The role of generalized utility theories in descriptive, prescriptive, and normative decision analysis*

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A number of new theories for decision making under risk have been proposed which relax some properties required by von Neumann-Morgenstern expected utility theory. This paper provides a framework for exploring the usefulness of these theories in the domains of descriptive, prescriptive, and normative decision analysis.

1. Introduction

A number of new theories for decision making under risk relax some requirements of von Neumann–Morgenstern [65] expected utility theory. This paper compares the characteristics of these generalized utility theories and those of expected utility theory and explores the usefulness of these characteristics in the domains of descriptive, prescriptive, and normative decision analysis. An overview of expected utility and generalized utility theories is in Machina [56] and reviews are in Fishburn [31], Machina [55], Sarin [61], and Weber and Camerer [67].

This section briefly discusses the three linked purposes of decision analysis. Section 2 evaluates the performance of expected utility based on normative, prescriptive, and descriptive criteria. Section 3 describes the preliminary evidence on the potential of generalized utility in descriptive decision analysis. Sections 4 and 5 evaluate generalized utility in normative and prescriptive uses. Section 6 presents a summary.

The classification of decision analysis activity into three distinct purposes will highlight some key distinctions among the desirable characteristics in the different domains. Of course, in any one preference theory or decision analysis application there may be a mixture of descriptive, prescriptive, and normative purposes. More discussion on the three purposes is in Brown [8] and Keller [41].

The goal of *normative decision analysis* is to develop models for optimal decision making which have logically, rationally, and morally compelling properties. Sets of compelling properties (or axioms) can be combined to identify various normative preference models, such as von Neumann–Morgenstern expected utility theory for decision making under risk. The appropriateness of these models can then be evaluated on the basis of mathematical correctness, elegance, parsimony, logical coherence, and philosophical arguments in favor of the normative appeal of the properties.

The purpose of *descriptive decision analysis* is to develop models of decision making which provide valid descriptions of actual decision making behavior. Such models can be evaluated on predictive ability, face validity, psychological insights into choice behavior, enhancement of understanding of cognitive processes, elegance.

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parsimony, etc. Much behavioral decision research has focused on identifying deviations of people's actual judgments or choices from those which would result from normative models. (Economists develop *positive* models which are intended to describe the behavior of economic agents, so in this sense such economic models are descriptive. However, economists also examine the validity of their positive models using criteria commonly used to evaluate normative decision models, such as logical coherence and suitability as a basis for further economic-theoretic modeling. Camerer [12] uses the term *productive* to describe the use of models to lay 'individual-level foundations for aggregate-level theory'.)

Prescriptive decision analysis bridges the gap between descriptive observations of the way people do make choices and the normative guidelines for how they should make choices: it prescribes techniques for aiding decision making. For example, finding that business executives participating in an experiment sometimes violated the normatively compelling principle of transitivity of preference orderings, MacCrimmon [57] tested a simple prescriptive technique to overcome these violations. When he verbally pointed out their intransitive orderings, many subjects chose to readjust their orderings and become transitive. Prescriptive decision analysis can be called decision engineering, since analysts are designing techniques to aid decision makers in making better decisions. Analysts must meet design specifications (normative goals), based on descriptive constraints of human judgmental abilities.

Prescriptive decision analysis should be judged with a more holistic focus than that used in evaluating normative decision analysis. As Budge [9] says, researchers should avoid the tendency to test a theory 'to destruction' by sequentially examining separate properties of the theory. Instead, the same theoretical structure should combine logical and ethical considerations with descriptive facts. The whole theory, rather than isolated axioms, should be the unit of analysis. Thus, the existence of appropriate and easily implemented methods for assessing preferences and a feasible analysis procedure will be just as important as logical coherence when a theory is used for prescriptive purposes. In addition, a prescriptive theory should take into account descriptive research to adjust for biased judgments of preferences, probabilities, or problem structure received from decision makers. Such a theory should also be designed to operate on the set of data available. For example, good choices should be prescribed even if data on alternative actions or states of nature are incomplete. Brown [8] believes 'that the validation of prescriptive technology should be primarily external, i.e., tools . . . (should be) tested by confrontation with the outside world', rather than focusing on tests of the internal logical consistency of models. He proposes five criteria for evaluating a prescriptive technique: technical soundness (the technique appropriately uses all relevant information and judgment), cost of delaying the decision while the technique is used, cognitive burden on the decision maker, acceptability to institution, and psychological acceptability.

