The Economics of Religious Communities: Social Integration, Discrimination and Radicalization*

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Abstract

This paper examines the economics of religious communities, characterizing the conditions under which a religious community becomes separated from mainstream society and radicalized. Blanket discrimination against all community members makes the community more cohesive and stricter. Targeted discrimination against actively religious members increases social integration on average, but can give rise to an extreme isolationist sect. Religious competition lowers total religious participation and rules out dynamic radicalization strategies by religious leaders. When blanket discrimination is endogenous, a religious leader can use a *niche construction* strategy to completely isolate and radicalize the community. Attempts to reduce targeted discrimination subsidize such strategies.

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1 Introduction

Multicultural societies can take divergent paths. Communities with distinct beliefs and practices can socially and culturally integrate into mainstream society or they can isolate themselves and reinforce cultural differences. For example, in response to Jewish Emancipation in 19th century Europe, which removed various formal and informal restrictions on Jewish participation in mainstream society, some Jewish communities embraced the new opportunities and integrated, while others resisted (Carvalho & Koyama 2016, Carvalho et al. 2017). Today, Muslims comprise the largest religious minority in most European nations. Concerns about social isolation and radicalization of Muslims have inspired bans on Islamic symbols such as minarets in Switzerland and full face veils (*niqab*) in France, Belgium, the Netherlands and Switzerland. These concerns have weighed heavily in elections in the United States, the Netherlands and France, as well as the 2016 Brexit referendum. They have also created sharp political cleavages in the European Union.

This paper examines the economics of religious communities. We analyze the degree to which a religious community integrates into mainstream society, its internal cohesion and susceptibility to radicalization. The path the community takes depends on both external pressures, such as outside economic opportunities and discrimination, as well as the internal organization of the community. Social integration by ethnic minorities has been linked to education (Constant & Zimmermann 2008, Constant et al. 2009), labor market conditions (Bisin et al. 2011*a*), discrimination (Bisin et al. 2011*b*, Eguia 2017), residential segregation (Bisin et al. 2016), social influence (Austen-Smith & Fryer 2005, Carvalho & Koyama 2017) and community size (Lazear 1999, Advani & Reich 2015). When turning to religious communities, special attention must be paid to the role of religious organizations in coordinating and mobilizing members of the community, as in the theory of religious clubs developed by Iannaccone (1992).[1] Doing so raises a distinct set of questions: Can religious communities avoid assimilation? How extreme can a religious organization be in its demands of members? When does a religious community remain cohesive and when does it fragment with extremists separating themselves into a strict, isolationist group? Can forward-looking religious leaders radicalize a community over time? What factors constrain extremism?

To address the questions just raised, we require a club model with five features:

[1]The religious club model has been further developed by Berman (2000), McBride (2008, 2010, 2015), Levy & Razin (2012), Aimone et al. (2013), Carvalho (2016, forthcoming*b*) and Carvalho & Koyama (2016). See Iyer (2016) for a review of the economics of religion literature and Carvalho (forthcoming*a*) in particular on religious clubs.

- *Heterogeneity.* Community members vary in their degree of religious commitment.
- *Exit option.* Community members have a well defined option to assimilate into mainstream society by not participating in a religious organization.
- *Increasing returns.* Joining a religious organization produces positive network externalities.[2]
- *Competition.* Religious communities must be compared under different competitive structures, e.g. monopoly versus free entry by religious entrepreneurs.
- *Forward-looking leaders.* Religious leaders use dynamic strategies to increase religious participation over time.

In addition, our equilibrium concept accounts for coalitional deviations, which are central to the formation and fragmentation of clubs. No existing model of religion incorporates all of these features.[3] By developing a religious club model with all five desired features, we are able to generate a number of new results, and better understand the conditions under which a community might socially isolate itself and radicalize.

First, a rising share of extremists—community members with high religious commitment—fragments the community, leading to either assimilation by moderate low-commitment types or schism.[4] Second, once a critical mass of extremists is reached, religious participation scales superlinearly with the share of extremists. Third, the effect of discrimination against community members in mainstream society (e.g. social stigma or labor market discrimination) depends crucially on the nature of discrimination. *Blanket discrimination* against all community members reduces social integration and makes the community more cohesive. In contrast, *targeted discrimination* against 'actively religious' members increases social integration by community members on average, but can give rise to an extreme isolationist sect. The distinction between targeted and blanket discrimination provides an economic explanation for why veiling among Muslim women is supposed to have risen following 9/11 (Haddad 2007). The cost of veiling depends on the difference between the stigma attached to being a veiled Muslim woman (targeted discrimination) and the stigma attached to being

[2]Increasing returns are implicit in the notion of a religious movement (see Binzel & Carvalho 2017).

[3]For example, Iannaccone's (1992) model has the first feature, McBride's (2008) model has the first, second and fourth features, and Carvalho and Koyama's (2016) model has the first, second and fifth features. See also Chen, McBride & Short (2018) who add heterogeneity in group objectives and fertility rates to the analysis. While strictness levels vary across groups in their model, they are fixed over time, ruling out the dynamic radicalization strategies examined in this paper.

[4]In keeping with the economics of religion (e.g. Iannaccone & Berman 2006, Berman 2009), 'extremist' and 'moderate' refer to levels of religious commitment, not religiously motivated violence.

an unveiled Muslim woman (blanket discrimination). Thus, veiling could increase after 9/11 if targeted discrimination was swamped by a rise in blanket discrimination. Fourth, competition among religious entrepreneurs in the community lowers religious participation in the community as a whole and raises social integration.

Finally, religious leaders can use various dynamic radicalization strategies. Suppose religious preferences are formed through intergenerational cultural transmission (e.g. Bisin & Verdier 2000). A religious monopolist can radicalize the community by initially forming a small, but extreme group. These extremists are used to transmit strong religious preferences to subsequent cohorts. Over time, the initially small club expands and becomes more extreme and isolated. When blanket discrimination is endogenous, a religious leader can use a *niche construction* strategy. An increase in religious strictness induces blanket discrimination against community members. Blanket discrimination insulates the community from outside options, enabling the leader to further raise strictness. Thus a religious leader can carve out a niche for the community, making it more extreme and cohesive. According to evolutionary biologists, niche construction strategies have played an important role in evolutionary history (Odling-Smee et al. 2003). For niche construction to work in our setting, targeted discrimination must be sufficiently low, so that a religious leader can raise strictness and still attract members. Hence attempts to reduce targeted discrimination can backfire and be exploited by religious leaders to intensify blanket discrimination against all community members. This process can end in a completely isolated and radical religious community. These conclusions depend crucially on there being a religious monopoly in the community. Dynamic radicalization strategies do not work under religious competition.

In addition to the results just described, this paper makes a number of technical and conceptual contributions to the economics of religion. First, standard club models of religion assume that religious leaders always screen out low-commitment types. When there are positive network externalities as in our model, screening emerges endogenously only under certain conditions which we characterize.[5] Second, the literature on religious competition uses Hotelling-style models in which individuals have fixed preferences over religious strictness and join the organization closest to their ideal point (Barros & Garoupa 2002, Montgomery 2003, McBride 2008, Iyer et al. 2014).[6] When there are positive externalities, however, the problem is more complex. Rather than an individual's ideal strictness being primitive, it

[5]Carvalho & Koyama (2016) examine endogenous screening without increasing returns or competition.

[6]For related examples in local public economics, see the Salop-style models of club formation developed by Haimanko et al. (2004) and Polborn (2008).

depends on the size of the club. Contrary to the existing literature we find that religious competition reduces religious participation. Competitive religious organizations cannot enforce strict participation requirements that push members to the point of indifference between joining and assimilating, as a monopolist does. Instead religious strictness is dictated by the preferences of an organization's members. This is a simple point, but one missing in the literature on religious markets (e.g. Iannaccone 1991). Third, we are the first to examine how discrimination affects the structure of a religious community.[7] Fourth, we provide the first dynamic characterization of religious schisms.[8] Such schisms are frequent phenomena in Protestant denominations (Stark & Finke 2000, Sutton & Chaves 2004). There have also been several important schisms in Islam (Maloney et al. 2010). Finally, we contribute to the emerging literature on the role of clubs in shaping cultural transmission (Carvalho 2016, Carvalho & Koyama 2016).[9] Unlike work on cultural leaders (Hauk & Mueller 2015, Verdier & Zenou 2015, Carvalho et al. 2017, Prummer & Siedlarek 2017, Verdier & Zenou 2018), group membership and participation intensity are endogenous in our work. Religious leaders do not directly transmit cultural traits, but indirectly shape cultural transmission through their effect on club membership, participation and club goods production.[10]

The paper is structured as follows. Section 2 presents a static model of a religious community under both religious monopoly and competition. Section 3 analyzes dynamic extensions of the model in which religious preferences and discrimination are endogenous. Section 4 concludes.

2 The Model

A religious community, embedded in a large (unmodeled) society, consists of a large but finite set of individual members N with cardinality n and a set of leaders, each with their own religious organization.

Individuals vary in their religious commitment. The set of high (low) commitment types is de-

[7]We follow work on ethnic groups by Bisin et al. (2011*b*) which distinguishes between conditional and unconditional discrimination.

[8]See Carvalho & Koyama (2016) and Anderson (2016) for static models of club splitting and Makowsky (2012) for a computational model. Sacks (2017) examines the dynamics of club splitting with an application to open-source software projects in developer communities. Chen & Sacks (2018) examine the dynamics of strategic alliances in nonprofits.

[9]This builds on seminal work by Bisin & Verdier (2000, 2001).

[10]See Iannaccone et al. (2011) and Rubin (2017) on the role of religious leaders in a political economy context.

noted by N_H (N_L) with cardinality $n_H \geq 1$ ($n_L \geq 1$). At times, we refer to low-commitment types as 'moderates' and high-commitment types as 'extremists'. The proportion of high-commitment types in the community is $p \equiv n_H/n$.

We begin by analyzing the monopoly case in which the community is served by a single religious organization. Religious competition is introduced in Section 2.2. Dynamic extensions of the model are presented in Section 3.

2.1 Religious Monopoly

There is one religious organization in the community—the club—governed by a leader. While community members cannot choose between religious organizations, they can choose not to participate in the club, instead spending all of their time outside the community. We call this *assimilation*.

Each individual $i \in N$ divides one unit of time between work/leisure outside the community and collective production of a religious club good in the community. The club good can include communal prayer and leisure, and various forms of 'social capital' formation (e.g. job contact networks, political activism and other forms of collective action). Denote the time spent by i on club goods production by x_i. A (pure) strategy profile is denoted by $x = (x_i)_{i \in N}$. Each unit of time spent outside of the community yields a payoff of $\pi > 0$.

The payoff function for a club member i is

$$u_i(x) = \pi\,(1 - x_i) + \theta_i \left(\sum_{j \in N} \omega_j x_j \right)^{1/2}. \tag{1}$$

The two terms in the payoff function are i's utility from outside activity and consumption of the club good, respectively. The club good is a function of the sum of weighted contributions by members. The weight ω_j is the 'mass' of agent j. We assume $\omega_j = 1/n$ for all $j \in N$, so that while the club good is collectively produced by members' contributions, no individual member has a large effect on production when n is large.[11] The value placed by i on the club good, $\theta_i > 0$, is referred to as i's religious commitment: $\theta_i = \theta_L$ for low-commitment types and $\theta_i = \theta_H$ for high-commitment types ($\theta_H > \theta_L > 0$).

[11]The argument in the second term of (1) is the average contribution if and only if all community members join the club.

