DIMENSION 4: GETTING SOMETHING FROM NOTHING

RON STERN
UNIVERSITY OF CALIFORNIA, IRVINE
MAY 6, 2010

JOINT WORK WITH RON FINTUSHEL
Topological n-manifold:

locally homeomorphic to $\mathbb{R}^n$
Topological n-manifold:
locally homeomorphic to $\mathbb{R}^n$

Smooth n-manifold:
locally diffeomorphic to $\mathbb{R}^n$

TOPOLOGICAL VS. SMOOTH MANIFOLDS
Topological n-manifold:
locally homeomorphic to $\mathbb{R}^n$

Smooth n-manifold:
locally diffeomorphic to $\mathbb{R}^n$

Low dimensions: $n=1,2,3$ topological $\iff$ smooth

---

TOPOLOGICAL VS. SMOOTH MANIFOLDS
Topological n-manifold:
locally homeomorphic to $\mathbb{R}^n$

Smooth n-manifold:
locally diffeomorphic to $\mathbb{R}^n$

Low dimensions: $n=1,2,3$ \text{ topological} $\iff$ \text{ smooth}

High dimensions: $n > 4$: smooth structures on topological manifolds determined up to finite indeterminacy by characteristic classes of the tangent bundle.

TOPOLOGICAL VS. SMOOTH MANIFOLDS
DIMENSION 4

Techniques from other dimensions do not apply
(in fact fail in a rather dramatic fashion)
DIMENSION 4

Techniques from other dimensions do not apply (in fact fail in a rather dramatic fashion)

IN FACT IT COULD BE THE CASE THAT EVERY CLOSED TOPOLOGICAL MANIFOLD HAS EITHER NO OR INFINITELY MANY SMOOTH STRUCTURES!

NO KNOWN COUNTEREXAMPLE
DIMENSION 4

Techniques from other dimensions do not apply
(in fact fail in a rather dramatic fashion)

IN FACT IT COULD BE THE CASE THAT
EVERY CLOSED TOPOLOGICAL MANIFOLD HAS
EITHER NO OR
INFINITELY MANY SMOOTH STRUCTURES!

NO KNOWN COUNTEREXAMPLE

HOW TO PROVE (OR DISPROVE) SUCH A WILD STATEMENT?

FOR SIMPLICITY LET’S ASSUME SIMPLY-CONNECTED
Classification of Simply-Connected Topological 4-Manifolds
Classification of Simply-Connected Topological 4-Manifolds

Intersection form of a 4-manifold
\[ H_2(X) \times H_2(X) \to \mathbb{Z} \]

Inner product space over the integers

Invariants: Rank = \( b_2 \)
Signature
Type (even, odd)
Classification of Simply-Connected Topological 4-Manifolds

Intersection form of a 4-manifold
\[ H_2(X) \times H_2(X) \rightarrow \mathbb{Z} \]
Inner product space over the integers

Invariants: Rank = $b_2$
Signature
Type (even, odd)

Freedman's + Donaldson's Theorem
Smooth compact simply connected 4-manifolds are determined up to homeomorphism by
(rank, signature, type)
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r\mathbb{CP}^2 \# s\overline{\mathbb{CP}^2}$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \mathbb{C}P^2 \# s \overline{\mathbb{C}P^2}$

$\mathbb{C}P^2$: Space of complex lines in $\mathbb{C}^3$

$H^2(\mathbb{C}P^2) = \mathbb{Z}$
generated by the class of a projective line $L$

$L.L = +1$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \mathbb{CP}^2 \# s \overline{\mathbb{CP}^2} \quad \mathbb{CP}^2$: Space of complex lines in $\mathbb{C}^3$

$$H^2(\mathbb{CP}^2) = \mathbb{Z}$$

generated by the class of a projective line $L$

$$L.L = +1$$

$\overline{\mathbb{CP}^2}$ is $\mathbb{CP}^2$ with the opposite orientation

$$H^2(\overline{\mathbb{CP}^2}) = \mathbb{Z}$$

generated by the class of a ‘line’ $E$, called an ‘exceptional curve’

$$E.E = -1$$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \mathbb{CP}^2 \# s \overline{\mathbb{CP}^2}$

