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Decision Trees

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A decision tree is a pictorial description of a well-defined decision problem. It is a graphical representation consisting of nodes (where decisions are made or chance events occur) and arcs (which connect nodes). Decision trees are useful because they provide a clear, documentable and discussible model of either how the decision was made or how it will be made.

The tree provides a framework for the calculation of the expected value of each available alternative. The alternative with the maximum expected value is the best choice path based on the information and mind-set of the decision-makers at the time the decision is made. This best choice path indicates the best overall alternative, including the best subsidiary decisions at future decision steps, when uncertainties have been resolved.

The decision tree should be arranged, for convenience, from left to right in the temporal order in which the events and decisions will occur. Therefore, the steps on the left occur earlier in time than those on the right.

Decision Nodes: Steps in the decision process involving decisions between several choice alternatives are indicated by decision nodes, drawn as square boxes. Each available choice is shown as one arc (or "path") leading away from its decision node toward the right. When a planned decision has been made at such a node, the result of that decision is recorded by drawing an arrow in the box pointing toward the chosen option. As an example of the process, consider a pharmaceutical company president's choice of which drug dosage to market. The basic dosage choice decision tree is shown in Figure 1. Note that the values of the eventual outcomes (on the far right) will be expressed as some measure of value to the eventual user (for example, the patient or the physician).

Chance Nodes: Steps in the process which involve uncertainties are indicated by circles (called

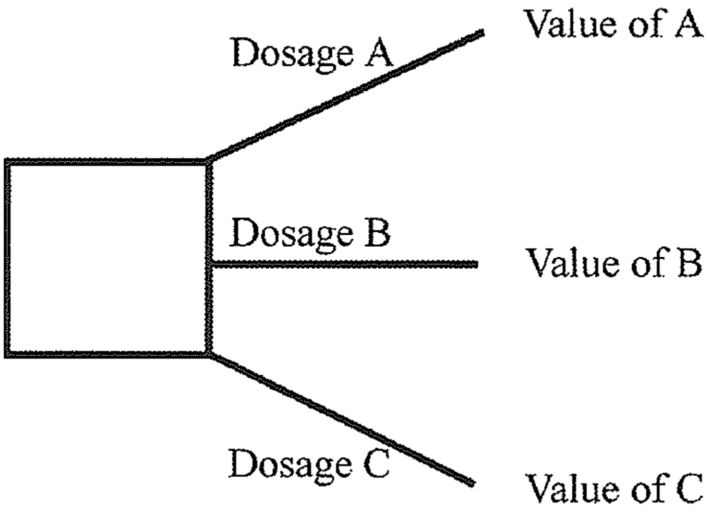

Fig. 1. The choice of drug dosage

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chance nodes), and the possible outcomes of these probabilistic events are again shown as arcs or paths leading away from the node toward the right. The results of these uncertain factors are out of the hands of the decision-maker; chance or some other group or person (uncontrolled by the decision-maker) will determine the outcome of this node. Each of the potential outcomes of a chance node is labeled with its probability of occurrence. All possible outcomes must be indicated, so the sum of the potential outcome probabilities of a chance node must equal 1.0. Using the drug dose selection problem noted above, the best choice of dose depends on at least one probabilistic event: the level of performance of the drug in clinical trials, which is a proxy measure of the efficacy of the drug. A simplified decision tree for that part of the firm's decision is shown in Figure 2. Note that each dosage choice has a subsequent efficacy chance node similar to the one shown, so the expanded tree would have nine outcomes. The probabilities (p_1, p_2, and p_3) associated with the outcomes are expected to differ for each dosage.

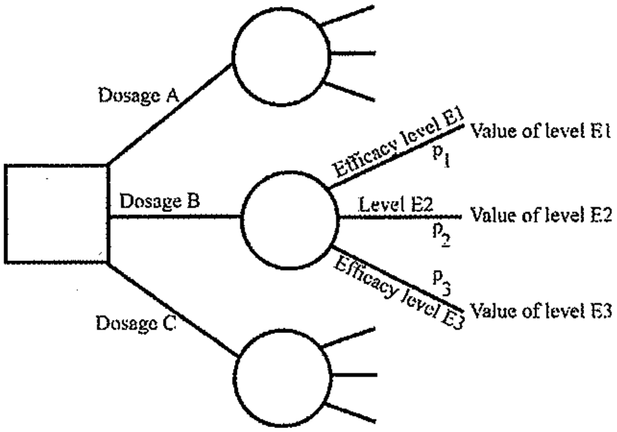

Fig. 2. The choice of drug dosage based on efficacy outcome

There are often several nodes in a decision tree; in the case of the drug dosage decision, the decision will also depend on the toxicity as demonstrated by both animal study data and human toxicity study data as well as on the efficacy data. The basic structure of this more complex decision is