Social and economic integration by i are both measured by $1-x_i$, interpreted respectively as the proportion of time i spends on social interactions with outgroup members and the time i spends on work to fund private consumption. The latter makes π the real wage rate faced by club members. These forms of social and economic integration are not only intrinsically important, but also drive other forms of integration, including adoption of mainstream cultural beliefs and practices, identification with the host society and intermarriage (Gordon 1964, Carvalho 2016).

The club faces the usual free-rider problem in collective production of the club good. As in seminal work by Iannaccone (1992), some intervention is required to mitigate this incentive problem. We assume that the club leader imposes a minimum participation requirement: to have access to the club good, an individual must devote at least s units of time to the production of the club good. Equivalently, s is a restriction on outside activity, which can be imposed indirectly by mandating various stigmatizing forms of dress, diet and sexual behavior which constrain social integration (see Iannaccone 1992, Aimone et al. 2013) or directly by monitoring time inputs (Carvalho 2016). We refer to s as the *strictness* of the club. If $x_i \geq s$, individual i is deemed a member of the club and receives the payoff described by (1). The set of club members is denoted by $M \equiv \{i : x_i \geq s\}$. These are the members of the community who are actively religious. The community is *cohesive* if all community members are also members of the club. Otherwise, it is *fragmented*.

We assume the payoff to outside activity by nonmembers is $\Delta\pi > 0$. The relative payoff to outside activity, Δ, is the value of a community member's outside option.[12] Under the economic integration interpretation, Δ is the relative wage faced by nonmembers. This is a generalization of past work, which assumes that an individual's payoff from outside activity is independent of club membership (i.e. $\Delta = 1$). If members of the club are discriminated against, Δ could be greater than one. If club members have access to a special production technology, Δ could be less than one.[13] As nonmembers are excluded from participation and consumption of the club good, the payoff to all nonmembers is $\Delta\pi$.

We assume the leader's objective is to maximize production of the club good:

$$G\left(x(s)\right) = \sum_{j \in N} x_j(s), \tag{2}$$

where individual contribution choices are written as functions of strictness s.[14]

[12] Later we will link Δ to discrimination against club members.

[13] See, for example, Bernstein (1992) on diamond trading networks among Orthodox Jews.

[14] Barros & Garoupa (2002) assume that religious leaders maximize the welfare of their members, while

The timing of the game is as follows:

Date 0. Nature partitions the community into low and high commitment types as described above.

Date 1. The club leader announces the minimum participation requirement s.

Date 2. Observing s, individuals simultaneously choose a division of one unit of time between outside activity and production of a club good in the community.

Date 3. The club good is produced and payoffs are received, as defined above.

We assume throughout that $n > (\theta_H/\theta_L)^2$ and $2\pi \geq \theta_H$.[15,16] The structure of the game is common knowledge.

2.1.1 Demand-Side Assimilation: Coordination Failure & Outside Options

Assimilation is an extreme form of social integration: i assimilates if he spends all his time outside the community, $x_i = 0$. There are three factors behind assimilation of community members in our model, two demand-side factors and one supply-side factor. By demand-side factors, we mean conditions under which community members cannot be induced to join the club at any level of strictness $s \in [0,1]$. By supply-side factors, we mean conditions under which a religious organization excludes community members who are willing to join at some level of strictness $s \in [0,1]$. The two demand-side factors identified in this subsection apply equally under competition as in the monopoly setting examined here.

First, clubs face a coordination problem in addition to the usual free-rider problem. *Coordination failure* is the first (demand-side) factor behind assimilation.[17]

Proposition 1. *There exists a subgame perfect equilibrium [SPE] in which* $M(s) = \emptyset$ *for all* $s \in [0,1]$ *if*

McBride (2008) assumes leaders maximize membership size. Our specification combines these and other motivations. Observing (2), religious leaders trade off membership size and participation intensity and are pushed (endogenously) toward maximizing members' welfare under religious competition.

[15]Assuming $n > (\theta_H/\theta_L)^2$ means the minimum participation constraint $x_i = s$ binds in equilibrium for all agents. This simplifies the presentation of the results.

[16]When $\theta_H > 2\pi$, we have to consider uninteresting boundary conditions, since the religious organization can always demand the maximum possible time contribution ($s = 1$) by high types and get it.

[17]See Kuran (1998) on cascades of ethnic/religious behavior due to increasing returns. Anderson (2016) studies the coordination problem agents face when attempting to establish a new club at a fixed cost.

$$\Delta \geq 1 + \frac{1}{n}\left(\frac{\theta_H}{2\pi}\right)^2. \tag{3}$$

Proofs of all propositions are in the Appendix.

It can be shown that for an open set of parameters satisfying (3), the complete assimilation SPE in Proposition 1 coexists with an SPE producing positive club membership. Having raised this coordination problem, we shall not pursue it any further in this paper. Coalitional deviations are central to the formation and fragmentation of clubs. Thus, we account for coalitional deviations by focusing on coalition-proof Nash equilibrium (CPE) as developed by Bernheim et al. (1987).[18]

For the subgame following announcement of strictness s, let $M(s)$ be the set of club members and $x(s)$ be the profile of contribution strategies.

Definition 1. Fix s. Consider the subgame following strictness choice s. A deviation by coalition $C \subseteq N$ from $x(s)$ is an alternative profile $x'(s)$ such that $x_i(s) = x'_i(s)$ unless $i \in C$. A coalitional deviation is profitable if

$$u_i\left(x(s)\right) < u_i\left(x'(s)\right) \tag{4}$$

for all $i \in C$. The deviation is *coalitionally stable* if there are no further profitable deviations from $x'(s)$ by any coalition $C' \subseteq C$.

Definition 2. The profile $x^*(s)$ implements a coalition-proof Nash equilibrium of the subgame induced by the leader's choice of s if no coalitionally stable deviation exists. $M^*(s)$ denotes the set of club members in this equilibrium.

Definition 3. A *religious equilibrium* (RE) of the entire game is a strategy profile $(s^*, x^*(s))$ in which $x^*(s)$ implements a coalition-proof Nash equilibrium for each s and s^* maximizes the leader's payoff:

$$G\left(x^*(s^*)\right) \geq G\left(x^*(s)\right) \text{ for all } s \in [0,1].$$

[18]While coalition-proofness rules out Pareto inefficient profiles of club membership (a coordination issue), it cannot select a Nash equilibrium with efficient contributions to the club good (a free rider problem), as no such Nash equilibrium exists in the absence of the minimum participation requirement s.

Let us now examine the independent role of the second demand-side factor, the value of the outside option determined by Δ and π. Consider a club which all community members join in equilibrium, i.e. $M^*(s) = N$. We refer to such a club as *inclusive*. Alternatively, consider club which only high-commitment types join, $M^*(s) = N_H$. We refer to such a club as *exclusive*. Under what conditions is there no choice of strictness s that can induce community members to form an inclusive or exclusive club?

Proposition 2. *Demand-side assimilation is determined by the value of the outside option.*

(i) *There exists an $s \in [0, 1]$ such that $M^*(s) = N$ only if*

$$\Delta \leq \underline{\Delta} \equiv 1 + \left(\frac{\theta_L}{2\pi}\right)^2. \qquad (IR_L)$$

Otherwise, L types assimilate: $M^*(s) \cap N_L = \emptyset$ for all $s \in [0, 1]$ in every RE.

(ii) *There exists an $s \in [0, 1]$ such that $N_H \subseteq M^*(s)$ only if*

$$\Delta \leq \max\{\underline{\Delta}, \overline{\Delta}\}, \text{ where } \overline{\Delta} \equiv 1 + p\left(\frac{\theta_H}{2\pi}\right)^2. \qquad (IR_H)$$

Otherwise, all types assimilate: $M^*(s) = \emptyset$ for all $s \in [0, 1]$ in every RE.

Thus a high value of the outside option, due to a large (common) payoff from outside activity π and/or large relative payoff from outside activity to nonmembers Δ, makes assimilation unavoidable. Observe that $\overline{\Delta} > \underline{\Delta}$ if and only if $p > (\theta_L/\theta_H)^2$. As we shall see, if the share of H types is sufficiently high, the club can induce H types to form an exclusive club even when it cannot stop L types from assimilating.

2.1.2 Supply-Side Assimilation: Screening

This brings us to the third factor driving assimilation in our model. We showed that assimilation may be unavoidable due to coordination failure and outside options. Even if the club leader could build an inclusive club, he may not wish to do so. That is, an inclusive club may not be the way to maximize total contributions. Instead, the leader may benefit from setting a high level of strictness, inducing assimilation by L types and extracting larger contributions from high-commitment H types. We refer to the formation of an exclusive club by design as *screening*. Moderates could be induced to participate, but are screened out. Whereas screening of low-commitment types is simply assumed in standard religious club models, screening here emerges endogenously under certain conditions.

In analyzing screening, we turn to the incentives of the club leader. As the leader wishes to maximize contributions, he will set strictness as high as possible. For $n > (\theta_H/\theta_L)^2$, all club members then devote the minimum required time s to collective production. For an inclusive club, the leader sets strictness up to $s = 1$ or the maximum strictness that satisfies the low type's participation constraint IR_L:

$$\underline{s} = \begin{cases} 1 & \text{if } \Delta \leq \frac{\theta_L}{\pi} \\ \left(\frac{\theta_L}{2\pi} + \sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)} \right)^2 & \text{if } \Delta \in \left(\frac{\theta_L}{\pi}, \underline{\Delta}\right] . \end{cases} \tag{5}$$

For an exclusive club, this means up to $s = 1$ or the maximum strictness that satisfies the high type's participation constraint IR_H:

$$\overline{s} = \begin{cases} 1 & \text{if } \Delta \leq \frac{\theta_H \sqrt{p}}{\pi} \\ \left(\sqrt{p}\frac{\theta_H}{2\pi} + \sqrt{p\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)} \right)^2 & \text{if } \Delta \in \left(\frac{\theta_H \sqrt{p}}{\pi}, \overline{\Delta}\right] . \end{cases} \tag{6}$$

If outside options are poor, in particular if $\Delta \leq (\theta_L/\pi)$, then the leader can demand that individuals devote all of their time to collective production ($s = 1$) and still induce all community members to participate. For higher values $\Delta \in (\theta_L/\pi, \underline{\Delta}]$, it may be possible to achieve cohesion, but not at full strictness $s = 1$. In this case, the leader of an inclusive club sets strictness so that IR_L binds, i.e. L types are indifferent between joining the club and assimilating. The same logic applies to the optimal strictness of an exclusive club $\overline{s}$. The main difference is that $\overline{s}$ is increasing in the share of high-commitment types p. Given increasing returns to membership, the larger an exclusive club, the more productive it is. Hence the larger is p (the size of an exclusive club), the further the club leader can push his members in terms of demands on their time. Note that $\overline{s} > \underline{s}$ if and only if $p > (\theta_L/\theta_H)^2$.

Having determined $\underline{s}$ and $\overline{s}$, we can precisely define the types of equilibria that can arise.

Definition 4. A *cohesive equilibrium* is an RE in which $M^*(s^*) = N$, $s^* = \underline{s}$ and $x_i^* = \underline{s}$ for all $i \in N$.

Definition 5. An *exclusive equilibrium* is an RE in which $M^*(s^*) = N_H$, $s^* = \overline{s}$ and $x_i^* = \overline{s}$ for $i \in N_H$ and zero for $i \in N_L$.

Proposition 2 tells us that assimilation by all community members is unavoidable if $\Delta > \max\{\underline{\Delta}, \overline{\Delta}\}$. For $\Delta \leq \max\{\underline{\Delta}, \overline{\Delta}\}$, the following proposition characterizes the set of RE.

Proposition 3. *The set of religious equilibria (RE) under monopoly is as follows.*

(i) $\Delta \leq \theta_L/\pi$: *There exists a cohesive RE with strictness* $s^* = 1$ *and contributions* $x_i = 1$ *for all* $i \in N$.