$\mathbb{CP}^2$: Space of complex lines in $\mathbb{C}^3$

$H^2(\mathbb{CP}^2) = \mathbb{Z}$

generated by the class of a projective line $L$

$L.L = +1$

$\overline{\mathbb{CP}^2}$ is $\mathbb{CP}^2$ with the opposite orientation

$H^2(\overline{\mathbb{CP}^2}) = \mathbb{Z}$

generated by the class of a ‘line’ $E$, called an ‘exceptional curve’

$E.E = -1$

Connected sum with $\overline{\mathbb{CP}^2}$ is called blowing up
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r\,\mathbb{C}P^2 \# s\,\overline{\mathbb{C}P^2}$

$\mathbb{C}P^2$: Space of complex lines in $\mathbb{C}^3$

$H^2(\mathbb{C}P^2) = \mathbb{Z}$

generated by the class of a projective line $L$

$L.L = +1$

$\overline{\mathbb{C}P^2}$ is $\mathbb{C}P^2$ with the opposite orientation

$H^2(\overline{\mathbb{C}P^2}) = \mathbb{Z}$

generated by the class of a ‘line’ $E$, called an ‘exceptional curve’ $E.E = -1$

Connected sum with $\overline{\mathbb{C}P^2}$ is called blowing up

Small Examples: Rational Surfaces $= \mathbb{C}P^2 \# k\overline{\mathbb{C}P^2}$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \mathbb{CP}^2 \# s \overline{\mathbb{CP}^2}$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r\ CP^2 \# s \overline{CP^2}$

GOAL: Do some surgery to alter smooth structure!
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \mathbb{C}P^2 \# s \overline{\mathbb{C}P^2}$

GOAL: Do some surgery to alter smooth structure!

OBSERVATIONS
If homeomorphic to complex surface of general type then $r = 2k + 1$
(Not diffeomorphic to $r \mathbb{C}P^2 \# s \overline{\mathbb{C}P^2}$)
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \, \mathbb{CP}^2 \# s \, \overline{\mathbb{CP}}^2$

GOAL: Do some surgery to alter smooth structure!

OBSERVATIONS

If homeomorphic to complex surface of general type then $r = 2k+1$
(Not diffeomorphic to $r \, \mathbb{CP}^2 \# s \, \overline{\mathbb{CP}}^2$)

The canonical class of a complex surface $X$, $H_2(X)$ satisfies:

- It is characteristic: $K_X \cdot y = y \cdot y \Mod 2$
- It is $K_X = -c_1(T^*(X))$, $K_X^2 = 3 \text{sign} + 2e$
- Adjunction Formula: For any complex curve $C$ of genus $g$ in $X$, $2g-2 = C \cdot C + K_X \cdot C$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r\ \mathbb{CP}^2 \# s\ \overline{\mathbb{CP}^2}$

GOAL: Do some surgery to alter smooth structure!

OBSERVATIONS

If homeomorphic to complex surface of general type then $r = 2k + 1$

$K_X^2 = 10k + 9 - s$ and has genus $10k + 10 - s$
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \mathbb{CP}^2 \# s \overline{\mathbb{CP}^2}$

**GOAL:** Do some surgery to alter smooth structure!

**OBSERVATIONS**

If homeomorphic to complex surface of general type then $r = 2k+1$

$$K_X^2 = 10k+9-s \text{ and has genus } 10k+10-s$$

The class $S = (3,3,3,\ldots,1,1,\ldots)$ in $(2k+1) \mathbb{CP}^2 \# s \overline{\mathbb{CP}^2}$ ($(k+1)$ 3’s) has $S^2 = 10k+9-s$ and is represented by an embedded torus, i.e. $g = 1$

(Seemed to be unknown)
EVERY CLOSED SMOOTH 4-MANIFOLD HAS INFINITELY MANY SMOOTH STRUCTURES?

WARMUP IDEA: Suppose $b_2$ is non-trivial and intersection form odd. So homeomorphic to $r \, \mathbb{CP}^2 \# s \, \overline{\mathbb{CP}^2}$

GOAL: Do some surgery to alter smooth structure!

