.2}$	\$0.81	\$3.05
averse	$EV > s^* > CE$	x	$p^{0.6}$	\$1.99	\$2.57
seeking	$CE > s^* > EV$	x	p^2	\$3.73	\$3.54
seeking	$CE > EV > s^*$	x	$0.35p^{40} + 0.65p$	\$3.28	\$2.62

Consider now the lotteries $X(t) = \delta_x + t\tilde{\varepsilon}$ with $E[\tilde{\varepsilon}] = 0$ (hence $EV(X(t)) = x$). Suppose that the function $V(Y_s)$ has a unique maximum. Then s^* is a continuous function of t. Therefore, if for $t = 0$, i.e., when $X(t) = \delta_x$, the solution of the maximization problem

$$\max_s V\left(x, \frac{s-a}{b-a}; \langle s, b\rangle, \frac{b-s}{b-a}\right)$$

is above (below) x, then it will also be above (below) x for a sufficiently small t. For the weighted utility and quadratic functional we therefore present examples based on degenerate lotteries of the form δ_x. By this we do not claim that interpretation 2 is valid even in the case when the lottery is degenerate. All we do is prove that because of continuity, Proposition 2 of [15] does not hold *even for non-degenerate lotteries* under this interpretation. If the decision maker is using a weighted utility functional, then he wants to maximize

$$WU(Y_s) = \frac{(s-a)w(CE(X))v(CE(X)) + (b-s)w(CE(\langle s, b\rangle))v(CE(\langle s, b\rangle))}{(s-a)w(CE(X)) + (s-a)w(CE(\langle s, b\rangle))}$$

where for every lottery Z, $CE(Z) = v^{-1}[WU(Z)]$. Suppose now that $X = \delta_x$, $a = 0$ and $b = 10$. The following four examples show that here, too, all four cases are possible.

Case					
Risk attitude	Order of EV and s^*	x	$w(x)$	$v(x)$	s_X^*
averse	$s^* > EV$	\$0.37	$e^{-0.1x}$	$\frac{x}{10}$	\$0.46
averse	$EV > s^*$	\$5.00	$e^{-0.3x}$	$\frac{x}{10}$	\$4.96
seeking	$s^* > EV$	\$1.00	$e^{0.1x}$	$\frac{x}{10}$	\$1.21
seeking	$EV > s^*$	\$4.00	$e^{0.2x}$	$\frac{x}{10}$	\$3.93

If the decision maker is using a quadratic utility functional, then he wants to maximize

$$Q(Y_s) = \left(\frac{s-a}{b-a}\right)^2 \varphi(\alpha, \alpha) + 2\left(\frac{s-a}{b-a}\right)\left(\frac{b-s}{b-a}\right)\varphi(\alpha, \beta) + \left(\frac{b-s}{b-a}\right)^2 \varphi(\beta, \beta)$$

where α satisfies $\varphi(\alpha, \alpha) = Q(X)$ and β satisfies $\varphi(\beta, \beta) = Q(\langle s, 10\rangle)$. Let

$$\varphi^1_{a,b}(x, y) = \frac{1}{4}\left[e^{ax} + e^{ay} + e^{(a-b)x}e^{by} + e^{bx}e^{(a-b)y}\right]$$

$$\varphi^2_{a,b}(x, y) = \frac{1}{4}\left[x^a + y^b + x^{a-b}y^b + x^b y^{a-b}\right]$$

Suppose, again, that $X = \delta_x$, $a = 0$, and $b = 10$, and consider the following four examples:

Case				
Risk attitude	Order of EV and s^*	x	$\varphi(x, y)$	s_X^*
averse	$s^* > EV$	$1.99	$1 - \varphi^1_{-3,-1.5}$	$2.05
averse	$EV > s^*$	$0.50	$\varphi^2_{1,0.1}$	$0.48
seeking	$s^* > EV$	$6.99	$\varphi^2_{6,2}$	$7.03
seeking	$EV > s^*$	$4.10	$\varphi^1_{0.2,1}$	$3.80

5. CONCLUDING REMARKS

In this paper we tested some of the implications of the Becker–DeGroot–Marschak mechanism. Although it is now clear that this mechanism does not necessarily reveal subjects' true certainty equivalents of lotteries if decision makers do not maximize expected utility, the mechanism may still be used to get some information about their preferences. Such an analysis was offered by Safra, Segal, and Spivak [15], but their results crucially depend on Karni and Safra's [8] specific interpretation of the mechanism as a two-stage lottery. It turns out that an alternative two-stage interpretation of the mechanism, suggested by Segal [17], yields different predictions from those of Safra, Segal and Spivak [15]. More specifically, the two interpretations differ in the prediction that the certainty equivalent of a lottery and its selling price should be on the same side of the lottery's expected value. Our experiment shows that although many subjects often behave that way, there is nevertheless a substantial proportion of responses with the certainty equivalent and the selling price on different sides of a lottery's expected value. This kind of behavior is consistent with Segal's interpretation [17], where the reduction of compound lotteries axiom is rejected, but not with Safra *et al.* [15], where the reduction axiom is used and the independence axiom is relaxed. It should be noted that many empirical results indicating nonexpected utility behavior can be modeled as violations of the reduction of compound lotteries axiom (see [18]). Our results may thus conform with other violations of expected utility theory.

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NOTES

[1] This is known as the preference reversal phenomenon. See next section for references.
[2] Holt [7] suggests that the preference reversal phenomenon may result from the fact that subjects play for real only a few of their choices. This argument was checked and rejected by Starmer and Sugden [19].
[3] Nevertheless, the strong connection between the Allais paradox and the preference reversal phenomenon prevails under this interpretation as well (see Wang [20]).
[4] Group A got 28 questions and Group B got 27 questions. To counteract possible order effects, subjects were randomly assigned to one of two random orderings of the questions and data from the two distinct orders have been pooled.

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