(ii) $\Delta \in (\theta_L/\pi, \underline{\Delta}]$: *There exists a unique threshold proportion of high-commitment types* $\hat{p} \in ([\theta_L/\theta_H]^2, 1)$, *which is strictly decreasing in* Δ.

If $p \leq \hat{p}$, *there exists a cohesive RE.*

If $p \geq \hat{p}$, *there exists an exclusive RE.*

(iii) $\Delta \in (\underline{\Delta}, \overline{\Delta}]$: *There exists an exclusive RE.*

There are no other RE in these cases.

First, note that a highly liberal club never forms: $s^* > 0$ whenever the club is nonempty. The equilibrium structure depends on the proportion of high-commitment types p and the value of the outside option determined by π and Δ. To understand this dependence, begin by considering $\Delta \leq \underline{\Delta}$.[19] In this case, the community is cohesive if the proportion of H types is sufficiently low, $p \leq \hat{p}$. If $p > \hat{p}$, the community fragments with L types assimilating and H types forming a stricter, more isolated group. The intuition is as follows. H types value the club good more highly than L types. To induce L types to join, strictness must be set relatively low $s \leq \underline{s}$. Alternatively, the leader could raise strictness and elicit larger contributions from the mass p of H types. An inclusive club's equilibrium output is $\underline{s}$, while an exclusive club's equilibrium output is $p\overline{s}$. Hence a religious leader prefers an exclusive club when the proportion of H types is greater than the relative strictness of an inclusive club: $p > \underline{s}/\overline{s} \equiv \hat{p}$. Thus, a religious community can fracture when it has a large share of extremists.

A complete picture of the equilibrium structure is presented in Figure 1. When p is low, $p \leq ([\theta_L/\theta_H])^2$, there are two possibilities. Either the community is cohesive if $\Delta \leq \underline{\Delta}$ or all community members assimilate. Otherwise, there are three possibilities. The community is

[19] This condition is satisfied for a relative payoff from outside activity of $\Delta = 1$ (the standard assumption in the literature) and for a range of Δ greater than one.

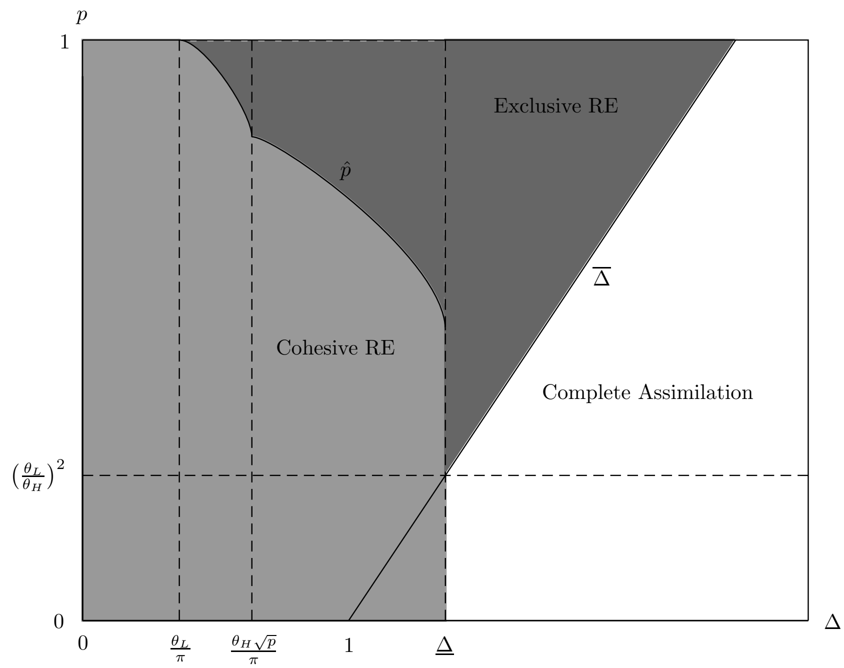


Figure 1: Equilibrium structure under monopoly.

cohesive if Δ is low, it fragments with the formation of an exclusive club if Δ is intermediate, and complete assimilation occurs if Δ is high ($\Delta > \overline{\Delta}$).

Figure 2 plots total religious participation as a function of the share of extremists p in case (ii) of Proposition 3. When p is low, an inclusive equilibrium is in place. All community members choose religious participation $\underline{s}$, so total religious participation is independent of p. As the proportion of extremists grows large ($p \geq \hat{p}$), the community fractures and an exclusive club is formed. Both the size of this club and its participation intensity $\overline{s}(p)$ are increasing in p. Hence the religious leader benefits from a more extreme community if and only if p is sufficiently large. Thenceforth, religious participation scales superlinearly with p.

2.1.3 The Role of Discrimination under Monopoly

The religious community in our model is embedded in a larger society. Let us now explore the relationship between community and society. This has implications for the contemporary debate about the social isolation and radicalization of Muslim communities. Muslim communities in Europe tend to exhibit lower levels of social and economic integration than other migrant groups.[20] For France, Algan, Landais & Senik (2012) show that immigrants with a

[20]Note that in some places integration rises sharply among second-generation Muslim immigrants (Algan, Bisin, Manning & Verdier 2012).

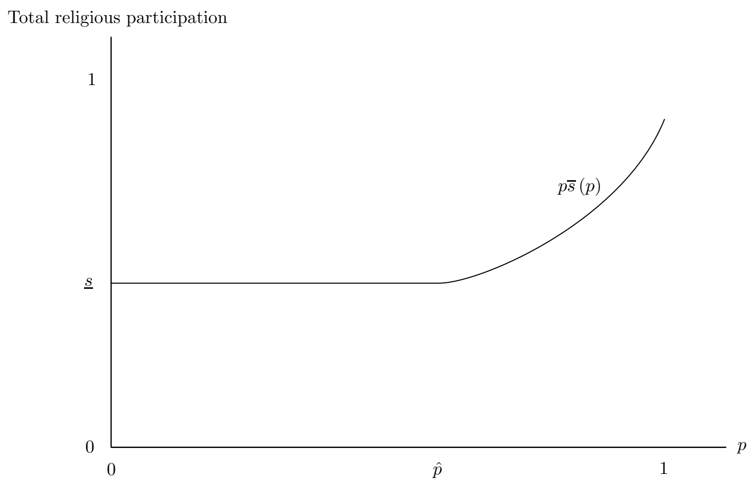


Figure 2: Total religious participation as a function of the proportion of extremists in case (ii) of Proposition 3.

Maghrebi background exhibit the lowest level of exogamy and convergence to native cultural values and practices, in particular those relating to religion and gender (e.g. fertility, spousal age gap). Constant et al. (2012) find that Turks are the least integrated immigrant group in Germany. They have weaker German national identity, speak German less proficiently, marry earlier, and have higher fertility rates. There is, however, evidence of convergence among second-generation Turkish immigrants in Germany.

There are many possible explanations for lower social and economic integration by Muslim communities, including higher levels of religiosity among Muslims, that is higher p and (θ_L, θ_H). One potential reason is discrimination by the host society. It is difficult to isolate religious discrimination from ethnic discrimination because, as in the case of Moroccans in France and Turks in Germany, religion and ethnicity largely coincide. Adida et al. (2010) solve this problem by focusing on Senegalese immigrants to France. Senegalese Muslims and Christians are similar along a number of social and economic dimensions. They examine response rates by employers to CVs with a typical (religiously neutral) Senegalese surname and either a conspicuously Muslim or Christian first name. All else equal, the candidate with the Muslim name is 2.5 times less likely to receive a job interview.[21]

We conceive of discrimination as actions taken by outgroup members that lower the payoff to outside activity by members of the religious community. This includes social stigma which negatively affects social interactions with outgroup members and labor market discrimination

[21]See also Duguet et al. (2010) for evidence of labor market discrimination against job seekers with a Moroccan background in France.

which lowers the expected wage faced by community members, as in the study of Adida et al. (2010).

Following Bisin et al. (2011b), we distinguish between two types of discrimination:[22] 'Blanket discrimination' is directed toward all community members regardless of their religious participation. 'Targeted discrimination' is directed only toward community members who are actively religious. Recall that π is the common component of the payoff to outside activity. Thus, π can be interpreted as the level of economic development in society. We shall show that the higher is economic development π, the greater the social integration of the religious community. As π is common to all community members regardless of whether they join the religious organization or assimilate, its reciprocal $\delta \equiv \pi^{-1}$ can also be understood as a measure of 'blanket discrimination'. In contrast, Δ is the relative payoff to outside activity for those who assimilate. Thus Δ is a measure of 'targeted discrimination' against actively religious community members.

As we shall see, the two forms of discrimination have different effects on social integration. The relationship is complex due to an interesting feature of the equilibrium structure that we will now bring to the fore: for p sufficiently large, there is a non-monotonic relationship between equilibrium strictness and both δ and Δ.

Define social integration (under monopoly) as the average time community members spend outside of the community:

$$\Psi^{M} \equiv 1 - \tfrac{1}{n} \sum_{j \in N} x_j^* \left(s^*\right).$$

We can state the following proposition.

Proposition 4. *Social integration depends on discrimination in the following manner.*

(i) *For $\Delta \leq \theta_L/\pi$, $\Psi^{\mathcal{M}} = 0$. For $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$, $\Psi^{\mathcal{M}} = 1$.*

(ii) *Suppose $\Delta \in \left(\theta_L/\pi, \max\left\{\underline{\Delta}, \overline{\Delta}\right\}\right)$. Social integration $\Psi^{\mathcal{M}}$ is decreasing in blanket discrimination δ and increasing in targeted discrimination Δ, and strictly so whenever $s^* < 1$.*

(iii) *For $p \in ([\theta_L/\theta_H]^2, 1)$ and $\Delta > 1$, there exist real numbers δ' and δ'' such that s^* is weakly increasing with respect to δ on the intervals $\delta \in [\delta', \delta'')$ and (δ'', ∞), with a discrete jump down from $\overline{s}$ to $\underline{s}$ at δ''.*

[22] See also Fouka (2017) for an empirical investigation.

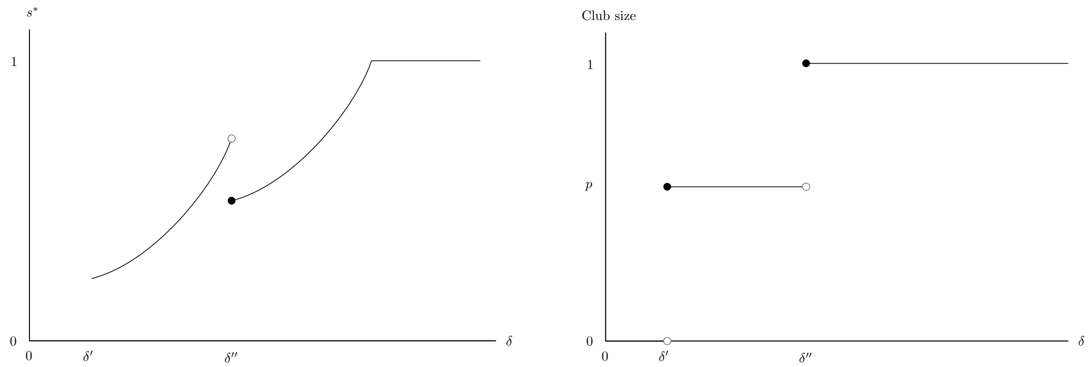
(iv) For $p \in ([\theta_L/\theta_H]^2, 1)$, there exists a real number Δ' such that s^* is weakly decreasing with respect to Δ on the intervals $\Delta \in [0, \Delta')$ and $(\Delta', \overline{\Delta}]$, with a discrete jump from $\underline{s}$ to $\overline{s}$ at Δ'.