Brown [8] expands the conception of 'prescriptive' to include developing decision making aids for more realistic decision environments. Normative models often assume the decision maker has complete knowledge of all the alternative actions, states of nature, payoffs and probabilities at a single point in time, at which the decision (or set of contingent, sequential decision plans) will be made. A broader conception of the modeling environment may lead, for example, to a focus on better problem structuring or to models allowing sequential restructuring of the problem. Keller and Ho's [45] work on problem structuring is an example of research which is in this very different but equally important genre. This broader notion of the proper role of prescriptive decision analysis will not be addressed further in this paper.

In light of the threefold purposes of decision analysis, a central question is to what extent researchers and decision technologists should attempt to use a single theory for dual or triple purposes. Much of the debate about the new generalized utility theories' characteristics can be clarified by clearly specifying the purposes of the proposed uses of the theories. The suitability of a theory should then be judged by the appropriate criteria for the chosen purpose(s).

2. The performance of expected utility in normative, prescriptive, and descriptive decision analysis

Expected utility theory is probably the most widely accepted normative theory for decision making under risk. Von Neumann and Morgenstern [65] axiomatized expected utility theory by showing that, if a set of apparently normatively appealing axioms hold, alternative actions can be ranked by their expected utilities. The expected utility is a weighted average of the utilities of the possible outcomes where the weights are the objective probabilities of each outcome. Savage's subjective expected utility model allows the derivation of a decision maker's own subjective probabilities for events, which are then used to compute the subjective expected utility of each alternative. Edwards [28, 29] and other psychologists have experimentally investigated a model wherein a person makes choices as if he or she transforms the objective probabilities into subjective probabilities, then computes expected utility via the resulting subjective probability weighting function.

2.1. Normative decision analysis and expected utility

Expected utility theory's original and primary purpose is normative. It is elegant, logically coherent, and parsimonious. It requires preference information from a person for only a few choices to identify a utility function which can be used to specify the normatively 'correct' choices for all possible related choice situations. The preference principles characterizing expected utility are generally compelling, although there is debate over whether the substitution (independence) axiom should hold normatively.

The *substitution principle* of expected utility theory requires that whenever some lottery A is preferred or indifferent to a lottery B, then the compound lottery $pA + (1-p)Z$ must be preferred or indifferent to the compound lottery $pB + (1-p)Z$. The compound lottery $pA + (1-p)Z$ is formed by having a p chance of getting lottery A and a $(1-p)$ chance of getting lottery Z, for any probability values p ranging from 0 to 1. So, a decision maker who prefers the sure \$3200 in option A in Fig. 1 over the risky option B also must prefer D over E, since D and E are formed by substituting lotteries A and B, respectively, into an otherwise identical lottery with a 10% chance of A or B and a 90% chance of Z (where Z is the degenerate lottery of getting \$0 for sure). Most people choose A over B and E over D. This most common response pattern violates the substitution principle, and thus expected utility, as will be discussed later in Section 2.3 on the descriptive usefulness of expected utility.

In a dynamic setting, expected utility theory has the property of *dynamic consistency*, i.e., if a person has option C at time 0 in Fig. 1, the planned choice between A' and B' made at time 0 should agree with the actual choice made at time 1. Notice that the planned choice of CA' (C then A') is strategically equivalent to D and the choice of CB' is equivalent to E [54]. By the substitution principle, if the actual choice is A' over B', then D is preferred over E, so the planned choice will be CA' over CB'.

Expected utility is *linear in probabilities*, since the expected utility

$$EU(pA + (1-p)B) = pEU(A) + (1-p)EU(B).$$

For this reason, it is sometimes called *linear expected utility*. In a Marschak triangle diagram graphically representing the set of all possible alternative actions with probability distributions over three fixed outcomes (see, e.g., [56]), this means that indifference curves are linear and parallel. Expected utility preferences are *separable* across mutually exclusive events, in the sense of *replacement separability* (the contribution of each outcome x_i and its probability p_i to the overall expected utility of an alternative action is independent of the other outcome/probability pairs) and *mixture separability* (the contribution of each outcome/probability pair to the overall expected utility can be broken down into the utility of x_i, multiplied by p_i).

