Effect of Probe Deflection on Dual-Probe Heat-Pulse Thermal Conductivity Measurements

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The dual-probe heat-pulse (DPHP) method is useful for measuring soil thermal properties; however, the probes of a DPHP sensor can deflect when inserted into the soil. Theoretical analysis has shown that measurements of thermal conductivity (λ) should be unaffected by deflection-induced changes in probe spacing. To verify this result, the conductivities of water, dry sand, and saturated sand were measured using DPHP sensors with probes subject to inward deflection, no deflection, and outward deflection. No error in λ was detected when probes were deflected inward by an amount that caused a 14% reduction in probe spacing. Outward deflection (15% increase in spacing) caused error in λ estimates, but the errors were small ($\leq$0.04 W m^{-1} K^{-1}) and likely to be of little consequence in most applications. We conclude that estimates of λ obtained with the DPHP method are largely unaffected by changes in probe spacing caused by deflection.

Abbreviations: DPHP, dual-probe heat pulse.

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Bristow et al. (1994) showed that the DPHP sensor of Campbell et al. (1991) can be used to measure soil thermal diffusivity (κ) and λ in addition to volumetric heat capacity (ρc). Briefly, the method involves measuring the temperature rise at a known distance from a cylindrical heater probe that contains resistance wire. A pulse of heat is introduced into the soil by passing electrical current through the wire. The temperature rise is measured as a function of time with a thermocouple or thermistor in a cylindrical probe oriented parallel to the heater probe. The thermal properties ρc, κ, and λ are then estimated from the magnitude and time of the maximum temperature rise.

Unfortunately, in some situations, the probes of a DPHP sensor can deflect when they are inserted into the soil. Deflection is problematic because errors in probe spacing—the distance between the heater and temperature probes—cause errors in ρc and κ estimates. The values of ρc and κ determined using the method of Bristow et al. (1994) are known to be quite sensitive to error in probe spacing (Kluitenberg et al., 1995; Liu et al., 2008). Interestingly, there is reason to believe that probe deflection should be far less problematic for estimates of λ. In fact, if we assume that deflection causes a change in probe spacing but influences the conductivity measurement in no other way, then probe deflection should theoretically produce no error whatsoever in λ estimates (Kluitenberg et al., 1995). This follows from the fact that λ estimates obtained using the method of Bristow et al. (1994) are independent of probe spacing (Noborio et al., 1996). As we discuss below, however, the effect of probe deflection may not be limited to causing changes in probe spacing. Thus, there is a need to experimentally verify the expectation that deflection should cause no error in λ estimates. We conducted a laboratory experiment designed to quantify error in λ estimates caused by probe deflection.

THEORY

Consider heat conduction from a line source of infinite length in homogenous, isotropic soil initially at a uniform temperature. If heat is released from the line source from time $t = 0$ to time $t = t_0$ at the constant rate per unit length q', the temperature rise ΔT at a fixed distance r_0 from the line source is

$$\Delta T(t) = -\frac{q'}{4\pi\lambda}\mathrm{Ei}\left(\frac{-r_0^2}{4\kappa t}\right); \quad 0 < t \leq t_0$$

$$\Delta T(t) = \frac{q'}{4\pi\lambda}\left\{\mathrm{Ei}\left[\frac{-r_0^2}{4\kappa(t-t_0)}\right] - \mathrm{Ei}\left(\frac{-r_0^2}{4\kappa t}\right)\right\}; t > t_0 \qquad [1]$$

where $-\mathrm{Ei}(-x)$ is the exponential integral of argument x. Bristow et al. (1994) used the time derivative of Eq. [1] to show that κ and ρc can be estimated from

$$\kappa = \frac{r_0^2}{4}\left\{\frac{1/(t_\mathrm{m}-t_0) - 1/t_\mathrm{m}}{\ln\left[t_\mathrm{m}/(t_\mathrm{m}-t_0)\right]}\right\} \qquad [2]$$