Hence blanket discrimination inhibits social integration of a religious community, while targeted discrimination promotes it. Social integration, however, depends on both equilibrium club membership and strictness. Separating the two, we find that equilibrium strictness is non-monotonic in both δ and Δ.[23] The effect of blanket discrimination on equilibrium strictness and membership, given targeted discrimination ($\Delta > 1$), is illustrated by Figure 3. Recall that $\delta \equiv \pi^{-1}$ is the common component of the payoff to outside activity and hence determines the opportunity cost of religious participation. For δ small, the opportunity cost is too high to attract moderate L types.[24] Instead, the religious organization forms a more extreme club composed exclusively of H types. As δ rises and the opportunity cost of religious participation falls, the organization can push H types further in terms of their participation (s^* rises). Eventually, the opportunity cost becomes low enough to make it worthwhile to attract L types and form a cohesive club. The discrete drop in strictness at δ'' to accommodate L types is shown in panel (a), while the jump in membership is shown in panel (b). Thenceforth, strictness of the club rises with blanket discrimination δ, until a cohesive community can be maintained at maximal strictness $s = 1$. In this case, we have complete social isolation of the religious community. Hence blanket discrimination inhibits social integration and produces a more cohesive and extreme religious community.

Now consider the effect of targeted discrimination Δ illustrated by Figure 4. Starting from a low level of Δ, a rise in Δ makes it more attractive to assimilate, prompting the religious organization to lower strictness s^* in order to keep L types in the club. This is depicted in panel (a) by the graph up to point Δ'. Once Δ is sufficiently high, however, it is not worthwhile to liberalize any further. Instead the religious leader benefits from raising strictness, inducing L types to assimilate and catering exclusively to H types. The discrete jump in strictness at point Δ' is shown in panel (a), while the assimilation of low commitment types

[23] A similar pattern of non-monotonicity in δ does emerge through a different mechanism in a religious club model by Carvalho & Koyama (2016). (They only analyze δ and make the standard assumption that $\Delta = 1$.) In their model, individuals can make both time and money contributions to a club, club goods are rival, and the club leader does not impose a minimum contribution requirement, but rather chooses a tax on outside activity. The fact that the same result emerges under substantially different assumptions suggests there is something robust about the pattern.

[24] For $\delta < \delta'$, complete assimilation is unavoidable.



(a) Equilibrium s (b) Equilibrium membership levels

Figure 3: The effect of blanket discrimination under monopoly ($\Delta > 1$)

is shown in panel (b).[25] Therefore, targeted discrimination increases overall rates of social integration in the community, but can fragment the community leading to the formation of a small and highly isolated religious group.

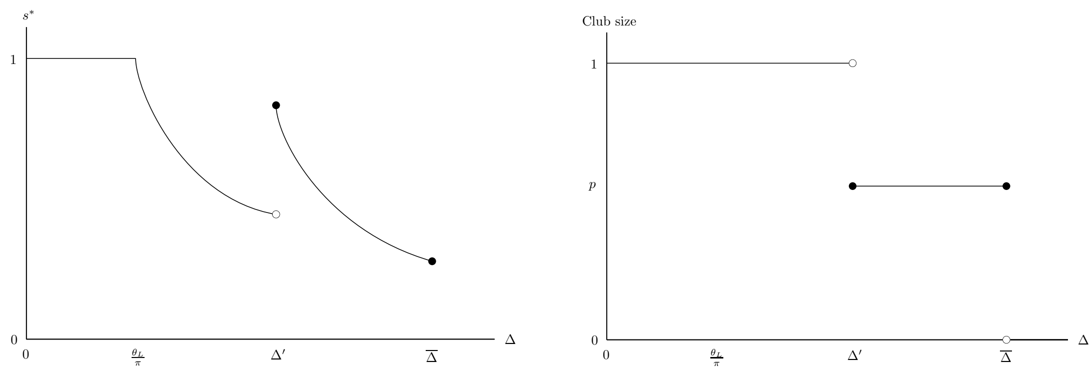
Why should we care about this effect given targeted discrimination raises social integration in aggregate? Berman (2009) shows that such close-knit groups are most at risk of transitioning from providing religious club goods into less benign forms of collective action. At the extreme end, terrorist organizations make extreme demands of members, including suicide attacks. They are also highly sensitive to defection. Due to their strictness and social isolation, small radical religious groups screen out all but the most committed types and elicit extreme contributions by members (Iannaccone 1992). This gives them an advantage in violent forms of collective action which require intensive participation and safeguards against defection and infiltration, as proposed by Berman & Laitin (2008) and Berman (2009).

2.2 Religious Competition

Let us now introduce competition among clubs. So far we have assumed that a single club exists. Either an individual participates in this club or is unaffiliated. Now entrepreneurs can freely enter the market and compete for members.

Consider a countable set of leaders $K = \{1, 2, \ldots\}$ indexed by k, each with his own club. We assume $|K| \geq 4$. The number of clubs that are 'active', i.e. attract one or more members, will be determined endogenously.

[25] As we know from Proposition 2, complete assimilation is unavoidable above $\overline{\Delta}$.



(a) Equilibrium s (b) Equilibrium membership levels

Figure 4: The effect of targeted discrimination under monopoly

At date 1, all leaders simultaneously announce strictness levels. The strictness profile is denoted by $\mathbf{s} = (s_1, s_2, \ldots s_k \ldots)$.

At date 2, each individual $i \in N$ chooses religious participation, where x_{ik} denotes i's contribution to club k. We make the standard assumption that an individual can only participate in one club, i.e. if $x_{ik} > 0$ then $x_{ik'} = 0$ for all $k' \neq k$. As before, i is a member of club k if $x_{ik} \geq s_k$. The set of such individuals is denoted by M_k. For each club k, the (pure) contribution profile is denoted by $x_k = (x_{ik})_{i \in M_k}$. A (pure) strategy profile is $\mathbf{x} = (x_k)_{k \in K}$.

The payoff function for individual $i \in M_k$ is:

$$u_i\left(x_i, \mathbf{x}_{-i}\right) = \pi\left(1 - x_{ik}\right) + \theta_i \left(\sum_{j \in N} \omega_j x_{jk}\right)^{1/2}. \tag{7}$$

If $x_{ik} < s_k$ for all $k \in K$, then an individual is excluded from all clubs and assimilates. The payoff to assimilation is again $\Delta\pi$. Each leader's objective is, as before, to maximize production of his club's good. Hence leader k's payoff is:

$$G_k\left(\mathbf{x}(\mathbf{s})\right) = \sum_{j \in N} \omega_j x_{jk}\left(\mathbf{s}\right). \tag{8}$$

The difference now is that the leader must compete for members. We need to adapt our equilibrium concept to account for this:

Definition 6. A *religious equilibrium* (RE) of the game under competition is a strategy profile $(\mathbf{s}^*, \mathbf{x}^*(\mathbf{s}))$ in which $x^*(\mathbf{s})$ implements a coalition-proof Nash equilibrium for each $\mathbf{s}$

and there is no profitable unilateral deviation for any leader:

$$G_k\left(\mathbf{x}^*(\mathbf{s}^*)\right) \geq G_k\left(\mathbf{x}^*(s_k, \mathbf{s}^*_{-k})\right), \text{ for all } s_k \in [0,1] \text{ and } k \in K.$$

Immediately, one can point to ways in which competition among religious entrepreneurs alters the equilibrium structure. Under monopoly, the community could only fragment through assimilation by L types. Under competition, there is a new form of fragmentation: schism.

Definition 7. A *schismatic equilibrium* is an RE in which all L types join one religious club and all H types join another.

In a static environment, this is simply a separating equilibrium. In section 3.2, we shall see how schism arises as part of a dynamic process.

Competition also alters the strictness of religious clubs. A religious monopolist pushes members to the point of indifference between joining and assimilating. Competition, combined with coalitional deviations, pushes leaders toward maximizing members' welfare. In a schismatic RE, each active club is homogeneous. The moderate club maximizes the welfare of its L-type members by setting strictness

$$\tilde{s}_L \equiv (1-p)\left(\frac{\theta_L}{2\pi}\right)^2. \tag{9}$$

The more extreme club maximizes the welfare of its H-type members by setting strictness

$$\tilde{s}_H \equiv p\left(\frac{\theta_H}{2\pi}\right)^2 > \tilde{s}_L. \tag{10}$$

In a cohesive RE, the active club's membership is heterogeneous, so there is no s that maximizes the welfare of all members. The club leader chooses s^* between the ideal strictness of L and H types, i.e. between

$$s_L \equiv \left(\frac{\theta_L}{2\pi}\right)^2 \text{ and } s_H \equiv \left(\frac{\theta_H}{2\pi}\right)^2. \tag{11}$$

The following proposition characterizes the set of RE for $\Delta \leq \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$. We know that complete assimilation is unavoidable for $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$.

Proposition 5. *Define* $\tilde{\Delta} \equiv 1 + (1-p)(\theta_L/2\pi)^2 \leq \underline{\Delta}$. *The set of religious equilibria (RE) under competition is as follows.*

(i) $\Delta \leq \tilde{\Delta}$: *There exists a unique threshold proportion of high-commitment types* $\tilde{p} \in ([\theta_L/\theta_H]^2, 1]$ *which is weakly decreasing in* Δ.

If $p \leq \tilde{p}$, *there exists a cohesive RE.*

If $p \geq \tilde{p}$, *there exists a schismatic RE.*

(ii) $\Delta \in \left(\tilde{\Delta}, \underline{\Delta}\right]$: *Again, there exists a unique threshold proportion of high-commitment types* $\tilde{p} \in ([\theta_L/\theta_H]^2, 1]$ *which is weakly decreasing in* Δ.

If $p \leq \tilde{p}$, *there exists a cohesive RE.*

If $p \geq \tilde{p}$, *there exists an exclusive RE.*

(iii) $\Delta \in (\underline{\Delta}, \overline{\Delta}]$: *There exists an exclusive RE.*

Strictness and participation levels are given by (9)-(11) *in the respective equilibria.*

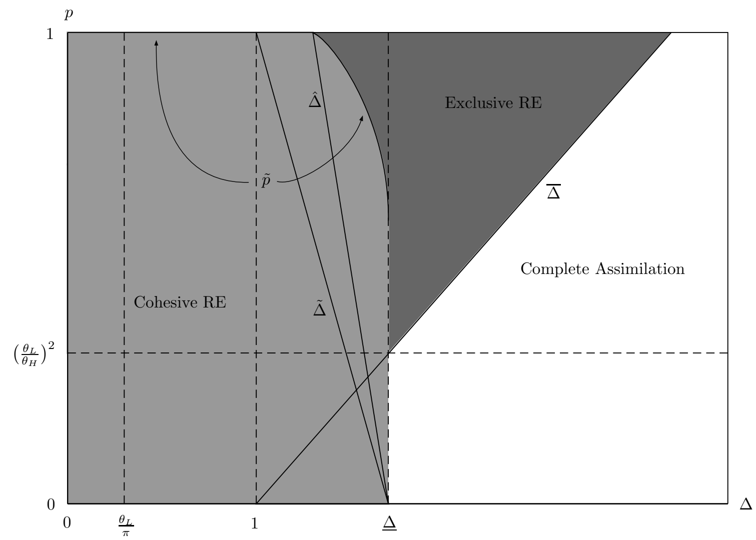
There are no other RE in these cases.

The equilibrium structure is more fully characterized by Figure 5.[26] When low and high commitment types are sufficiently similar ($\theta_L/\theta_H \geq 1/2$), three types of equilibria arise. Roughly speaking, when outside options are poor (Δ small), the unique RE is cohesive. When Δ is intermediate and the proportion of H types p is large, the unique RE is exclusive. L types choose to assimilate rather than form an exclusive club of their own. For large Δ, even H types assimilate.