Expected utility also satisfies *consequentialism* [54]. At any point in time we can focus on the consequences from now on (choices, states, probabilities, and outcomes) and we do not need

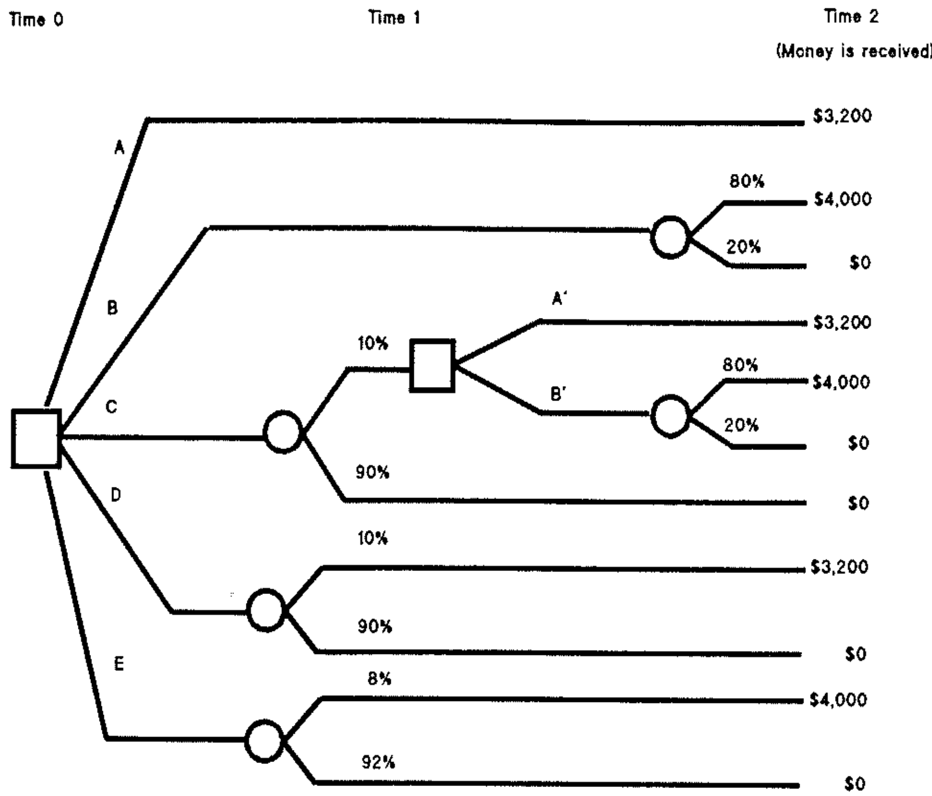

Fig. 1. Decision tree.

to know where we have come from or what other probability or choice branches were previously available. Thus, the analysis of the expected utility of alternative actions can be carried out by '*folding back*' a decision tree representation of the choices and states, by computing the maximum expected utility based on the options and states remaining at any one point in time.

Additional characteristics of expected utility include the transitivity, common-consequence (sure-thing), reduction-of-compound-lotteries, and betweenness principles. Expected utility orderings are consistent with first-order stochastic dominance rankings. Finally, expected utility requires ambiguity indifference (i.e., indifference between two risky options which are identical except that one option has a non-vague subjective probability p for an event and the other has the same subjective probability for a corresponding event, but the probability p is ambiguous; see [30]).

A question not directly addressed by utility theory is the choice of risk attitude. Under expected utility theory, a person is labeled risk averse if a sure monetary amount (such as the $3200 in Option A in Fig. 1) is preferred over a lottery (such as Option B) whose expected monetary value is equal to that sure amount. This labeling scheme is misleading, because it mixes attitude towards risk with strength of preference for different outcomes. An unresolved question is whether risk attitude should be a by-product of assessment judgments (as it is in utility theory assessment procedures) or it should be a conscious decision. For example, a person might choose to be relatively risk neutral [27, 44] over a certain range of outcomes. Utility theory can accommodate either approach, since only the assessment procedures need to be modified to guarantee a specific risk attitude prior to the calibration of the utility function. A related question is whether an S-shaped utility function, with a point of inflection at a target or reference level should be allowed in normative or prescriptive decision analysis. Some people (especially economists) argue that a person should retain either risk aversion, proneness, or neutrality over all outcome domains.

2.2. Prescriptive decision analysis and expected utility

Expected utility is well-suited as a prescriptive model for decision making under risk. First, its strong normative appeal has made it popular among decision analysts. Second, the required analysis is straightforward, assuming all required information on alternative actions, states, probabilities, and outcomes are available. Decision trees can be constructed by hand or with existing computer software (such as Arborist, Supertree, etc.) and then analyzed by folding back the tree. Third, multiattribute utility models exist [40], so a more accurate description of problem attributes is possible. Fourth, many applications have already been successfully carried out, so the theory has been field tested. Reviews of applications are in Keeney and Raiffa [40], Howard et al. [38], and von Winterfeldt and Edwards [66].

