1 PROPERTIES OF UTILITY THEORIES AND RELATED EMPIRICAL PHENOMENA

L. Robin Keller

Introduction

Expected utility theory, probably the most widely accepted normative theory for decision making under risk, has several required properties. Since different sets of axioms can be combined to result in the expected utility model, the term *property* can refer to either an axiom or a characteristic resulting from combinations of axioms. Since most properties are seen as appropriate components of a normative theory of choice, they could be referred to as *principles* or *desiderata* to emphasize their normative status (see Howard, 1992). But, not all properties hold consistently in choices made by experimental subjects. The resulting conflict between the normative appeal of expected utility theory and its shortcomings as a descriptive model of choice has been a motivating force in the development of generalized utility theories which relax the requirement that various properties hold.

The purpose of this chapter is to provide an overview of the properties and related experimental phenomena and their link with developments in generalized utility theories. The chapter thus serves as an introduction for the issues raised by the remaining chapters in this volume. The chapter is

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organized as follows. The second section briefly describes expected utility theory and lists a number of generalized utility theories. The third section contains some key properties of expected utility theory, with highlights of related experiments and generalized utility theories not requiring those properties. The fourth section continues the discussion of consequentialism, dynamic consistency, and substitution property violations. A summary follows in the last section.

Expected Utility and Generalized Utility Theories

This section contains a brief discussion of expected utility and generalized utility theories. Fishburn (1988) and Machina (1987b) present details on the different theories, and reviews are in Fishburn (1989), Machina (1987a), Sarin (1989), and Weber and Camerer (1987).

Expected Utility Theory

von Neumann and Morgenstern (1947) axiomatized expected utility theory by showing that, if a set of apparently normatively appealing axioms hold, alternative actions can be ranked by their expected utilities. The expected utility of an alternative action is the weighted average of the utilities of the possible outcomes where the weights are the objective probabilities of each outcome. Savage's (1954) subjective expected utility model allows the derivation of a decision maker's own subjective probabilities for events, which are then used to compute the subjective expected utility of each alternative. Edwards (1955, 1962) and other psychologists have experimentally investigated a model wherein a person makes choices as if he or she transforms the objective probabilities into subjective probabilities, then computes expected utility via the resulting subjective probability weighting function. Many prescriptive applications of expected utility theory have been carried out, especially for problems with multiple attributes in which multiattribute utility theory is used (Keeney and Raiffa, 1976). Keeney (1992) discusses the choice of axioms to guide prescriptive decision analysis, along with other prescriptive issues.

A fairly large body of experimental evidence, stimulated by the paradox introduced by Allais (1953), shows that subjects systematically make choices that violate properties required by expected utility. This evidence shows that expected utility is not a fully valid descriptive model of choice under risk. Representative experimental studies are cited later in the third section, throughout the discussion of utility properties. Also, a few recent experiments have gone further and actually assessed subjects' expected utility to determine the percentage of choices correctly predicted. The preliminary evidence shows that assessed and/or fitted expected utility functions predict choices moderately well compared to generalized utility models, but with room for improvement. Currim and Sarin (1989, 1990) compared experimental subjects' assessed expected utility models with their prospect theory, weighted utility, and lottery dependent utility models; and Daniels and Keller (1990) assessed expected utility and lottery dependent utility models. Overall, expected utility did about as well as the generalized utility models in predicting choices on a hold-out sample of paired comparison choices, even when the problems were structured to induce expected utility property violations. However, the potential for improved predictive performance by generalized utility models may still be achieved. For example, Daniels and Keller (1992) have explored a choice-based assessment mechanism in which lottery dependent expected utility appears to perform better than expected utility. Also, Shafir et al. (1989) proposed an advantage model of choice that outperformed two special cases of expected utility.

There are at least three different categories of responses to the descriptive violations of expected utility. One is to argue that expected utility theory's purpose is normative and to reclarify conditions under which expected utility is an appropriate model for prescriptive use and when it is not, such as when distributional equity is involved. Keeney (1992) and Howard (1992) discuss the use of expected utility in prescriptive applications, and Keeney discusses its inapplicability in portions of problems requiring equity considerations.

Another response to descriptive violations, followed in Keller (1985a,b), is to develop prescriptive techniques, such as visual problem representations, to aid decision makers to conform with expected utility theory. The stream of research attempting to develop unbiased utility assessment procedures also follows this general approach. Keller (1989b) contains a discussion of the problems of descriptive violations of expected utility when it is to be used as a prescriptive model.