OBSERVATIONS
If homeomorphic to complex surface of general type then $r = 2k+1$

$K_x^2 = 10k+9-s$ and has genus $10k+10-s$

The class $S = (3,3,3,...,1,1,...)$ in $(2k+1) \, \mathbb{CP}^2 \# s \, \overline{\mathbb{CP}^2}$ ($(k+1)$ 3’s) has $S^2 = 10k+9-s$ and is represented by an embedded torus, i.e. $g = 1$
(Seemed to be unknown)

GOAL: Surger to increase genus of $S$
Log Transformation (Surgery)

$T : \text{torus in } X \text{ with } T.T=0$

Neighborhood $T \times D^2 = S^1 \times (S^1 \times D^2)$
Log Transformation (Surgery)

\( T : \) torus in \( X \) with \( T \cdot T = 0 \)

Neighborhood \( T \times D^2 = S^1 \times (S^1 \times D^2) \)

\( p/q \) - Surgery:

Cut out and reglue so that \( \partial D^2 \) is glued to the curve

\[ pt \times (q[S^1] + p[\partial D^2]) \]
Log Transformation (Surgery)

\[ T : \text{torus in } X \text{ with } T \cdot T = 0 \]

Neighborhood \( T \times D^2 = S^1 \times (S^1 \times D^2) \)

\( p/q \) - Surgery:

Cut out and reglue so that \( \partial D^2 \) is glued to the curve

\[ pt \times (q[S^1] + p[\partial D^2]) \]

- In general, depends on choice of splitting
- If \( T \) is a complex curve and \( \pi_1(X) = 0 \), then this only depends on \( p \). Called \( p \)-log transform

Process creates a multiple torus

\[ S^1 \times \]
Exotic Rational Elliptic Surfaces

\[ E(1) = \mathbb{CP}^2 \# 9\bar{\mathbb{CP}}^2 = \text{Rational elliptic surface} \]
Exotic Rational Elliptic Surfaces

\[ E(1) = \mathbb{CP}^2 \# 9\overline{\mathbb{CP}}^2 = \text{Rational elliptic surface} \]

Dolgachev Surface:

\[ E(1)_{2,3} \text{ the result of multiplicity 2 and 3- log transforms} \]

Donaldson’s Theorem. \( E(1)_{2,3} \) is homeo but not diffeo to \( E(1) \)
Exotic Rational Elliptic Surfaces

\[ \text{E}(1) = \mathbb{CP}^2 \# 9\overline{\mathbb{CP}^2} = \text{Rational elliptic surface} \]

Dolgachev Surface:
\[ \text{E}(1)_{2,3} \text{ the result of multiplicity 2 and 3-log transforms} \]

Donaldson’s Theorem. \(\text{E}(1)_{2,3}\) is homeo but not diffeo to \(\text{E}(1)\)

Knot Surgery:
Remove neighborhood of torus \(T\) with \(T.T=0\)
Replace with \(S^1 \times (S^3 - \text{Nbd(Knot)})\) Get \(X_K\)
Exotic Rational Elliptic Surfaces

\[ E(1) = \mathbb{CP}^2 \# 9 \overline{\mathbb{CP}}^2 = \text{Rational elliptic surface} \]

Dolgachev Surface:
\[ E(1)_{2,3} \text{ the result of multiplicity 2 and 3- log transforms} \]

Donaldson's Theorem. \( E(1)_{2,3} \) is homeo but not diffeo to \( E(1) \)

Knot Surgery:
- Remove neighborhood of torus \( T \) with \( T.T=0 \)
- Replace with \( S^1 \times (S^3-\text{Nbd(Knot)}) \) Get \( X_K \)

F\&Stern: If \( \pi_1(X)=0, \pi_1(X-T)=0, \) and \( T \) is homologically nontrivial, then \( X_K \) is homeo not diffeo to \( X \).