A primary problem with expected utility as a prescriptive decision analysis technique is *assessment of the utility function*. Assessed functions can often vary systematically with the response method. In a common assessment procedure, the decision maker adjusts a sure monetary amount to determine the *certainty equivalent* which attains indifference with a lottery. This method can result in a different utility function than the one the same person would get if a probability is adjusted to determine a *probability equivalent* which matches one option to an indifferent option [36, 37]. Further, certainty equivalents may vary depending on the way they are assessed. In the Becker–deGroot–Marschak [2] mechanism for promoting 'correct' certainty equivalent responses, the experimenter offers to buy the lottery from the subjects if a randomly generated offer price exceeds their stated minimum selling price. In preliminary experimental work, Uzi Segal and I have found that assessment of certainty equivalents for a lottery via this mechanism can lead to different certainty equivalent values (when the random offer price is drawn from a larger range of possible prices), as implied by at least one generalized utility theory [60].

Since people often violate the substitution principle, another assessment problem can occur. For example, in the Fig. 1 choice situation, people often appear risk averse by choosing option A to get a sure \$3200 over the risky option B which has an 80% chance of \$4000 (or else \$0). The expected utility function assessed with such question responses is concave, reflecting risk aversion. However, if expected utility is assessed over the same range with questions containing low probabilities for the positive, non-zero outcomes, such as D *versus* E in the figure, the function would be convex, reflecting risk proneness, if E is preferred over D. Such a preference for A and E violates the substitution principle of expected utility. The problem for prescriptive decision analysis is not that expected utility is violated, since the analysis process of applying an assessed utility function to a problem will guarantee that the substitution principle and expected utility are obeyed. Rather, the problem is that assessment questions, which by expected utility standards should yield identical utility functions, can produce widely varying utility functions, which may even switch from risk aversion to risk proneness.

Another assessment problem is the discrepancy between preferences when elicited with paired comparison and direct rating methods. This discrepancy is called the *preference reversal phenomenon* [49, 35]. Although more research needs to be done on the effects of response modes on expressed preferences to support practical use of preference assessment technologies, two approaches seem promising. First, Tversky et al. [64] have introduced a contingent weighting model to represent the difference in inferred preferences resulting from different response modes. Also, Bostic et al. [7] found that a choice-based sequential procedure for discovering certainty equivalents holds promise for eliminating the systematic overstating of the value for lotteries with a moderate probability of a large gain.

Another issue in prescriptive decision analysis is determining *how far to aid a decision maker* in restructuring the problem and the relevant preferences. If a person's subjective probability of an injury when not wearing seat belts is too low, seat belts might not be worn when they 'should' be, based on the objective probability of injury. In this case, an analyst may wish to point out objective probability information. Similarly, a person may violate principles required by expected utility. An analyst is confronted with deciding how far to push a person to conform with expected utility before allowing the person to violate these principles and use a generalized utility model for guiding choice. Also, altering the framing of a decision situation by adjusting the perceived status quo (or reference) level can greatly alter a decision maker's perspective and risk taking behavior. A reasonable prescription is to limit the number of times a decision maker resets the reference level, as suggested by von Winterfeldt and Edwards [66, pp. 373–377].

2.3. *Descriptive decision analysis and expected utility*

The descriptive validity of expected utility has been strongly challenged in two types of laboratory experiments. Most experiments have examined patterns of choices to demonstrate violations of expected utility principles. A few recent experiments have gone further and actually assessed subjects' expected utility to determine the percentage of choices correctly predicted.

First, a fairly large body of experimental evidence shows that subjects systematically make choices which violate principles required by expected utility. Substitution (or common-ratio or independence) principle and sure-thing (or common consequence) principle violations have been shown by, e.g., MacCrimmon and Larsson [58], Kahneman and Tversky [39], and Keller [42]. Violations of the reduction of compound lotteries principle [43] and the betweenness principle [20, 21, 22] have also been shown. Aversion to ambiguity in probabilities [30] has also been demonstrated in experiments and models have been proposed to accommodate non-indifference to ambiguous probabilities, but they will not be addressed in this paper.