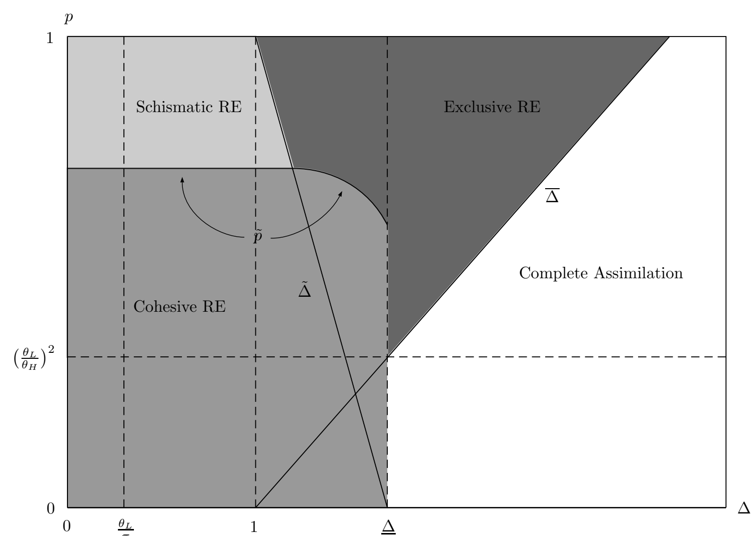
When low and high commitment types are sufficiently distinct ($\theta_L/\theta_H < 1/2$), a fourth type of equilibrium arises. When Δ is small and the proportion of H types p is large, the unique RE is schismatic. Rather than forming an inclusive club or assimilating, L and H types sort into two exclusive clubs. This schism fragments the community where it would have been cohesive under monopoly. We will compare the levels of social integration under monopoly and competition in section 2.2.2.

Despite differences in the equilibrium structure, the role of extremists is the same as in the monopoly case. Whether through schism or formation of a single exclusive club, a large

[26]The additional details in the Figure are proved in the Appendix along with Proposition 5.



(a) $\frac{\theta_L}{\theta_H} \geq \frac{1}{2}$



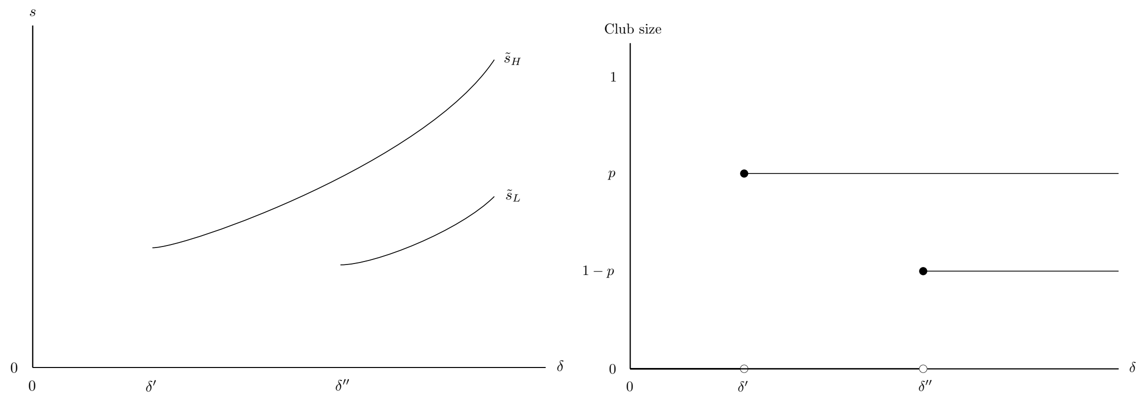
(b) $\frac{\theta_L}{\theta_H} < \frac{1}{2}$

Figure 5: Equilibrium structure under competition

share p of extremist H types fragments the community. In addition, once a critical mass is reached and fragmentation occurs, religious participation scales superlinearly with p.

2.2.1 The Role of Discrimination under Competition

Competition alters some, but not all, of the effects of discrimination on social integration. Social integration under competition is once again the average time community members



(a) Equilibrium s (b) Equilibrium membership levels

Figure 6: The effect of blanket discrimination under competition. *Case*: p large, $\frac{\theta_L}{\theta_H} < \frac{1}{2}$

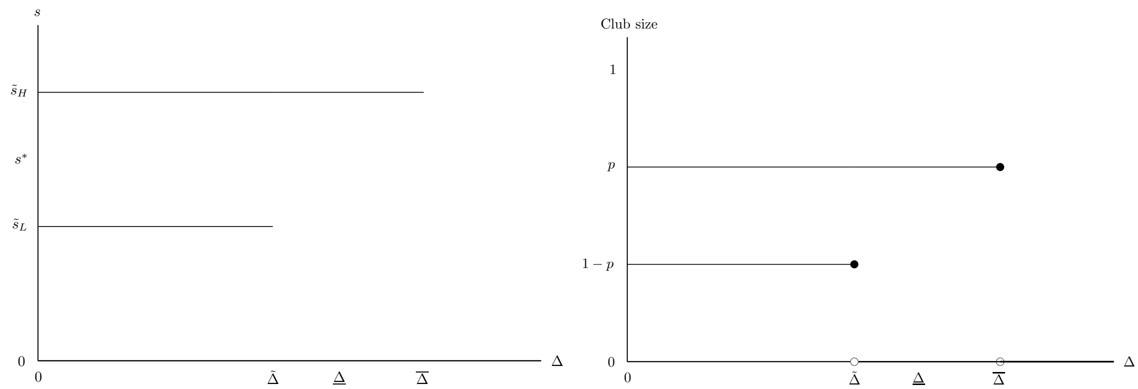
spend outside of the community:

$$\Psi^C \equiv 1 - \tfrac{1}{n} \sum_{k \in K} \sum_{j \in N} x^*_{jk}\left(\mathbf{s}^*\right).$$

To see the effects of competition most clearly, consider the case of p large and $\theta_L/\theta_H > 1/2$, so that a schismatic RE exists.

The effect of blanket discrimination δ is illustrated by Figure 6. As δ rises and the opportunity cost of religious participation falls, H types are *willing to* participate more intensively. That is, the welfare-maximizing strictness for H types, $\tilde{s}_H$, rises. Eventually, the opportunity cost becomes low enough to make it worthwhile for L types to form their own exclusive club at lower strictness $\tilde{s}_L$. The formation of an exclusive L type club at δ'' appears in panel (b). Thenceforth, the strictness levels of both exclusive clubs rise with δ. As under monopoly, blanket discrimination inhibits social integration.

The effect of targeted discrimination Δ is illustrated by Figure 7. Consider the extensive margin. For Δ small, a schismatic equilibrium arises. At $\tilde{\Delta}$, assimilation becomes sufficiently attractive for L types and the exclusive L type club dissolves. Note that $\tilde{\Delta} < \underline{\Delta}$, so this requires a lower level of targeted discrimination than under monopoly. At some $\Delta \in (\tilde{\Delta}, \overline{\Delta})$, even H types assimilate. Again this requires, a lower level of targeted discrimination than under monopoly. Hence targeted discrimination promotes social integration at the extensive margin even more effectively than under monopoly. The same is not true of the intensive margin. Under monopoly, religious organizations pushed members to the point of indifference between joining and assimilating. This point was determined by Δ. In contrast, competition aligns strictness choices with members' preferences, which take into account the



(a) Equilibrium s (b) Equilibrium membership levels

Figure 7: The effect of targeted discrimination under competition. *Case*: p large, $\frac{\theta_L}{\theta_H} < \frac{1}{2}$

cost of participation, so the participation constraint almost nowhere binds. Hence targeted discrimination has no effect on social integration at the intensive margin under competition.

2.2.2 Social Integration: Monopoly vs Competition

Compared to the monopoly case, social integration is greater under competition.

Proposition 6. $\Psi^{\mathcal{M}} \leq \Psi^{\mathcal{C}}$, *and strictly so for an open set of parameters including* $(\underline{\Delta}, \overline{\Delta}]$.

The main prediction of the religious markets literature is that religious competition increases overall religious *affiliation* (e.g. Iannaccone 1991, McBride 2008). We focus on religious *participation* and show that competition lowers overall participation by community members. We have already alluded to the intuition behind our result, but it warrants repeating. Religious organizations in our model maximize religious participation by members, rather than members' welfare. Since they do not internalize the cost of participation by members, religious organizations wish to push members beyond the welfare-maximizing level of participation. Competition limits how strict religious organizations can be, tying equilibrium participation to the preferences of individual members rather than the incentives of religious leaders. However, the proof is more involved than this simple intuition suggests. One cannot simply compare the strictness of inclusive and exclusive clubs under monopoly and competition, but has to account for which types of clubs arise in each setting for a given set of parameters. For example, it could be that an exclusive club forms under monopoly while schism occurs under competition. The monopolist club is stricter than the H-type club under competition. But including the schismatic L-type club, it could be that overall religious participation is higher under competition. We find this not to be the case.

3 Dynamic Social Integration and Radicalization

Thus far, we have analyzed a static model in which religious commitment (p), blanket discrimination (δ) and targeted discrimination (Δ) are exogenous. We now embed this stage game in an infinite-horizon model in which religious participation determines future levels of religious commitment and discrimination. First, we study the evolution of religious commitment over time, with discrimination held constant. Then we allow blanket discrimination to co-evolve with religious commitment.

Time is discrete and indexed by $t = 1, 2, \ldots$ At the beginning of the game ($t = 0$), nature partitions the community as described at date 0 in Section 2. Denote by $p^0 = n_H^0/n$ the initial proportion of high-commitment types in the population. Individuals live for one period. Religious leaders have an infinite horizon, maximize the sum of discounted stage-game payoffs defined by (2), with (common) discount factor $\mu \in [0, 1]$. The remaining structure of the game depends on which parameters are treated as endogenous and whether the religious leader faces competition.

Markov perfection is added to the concept of religious equilibrium specified in Definitions 3 and 4 to produce *Markov perfect religious equilibria* (MPRE). Since community members are short-lived, Markov perfection only constrains the behavior of religious leaders (who are forward looking), and only in the competitive case.

3.1 Endogenous Religious Preferences

In this subsection, we allow the proportion of high-commitment types to evolve (endogenously) holding both blanket and targeted discrimination fixed. Rather than the population dynamics being driven by fertility as in Berman (2000) and McBride (2015), they are driven by cultural transmission. As we shall see, the dynamics are qualitatively different under religious monopoly than under competition.

3.1.1 Monopoly

Consider a religious monopolist. Each stage t is composed of dates 1-3 as described in Section 2, with the addition of a fourth date at which religious preferences are formed through cultural transmission. At date 4, each individual i asexually reproduces a single child and then dies. The distribution of religious preferences in the next generation $t+1$ is determined by religious participation in period t. Although n remains fixed n_L and n_H are now functions of t. We write n_L^t and n_H^t, with $p^t = n_H^t/(n_L^t + n_H^t)$.