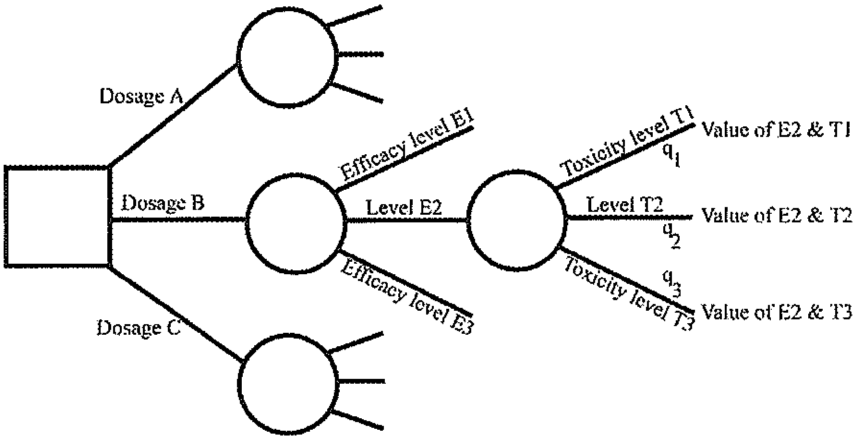

Fig. 3. The choice of drug dosage based on uncertain efficacy and toxicity

shown in Figure 3. The completely expanded tree has 27 eventual outcomes and associated values. Notice that although not always the case, here the probabilities (q_1, q_2, and q_3) 0of the toxicity levels are independent of the efficacy level.

One use of a decision tree is to clearly display the factors and assumptions involved in a decision. If the decision outcomes are quantified and the probabilities of chance events are specified, the tree can also be analyzed by calculating the expected value of each alternative. If several decisions are involved in the problem being considered, the strategy best suited to each specific set of chance outcomes can be planned in advance.

Probabilities: Estimates of the probabilities for each of the outcomes of the chance nodes must be made. In the simplified case of the drug dose decision above, the later chance node outcome probabilities are modeled as being independent of the earlier chance nodes. While not intuitively obvious, careful thought should show that the physiological factors involved in clinical efficacy must be different from those involved in toxicity, even if the drug is being used to treat that toxicity. Therefore, with most drugs, the probability of high human toxicity is likely independent of the level of human efficacy. In the more general non-drug situations, however, for sequential steps, the latter probabilities are often dependent-conditional probabilities, since their value depends on the earlier chance outcomes.

For example, consider the problem in Figure 4, where the outcome being used for the drug dose decision is based on the eventual sales of it. The values of the eventual outcomes now are expressed as sales for the firm.

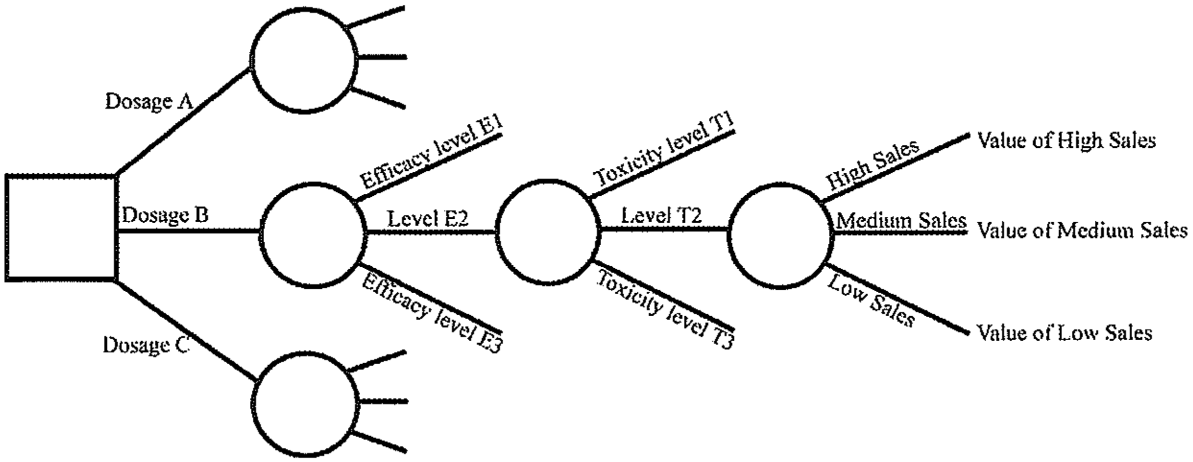

Fig. 4. The choice of dosage based on efficacy and toxicity and their eventual effect on sales