Second, preliminary evidence shows that assessed and/or fitted expected utility functions predict choices moderately well, but with much room for improvement. Currim and Sarin [23] assessed experimental subjects' expected utility and prospect theory models, and Daniels and Keller [24] assessed expected utility and lottery dependent utility models. In both cases, expected utility did about as well as the two generalized utility models in predicting choices on a hold-out sample of paired comparison choices, even when the problems were structured to induce substitution or sure-thing principle violations.

When examining predictive performance it is important to examine how much utility is lost by using a specific model. A model may not accurately predict all choices, but it may predict the correct choices whenever there is a big difference in the perceived value of the two options. For example, Daniels and Keller [24] calculated the utility difference between predicted and actual choices in addition to tabulating the number of correct predictions by a model. Future experimental work should measure the magnitude of the potential mistakes in prediction resulting from a specific model.

Part of the appeal of the expected utility model is its simplicity, with preference defined on the probability distribution over outcomes. Actual decision making depends on many additional factors, such as fear, regret, context, memory capacity, processing capacity, and framing effects. Perhaps entirely different models should be used for describing choice. An important criterion for evaluating descriptive research is the potential for insights on behavior coming from the model. A risk averse expected utility function may say something about a person's psychological attitude toward risk, but it is confounded with strength of preference. A model which may provide more insight about peoples' thought processes, for example, is Lopes' [51] two-factor theory for risky choice, which combines a dispositional factor (desire for security versus potential) with an aspiration level factor.

There are at least two different categories of responses to the descriptive violations of expected utility. One response, followed in Keller [42, 43] is to develop prescriptive techniques, such as visual problem representations, to aid decision makers to conform with expected utility theory. The other response is to develop new descriptive models which are empirically valid. Some will say that the generalized expected utility models which relax the substitution principle, or other principles, are designed for the purpose of creating a descriptively valid model. If so, then the generalized models should be judged by the entire set of criteria used for descriptive decision analysis. Others may argue that there is a combination of descriptive and normative purposes behind the new generalizations of utility theory, and they should be judged both descriptively and normatively. Still others argue that *if* generalized utility theories are normatively appropriate, then perhaps they should be used for prescribing decisions, and they should then be evaluated on the basis of prescriptive criteria. The next three sections examine the properties of the new generalizations of expected utility from the perspective of one of the three purposes of decision analysis. Potential users of the generalized utility technology should take into account the criteria and issues to be discussed when choosing among the existing or still-to-be developed preference modeling approaches.

3. The performance of generalized utility in descriptive decision analysis

Many generalized utility theories have been recently proposed as variants of expected utility theory. Some of these theories include prospect theory [39]; weighted utility [13, 17, 18] and the related skew-symmetric bilinear utility [32, 33] and regret theory [6, 50]; lottery dependent utility [4]; approximate expected utility [48]; expected utility with rank dependent probabilities (Quiggin's [59] anticipated utility, Yaari [68], Luce and Narens' [52] dual bilinear utility); general quadratic utility [16, 53, n. 45]; implicit expected utility [14, 25]; and ordinal independence [62, 34].

Since their development was primarily motivated by descriptive violations of expected utility theory principles, most generalized theories are designed to account for these violations. Thus, they generally have the potential to describe choices which have been observed in laboratory settings. This potential is usually first demonstrated theoretically by showing the model is mathematically able to match non-expected utility choices. Next, new data are collected for existing or new questions to show the preference patterns the new models are theoretically capable of predicting; e.g., Chew and Waller [19] followed this approach to evaluate weighted utility theory.

Camerer [12] contrasted several generalized utility theories on the basis of implied preference patterns and collected experimental subjects' choices. The theories included weighted utility, and the related skew-symmetric bilinear utility and regret theory; implicit expected utility theory; the fanning-out hypothesis of Machina [53]; lottery dependent expected utility; prospect theory; and expected utility with rank dependent probabilities. Camerer examined sets of choices to gather evidence on subjects' indifference curves. Indifference curves which are parallel straight lines in the Marschak triangle conform with expected utility, and non-parallel indifference curves violate expected utility. The predominant patterns of choices violating the substitution and sure-thing principles can be represented by preference models which allow indifference curves to fan out. Fanning-in indifference curves correspond to violations of the common consequence principle (and thus expected utility), but not in the most common way. Camerer found evidence of both fanning-out and fanning-in of indifference curves. No one existing theory could explain all the preference data, but prospect theory and the fanning-out hypothesis matched most of the data.