Cultural transmission occurs as follows. Denote the community-average level of participation in period t by

$$F\left(p^t\right) = \frac{1}{n} \sum_{j \in N} x_j^* \left(s^t, p^t\right). \tag{12}$$

The club-average level of participation in period t is

$$\tilde{F}\left(p^t\right) = \frac{1}{|M^*\left(p^t\right)|} \sum_{j \in N} x_j^* \left(s^t, p^t\right). \tag{13}$$

The distribution of types at time $t+1$ is determined by the weighted average of the community-average and club-average participation levels:

$$p^{t+1} = (1-\alpha) F\left(p^t\right) + \alpha \tilde{F}\left(p^t\right), \tag{14}$$

where $\alpha \in [0,1]$ is the weight on the club-average. Note that F and $\tilde{F}$ differ only when extremist H types form an exclusive club. The cultural dynamic (14) can be thought of as the expected motion of an underlying (stochastic) cultural transmission process. For example, for $\alpha = 0$, a child is matched with an adult member of the community uniformly at random. With probability equal to the adult's participation level, the child acquires H religious commitment. For $\alpha > 0$, a child is more likely to be matched with an actively religious club member than an unaffiliated community member. Hence we refer to α as the degree of *extremist influence* in the community. This is broadly consistent with the cultural transmission models of Bisin & Verdier (2000, 2017), Carvalho et al. (2017) and Chen, McBride & Short (2018).

Community members are short-lived and do not have dynastic preferences: each i maximizes his own payoff given by (1). Since we focus on Markov perfect equilibria, the (long-lived) leader's problem is defined recursively by the Bellman equation

$$V\left(p^t\right) = \max_{s^t \in [0,1]} \left\{ \sum_{j \in N} \omega_j x_j \left(s^t, p^t\right) + \mu V\left(p^{t+1}\right) \right\}. \tag{15}$$

We first study the dynamics for $\alpha = 0$. As established in Proposition 1, each community member contributes either s or zero. Given this, the following proposition describes the leader's equilibrium strategy.

Proposition 7. *Suppose $\alpha = 0$. In every MPRE:*

(i) $\Delta \leq \delta\theta_L$: *the community is cohesive,* $s^t(p^t) = 1$ *and* $p^t = 1$ *for all* t.

(ii) $\Delta \in (\delta\theta_L, \underline{\Delta}]$:

$$s^t\left(p^t\right) = \begin{cases} \underline{s} & \text{if } p^t \leq \hat{p} \\ \overline{s}\left(p^t\right) & \text{if } p^t > \hat{p}. \end{cases}$$

If $p^0 > [\Delta(\pi/\theta_H)]^2$, *then* $s^0\left(p^0\right) = \overline{s}\left(p^0\right) = 1$ *and* $p^t = p^0$ *for all* t. *Otherwise,* p^t *is decreasing in* t *and there exists a finite time* T *such that an inclusive club forms and* $p^t = \underline{s}$ *for all* $t \geq T$.

(iii) $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\left(p^0\right)\right]$:

$$s^t\left(p^t\right) = \begin{cases} \overline{s}\left(p^t\right) & \text{if } \Delta \leq \overline{\Delta}\left(p^t\right) \\ s \in [0,1] & \text{if } \Delta > \overline{\Delta}\left(p^t\right). \end{cases}$$

If $p^0 > [\Delta(\pi/\theta_H)]^2$, *then* $s^0\left(p^0\right) = \overline{s}\left(p^0\right) = 1$ *and* $p^t = p^0$ *for all* t. *Otherwise,* p^t *is decreasing in* t *and there exists a finite time* T *such that complete assimilation occurs and* $p^t = 0$ *for all* $t \geq T$.

(iv) $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\left(p^0\right)\right\}$: *complete assimilation occurs and* $p^t = 0$ *for all* $t \geq 1$.

The evolution of religious commitment in the community depends on the levels of targeted discrimination (Δ) and blanket discrimination ($\delta = 1/\pi$). The process with $\alpha = 0$ tends toward either a cohesive community or complete assimilation. An exclusive club can only be sustained under two special conditions: (a) there exists a $p' \in [\hat{p}, 1)$ such that $\overline{s}(p') = 1$ and (b) $p^0 > p'$. In this case, the proportion of H types is invariant over time, though any perturbation will permanently change this proportion.[27]

Otherwise, the following occurs: When the ratio of targeted to blanket discrimination is less than the religious commitment of L types, $\Delta/\delta \leq \theta_L$, a cohesive community can be maintained at full strictness for all p. Hence, by (14), $p^t = 1$ for all $t \geq 1$. Next consider

[27] Formally, p^0 is Lyapunov stable, but not asymptotically stable in this case.

$\Delta \in (\delta\theta_L, \underline{\Delta}]$. If the initial proportion of H types is large ($p^0 > \hat{p}$), the religious leader begins by forming an exclusive club. But if $\overline{s}(p^0) < 1$, this club atrophies: $p^0 > p^1 > p^2 \ldots$ There is a finite time T at which the threshold $\hat{p}$ is crossed. At that point, the leader forms an inclusive club and the process immediately stabilizes with $p^t = \underline{s}$ for all $t > T$. When $p^0 \leq \hat{p}$, an inclusive club is formed immediately and $p^t = \underline{s}$ for all $t > 1$. Finally, when $\Delta > \min\left\{\underline{\Delta}, \overline{\Delta}\right\}$, complete assimilation occurs and religious commitment in the community collapses.

When extremists have disproportionate influence, i.e. $\alpha > 0$, an exclusive club is more stable.

Proposition 8. *Suppose $\Delta < \theta_H/\pi$ and at least one of the following holds:*

(i) $\Delta < 1$ *and* $\theta_H \geq \pi$,

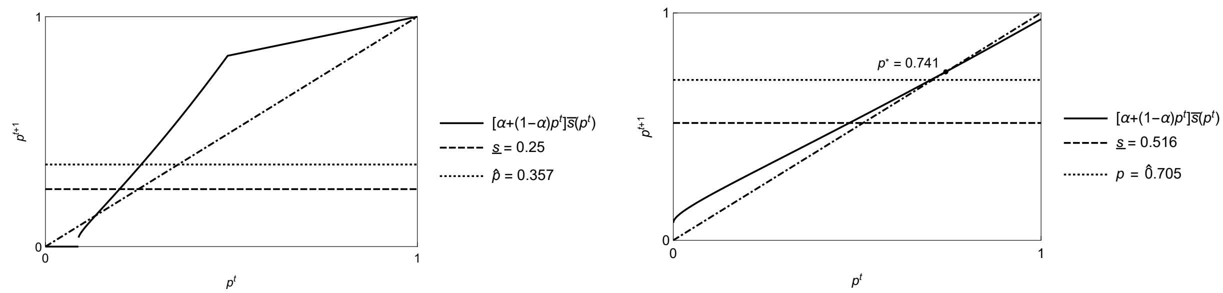
(ii) $\Delta \leq \underline{\Delta}$ *and* $\underline{s}\left(\frac{\theta_H}{\pi} - 1\right) > (\Delta - 1)$,

(iii) $p^0\left(\frac{\theta_H}{\pi} - 1\right) > (\Delta - 1)$.

Then for α and μ sufficiently large, there exists a finite time T such that an exclusive club forms in all $t > T$ and $\lim_{t\to\infty} p^t = 1$.

When the religious leader is sufficiently patient (μ large) and extremist members strongly influence cultural transmission (α large), there is an open set of parameters under which an exclusive club forms at some finite time and persists thereafter. Conditions (i) and (ii) apply independently for all initial conditions $p^0 \in [0, 1]$. Moreover, under the condition $\Delta < \theta_H/\pi$, this exclusive club eventually consumes the entire community, $\lim_{t\to\infty} p^t = 1$, at full strictness $s = 1$.

To illustrate how this *radicalization strategy* works, see Figure 8. In the dynamic depicted in Panel (a), if religious commitment starts out too low (p^0 low), the leader forms an inclusive club at low strictness $\underline{s}$. In the next period, $p^1 = \underline{s}$. At this point, there are enough extremists to form an exclusive club. Though the leader must forgo some religious participation today to do so, he uses the H types in this exclusive club to produce more high-commitment types in the community over time (since $\alpha > 0$). This also raises the intensity of religious participation for each H type. Eventually, the exclusive club is so successful that it attracts all members of the community, reproducing an inclusive club that is far more extreme than the original.



(a) $\alpha = 0.667$, $\Delta = 1.05$, $\theta_L = 0.6$ and $\theta_H = 1.5$.

(b) $\alpha = 0.9$, $\Delta = 0.915$, $\theta_L = 0.6$ and $\theta_H = 0.9$. $\overline{s}(p^*) = 0.76$.

Figure 8: Representative equilibrium paths for $\alpha > 0$ and π normalized to 1.

Panel (b) shows that the condition $\Delta < \theta_H/\pi$ imposed in Proposition 8 is not necessary: an exclusive club can persist under other conditions at strictness $s < 1$. In this example, the dynamic converges to a state in which approximately 74% of community members are club members and spend 76% of their time on club activity.

These radicalization strategies designed to transit from a community with low religious commitment to one with high religious commitment work only when extremists have disproportionate influence ($\alpha > 0$) and the leader is sufficiently patient. Otherwise, as we have seen in Proposition 7, either a moderate, inclusive club is formed or complete assimilation occurs.

3.1.2 Competing Clubs

To account for multiple clubs, we augment community-average religious participation $F\left(p^t\right)$ and club-average religious participation $\tilde{F}\left(p^t\right)$ as follows

$$F\left(p^t\right) = \frac{1}{n}\sum_{k\in K}\sum_{j\in N} x^*_{jk}\left(s^t, p^t\right) \tag{16}$$

$$\tilde{F}\left(p^t\right) = \frac{1}{\sum_{k\in K}\left|M^*_k\left(p^t\right)\right|}\sum_{j\in N} x^*_{jk}\left(s^t, p^t\right), \tag{17}$$

where $M^*_k\left(p^t\right)$ is the set of members of club k. The dynamic for p^{t+1} is as in (14) with the updated $F\left(p^t\right)$ and $\tilde{F}\left(p^t\right)$.

Competitive forces mean that each religious entrepreneur acts as if he is myopic ($\mu = 0$). The reason is that consumers live for one period and do not have dynastic preferences. Therefore if a religious club does not (approximately) maximize its members' stage-game payoff, then another entrepreneur could step in and steal its members by setting the welfare-maximizing level of strictness for that period. We formally state this result and describe the ensuing religious dynamics in the following proposition.

Proposition 9. *For each p^t, the stage-game outcome of an MPRE is the same as the corresponding RE described by Proposition 5.*

(i) *If $\Delta \leq \underline{\Delta}$, then there exists a finite time $T \geq 0$ such that a cohesive club forms and $p^t = s^*$ for all $t \geq T$.*

(ii) *If $\Delta > \underline{\Delta}$, then there exists a finite time $T \geq 0$ such that there is complete assimilation and $p^t = 0$ for all $t \geq T$.*

Although the dynamics always settle into either cohesion or complete assimilation, the transition can be quite complex and pass through multiple states. To illustrate, suppose that $\theta_L/\theta_H < 1/2$, $p^0 > \tilde{p}$ and $\Delta \in (\tilde{\Delta}(1), \tilde{\Delta}(\tilde{p}))$.[28] At the outset, the MPRE implements an exclusive extremist club with strictness $\tilde{s}_H$. Under exclusivity, p^t is falling so $p^0 > p^1 > p^2 \dots$ By construction, $\tilde{\Delta}$ is strictly decreasing in p. Therefore there exists a finite time $T' < T$, where at time $T' - 1$, $\Delta \geq \tilde{\Delta}(p^{T'-1})$ and at time T', $\Delta \leq \tilde{\Delta}(p^{T'})$. Beginning at this time, MPRE implements a schismatic equilibrium. A religious entrepreneur enters and captures the moderate individuals with strictness $\tilde{s}_L$ while the extremist remain in their original club. Under schism, p^t continues to fall.[29] Eventually, time T is reached where $p^T < \tilde{p}$, at which point a religious entrepreneur creates a cohesive club with strictness s^*, attracting the entire community. Once this cohesive club is formed, $p^t = s^* < \tilde{p}$ for all subsequent t. Thus the dynamics can begin with exclusion, transition to schism and finally settle into cohesion.[30]

Our results bear on a famous debate between David Hume and Adam Smith. Hume proposed that a state-funded religious monopoly would limit religious extremism by bribing the clergy into indolence (Hume 1983, vol. 3, sec. 29). Smith argued to the contrary that religious competition would produce fragmentation and avert power struggles among large religious rivals (Smith 2003). We have provided here an alternative argument for religious competition:

[28] Visually, these initial conditions correspond to a vertical cut of Figure 5(b) at Δ just slightly larger than 1 and p^0 just slightly below 1.