(The Seiberg-Witten invariant of \( X \) gets multiplied by the Alexander polynomial of \( K \))
Exotic Rational Elliptic Surfaces

\[ E(1) = \mathbb{CP}^2 \# 9\overline{\mathbb{CP}}^2 = \text{Rational elliptic surface} \]

Dolgachev Surface:
\[ E(1)_{2,3} \text{ the result of multiplicity 2 and 3- log transforms} \]

Donaldson's Theorem. \( E(1)_{2,3} \) is homeo but not diffeo to \( E(1) \)

Knot Surgery:
- Remove neighborhood of torus \( T \) with \( T \cdot T = 0 \)
- Replace with \( S^1 \times (S^3 - \text{Nbd(Knot)}) \) Get \( X_K \)

F&Sterne: If \( \pi_1(X) = 0, \pi_1(X-T) = 0, \) and \( T \) is homologically nontrivial, then \( X_K \) is homeo not diffeo to \( X \).

**PROBLEM:** \( \mathbb{CP}^2 \# k\overline{\mathbb{CP}}^2 \), \( k < 9 \), have no such tori.
KEY TO KNOT SURGERY CONSTRUCTION

Find embedded torus $T$ that is non-trivial in homology
KEY TO KNOT SURGERY CONSTRUCTION

Find embedded torus $T$ that is non-trivial in homology

$\mathrm{Nbd\ of\ fiber\ torus\ in\ } S^1 \times B_T$ 

Bing double of

$1/1$ and $1/n$ surgeries on $B_T$ give family of manifolds homeo but not diffeo to $E(1)$
KEY TO KNOT SURGERY CONSTRUCTION

Find embedded torus $T$ that is non-trivial in homology

Nbd of fiber torus in $B_T$ Bing double of $\mathbb{S}^1 \times \mathbb{S}^1$

$1/1$ and $1/n$ surgeries on $B_T$ give family of manifolds homeo but not diffeo to $E(1)$

$-1$ surgery

Knot surgery theorem proved by studying change in SW invariant when unknotting $K$
KEY TO KNOT SURGERY CONSTRUCTION

Find embedded torus $T$ that is non-trivial in homology

$B_T$  Bing double of

$S^1 \times \mathbb{R}$

nullhomologous tori:

1/1 and 1/n surgeries on $B_T$ give family of manifolds homeo but not diffeo to $E(1)$

Knot surgery theorem proved by studying change in SW invariant when unknotting $K$
KEY TO KNOT SURGERY CONSTRUCTION

Find embedded torus $T$ that is non-trivial in homology

Nbd of fiber torus in $S^1 \times \mathbb{R}$

$B_T$, Bing double of nullhomologous tori:

1/1 and 1/n surgeries on $B_T$ give family of manifolds homeo but not diffeo to $E(1)$

$-1$ surgery

Knot surgery theorem proved by studying change in SW invariant when unknotting K

KEY IDEA

Santeria Surgery: Surgery on nullhomologous tori
KEY TO KNOT SURGERY CONSTRUCTION

Find embedded torus $T$ that is non-trivial in homology.

Nbd of fiber torus in $S^1 \times \mathbb{R}^3$

$B_T$ Bing double of nullhomologous tori:

1/1 and 1/n surgeries on $B_T$ give family of manifolds homeo but not diffeo to $E(1)$

Knot surgery theorem proved by studying change in SW invariant when unknotting $K$

KEY IDEA

Santeria Surgery: Surgery on nullhomologous tori

How do we find them when no minimal genus essential tori are present?
Increasing Genus by Surgery

\[ T^2 \times D^2 = S^1 \times \]

Standard surgeries on \( B_T \) turn it into \( T^2 \times T_0 \)

\( T_0 = T^2 - D^2 \)
Increasing Genus by Surgery

\[ T^2 \times D^2 = S^1 \times \]

Standard surgeries on \( B_T \) turn it into \( T^2 \times T_0 \)

\[ T_0 = T^2 - D^2 \]

This increases genus of surfaces normal to \( B_T \) by one for each normal crossing
CP$^2$

No reasonable $T^2 \times D^2$ in $CP^2$
No reasonable $T^2 \times D^2$ in $CP^2$

All we really need is

$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$
No reasonable $T^2 \times D^2$ in $\mathbb{C}P^2$

All we really need is

$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$

Can we find $A$?
No reasonable $T^2 \times D^2$ in $\mathbb{CP}^2$

All we really need is
$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$

Can we find $A$?