Although the generalized theories often have the potential, in principle, to match non-expected utility choices, for successful use as a descriptive model to predict choice, the predictive performance must also be examined. This has yet to be attempted for the majority of the models. In fact, many models still do not have a precise enough form that a preference function can be assessed or fitted so predictions on choices between arbitrary options can be made. For example, Leland [48] posits the existence of an approximate utility function with a step function form, so that sometimes two close outcomes are identical in 'approximate' utility, but he does not show how to assess such a function. Currim and

Sarin [23] have elicited prospect theory functions and Daniels and Keller [24] elicited lottery dependent utility theory functions. These first investigations showed that the two generalized models did about the same as expected utility in predicting choices when assessed with the usual certainty or probability equivalent procedures, but that there was much room for improvement if specialized assessment procedures could be developed. The generalized models did predict patterns of preferences violating expected utility, but they did not always predict the specific choices for a subject. Further, the problem of variation in assessed functions arising due to different response modes (which was discussed under prescriptive expected utility) remains unsolved.

It is important to note that descriptive models may not need to be assessed to meet certain uses. For example, economists, including Machina and Leland, have developed theoretical generalized utility models whose general properties can be used in further economic-theoretic modeling without specifying a precise functional form. However, in other cases, the theory must be precise enough to allow prediction of choices among any set of arbitrarily chosen options. For example, suppose a firm wants to predict the market share for alternative warranties for consumer durables. Using a theory facilitating preference assessment, survey respondents could answer simple questions to assess their preference functions, which could then be used to predict their choices among the alternative warranty policies. In the latter case, a model, such as expected utility, may be preferred because of good predictive performance and ease of data collection even if it requires preference properties, such as the substitution principle, which are known to be descriptively violated. Thus, expected utility theory should not be ruled out as a useful predictive theory until it is replaced with a theory that clearly does better in predicting arbitrary choices.

The generalized utility theories share with expected utility the advantage and disadvantage of requiring only information on the probability distribution over outcomes and on a person's preference judgments. Such models may not capture the richness needed in operational contexts. Prospect theory does enrich the domain of the model by adding a preliminary stage in which a problem is framed and encoded for subsequent analysis, but the specifics of this preliminary stage need to be further developed.

4. The performance of generalized utility in normative decision analysis

The potential performance of generalized utility in normative decision analysis will be determined on the basis of the normative acceptability of the characteristics of various non-expected utility models. The characteristics of expected utility will generally be used as a baseline for the philosophical debate on normatively desired characteristics. For example, the original version of prospect theory may violate the normatively compelling property of *first-order stochastic dominance preference* [54, footnote 17], which is satisfied by expected utility and some generalized utility models. (A new rank-dependent form of prospect theory is under development which does not violate stochastic dominance.) This section contains the normative arguments for and against different characteristics. Of special concern when *substitution principle violations for static lotteries are allowed*, is whether *dynamic* consistency and/or *consequentialism* should hold.

Machina [54] argues that non-expected utility models which would be used in economic theory should have the arguably normative properties of dynamic consistency and non-consequentialism. (See also Chew and Epstein [15].) Non-consequentialism means that the choice between A' and B' at time 1 in Fig. 1 cannot be made without knowing that there was a previous 10% probability of arriving at the choice node at time 1, and a 90% probability of the outcome \$0 which might have happened had Option C been chosen at time 0. Such a dynamically consistent non-expected utility model would not always obey the substitution principle applied to static single stage lotteries, and could thus model the simultaneous preference among single stage lotteries of A over B but E over D in the figure. However, using a dynamically consistent non-expected utility model, under option C the planned choice between A' and B' at time 0 in the decision tree in Fig. 1 would have to agree with the actual choice made at time 1. A decision maker with these preferences would be classified as a *gamma-type* according to Machina's [54] categorization of decision makers into *alpha*, *beta*, *gamma*, and *delta* types, as shown in Fig. 2. *Alpha-types* use expected utility and thus obey the substitution principle, consequentialism, and dynamic consistency. *Betas*, *gammas*, *deltas* (and an added type: *epsilons*) sometimes violate the substitution principle for static lotteries.