The probability of high sales depends on the efficacy as well as on the toxicity, so the dependent-conditional probability of high sales is the probability of high sales given that the efficacy is level 2 and toxicity is level 2, which can be written as $p_{(\text{High Sales}\,|\,\text{Efficacy level 2 and Toxicity level 2})}$.

Outcome Measures: At the far right of the tree, the possible outcomes are listed at the end of each branch. To calculate numerical expected values for alternative choices, outcomes must be measured numerically and often monetary measures will be used. More generally, the "utility" of the outcomes can be calculated. Single or multiple attribute utility functions have been elicited in many decision situations to represent decision makers' preferences for different outcomes on a numerical (although not monetary) scale.

The Tree As An Aid in Decision Making: The decision tree analysis method is called "fold-back" and "prune." Beginning at a far right chance node of the tree, the expected value of the outcome measure is calculated and recorded for each chance node by summing, over all the outcomes, the product of the probability of the outcome times the measured value of the outcome. Figure 5 shows this calculation for the first step in the analysis of the drug dose decision tree.

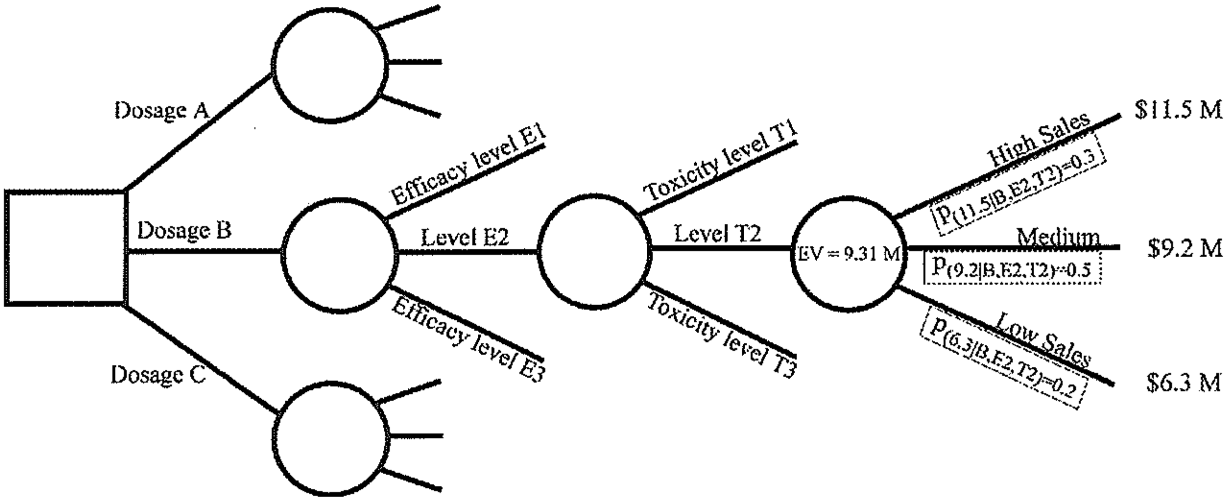

Fig. 5. The first step, calculating the expected value of the chance node for sales:

$$EV = 0.3(11.5) + 0.5(9.2) + 0.2(6.3) = 9.31$$

This step is called "folding back the tree" since the branches emanating from the chance node are folded up or collapsed, so that the chance node is now represented by its expected value. This is continued until all the chance nodes on the far right have been evaluated. These expected values then become the values for the outcomes of the chance or decision nodes further to the left in the diagram. At a decision node, the best of the alternatives is the one with the maximum expected value, which is then recorded by drawing an arrow towards that choice in the decision node box and writing down the expected value associated with the chosen option. This is referred to as "pruning the tree," as the less valuable choices are eliminated from further consideration. The process continues from right to left, by calculating the expected value at each chance node and pruning at each decision node. Finally the best choice for the overall decision is found when the last decision node at the far left has been evaluated.

See **Decision analysis; Decision making; Decision problems; Group decision making; Multi-attribute utility theory; influence diagrams; preference theory; utility theory; Bayesian decision theory.**

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