and

$$\rho c = \frac{q'}{4\pi\kappa\Delta T_\mathrm{m}}\left\{\mathrm{Ei}\left[\frac{-r_0^2}{4\kappa(t_\mathrm{m}-t_0)}\right] - \mathrm{Ei}\left(\frac{-r_0^2}{4\kappa t_\mathrm{m}}\right)\right\} \qquad [3]$$

where ΔT_m is the maximum temperature rise and t_m is the time at which ΔT_m occurs. Whereas Bristow et al. (1994) presented the expression for conductivity in the form $\lambda = \kappa \rho c$, it can also be written in the form (Noborio et al., 1996)

$$\lambda = \frac{q'}{4\pi \Delta T_m}\left(\mathrm{Ei}\left\{ \frac{-\ln\left[t_m/(t_m - t_0)\right]}{t_0/t_m} \right\} - \mathrm{Ei}\left\{ \frac{-\ln\left[t_m/(t_m - t_0)\right]}{t_0/(t_m - t_0)} \right\} \right) \quad [4]$$

in which r_0 does not appear. This expression implies that the influence of r_0 is reflected entirely in the values of t_m and ΔT_m. Any change in r_0 causes corresponding changes in t_m and ΔT_m, but these changes compensate for each other in such a way that λ remains constant.

MATERIALS AND METHODS

The seven DPHP sensors used for this experiment were identical to those described in Basinger et al. (2003). Each consisted of two probes (27-mm exposed length) mounted in a polyvinyl chloride block (i.e., sensor body). The probes were parallel, with a center-to-center spacing of ~6 mm. One probe contained a thermistor and the other contained a heating element with a resistance of $R = 820\ \Omega\ \mathrm{m}^{-1}$. The probes were fabricated from lengths of stainless steel tubing (0.84-mm i.d., 1.27-mm o.d.) that were filled with thermally conductive epoxy. Basinger et al. (2003) provided additional details regarding the sensor components and construction.

The probes of all sensors were subjected to treatments of inward deflection, no deflection, and outward deflection. The probes were deflected inward by placing a 3-mm-long section of rubber tubing (3.1-mm i.d., 1.5-mm wall thickness) around the distal ends of the heater and temperature probes (Fig. 1A). This resulted in a center-to-center spacing of ~4.2 mm (caliper measurement) at the distal end of the probes and ~5.2 mm at the location of the thermistor. Outward deflection, achieved with a 2-mm-thick Plexiglas spacer (Fig. 1B), resulted in a center-to-center spacing of ~8.2 mm at the distal ends of the probes and ~6.8 mm at the location of the thermistor. None of the sensors failed during the experiment.

Conductivity measurements were made in agar-immobilized water and in oven-dried and water-saturated Tottori dune sand (Mori et al., 2003). These media were chosen because they provided a wide range of conductivities and they allowed sample preparation without causing deflection in addition to that imposed. All measurements were done in 600-mL beakers with the sensors (one per beaker) suspended at a height that left the entire sensor (probes and sensor body) fully immersed after sample preparation was completed. For the measurements in water, beakers were filled with 500 mL of agar-immobilized water (4 g L^{-1}). For the measurements in sand, 815 g of oven-dried sand was added to each beaker to yield a soil volume of 500 cm^{-3} when packed to the target bulk density of 1.63 g cm^{-3}. To prepare the saturated sand, the beakers were filled with 325 mL of water before adding the oven-dried sand. Excess water was removed after packing was completed. Packing was done by repeatedly tapping the rim of the beaker with a rubber mallet until the desired density was achieved.