Machina is concerned that economic researchers will not accept a model which can potentially predict dynamically inconsistent choices. This behavior arises by being a consequentialist and isolating the focus at time 1 only on A' and B', perhaps choosing A' over B', having planned on CB' over CA' originally. The argument against dynamic inconsistency is normative. It hinges on the possibility that a person can be made to 'make book' against his/her own choices, making the person into a perpetual money pump, cycling among options to eventual ruin. Adding to this normative argument the descriptive observation that such money pumps are not observed in economic markets, Machina [54] rejects dynamic inconsistency. Thus, he rejects *beta-type* preferences (consequentialist, not dynamically consistent, substitution principle violators) and, implicitly, *epsilon-type* preferences (which only differ from betas on not being consequentialists).

As an aside, it seems that a better approach to economic modeling, due to the need for descriptive validity, would be to continue the search for mathematically tractable theories which are descriptively valid, both for individual judgment behavior and for the observed aggregate market behavior. First, examination of the market behavior may reveal isolated judgments which can be shown to be dynamically inconsistent. For example, perhaps money pumps have not been found because our model of the decision situation is too simplified. Since people do not purchase houses frequently, it would be difficult to observe a person repeatedly buying and selling houses, cycling down to eventual ruin. But, it may be possible to find a person isolating the house selling problem at the current stage (to sell or rent) rather than recalling the previous probability of getting to the stage of having to move and the other previous probabilistic branches. Such a person may choose the option of selling (which may reflect risk aversion if the selling price is a sure amount), even though the planned choice three years previously (prior to gaining information about the need to move to a new job) might have been to rent (if and when a move had to be made). Renting may have a wider distribution of possible income flows depending on the renter availability and possible damages, and thus may be a risk prone choice.

I believe that non-expected utility models were developed in response to both types of substitution principle violations, those for static choices and for dynamic (multiple-stage) choices. Since experimental evidence suggests that this is how people see the problem and make their choices, a descriptively valid model of decision making under risk should definitely allow the planned choice to differ from the actual choice. Further, a good argument can be made that a normative model should allow the difference between planned and actual choices if a reasonable decision maker chooses, upon reflection, to make different choices. Sarin [61] presents the philosophical debate over whether dynamic consistency should hold in normative models and argues that a

	Dynamic consistency	
	Inconsistent $CA' \leq CB'$ and $A' > B'$ occurs	Consistent $CA' > CB' \leftrightarrow A' < B'$
Consequentialist $A > B \leftrightarrow A' > B'$	Beta	Delta
Not consequentialist $A \leq B$ and $A' > B'$ occurs	Epsilon	Gamma

Fig. 2. Classification of decision makers who violate substitution principle for static lotteries ($A > B$ and $D \leq E$ occurs). (Notes: $>$ and $\leq$ indicate preference order; A, A', B, B', C, D, and E are options in Fig. 1; Alpha-type (expected utility) preferences obey substitution principle, consequentialism, and dynamic consistency.)

decision maker may wish to violate dynamic consistency.

For example, the lottery dependent utility theory of Becker and Sarin [4] will allow planned choices to differ from actual. Applying their model to option C in Fig. 1's decision tree problem, at time 0, a *beta-type* consequentialist who is not dynamically consistent might note that CB' is strategically equivalent to E and choose the *planned choice* CB' over CA', which is strategically equivalent to D. Then, whenever the decision node at time 1 arises, this consequentialist beta-type revises the tree and only compares A' and B', and may choose A' as the *actual choice*.

Whether a particular generalized model represents dynamically consistent choices may depend not on the model per se, but on the way it is applied to choice situations and how the decision maker frames and reframes choices over time. For example, Becker and Sarin [5] show how to analyze the utility of alternatives using a modified approach for folding back a decision tree. Following this analysis procedure yields *delta-type* preferences which are dynamically consistent since planned choice equals actual choice, because plans are always made by working backwards through the entire tree. This procedure is also consequentialist, since folding back the decision tree to determine choice is done by isolating focus on the current and future stages only. Machina [54] presents three arguments against delta-type people: (1) strategically equivalent lotteries will not be indifferent [46, 47]; (2) aversion to costless information in decision trees, and (3) folding 'back is only appropriate when the objective function is separable across the various subdecisions of a problem'. Further debate should be conducted on the merits of dynamic consistency and consequentialism as normative principles, starting with an investigation of these three criticisms.

5. The performance of generalized utility in prescriptive decision analysis

If generalized utility models are to be used in aiding decision making, they should be *assessable*, have *feasible analysis procedures*, and recommend *problem framings with face validity*, i.e., problem framings which are acceptable to the decision maker. These practical concerns should be weighed against the concerns of the normative appropriateness of the models, which were discussed above.