A final response is to develop new models, including the generalized utility models, that may be descriptively valid and that might be used prescriptively in special settings. Miyamoto (1992) introduces his generic utility theory, designed as a general framework for descriptive multiattribute utility modeling.

Generalized Utility Theories

Many generalized utility theories have been recently proposed as variants of expected utility theory. Weber and Camerer (1987, see Figure 10) provide a concise summary of the relationship of expected utility with the various generalized theories. Some of the theories are described in chapters of this volume. Representative theories include prospect theory (Kahneman and Tversky (1989)); weighted utility (Chew and MacCrimmon (1979a,b)), (Chew (1983)) and the related skew-symmetric bilinear utility (Fishburn (1983, 1984)) and regret theory (Bell 1982)), Loomes and Sugden (1982)); lottery dependent utility (Becker (1986), Becker and Sarin (1987)); approximate expected utility (Leland (1988)); expected utility with rank dependent probabilities (Quiggin's (1982) anticipated utility); Yaari (1987), Luce and Narens' (1985) binary

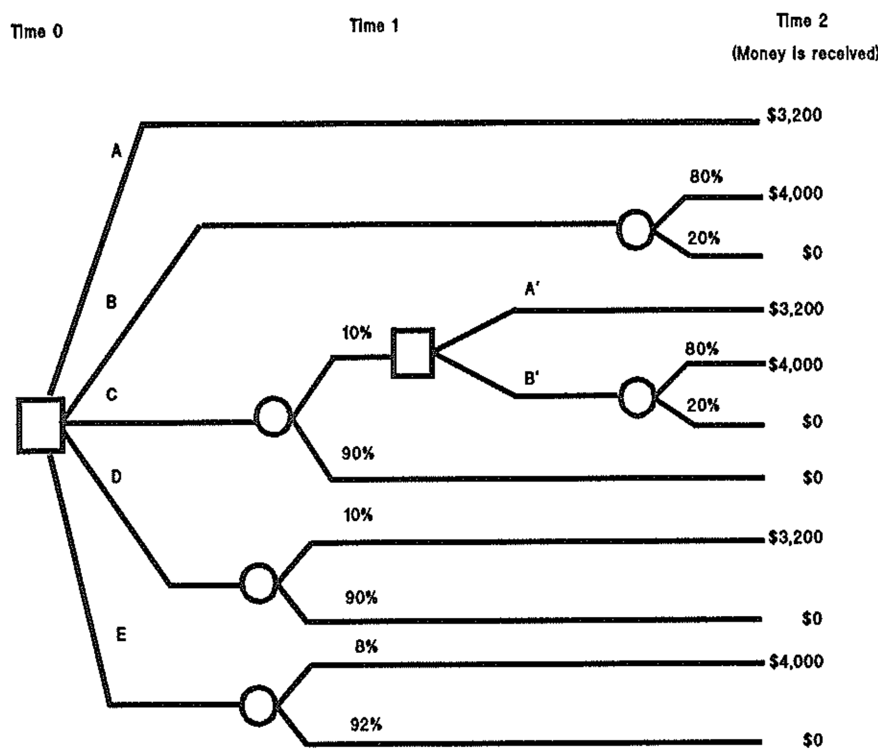

Figure 1–1. Decision tree.

rank dependent (or dual bilinear) utility; general quadratic utility (Chew, Epstein, and Segal (1988), Machina (1982, see note 45)); implicit expected utility (Chew (1985), Dekel (1986)); and ordinal independence (Segal (1984), Green and Jullien (1988)).

Since their development was primarily motivated by descriptive violations of expected utility theory properties, most generalized theories are designed to account for these violations. Thus, they generally have the potential to describe choices that have been observed in laboratory settings. This potential is usually first demonstrated theoretically by showing that the model is mathematically able to match nonexpected utility choices. Next, new data are collected for existing or new questions to show the preference patterns the new models are theoretically capable of predicting; for example, Chew and Waller (1986) followed this approach to evaluate weighted utility theory. LaValle (1992) discusses some limitations on the use of generalized utility theories in prescriptive analysis, and Keller (1989b) discusses the role of generalized utility theories in descriptive, prescriptive, and normative decision analysis.

Properties of Expected Utility Theory

This section contains a discussion of properties required by expected utility theory. Some properties serve as axioms in certain axiomatic developments of the theory, others result from combinations of axioms or from the expected utility model in general.

