

Evolution & Learning in Games

Econ 243B

Jean-Paul Carvalho

Lecture 11.

Cultural Transmission

Motivation

- ▶ Prior to the 1960s, it was conventional wisdom among social scientists that immigrants from various ethnic and religious backgrounds would assimilate into American culture.
- ▶ It became obvious that this was not occurring not only in America, but elsewhere.
- ▶ High rates of homogamy persisted along with distinctive cultural traits:
 - ▶ Basque and Catalan culture in Spain,
 - ▶ Ultra-Orthodox Judaism in New York,
 - ▶ Conservatives and liberals in the United States.

The Bisin-Verdier Model

- ▶ Consider a simple baseline model of cultural transmission.
- ▶ Agents form a continuum and can have either cultural trait a or b .
- ▶ Each parent (asexually) produces one child, socializes them and then dies.
- ▶ With probability τ^i a parent with trait $i \in \{a, b\}$ successfully passes on her trait to her child (**vertical transmission**).
- ▶ With probability $1 - \tau^i$ the child is matched at random with someone from her parent's generation and acquires their trait (**oblique transmission**).

Dynamics: Exogenous Socialization

- ▶ Let q equal the proportion of type a individuals in the population.
- ▶ The probability that a type b individual has a type a child is $(1 - \tau^b)q$.
- ▶ The probability that a type a individual has a type b child is $(1 - \tau^a)(1 - q)$.
- ▶ In continuous time the dynamic is:

$$\begin{aligned}\dot{q} &= \underbrace{(1 - q)(1 - \tau^b)q}_{\text{inflow}} - \underbrace{q(1 - \tau^a)(1 - q)}_{\text{outflow}} \\ &= (\tau^a - \tau^b)q(1 - q).\end{aligned}\tag{1}$$

The Melting Pot

- ▶ We have a melting pot, i.e. a monomorphic cultural equilibrium:
 - ▶ $q = 1$ is globally asymptotically stable if $\tau^a > \tau^b$.
 - ▶ $q = 0$ is globally asymptotically stable if $\tau^b > \tau^a$.
- ▶ How can we get cultural diversity, i.e. a polymorphic cultural equilibrium?

Endogenous Socialization

- ▶ Bisin and Verdier's contribution is to introduce a choice of socialization effort. For example:
 - ▶ teaching,
 - ▶ school choice,
 - ▶ residential choice,
 - ▶ homogamy.

Imperfect Empathy

- ▶ To model socialization choice, parents need to have preferences over the traits that their children can acquire.
- ▶ Parents are **imperfectly empathetic**: they evaluate their children's behavior based on their own preferences.
- ▶ Formally, a parent with trait i gets a payoff of V^{ij} if their child acquires trait j , where $V^{ii} > V^{ij}$ whenever $i \neq j$.

Objective Functions

- ▶ A parent with trait a in state q has utility function:

$$U^a(q) = [\tau^a + (1 - \tau^a)q]V^{aa} + (1 - \tau^a)(1 - q)V^{ab} - c(\tau^a).$$

They choose socialization effort τ^a at cost $c(\tau^a)$ to maximize this function.

- ▶ A parent with trait b in state q has utility function:

$$U^b(q) = [\tau^b + (1 - \tau^b)(1 - q)]V^{bb} + (1 - \tau^b)qV^{ba} - c(\tau^b).$$

First-Order Conditions

- ▶ The FOC for an a type is:

$$(1 - q)(V^{aa} - V^{ab}) = c'(\tau^a).$$

- ▶ The FOC for a b type is:

$$q(V^{bb} - V^{ba}) = c'(\tau^b).$$

Optimal Socialization Effort

Proposition 1. Optimal socialization effort varies as follows:

- (i) τ^i is strictly increasing in 'cultural intolerance' $V^{ii} - V^{ij}$,
- (ii) τ^a is strictly decreasing in q ,
- (iii) τ^b is strictly increasing in q ,
- (iv) $\tau^a > \tau^b$ if and only if $q < \frac{V^{aa} - V^{ab}}{V^{aa} - V^{ab} + V^{bb} - V^{ba}}$.

Hence 'minorities' expend more effort on socialization.

Dynamics: Exogenous Socialization

Population dynamics are given by (1) except that now τ is endogenous.