Models relaxing the substitution principle need not be harder to assess. Currim and Sarin [23] have shown how to assess prospect theory in an experimental comparison of the predictive performance of prospect theory and expected utility. Keller and Daniels [24] have assessed lottery dependent expected utility theory of Becker and Sarin [4] and expected utility theory. In both cases expected utility theory did about as well as the generalized theories.

However, if very complicated mathematical forms are needed, if assessment questions are too lengthy or complicated, or if different functions must be assessed in local regions, a generalized utility model may not be practical for actual applications. Further, with the level of assessment precision attainable with our current assessment procedures, theoretically different models may differ little in their prescribed choices. For example, a specific decision maker's concave expected utility function assessed with an exponential form and with a power form may be indistinguishable over the range of relevant outcomes and for the attainable level of assessment precision, even though the two functional forms imply different preference attitudes. More research needs to be done to determine the likely difference between generalized utility models in prescriptive performance.

The feasibility of the analysis of generalized utility models requires that an assessed or fitted model can be obtained, that sufficient data on the problem structure (probabilities, states, alternatives, and outcomes) is available, and that computation of maximum utility is possible (by hand or computer). Some generalized utility models are criticized because their analysis procedure does not allow folding back the decision tree as is possible under expected utility, which is consequentialist and focuses only on the current and future time periods to make a choice among current options [47]. However, as described earlier, Becker and Sarin [5] show how their generalized utility model, lottery dependent utility, can be used in a modified folding back procedure. Further, with the availability of computers, the criterion of 'ease of hand calculation' can now be replaced by the 'availability of computer programmable computational procedures', so not being able to fold back a tree is not an insurmountable analytic problem.

Since the choice of the way to frame the current decision problem can alter the choice prescribed by generalized utility models, framing issues are of considerable practical concern. Should you frame your life decisions as being at the actual current decision point or at the initial life planning point (say at age 12)? If you model your problem as if you are at the initial point you may plan to be risk prone with respect to the choice of a job after college graduation, but at the actual job choice point you may be risk averse. Also, when to stop elaborating the decision tree into the future [46] must be decided. Some will argue that these framing problems should be avoided by retaining the expected utility model for prescriptive decision analysis. This requires, for example, that at time 0 in Fig. 1, if A is better than B, then D is better than E, and CA' is better than CB'; and at time 1, A' is better than B'. More philosophical debate is needed to resolve the issue of which models are prescriptively useful, given that different models' choice prescriptions remain invariant among different sets of transformations of problem frames.

I am now inclined to allow decision makers to choose among possible problem structures, after showing them the alternative framings which are supposedly equivalent (from an expected utility perspective). So, if a house seller perceives the current decision situation as a choice between sell or rent and chooses to ignore previous branches in the decision tree, allow it. Then aid choice at that point, possibly with a generalized utility model, even if the choice (of, say, the risk averse option of selling) may be seen as dynamically inconsistent if framed in the context of a five-year planning period, starting three years previously. Back then, since the chance of ending up at this decision node was probably low, the planned choice might have been the risk prone choice of renting. Such an approach allows the decision maker to identify the psychologically relevant problem structure for the current problem. This approach must be contrasted with one of the alternatives, which is to alter the person's world view sufficiently so that all current choices are seen as only one of the myriad of possible choices which could have been presented, rather than the relatively circumscribed pseudo-certain frame of a current choice that is probably more common. The alternative approach edges on altering culturally-based perceptions of time and fate, and should be examined carefully.

6. Summary

This paper has provided a framework for examining the potential of generalized utility theories for use in descriptive, normative, and prescriptive decision analysis. The decision analysis community should evaluate the relative value of alternative directions for future research and technology development in meeting these three linked purposes for decision analysis.

A major gap now exists in which the many models which can potentially represent non-expected utility preferences have yet to be evaluated on the basis of predictive performance. Further, more philosophical debate is needed to resolve the issue of the allowable transformations of problem frames which yield identical prescriptions by preference models.

Also, behavioral decision research has up to now focused on investigating deviations from normative models. An entirely new paradigm, containing psychologically relevant variables such as disposition to seek security, and variables which can actually be controlled in the decision environment may yield more benefits for understanding how people make unaided decisions. For example, in medical decision making, a descriptive model could include format, context, and interaction with medical professionals to represent patients' choices between risky surgical and medical alternatives.

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