Substitution

The *substitution* property of expected utility theory requires that whenever some lottery A is preferred or indifferent to a lottery B, then the compound lottery $pA + (1 - p)Z$ must be preferred or indifferent to the compound lottery $pB + (1 - p)Z$, which is formed by substituting B in place of A in the compound lottery. The compound lottery $pA + (1 - p)Z$ is constructed by having a p chance of getting lottery A and a $(1 - p)$ chance of getting lottery Z, for any probability values p ranging from 0 to 1. This property is also called *common-ratio* (Kahneman and Tversky, 1979) and *independence* (Segal, 1992).

Figure 1–1 contains a decision tree with a set of alternative actions that will be used to illustrate examples in this chapter. A decision maker who prefers the sure \$3,200 in option A in the figure over the risky option

B (with an 80 percent chance of \$4,000 or else \$0) also must prefer D over E, according to the substitution property. This is because D and E are formed by substituting lotteries A and B, respectively, into an otherwise identical lottery with a 10 percent chance of A or B and a 90 percent chance of Z (where Z is the degenerate lottery of getting \$0 for sure). Most people choose A over B and E over D. This most common response pattern violates the substitution property, and thus expected utility. Substitution property violations have been shown by, for example, MacCrimmon and Larsson (1979), Kahneman and Tversky (1979), and Keller (1985a).

Luce (1992) points out that most tests of the substitution property confound *monotonicity* with an assumed *accounting equivalence*. The monotonicity property requires that "if a consequence in a gamble is replaced by a more preferred consequence (where this more preferred consequence may itself be a gamble), then the resulting gamble is preferred to the original one" (Luce, 1992). The hidden assumption above (and in experimental tests of the property) is the simultaneous application of monotonicity and an accounting equation requiring that a person equates the compound lottery (that has a 10 percent chance of B (\$4,000, 80 percent; \$0, 20 percent) or else \$0) with the corresponding simple lottery E, (\$4,000, 8 percent; \$0, 92 percent). This accounting equation can be called the *reduction of compound lotteries* property or the *economic equivalence* property (Sarin, 1992). Keller (1985b) found evidence of violations of the reduction of compound lotteries property.

Generalized utility theories usually allow the substitution property to be violated and usually retain some of the other properties required by expected utility. Segal (1992) argues, however, that the independence (substitution) property could be retained in a generalized utility model, if the reduction of compound lotteries property were relaxed.

Sure-thing

The *sure-thing* property of expected utility requires that whenever some lottery D, formed by reducing the compound lottery $pA + (1 - p)Z$, is preferred over E, the reduced compound lottery corresponding to $pB + (1 - p)Z$; then D' must be preferred over E', where D' and E' are formed by replacing the common consequence Z with a new "sure-thing" consequence Z', which is commonly received in both D and E, respectively. The Allais (1953) Paradox is the prototypical example of sure-thing property violations. Howard (1992) argues that violating the sure-thing property is not rational. Sure-thing (or *common consequence*) principle violations have been shown by, for example, MacCrimmon and Larsson (1979), Kahneman and Tversky (1979), and Keller (1985a). LaValle (1992) examines the role of this "sure-thing substitution" property in utility theories.

Linearity in Probabilities

Expected utility is *linear in probabilities*, since the expected utility of a compound lottery $U(pA + (1 - p)B)$ is equal to $pU(A) + (1 - p)U(B)$. For this reason, it is sometimes called *linear expected utility*. In a Marschak triangle diagram graphically representing the set of all possible alternative actions with probability distributions over three fixed outcomes (see Machina (1987b)), this means that indifference curves for expected utility are linear and parallel. (The substitution property leads to these parallel straight lines.) Nonparallel indifference curves violate expected utility. The predominant patterns of choices violating the substitution and sure-thing properties can be represented by preference models that allow indifference curves to fan out (Machina (1982)).

Fanning-in indifference curves correspond to violations of the sure-thing property, but not in the most common response pattern. Camerer (1989) examined sets of choices to gather evidence on subjects' indifference curves and found evidence of both fanning out and fanning in of indifference curves. No one existing theory could explain all the preference data, but prospect theory and the fanning-out hypothesis matched most of the data. Camerer (1992) provides details of tests of generalized utility theories conducted by himself and others.