Proposition 2. The process converges to the interior steady state $q^* = \frac{V^{aa} - V^{ab}}{V^{aa} - V^{ab} + V^{bb} - V^{ba}}$ from any $q \in (0, 1)$.

Therefore, a polymorphic cultural distribution emerges from almost every initial state.

Generalizing the Analysis

- ▶ How can we extend the Bisin and Verdier framework to n traits?
- ▶ What is the relationship between Bisin-Verdier style cultural evolution and standard dynamics in evolutionary game theory?

A More General Analysis

(Montgomery 2010 AEJ:Micro)

- ▶ Agents form a continuum and possess one of n cultural traits, $i \in \{1, \dots, n\}$.
- ▶ Each parent (asexually) produces one child, socializes them and then dies.
- ▶ A parent with trait i will have a child with trait $j \neq i$ with probability:

$$P_{ij} = (1 - \tau_i)q_j \quad (2)$$

and a child with trait i with probability:

$$P_{ii} = \tau_i + (1 - \tau_i)q_i. \quad (3)$$

The Cultural Evolutionary Dynamic

- ▶ In discrete-time:

$$q_i(t+1) = \sum_j q_j(t) P_{ji}. \quad (4)$$

- ▶ Substituting (2) and (3) into (4):

$$\begin{aligned} q_i(t+1) &= q_i(t) [\tau_i + (1 - \tau_i)q_i(t)] + \sum_{j \neq i} q_j(t)(1 - \tau_j)q_i(t) \\ &= q_i(t)\tau_i + (1 - \tau_i)q_i(t)^2 + q_i(t) \sum_{j \neq i} q_j(t)(1 - \tau_j) \\ &= q_i(t) + q_i(t) \left[\tau_i - \sum_j q_j(t)\tau_j \right]. \end{aligned} \quad (5)$$

The Cultural Evolutionary Dynamic

- ▶ Taking the continuous-time limit, we have:

$$\dot{q}_i = q_i \left[\tau_i - \sum_j q_j \tau_j \right] \quad (6)$$

for all $i = 1, \dots, n$.

- ▶ Clearly, when the τ s are exogenous, the dynamic converges to a monomorphic distribution centered on trait $i \{ \tau_i \}_{i=1}^n$.

Endogenous Socialization

- ▶ Let us proceed along the lines of Bisin and Verdier (2000) except with n traits and a quadratic socialization cost:

$$\max_{\tau_i} \sum_j P_{ij} V_{ij} - \frac{1}{2} (\tau_i)^2, \quad (7)$$

where V_{ij} is an i type's payoff from having a child with trait j .

- ▶ The FOC is:

$$\begin{aligned} \tau_i^* &= (1 - q_i) V_{ii} - \sum_{j \neq i} q_j V_{ij} \\ &= V_{ii} - \sum_j q_j V_{ij} \\ &= \sum_j q_j [V_{ii} - V_{ij}] \\ &\equiv \sum_j q_j \Delta_{ij}, \end{aligned} \quad (8)$$

where Δ_{ij} is an i type's intolerance toward j .

The Replicator Dynamic

- ▶ Substituting into the dynamic (6), we have:

$$\dot{q}_i = q_i \left[\sum_j q_j \Delta_{ij} - \sum_j q_j \sum_k q_k \Delta_{jk} \right] \quad (9)$$

for all $i = 1, \dots, n$.

- ▶ Interpreting Δ_{ij} as the payoff from playing strategy i against j , this becomes a well-known evolutionary dynamic:
 - ▶ The **replicator dynamic** operating on a particular population game.
 - ▶ It is as if individuals directly choose their own trait to maximize their expected payoff, determined by their expected distaste for traits drawn randomly from the population.

Convergence Results

- ▶ Hence we can exploit standard results from evolutionary game theory on the replicator dynamic to study cultural evolution.
- ▶ Suppose that $\Delta_{ij} = \Delta_i$ for all $j \neq i$ (and $\Delta_{ii} = 0$), i.e. each group is intolerant of all other traits to an equal degree.
- ▶ Then this is a **strictly stable** game:
 - ▶ There is a unique Nash equilibrium (distribution of traits), which is globally asymptotically stable.
 - ▶ Two or more traits are active in this equilibrium.
 - ▶ Every trajectory of the replicator dynamic in the interior of the n -dimensional simplex converges to this state.
- ▶ More generally, we can cast this as a potential game and exploit the corresponding results on such games.