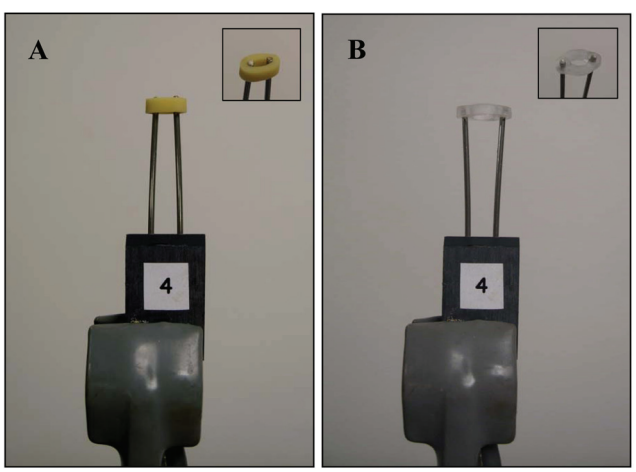

Fig. 1. Photographs showing dual-probe heat-pulse sensors with (A) probes deflected inward and (B) probes deflected outward.

The data acquisition and control system consisted of a datalogger, multiplexer, and relay driver (CR3000, AM16/32B, and SDM-CD16AC, respectively; Campbell Scientific, Logan, UT). Heating was achieved by applying ~12.9 V to the heater element for 8 s, which yielded a nominal value for q' of 110 W m^{-1}. The heating rate for each measurement was calculated as $q' = I^2R$, where I is the current in the heater circuit. The current was measured by sampling the voltage drop across a 1-Ω shunt resistor at a frequency of 2 Hz. Thermistor output was measured and converted to temperature as described in Mori et al. (2003). The temperature was measured at a frequency of 2 Hz from time $t = 0$ (i.e., the time at which heating was initiated) until $t = 180$ s. The temperature rise, ΔT, was calculated by subtracting the initial temperature (the mean of 10 observations) measured a few seconds before the onset of heating. All experiments were conducted in a room maintained at 20 ± 1°C.

Thermal conductivity was determined by evaluating Eq. [4] with $t_0 = 8$ s and measured values of q', ΔT_m, and t_m. The values of ΔT_m and t_m were identified after applying a nine-point, third-order smoothing filter (Savitzky and Golay, 1964) to the $\Delta T(t)$ data. Measurements were repeated five times (with an interval of 75 min between measurements) and the results were averaged to obtain the λ estimate for each combination of sensor, medium, and deflection treatment. This yielded a total of 63 conductivity observations (7 sensors × 3 deflection treatments × 3 media). Summary statistics were computed by treating the sensors as replicates. For each medium, two-tailed, paired t-tests ($P < 0.05$) were used to compare the mean conductivities for the inward and outward deflection treatments with the mean conductivity for the no-deflection treatment. Paired tests were used because all the deflection treatments were applied to the same set of seven sensors, and thus it was necessary to account for the possibility of sensor effects.

To quantify the effect of the deflection treatments on the probe spacing, estimates of r_0 were obtained from the values of ΔT_m and t_m for the agar-immobilized water. Probe spacing was determined by solving Eq. [3] with $\rho c = 4.174$ MJ $\mathrm{m}^{-3}\ \mathrm{K}^{-1}$ and $\kappa = 1.44 \times 10^{-7}\ \mathrm{m}^2\ \mathrm{s}^{-1}$. The values of r_0 obtained in this way are to be considered *apparent* or *effective* measures of probe spacing (Mori et al., 2003; Ham and Benson, 2004). Typically, they are not equivalent to the physical center-to-center spacing of the probes at the midpoint of the temperature probe (i.e., where the temperature is measured).

RESULTS AND DISCUSSION

The measurements in agar-immobilized water yielded mean values of apparent probe spacing (r_0) of 4.99, 5.81, and 6.66 mm for inward deflection, no deflection, and outward deflection, respectively (standard deviations ≤ 0.078 mm). Inward deflection caused a reduction in r_0 of 0.82 mm (14%) and outward deflection caused an increase in r_0 of 0.85 mm (15%). Relative to the results for no deflection, inward deflection increased the magnitude of ΔT_m and caused it to occur earlier in time for all three media (Table 1; Fig. 2). Outward deflection decreased

Table 1. Maximum temperature rise (ΔT_m), time of the maximum temperature rise (t_m), and thermal conductivity (λ) for oven-dried sand, agar-immobilized water, and water-saturated sand. Results were obtained with dual-probe heat-pulse sensors that had probes subjected to inward deflection, no deflection, and outward deflection. Values of λ were determined from Eq. [4].