Betweenness

The *betweenness* property states that if lottery A is preferred over B, then the compound lottery $pA + (1 - p)B$ is "in-between" the original lotteries in the preference ordering. Camerer (1992) discusses betweenness, showing that it is a special case of the substitution property in which Z is fixed at A or B. Betweenness implies indifference curves that are straight lines. Coombs (1969, 1975) and Coombs and Huang (1970) found violations of betweenness by observing orderings of original gambles and the simple lottery formed by reducing the compound lottery (assuming the reduction of compound lotteries property holds). They then proposed portfolio theory as a preference model that can capture betweenness violations. Camerer (1992) reports on other betweenness tests.

First-order stochastic dominance preference

Expected utility orderings are consistent with *first-order stochastic dominance* rankings. Assume that more is better of the attribute (for example, money), and that outcomes are labeled in ascending order of preference, so x_i is less than or equal to x_{i+1} and y_i is less than or equal to y_{i+1}, for all i. Then alternative X dominates alternative Y in the table below if x_k is preferred to y_k for some k and x_i is preferred or indifferent to y_i for the remaining i not equal to k.

	Probabilities of States				
	p_1	p_2	p_3	...	p_n
Alternative X	x_1	x_2	x_3	...	x_n
Alternative Y	y_1	y_2	y_3	...	y_n

More generally, X, dominates a different alternative Y by first order stochastic dominance, if the probability of getting an outcome less than w with alternative X (this probability is the sum of the probabilities p_i for all i's such that x_i is less than w) is less than or equal to the corresponding probability for alternative Y, for all possible levels of w (Fishburn (1988) and Bunn (1984)). This can be generalized to alternatives specified by any continuous probability distribution over outcomes. Luce (1992) suggests the term *likelihood dominance* to generalize stochastic dominance to cases when probabilities of events are unknown.

Luce points out (personal correspondence) that first-order stochastic dominance covers two generally nonequivalent concepts that need to be distinguished. First, monotonicity requires that a gamble formed by replacing a less preferred consequence with a more preferred one is preferred over the original gamble. Second, in the context of a two-outcome gamble, if a new gamble is formed by making the better consequence more likely, then the new gamble should be preferred over the original one. A theory can violate one of these concepts and not the other.

The original version of prospect theory may violate the normatively compelling property of *first-order stochastic dominance preference* (Machina

(1989, see note 17)), which is satisfied by expected utility and some generalized utility models. (A new rank-dependent form of prospect theory is under development that does not violate stochastic dominance.)

Ambiguity Indifference

Expected utility requires *ambiguity indifference.* This means that indifference must hold between two risky options that are identical except that one option has a non-vague subjective probability p for an event, and the other has the same subjective probability for a corresponding event, but the probability p is ambiguous; see Ellsberg's (1961) Paradox. Aversion to ambiguity in probabilities has been demonstrated in experiments and models have been proposed (Sarin, 1992) to accommodate nonindifference to ambiguous probabilities. Howard (1992) presents an argument that ambiguity aversion is irrational and that decision makers should be ambiguity indifferent.

Fixed Reference Level

Under expected utility theory, the status quo (or perceived reference level) is assumed to remain fixed throughout the period or epoch (see Howard, 1992) in which the model is to be used. For example, a single attribute utility function over a monetary attribute might be assessed over total assets, and the function would not be allowed to change from day to day, even though total assets change. Specifically, expected utility is not modeled as a function of changes in assets from the status quo. However, experiments show that people often react quite asymmetrically to incremental changes that are perceived as gains or losses with respect to the current perceived status quo or some target or reference level. This asymmetry has motivated the development of generalized utility models that treat gains and losses differently (for example, Kahneman and Tversky's (1979) prospect theory, which is a type of rank and sign dependent utility function; see Luce, 1992).

Note that an expected utility function can have different risk attitudes in the gain and loss domains, but the reference level must remain fixed. An S-shaped function, with a point of inflection at a target or reference level can represent risk aversion (concavity) in the gain domain and risk proneness (convexity) in the loss domain. However, some people (especially economists) argue that a person should retain either risk aversion, proneness, or neutrality over both gain and loss domains. Even if expected utility is represented with an S-shaped function, the reference level must remain fixed throughout the decision period. A reasonable prescription is to limit the number of times a decision maker resets the reference level, thus requiring a new decision model, as suggested by von Winterfeldt and Edwards (1986, pp. 373–377).