[29] This statement, along with p^t falling under exclusion, is proven in the proof of Proposition 9.

[30] As schism is not persistent in our model, other factors, omitted here, are required to stabilize long-lasting schisms such as those in Christianity, Islam (Maloney et al. 2010) and Judaism (Carvalho & Koyama 2016). The obvious candidates are coordination failure (i.e. failure to form an inclusive club through coalitional deviations), switching costs, and endogenous fertility (McBride 2015).

Competition ties strictness choices to members' preferences, forcing religious entrepreneurs to act in a myopic fashion and eliminating the dynamic radicalization strategy described in Proposition 8. The next section examines a further radicalization strategy which is observed under religious monopoly, but not under religious competition.

3.2 Endogenous Discrimination

We now make blanket discrimination δ^t endogenous and examine its co-evolution with religious commitment p^t. We have seen that religious participation is increasing in blanket discrimination under both monopoly and competition. We are interested here in how endogenous blanket discrimination enables religious leaders to use dynamic strategies that raise religious participation over time. Thus we restrict attention to the monopoly case. As we have seen in Section 3.1.2, competition rules out such radicalization strategies.

We assume the religious leader can affect blanket discrimination through his choice of strictness. In particular, blanket discrimination δ^t is determined in a similar manner to religious commitment p^t, as a weighted average of community-average religious participation $F(\cdot)$ and club-average religious participation $\tilde{F}(\cdot)$:

$$\delta^{t+1} = \lambda\left[(1-\alpha)F(p^t,\delta^t) + \alpha\tilde{F}(p^t,\delta^t)\right], \tag{18}$$

where λ is the degree to which the broader society responds to religious participation in the community with blanket discrimination against community members. We call λ the *propensity for blanket discrimination.*

A state $(\delta, p) = (0, 0)$ with no blanket discrimination and no H types is referred to as *full tolerance.* A state $(\delta, p) = (\lambda, 1)$ with maximal blanket discrimination ($\delta = \lambda$) and complete radicalization ($p = 1$) is referred to as *zero tolerance.*

Proposition 10. *The asymptotic behavior of* (p^t, δ^t) *is as follows.*

(i) *Full tolerance is locally asymptotically stable if and only if* $\Delta > 1$. *In such a state, there is complete assimilation* ($\Psi = 1$).

(ii) *Suppose* $\alpha > 0$. *Zero tolerance is locally asymptotically stable if and only if* $\lambda > \Delta/\theta_H$. *In such a state, the community is completely isolated* ($\Psi = 0$).

(iii) *If in the initial state* (δ^0, p^0) *there is positive religious participation* ($\Psi < 1$)*, then there exists a value* $\lambda' \geq 1$ *such that if* $\lambda > \lambda'$*, the dynamic converges in finite time to zero tolerance:* $(\delta^t, p^t) = (\lambda, 1)$.

When targeted discrimination Δ is sufficiently high, a fully tolerant state with zero blanket discrimination is asymptotically stable. When targeted discrimination Δ is sufficiently low relative to the propensity for blanket discrimination λ, then a zero tolerance state with maximal blanket discrimination, complete radicalization and zero social integration is asymptotically stable.[31] Moreover, when λ is sufficiently large, society converges to a zero tolerance state from almost all initial states in which there is not complete assimilation.

The intuition behind Proposition 10 is as follows: By raising strictness today, a religious leader can induce blanket discrimination against community members. Blanket discrimination lowers the opportunity cost of religious participation by 'taxing' outside activity for all community members (active or otherwise). In this way, the religious leader can boost future religious participation. This strategy can be thought of as a form of *niche construction* (Odling-Smee et al. 2003)—by inducing blanket discrimination against community members, a religious monopolist shields itself from competition from secular alternatives outside the community. This radicalization strategy works only under certain conditions. First, the broader society must respond to high religious strictness with blanket discrimination, not targeted discrimination. Second, targeted discrimination must be sufficiently low relative to the propensity for blanket discrimination. Otherwise, the leader would be unable to raise strictness high enough to trigger increases in blanket discrimination over time. Hence attempts to reduce targeted discrimination against actively religious members can backfire and produce maximal blanket discrimination against community members, leading to a completely isolated, radical community. This, (partially) reinforces our static results: greater targeted discrimination can avert extreme forms of religious participation and social isolation of the community.

4 Conclusion

This paper has examined the economics of religious communities. We have made a number of contributions to the religious clubs literature by modeling the degree to which a religious

[31] Note that for $\alpha = 0$ this state is Lyapunov stable, but not asymptotically stable.

community integrates into mainstream society, its internal cohesion and susceptibility to radicalization. We have found that religious participation scales superlinearly with the share of extremists, once a critical mass is reached. A rising share of extremists fractures the religious community. Blanket and targeted discrimination have different effects on community structure, but both can produce some form of extremism. A religious monopolist can use extremist members to radicalize the community over time. A monopolist may also boost participation by cultivating blanket discrimination against community members, a form of niche construction. This is possible only when targeted discrimination is low relative to the propensity for blanket discrimination. Religious competition rules out such radicalization strategies.

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Appendix

Proof of Proposition 1.

Proof. Suppose $x_j = 0$ for all $j \neq i$. The best i can do by joining is to choose:

$$\arg\max_{x_i \in [0,1]} \pi (1 - x_i) + \theta_i \left(\frac{x_i}{n}\right)^{1/2} = \frac{\theta_i^2}{4\pi^2 n}. \tag{19}$$

This yields a payoff of at most

$$\pi + \frac{\theta_H^2}{4\pi n}, \tag{20}$$

i.e. when $\theta_i = \theta_H$. (20) is no greater than the assimilation payoff $\Delta\pi$ if

$$\Delta \geq \hat{\Delta} \equiv 1 + \frac{1}{n}\left(\frac{\theta_H}{2\pi}\right)^2. \tag{21}$$

Hence in this case complete assimilation is a Nash equilibrium for all $s \in [0, 1]$.

□

Proof of Proposition 2.

Proof. (i) Consider a cohesive equilibrium. Suppose for the moment that $x_i(s) = s$ for all $i \in N$. The payoff to i from joining this inclusive club is

$$\pi(1 - s) + \theta_i (s)^{1/2}, \tag{22}$$

which is maximized at

$$s = \left(\frac{\theta_i}{2\pi}\right)^2, \tag{23}$$

yielding a maximum of

$$\pi + \frac{1}{\pi}\left(\frac{\theta_i}{2}\right)^2. \tag{24}$$

Hence there exists an s such that $M^*(s) = N$ only if the assimilation payoff $\Delta\pi$ exceeds (24) for L types, or

$$\Delta \leq \underline{\Delta} \equiv 1 + \left(\frac{\theta_L}{2\pi}\right)^2. \tag{25}$$

To show that $x_i(s) = s$ for all $i \in N$, differentiate $\pi(1 - x_i) + \theta_i \left(\frac{x_i + (n-1)s}{n}\right)^{1/2}$ with respect to x_i and evaluate as above at the maximizer $x_i = s = \left(\frac{\theta_L}{2\pi}\right)^2$ for L types:

$$\frac{\theta_i}{2n}\left(\frac{\theta_L}{2\pi}\right)^{-1} - \pi \tag{26}$$

$$= \pi\left(\frac{\theta_i}{\theta_L n} - 1\right). \tag{27}$$

This is negative for $\theta_i = \theta_H$ (and hence θ_L) if $n > \frac{\theta_H}{\theta_L}$, which holds because $n > \left(\frac{\theta_H}{\theta_L}\right)^2$ by assumption. The result follows.

(i) Consider an exclusive equilibrium. Suppose for the moment that $x_i(s) = s$ for all $i \in N_H$.

The payoff to i from joining this exclusive club is

$$\pi(1-s) + \theta_H (ps)^{1/2}, \tag{28}$$

which is maximized at

$$s = p\left(\frac{\theta_H}{2\pi}\right)^2, \tag{29}$$

yielding a maximum of

$$\pi + \frac{p}{\pi}\left(\frac{\theta_H}{2}\right)^2. \tag{30}$$

Hence there exists an s such that $M^*(s) = N_H$ only if the assimilation payoff $\Delta\pi$ exceeds (30), or

$$\Delta \geq \overline{\Delta} \equiv 1 + p\left(\frac{\theta_H}{2\pi}\right)^2. \tag{31}$$

To show that $x_i(s) = s$ for all $i \in N$, differentiate $\pi(1-x_i) + \theta_i\left(\frac{x_i+(n-1)s}{n}\right)^{1/2}$ with respect to x_i and evaluate as above at the maximizer $x_i = s = p\left(\frac{\theta_H}{2\pi}\right)^2$ for H types:

$$\begin{aligned} & \frac{\theta_H}{2n\sqrt{p}}\left(\frac{\theta_H}{2\pi}\right)^{-1} - \pi & (32)\\ = \; & \pi\left(\frac{1}{n\sqrt{p}} - 1\right). & (33)\end{aligned}$$

This is non-positive if $\sqrt{p}n \geq 1$ or, recalling that $p \equiv \frac{n_H}{n}$, if $\sqrt{n_H n} \geq 1$, which holds by assumption that $n_H \geq 1$. The result follows. □

Proof of Proposition 3.

Proof. By the same argument in the proof of Proposition 2, we can show that $x_i = s^* \in \{\underline{s}, \overline{s}\}$ for all $i \in M^*(s^*)$.

Now consider membership choices. Due to increasing returns, either all L types join or none do. Likewise for H types. As $\theta_H > \theta_L$, if all L types join, so do all H types. Hence $M^*(s) \in \{\emptyset, N_H, N\}$.

Suppose there exists an s' such that $M^*(s') \neq \emptyset$. The club leader will never set s such that $M^*(s) = \emptyset$, as $G(s) = 0$ in this case, a minimum of his objective function. Combining this fact with Proposition 2, $M^*(s^*) = \emptyset$ if and only if $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$.

Now suppose that $\Delta \leq \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$, so that $M^*(s^*)$ equals N_H or N. We have established that $s^* = \overline{s}$ in the first case and $s^* = \underline{s}$ in the second case.

Given $x_i = s$ for all $i \in M(s)$, the club leader prefers an inclusive club if and only if

$$\underline{s} \geq p\overline{s}. \tag{34}$$

Case 1: $\Delta \leq \theta_L/\pi$ or $p \leq (\theta_L/\theta_H)^2$. By (5) and (6), $\underline{s} \geq \overline{s}$. Hence all types are willing to join the club at $s = \underline{s}$ and (34) is satisfied, so the leader prefers an inclusive club.

Case 2: $\Delta > \theta_L/\pi$ and $p > (\theta_L/\theta_H)^2$. First, $\underline{s} = \overline{s}$ for $p = (\theta_L/\theta_H)^2$ by (5) and (6). Hence (34) holds. Second $\underline{s} < \overline{s} = p\overline{s}$ for $p = 1$. Third, $\overline{s}$ is increasing in p and $\underline{s}$ is independent of p. Therefore, there exists a unique $\hat{p} \in \left([\theta_L/\theta_H]^2, 1\right)$, at which (34) binds. The leader prefers an inclusive club if and only if $p \leq \hat{p}$. By Lemma 1 below, $\hat{p}$ is strictly decreasing in Δ.