Medium	Deflection treatment	ΔT_m		t_m		λ	
		Mean	SD	Mean	SD	Mean	SD
		K		s		W m^{-1} K^{-1}	
Dry sand	inward	3.01	0.105	37.5	0.71	0.26	0.0082
	none	2.26	0.058	47.8	1.16	0.26	0.0053
	outward	1.68	0.040	58.0	1.24	0.29*	0.0076
Water	inward	1.00	0.042	47.7	1.64	0.60	0.0053
	none	0.74	0.027	62.7	1.83	0.60	0.0069
	outward	0.56	0.017	82.4	1.74	0.59	0.0069
Saturated sand	inward	1.44	0.089	14.6	0.54	1.85	0.0376
	none	1.08	0.055	17.3	0.51	1.89	0.0248
	outward	0.81	0.035	21.7	0.77	1.85*	0.0373

* For a given medium, treatment means differ significantly from the mean for the no-deflection treatment at the $P < 0.05$ level.

the magnitude of ΔT_m and caused it to occur later in time for all three media (Table 1; Fig. 2).

Although probe deflection caused rather substantial changes in ΔT_m and t_m, these changes should theoretically have no effect on λ estimates if deflection produced no effects other than changes in the probe spacing. This was found to be the case for inward deflection in all media and for outward deflection in water but not for outward deflection in dry and saturated sand (Table 1). Outward deflection caused the conductivity of the dry sand to be overestimated by 0.03 W m^{-1} K^{-1} (11.5%), and it caused the conductivity of the saturated sand to be underestimated by 0.04 W m^{-1} K^{-1} (2%). Although the error for the dry sand is substantial on a percentage basis, absolute errors of 0.03 to 0.04 W m^{-1} K^{-1} are unlikely to be of any physical significance for most applications.

Considering other known sources of error associated with the DPHP method, there is reason to believe that a change in probe spacing is not the only mechanism by which probe deflection might influence λ estimates. This is particularly true for the case of outward deflection, because the increase in probe spacing delays the arrival of the heat pulse (Fig. 2). As a result, there is greater opportunity for the measurement to be influenced by the presence of the sensor body (Ham and Benson, 2004) and the finite length of the heater probe (Bristow et al., 1994; Kluitenberg et al., 1995). For the experimental results presented here, there was also greater opportunity for measurements to be influenced by the Plexiglas spacer (Fig. 1B) and any packing artifacts that may have been caused by the presence of the spacer. Clearly, the small differences in conductivity estimates suggest that the 15% increase in apparent probe spacing due to outward deflection was small enough that these potential sources of error had minimal influence.

Estimates of λ can also be obtained by using least squares minimization to fit Eq. [1] to time series of $\Delta T(t)$ data (Mori et al., 2003). Upon making the substitution $\gamma = r_0^2/\kappa$ in Eq. [1], least-squares minimization can be performed to solve for λ and γ. Because γ can be estimated without specifying a value for r_0, it follows that λ estimates obtained in this way should theoretically be independent of r_0. To verify this result, pairs of λ and γ values were estimated by fitting Eq. [1] to the 315 different $\Delta T(t)$ curves (3 media × 3 deflection treatments × 7 sensors × 5 repetitions). Although the mean conductivities (data not shown) were 0 to 6% greater than those presented in Table 1, the results led to identical conclusions regarding the effects of deflection on the λ estimates.