$\mathbb{CP}^2 = \{(x,y,z) \in \mathbb{C}^3 -(0,0,0)\}/(x,y,z) \sim (tx,ty,tz) \mid t \in \mathbb{C}^*$

$(x:y:z)$ Homogeneous coordinates

$\mathbb{CP}^2 = \text{Union of coordinate nbds of 3 special points } (1:0:0) \text{ (0:1:0) and (0:0:1)}$
All we really need is

\[ A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0 \]

Can we find A?

\[ CP^2 = \{(x,y,z) \in \mathbb{C}^3 \setminus (0,0,0)\}/(x,y,z) \sim (tx,ty,tz) \mid t \in \mathbb{C}^* \]

(x:y:z) Homogeneous coordinates

\[ CP^2 = \text{Union of coordinate nbds of 3 special points} (1:0:0) (0:1:0) \text{ and } (0:0:1) \]

Each coord nbd \( \cong B^4 \)
The Manifold $A$ 

$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$
The Manifold $A$

$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$

In terms of handles attached to $B^4$:

$$B^4 = \text{intersection of two circles}$$
The Manifold $A$

$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$

In terms of handles attached to $B^4$:

$$B^4 = \text{untwisted string}$$

$$A = \text{unspecialized knot}$$
The Manifold $A$

$$A = S^1 \times (S^1 \times D^2) - \text{pt} \times T_0$$

In terms of handles attached to $B^4$:

$B^4 = S^1 \times S^1$

$A$ results from attaching $n=0$ handles to $B^4$ where $n=0$
More about A

\[ B^4 = \quad A = \]
More about $A$

$B^4 = \bullet$

Adding 1-handles = subtracting 2-handles
More about $A$

$B^4 = \quad A =$

Adding 1-handles = subtracting 2-handles

add 2-hdles   subtract 2-hdles

$B^4$       $A$
More about $A$

$B^4 = \quad \quad A = \quad \quad 0$

Adding 1-handles = subtracting 2-handles

$B^4 =$ add 2-handles

$A =$ subtract 2-handles
More about A

\[ B^4 = \begin{array}{c} \text{0} \\ \text{0} \end{array} \quad A = \begin{array}{c} \text{0} \\ \text{0} \end{array} \]

Adding 1-handles = subtracting 2-handles

Add 2-hdles

Subtract 2-hdles
More about $A$

$B^4 = \begin{array}{c}
0
\end{array}$

$A = \begin{array}{c}
0
\end{array}$

Adding 1-handles = subtracting 2-handles

IDEA IS CLEAR: Cover $r \text{CP}^2 \# s \text{CP}^2$ with 4-balls and do alot of "Pushing and shoving"
More about $\mathbb{C}P^2$

Ruled surface $F_1$
$S^2$-bundle over $S^2$
More about $\mathbb{CP}^2$

Ruled surface $F_1$

$S^2$-bundle over $S^2$

Complement of $s_-$ and $f$ is a 4-ball containing disks of relative self-intersections 0 and +1
More about $\mathbb{CP}^2$

Ruled surface $F_1$
$S^2$-bundle over $S^2$

Complement of $s_-$ and $f$ is a 4-ball containing disks of relative self-intersections 0 and +1

This is coordinate nbd in $\mathbb{CP}^2$
$\text{Nbd}=B^4$ where $n=+1$

Union of 3 coord nbds is $\mathbb{CP}^2$
Pinwheel Structures

Previous example - kind of 'triple sum'
Remove transverse surface from 4-mfds
Glue together along pieces of the boundary

\[ K_{\mathbb{CP}^2} = -3L = \text{union of 3 2-spheres} \]
Pinwheel Structures

Previous example - kind of ‘triple sum’
Remove transverse surface from 4-mfds
Glue together along pieces of the boundary

\[ K_{\mathbb{C}P^2} = -3L = \text{union of 3 2-spheres} \]

Problem: Twisting of the bundle $F_1$

Push and shove gives instead of $A$
Resolution of our problem: Blow up each coordinate neighborhood

Blow up of $CP^2 \# 3CP^2$ is

\[ \begin{array}{c}
\begin{array}{c}
\text{Blow up of} \\
0 \\
+1
\end{array}
\end{array} \]

is

\[ \begin{array}{c}
\begin{array}{c}
\text{Blow up of} \\
0 \\
-1
\end{array}
\end{array} \]