An issue not directly addressed by expected utility theory is the choice of risk attitude. Under expected utility theory, a person is labeled risk averse if a sure monetary amount (such as the $3,200 in Option *A* in figure 1–1) is preferred over a lottery (such as Option *B*) whose expected monetary value is equal to that sure amount. This labeling scheme is misleading because it mixes attitude toward risk with strength of preference for different outcomes. For example, suppose a student feels the increase in value of getting a grade of A rather than a B- is equal to the increase in value of getting a B- rather than a C. Thus, the strength of the preference increase in going from a C to a B- is the same as from a B- to an A for this student. Then, if the person is indifferent between a B- for sure or a 50 percent chance of an A and a 50 percent chance of a C, (s)he displays relative risk neutrality (Dyer and Sarin (1982), Keller (1985c)). This is because (s)he is risk neutral, relative to her/his strength of preference for outcomes. But, following the conventional labeling of risk attitude, the student is indicating risk aversion since, using the standard 4.0 grading scale, the expected grade points of the risky option are $0.5(4.0) + 0.5(2.0) = 3.0$, which is equivalent to a B grade. The person preferred a B- over an option with the expected grade points = B, so (s)he is seen as giving up a risk premium of from B to B- to avoid the risk. However, aversion to risk might have not entered into this student's thinking, since B- was seen as halfway in between an A and a C in value, but not in the underlying grade point scale. Risk attitudes and strength of preference notions have not been clarified for most generalized utility theories. However, the value function in Kahneman and Tversky's (1979) prospect theory is usually interpreted as a strength of preference function measuring preferences under certainty.

An unresolved question is whether risk attitude should be a by-product of assessment judgments (as it is in expected utility theory assessment procedures) or it should be a conscious decision. For example, a person might choose to be relatively risk neutral over a certain range of outcomes. Expected utility theory can accommodate either approach since only the assessment procedures need to be modified to guarantee a specific risk attitude prior to the calibration of the utility function.

Since the choice of whether to frame the current decision problem's outcomes as gains or losses with respect to the reference level can alter the choice prescribed by a utility model, framing issues are of considerable practical concern. A related question is the choice of the temporal beginning point of the problem, as modeled by a decision tree. Should you frame your life decisions as being at the actual current decision point or at the initial life planning point (say at age 12)? LaValle (1992) addresses this issue. Also, when to stop elaborating the decision tree into the future (LaValle, 1989, 1992) and must be decided. LaValle suggests replacing the standard terms such as *consequences* or *outcomes* for the endpoints in the tree with the term *Positions*, to emphasize that an endpoint today is "the first day of the rest of your life." Howard (1992) prefers the term *prospect*. Thus, the determination of the appropriate small-world (in Savage's terms) for the current decision problem is a key problem and can be more critical for generalized utility models than for expected utility.

Separability

Expected utility preferences are *separable* across mutually exclusive events (Machina, 1989), in the sense of *replacement separability* (the contribution of each outcome x_i and its probability p_i to the overall expected utility of an alternative action is independent of the other outcome/probability pairs) and *mixture separability* (the contribution of each outcome/probability pair to the overall expected utility can be broken down into the utility of x_i, multiplied by p_i). LaValle (1992) discusses problems for generalized utility theories that are nonseparable if they are to be used for normative or prescriptive uses.

Dynamic Consistency

In a dynamic (multiple-stage) setting, expected utility theory has the property of *dynamic consistency*, that is, if a person has option C at time 0 in figure 1–1, the planned choice between A' and B' made at time 0 should agree with the actual choice made at time 1. Notice that the planned choice of CA' (C then A') is strategically equivalent to D and the choice of CB' is equivalent to E (Machina, 1989). By the substitution property, if the actual choice is A' over B', then D is preferred over E, so the planned choice will be CA' over CB'. Howard (1992) emphasizes in his related notion of *sequential consistency* that thoughts during this current epoch about planned actions should be consistent, but he says that at the future time the person is free to make any choice.

Consequentialism

Expected utility also satisfies *consequentialism* (Machina (1989), Hammond (1988)). At any point in time we can focus on the consequences from now on (choices, states, probabilities, and outcomes), and we do not need to know where we've come from or what other probability or choice branches were previously available. Thus, the analysis of the expected utility of alternative actions can be carried out by "folding back" a decision tree representation of the choices and states and by computing the maximum expected utility, based on the options and states remaining at any one point in time. Sarin (1992) uses the term *principle of optimality* to refer to consequentialism, emphasizing the notion that at the current choice point our preference order over current options does not vary with the probability that we would have ended up at this choice point.