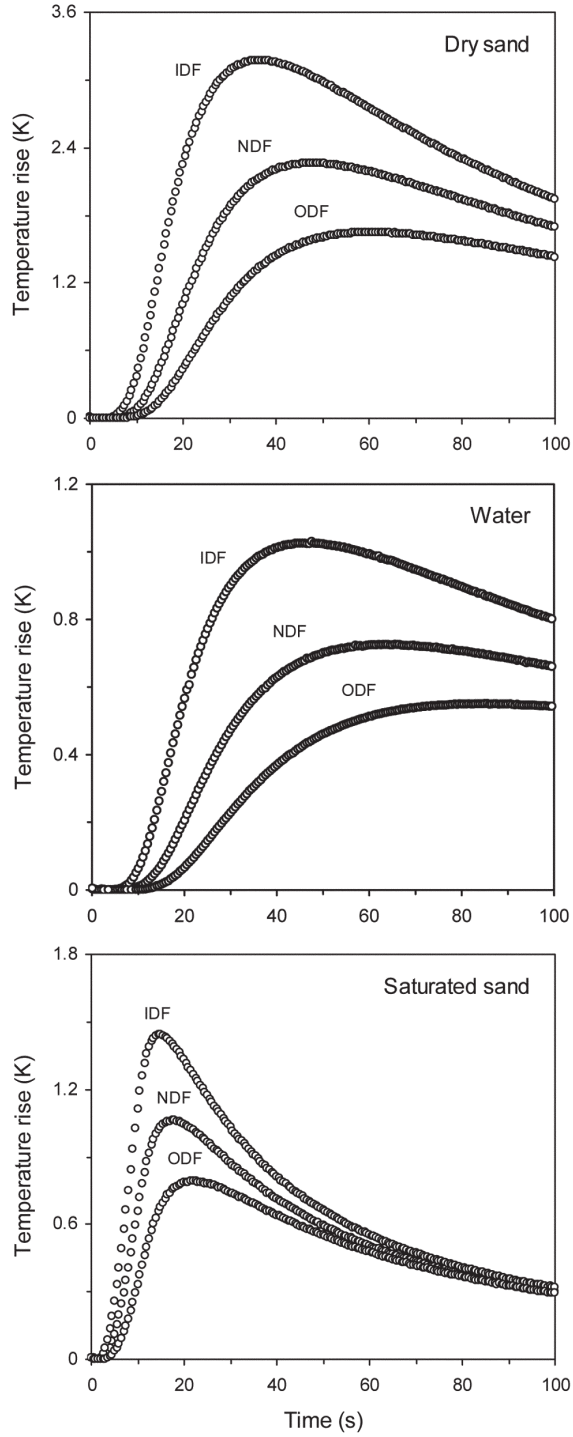

Fig. 2. Temperature rise as a function of time from dual-probe heat-pulse (DPHP) measurements in oven-dried sand, agar-immobilized water, and water-saturated sand. All results are for the same DPHP sensor, with probes subjected to inward deflection (IDF), no deflection (NDF), and outward deflection (ODF).

We examined the cases of inward and outward deflection in this work because these are the two types of deflection we have encountered in field work with sensors similar to those of Basinger et al. (2003). The extent of the imposed deflection treatments was purposefully chosen to exceed the extent of deflection that we have encountered in practice. We acknowledge, however, that deflection geometry could be more complicated than the cases we have considered. Liu et al. (2008) provided a theoretical treatment of more complex deflection geometry.

Probe deflection could also be more extreme, especially for sensors with probes of smaller diameter.

CONCLUSIONS

We conclude that estimates of thermal conductivity obtained with the DPHP method of Bristow et al. (1994) are largely unaffected by changes in probe spacing and other errors resulting from inward or outward deflection of the heater and temperature probes. No error in λ estimates was detected when the probes of the DPHP sensors were deflected inward by an amount that caused a 14% reduction in apparent probe spacing. Although errors in λ estimates were detected when probes were deflected outwards (15% increase in apparent probe spacing), the errors were small ($\leq$0.04 W m^{-1} K^{-1}) and likely to be of little consequence in most applications. Our conclusions regarding the influence of probe deflection were the same regardless of whether λ was estimated from the magnitude and time of the maximum temperature rise or from a time series of $\Delta T(t)$ data.

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