**Objectives**

- Present an analytical model of benthic exchange and reaction
- Pumping of solute across bedforms
- Advection-dominated mass transport
- First-order reaction in sediment
- Evaluate model
  - Compare to numerical solution
  - Derive solution for solute flux into the sediment
- Application
  - Correlations for mass transfer coefficient
  - Trade-off between volume of water processed in the sediment and extent of reaction

**Solution approach (neglect dispersion & diffusion)**

Streamline geometry

\[ C_I = \frac{C_f}{C_0} \]

Predicted concentration field (analytical)

\[ C(y) = C_I(l) \exp \left( \frac{-r_y \sin \theta}{2 \lambda \pi \cos \theta} \right) \]

Mass-transfer-limited flux

\[ J_{\text{mass}} = -C_0 u_m \left( \frac{\theta \pi}{\lambda} \right) \]

Variable definitions

- \( u_m \): mass velocity
- \( \theta \): horizontal distance
- \( \lambda \): bed form wavelength
- \( \pi \): unit cell

**Elliot and Brooks Velocity Model**

- Normalize by wave number
  - \( \bar{x} = 2\pi x / \lambda \)
  - \( \bar{y} = 2\pi y / \lambda \)

- Velocity field
  - \( u_x = \text{u}_x \)
  - \( u_y = \text{u}_y \)

- Pressure head
  - \( \bar{h} = \text{h}_x \sin \pi \bar{x} \)

- Bed geometry
  - \( \bar{h} = \text{h}_x \sin \pi \bar{x} \)

- Maximum pressure head
  - \( u_m = -2\pi K_b h_n / \lambda \)

**Applications**

- Numerical and analytical solutions near-identical
- Numerical simulation carried out with COMSOL
- Mechanical dispersion & molecular diffusion negligible
- Dirichlet b.c. at surface causes gradient artifact

**Ecosystem services**

- Flux into sediment = mass removal in sediment
- Mass removal in sediment = ecosystem service (N, P, C processing; CEC removal)
- Model identifies trade-off between volume of water processed by hyporheic zone and extent of reaction in sediment (hard to optimize both)

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