Some generalized utility models are criticized because their analysis procedure does not allow folding back the decision tree as is possible under expected utility (LaValle and Wapman, 1986). However, Becker and Sarin (1989) show how their generalized utility model, lottery dependent utility, can be used in a modified folding back procedure. It might be possible to modify their approach for other generalized theories.

Equivalence of Extensive and Normal Forms of Decision Tree

Reducing a multiple-stage extensive-form decision tree to one in normal form with a set of options from which to choose, followed by a single chance stage with a set of possible states, will not lead to different decisions under expected utility. This is a result of applying the reduction of compound lotteries property. But, with generalized utility theories, different choices may result with the normal and extensive forms. Thus, expected utility has the property of *invariance* (LaValle, 1992), that is, the way the tree is drawn should not affect the optimal choice as long as the same real options are present and the same use of information is made. In other words, *strategically equivalent* representations should have the same preference rankings. Luce (1992) discusses the implications of this *required indifference between formally equivalent framings of a gamble*.

Transitivity

Many preference theories, including expected utility, require that if A is preferred over B and B over C, then, by *transitivity*, A should be preferred over C. Fishburn (1988) discusses nontransitive nonlinear utility theories. Luce (1992) summarizes key experimental evidence of transitivity violations, including the preference reversal phenomenon (Grether and Plott, 1979; Lichtenstein and Slovic, 1971). However, these studies assume equivalence between judged and choice indifferences, which is now being questioned (Bostic, Herrnstein and Luce, 1990; Tversky, Sattath and Slovic, 1988). MacCrimmon (1965), finding that business executive subjects sometimes violated transitivity, verbally pointed out their intransitive orderings, and many chose to readjust their orderings and become transitive. However, subjects often wish to persist in violations of other expected utility properties, especially substitution, sure-thing, and ambiguity indifference.

Discussion of Consequentialism, Dynamic Consistency, and Substitution Violations

This section contains a discussion of generalized utility theory violations of the consequentialism, dynamic consistency, and substitution properties. The discussion is motivated by questions about the potential usefulness of generalized expected utility models. A special concern when substitution property violations for static (one-chance stage) lotteries are allowed, is whether *dynamic consistency* and/or *consequentialism* should hold in nonexpected utility models for use in economic theory as positive models that are descriptively accurate and can be used to make economic predictions.

Machina (1989) argues that models violating the substitution property for static lotteries should have the properties of dynamic consistency and nonconsequentialism to be useful in economic theory (see also Chew and Epstein (1989)). Nonconsequentialism means that the choice between A' and B' at time 1 in Figure 1–1 cannot be made without knowing that there was a previous 10 percent probability of arriving at the choice node at time 1, and a 90 percent probability of the outcome \$0 which might have happened had Option C been chosen at time 0. Such a dynamically consistent nonexpected utility model would not always obey the substitution property applied to static single stage lotteries and could thus descriptively model the simultaneous preference among single stage

Table 1–1. Classification of Decision Makers Who Violate Substitution Principle for Static Lotteries ($A > B$ and $D \leqslant E$ Occurs)

	Dynamic Consistency	
	Inconsistent $CA' \leqslant CB'$ *and* $A' \geqslant B'$ *occurs*	*Consistent* $CA' \geqslant CB' \leftrightarrow A' \geqslant B'$
Consequentialist $A \geqslant B \leftrightarrow A' \geqslant B'$	Beta	Delta
Not Consequentialist $A \leqslant B$ and $A' \geqslant B'$ occurs	Epsilon	Gamma

Notes:
$\geqslant$ and $\leqslant$ indicate preference order.
A, A', B, B', C, D, and E are options in figure 1–1.
Alpha-type (expected utility) preferences obey substitution priciple, consequentialism, and dynamic consistency.

lotteries of A over B but E over D in the figure. However, using a dynamically consistent nonexpected utility model, under option C the planned choice between A' and B' at time 0 in the decision tree in figure 1–1 would have to agree with the actual choice made at time 1. A decision maker with these preferences would be classified as a gamma-type according to Machina's (1989) categorization of decision makers into *alpha, beta, gamma*, and *delta* types, as shown in table 1-1. Alpha-types use expected utility and thus obey the substitution property, consequentialism, and dynamic consistency. Betas, gammas, deltas (and an added type: *epsilons*) sometimes violate the substitution property for static lotteries.