$\subset A$
Resolution of our problem: Blow up each coordinate neighborhood

Blow up of of

is

3 push and shove moves give 3 copies of A
Exotic Manifolds

\[ R = \mathbb{CP}^2 \# 3 \overline{\mathbb{CP}^2} \] contains 3 disjoint copies of \( A \), one in each blown up coord nbd \( C_i \)
Exotic Manifolds

\[ R = \mathbb{CP}^2 \# 3 \mathbb{CP}^2 \] contains 3 disjoint copies of \( A \), one in each blown up coord nbd \( C_i \)

In each \( C_i \), the Bing tori \( BT \) are transverse to each of the 6 disks comprising \( K_R = -3L + E_1 + E_2 + E_3 \)
Exotic Manifolds

\[ R = \mathbb{CP}^2 \# 3 \mathbb{CP}^2 \] contains 3 disjoint copies of A, one in each blown up coord nbd \( C_i \).

In each \( C_i \), the Bing tori BT are transverse to each of the 6 disks comprising \( K_R = -3L + E_1 + E_2 + E_3 \).

\[ \Rightarrow \text{Surgeries on the Bing tori increase the genus of the canonical class by 6} \]
Exotic Manifolds

\[ \text{R} = \mathbb{C}P^2 \# 3 \mathbb{C}P^2 \] contains 3 disjoint copies of A, one in each blown up coord nbd \( C_i \).

In each \( C_i \), the Bing tori BT are transverse to each of the 6 disks comprising \( K_R = -3L + E_1 + E_2 + E_3 \).

\[ \Rightarrow \] Surgeries on the Bing tori increase the genus of the canonical class by 6.

These surgeries give rise to a symplectic manifold \( X \) homeo to \( R \) but not diffeo to it, because its canonical class has genus 7 instead.
R=CP^2 #3CP^2 contains 3 disjoint copies of A, one in each blown up coord nbd C_i

In each C_i, the Bing tori BT are transverse to each of the 6 disks comprising \( K_R = -3L + E_1 + E_2 + E_3 \)

\[ \Rightarrow \text{Surgeries on the Bing tori increase the genus of the canonical class by 6} \]

These surgeries give rise to a symplectic manifold X homeo to R but not diffeo to it, because its canonical class has genus 7 instead.

More surgeries give \( \infty \) family, rest not symplectic
Th’m. (F & Stern) For $k=2,...,7$ and $k=9$, there are nullhomologous tori embedded in $\mathbb{CP}^2 \# k\overline{\mathbb{CP}^2}$ upon which Santeria surgery gives rise to an infinite family of mutually nondiffeomorphic minimal 4-manifolds homeomorphic to $\mathbb{CP}^2 \# k\overline{\mathbb{CP}^2}$
**Theorem and Conjecture**

**Th’m.** (F & Stern) For \( k=2,\ldots,7,8,9 \) there are nullhomologous tori embedded in \( \mathbb{CP}^2 \# k\overline{\mathbb{CP}^2} \) upon which Santeria surgery gives rise to an infinite family of mutually nondiffeomorphic minimal 4-manifolds homeomorphic to \( \mathbb{CP}^2 \# k\overline{\mathbb{CP}^2} \)

**Wild Conjecture.** Every simply connected smooth 4-manifold is obtained by surgery on tori in a connected sum of copies, with either orientation, of \( S^4, \mathbb{CP}^2, \overline{\mathbb{CP}^2}, S^2 \times S^2 \), and the K3-surface.
IN DIMENSION 4

WHAT CHANGES SMOOTH STRUCTURES????
IN DIMENSION 4
WHAT CHANGES SMOOTH STRUCTURES????

WORM HOLE SURGERY
There,
I've got his nullhomologous torus.

I can improve this manifold by surgery.

I'll just replace his torus with a twist.

Much better after surgery!
I'll just replace his torus with a twist.

Much better after surgery!
It looks different, but after all this work ... it has the same Seiberg-Witten invariant.
it looks different, but after all this work ...

It has the same Seiberg-Witten invariant.

Let's try again.
YES, NO DOUBT A DIFFERENT SEIBERG-WITTEN INVARIANT