Machina is concerned that economic researchers will not accept a model that can potentially predict dynamically inconsistent choices. This behavior arises by being a consequentialist and isolating the focus at time 1 only on A' and B', perhaps choosing A' over B', having planned on CB′ over CA′ originally. The argument against dynamic inconsistency is normative. It hinges on the possibility that a person can be made to "make book" against his/her own choices, making the person into a perpetual money pump, cycling among options to eventual ruin. Adding to this normative argument the descriptive observation that such money pumps are not observed in economic markets, Machina (1989) rejects dynamic inconsistency. Thus, he rejects beta-type preferences (consequentialist, not dynamically consistent, substitution property violators) and, implicitly, epsilon-type preferences (which differ from betas only in not being consequentialists). So, Machina rejects the two types of preferences that may be descriptively most common. Upon reflection, I believe that I tend to be a beta-type in casual decision making. LaValle (1992) suspects most people are epsilon-types in casual decisions.

It seems that a better approach to economic modeling, due to the need for descriptive validity, would be to continue the search for mathematically tractable theories that are descriptively valid, both for individual judgment behavior and for the observed aggregate market behavior. I believe that nonexpected utility models were developed in response to both types of substitution property violations, those for static choices and those for dynamic (multiple-stage) choices. Since experimental evidence suggests that this is how people see the problem and make their choices, a descriptively valid model of decision making under risk should definitely allow the planned choice to differ from the actual choice, violating dynamic consistency. Since economic models rely on descriptively accurate models of unaided consumer decision making, dynamic inconsistency should be allowed in those models (Keller, 1989b).

Sarin (1989) presents the philosophical debate over whether dynamic consistency should hold in *normative* models and suggests that a decision maker may wish to violate dynamic consistency in some limited prescriptive settings. Sarin (1992) further argues that although recent generalizations of utility theory can descriptively model such dynamically inconsistent choices, they do not form a coherent normative theory for decision analysis. For example, the lottery dependent utility theory of Becker and Sarin (1987) will allow planned choices to differ from actual. Applying their model to option C in figure 1–1's decision tree problem, at time 0, a beta-type consequentialist who is not dynamically consistent might note that CB' is strategically equivalent to E and thoose the *planned choice* CB' over CA', which is strategically equivalent to D. Then, whenever the decision node at time 1 arises, this consequentialist beta-type revises the tree and only compares A' and B', and may choose A' as the *actual choice*.

Whether a particular generalized model represents dynamically consistent choices may depend not on the model per se, but on the way it is applied to choice situations and how the decision maker frames and reframes choices over time. (LaValle (1992) addresses the problems encountered by nonseparable utility theories with respect to the framing of the decision horizon.) As an example of one way to apply a generalized utility model, Becker and Sarin (1989) show how to analyze the lottery dependent expected utility of alternatives using a modified approach for folding back a decision tree. Following this analysis procedure yields delta-type preferences that are dynamically consistent (since planned choice equals actual choice), because plans are always made by working backwards through the entire tree. This procedure is also consequentialist, since folding back the decision tree to determine choice is done by isolating focus on the current and future stages only. However, Machina (1989) presents three arguments against such delta-type preferences:

1. strategically equivalent lotteries will not be indifferent (LaValle (1989), LaValle and Wapman (1986));
2. delta-types can display aversion to costless information in decision trees (Wakker, 1988); and
3. folding "back is only appropriate when the objective function is separable across the various subdecisions of a problem."

Summary

This chapter contains an overview of properties required by expected utility theory and experiments investigating descriptive violations of these properties. The properties are known by a variety of terms and are not mutually exclusive. Their intertwining makes difficult the task of sorting out the implications and potential applications of different theories that relax certain properties. The remaining chapters in this volume take on this task.

The potential contributions of the new generalized utility theories have been obscured by some confusion over the purposes and possible uses of various theories (Keller (1989a,b)). Despite the muddied waters, the consensus remains that expected utility is a coherent normative theory for decisions under risk, but not all its properties are descriptively valid. Further, the new generalized utility theories' contributions will be primarily for descriptive or predictive purposes. However, in special cases, the generalized utility theories may be used for prescriptive guidance of choice under risk. Also, in economic theoretic modeling, positive models are needed that are descriptively accurate and mathematically tractable so economic predictions can be made. More investigation of generalized utility models and their properties is needed to find appropriate economic theoretic models.

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