Now we check incentive compatibility. By construction of (5), an inclusive club can be implemented at $\underline{s}$ if and only if $\Delta \leq \underline{\Delta}$ (IR_L).

For an exclusive club to be incentive compatible at $\overline{s}$, $\Delta \leq \overline{\Delta}$ (IR_H). In addition, there must be no profitable coalitional deviation by low types. The most profitable involves I_L joining, contributing $x_i = \overline{s}$, and forming an inclusive club. Recall that $\underline{s}$ is the maximum strictness L types would tolerate in an inclusive club. For $p > (\theta_L/\theta_H)^2$, $\underline{s} < \overline{s}$. Therefore, an exclusive club is incentive compatible when $\Delta \leq \overline{\Delta}$ and $p > (\theta_L/\theta_H)^2$.

We know the leader prefers an exclusive club to complete assimilation. Therefore an exclusive club forms if $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\right]$. This establishes the proposition. $\square$

Lemma 1. *$\hat{p}$ is strictly decreasing in Δ on the domain $\left(\frac{\theta_L}{\pi}, \underline{\Delta}\right]$.*

Proof. By definition,

$$\hat{p}\overline{s}(\hat{p}) = \underline{s}.$$

Suppose $\overline{s}(\hat{p}) = 1$. Then

$$\frac{d\hat{p}}{d\Delta} = \frac{d\underline{s}}{d\Delta},$$

which is negative for $\Delta > \theta_L/\pi$ by inspection of (5).

Now suppose $\overline{s}(\hat{p}) < 1$. Then

$$\begin{aligned}
\frac{d\hat{p}}{d\Delta}\overline{s} + \hat{p}\left[\frac{\partial\overline{s}}{\partial\Delta} + \frac{\partial\overline{s}}{\partial\hat{p}}\frac{d\hat{p}}{d\Delta}\right] &= \frac{d\underline{s}}{d\Delta} \\
\frac{d\hat{p}}{d\Delta}\left[\overline{s} + \hat{p}\frac{\partial\overline{s}}{\partial\hat{p}}\right] &= \frac{d\underline{s}}{d\Delta} - \hat{p}\frac{\partial\overline{s}}{\partial\Delta} \\
&= \frac{\hat{p}\sqrt{\overline{s}}}{\sqrt{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}} - \frac{\sqrt{\underline{s}}}{\sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}},
\end{aligned} \tag{35}$$

so that $d\hat{p}/d\Delta$ is negative if and only if the RHS of (35) is negative. As $\hat{p} = \underline{s}/\overline{s}$, (35) is equivalent to

$$\begin{aligned} \hat{p} &< \sqrt{\hat{p}}\sqrt{\frac{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}} \\ \hat{p} &< \frac{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}. \end{aligned} \tag{36}$$

Since $\hat{p} \in \left([\theta_L/\theta_H]^2, 1\right)$, (36) is satisfied. □

Proof of Proposition 4.

Proof. (i) By construction of $\underline{\Delta}$ and $\overline{\Delta}$, complete assimilation occurs for $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$, since neither L nor H types could benefit from joining a club by Proposition 2. Hence $\Psi^{\mathcal{M}} = 1$.

By proposition 3(i), for $\Delta \leq \theta_L/\pi$, $M^* = N$ and $s^* = 1$. Hence $\Psi^{\mathcal{M}} = 0$.

(ii) For $\Delta \in (\theta_L/\pi, \underline{\Delta})$,

$$\Psi^{\mathcal{M}} = 1 - \max\{\underline{s}, p\overline{s}\}. \tag{37}$$

For $\Delta \in \left[\underline{\Delta}, \overline{\Delta}\right)$,

$$\Psi^{\mathcal{M}} = 1 - p\overline{s} \tag{38}$$

by Proposition 3(iii).

By (5) and (6), whenever less than one, $\underline{s}$ is a continuous and strictly increasing function of δ and a continuous and strictly decreasing function of Δ. The same goes for $\overline{s}$. Therefore, (37) and (38) are decreasing in δ and increasing in Δ, and strictly so when $s^* < 1$.

(iii) Fix $\Delta > 1$. Define $\underline{\delta} \equiv \frac{2}{\sqrt{p}\theta_H}\sqrt{\Delta - 1}$. Define also $\overline{\delta} \equiv \frac{2}{\theta_L}\sqrt{\Delta - 1}$.

By hypothesis, $p > (\theta_L/\theta_H)^2$. Therefore, $\underline{\delta} < \overline{\delta}$ and $\underline{\Delta} < \overline{\Delta}$.

By inspection of (IR_H), for $\delta < \underline{\delta}$, we have $\Delta < \underline{\Delta} < \overline{\Delta}$. Hence there is complete assimilation.

By inspection of (IR_L), for $\delta \in \left(\underline{\delta}, \overline{\delta}\right)$, we have $\underline{\Delta} < \Delta < \overline{\Delta}$. Hence the unique RE is exclusive by Proposition 3(iii).

For $\delta \in \left(\overline{\delta}, \theta_L/\Delta\right)$, we know from Proposition 3(ii) there exists an exclusive RE if and only if $p \geq \hat{p}$ and a cohesive RE if and only if $p \leq \hat{p}$. By Lemma below, $\hat{p}$ is strictly decreasing in π and hence strictly increasing in δ on the domain $\left(\overline{\delta}, \theta_L/\Delta\right)$. Hence there exists a unique δ at which the RE switches from exclusive at strictness $\overline{s}$ to cohesive at strictness $\underline{s}$.

For $p > (\theta_L/\theta_H)^2$, $\overline{s} > \underline{s}$. In addition, $\underline{s}$ and $\overline{s}$ are weakly decreasing in Δ. The result follows.

(iv) By inspection of Figure 1, for $p \in ([\theta_L/\theta_H]^2, 1)$, there exists a unique Δ at which the RE switches from cohesive at strictness $\underline{s}$ to exclusive at strictness $\overline{s}$. Again, for $p > \left(\frac{\theta_L}{\theta_H}\right)^2$, $\overline{s} > \underline{s}$. In addition, $\underline{s}$ and $\overline{s}$ are weakly decreasing in Δ. This establishes the proposition. □

Lemma 2. *Suppose $\Delta > 1$. $\hat{p}$ is strictly decreasing in π on the domain $\left(\overline{\delta}, \theta_L/\Delta\right)$.*

Proof. By definition,

$$\hat{p}\overline{s}\left(\hat{p}\right) = \underline{s}.$$

Suppose $\overline{s}\left(\hat{p}\right) = 1$. Then

$$\frac{d\hat{p}}{d\pi} = \frac{d\underline{s}}{d\pi},$$

which is negative for $\pi \geq \theta_L/\Delta$ by inspection of (5).

Now suppose $\overline{s}\left(\hat{p}\right) < 1$. Then

$$\begin{aligned}\frac{d\hat{p}}{d\pi}\overline{s}\left(\hat{p}\right) + \hat{p}\left[\frac{\partial\overline{s}\left(\hat{p}\right)}{\partial\pi} + \frac{\partial\overline{s}\left(\hat{p}\right)}{\partial\hat{p}}\frac{d\hat{p}}{d\pi}\right] &= \frac{d\underline{s}}{d\pi} \\ \frac{d\hat{p}}{d\pi}\left[\overline{s}\left(\hat{p}\right) + \hat{p}\frac{\partial\overline{s}\left(\hat{p}\right)}{\partial\hat{p}}\right] &= \frac{d\underline{s}}{d\pi} - \hat{p}\frac{\partial\overline{s}\left(\hat{p}\right)}{\partial\pi}.\end{aligned}$$

Therefore

$$\text{sign}\left\{\frac{d\hat{p}}{d\pi}\right\} = \text{sign}\left\{\frac{d\underline{s}}{d\pi} - \hat{p}\frac{\partial\overline{s}\left(\hat{p}\right)}{\partial\pi}\right\},$$

where

$$\begin{aligned}\frac{d\underline{s}}{d\pi} &= -\frac{\theta_L\underline{s}}{\pi^2\sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}} \\ \hat{p}\frac{\partial\overline{s}\left(\hat{p}\right)}{\partial\pi} &= -\frac{\theta_H\hat{p}\overline{s}\left(\hat{p}\right)\sqrt{\hat{p}}}{\pi^2\sqrt{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}}.\end{aligned}$$

Thus $d\hat{p}/d\pi < 0$ if

$$\begin{aligned}-\frac{\theta_L\underline{s}}{\pi^2\sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}} &< -\frac{\theta_H\hat{p}\overline{s}\left(\hat{p}\right)\sqrt{\hat{p}}}{\pi^2\sqrt{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}} \\ \frac{\theta_L}{\sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}} &> \frac{\theta_H\sqrt{\hat{p}}}{\sqrt{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}} \\ \theta_L\sqrt{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)} &> \theta_H\sqrt{\hat{p}}\sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}.\end{aligned}$$

Squaring both sides yields

$$\begin{aligned}\theta_L^2\left(\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)\right) &> \theta_H^2\hat{p}\left(\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)\right) \\ -\theta_L^2(\Delta - 1) &> -\theta_H^2\hat{p}(\Delta - 1) \\ \hat{p}(\Delta - 1) &> \left(\frac{\theta_L}{\theta_H}\right)^2(\Delta - 1).\end{aligned}$$

Recall that $\hat{p} > (\theta_L/\theta_H)^2$ and $\Delta > 1$ by hypothesis. Hence the inequality holds. $\square$

Proof of Proposition 5 and Figure 5.

Proof. We shall establish the proposition by identifying the conditions under which each class of RE exists. We make use of the fact that x^*_{ik} equals s_k or zero for all $i \in N$ and $k \in K$.

Define $\tilde{\Delta} \equiv 1 + (1-p)(\theta_L/2\pi)^2 \leq \underline{\Delta}$. Note that if $\Delta < \tilde{\Delta}$, L types prefer schism to assimilation. Therefore, the RE is either cohesive or schismatic. If $\Delta > \tilde{\Delta}$, L types prefer assimilation to schism. Therefore, the RE is either cohesive or exclusive.

Cohesive RE. Let $s^* \in [s_L, s_H]$ be the strictness of the unique active group. The equilibrium payoff to i is:

$$\pi\big(1 - s^*\big) + \theta_i\big(s^*\big)^{1/2}. \tag{39}$$

Due to increasing returns, if there is a profitable deviation by a subset of type θ agents, then there is an even more profitable deviation by the full set of type θ agents N_θ, $\theta = L, H$. Thus only three types of deviations need to be ruled out: (I) another leader attracts all agents to form a new inclusive club, (II) at least one other leader forms an exclusive club by attracting all individuals of type θ only, and (III) at least one type θ chooses assimilation.

To be profitable, a type-I deviation requires there be an $s \in [0,1]$ such that:

$$\pi\big(1 - s^*\big) + \theta_L\big(s^*\big)^{1/2} \;<\; \pi\big(1 - s\big) + \theta_L s^{1/2}, \tag{40}$$

and

$$\pi\big(1 - s^*\big) + \theta_H\big(s^*\big)^{1/2} \;<\; \pi\big(1 - s\big) + \theta_H s^{1/2}. \tag{41}$$

Let $s^* \in [s_L, s_H]$ (the interval is defined by (11)). Because the RHS of (40) is strictly concave and maximized at s_L, (40) is violated for $s \geq s^*$. Because the RHS of (41) is strictly concave and maximized at s_H, (41) is violated for $s \leq s^*$. If $s^* \notin [s_L, s_H]$, both (40) and (41) hold. Hence no such deviation is profitable if and only if $s^* \in [s_L, s_H]$.

Now consider a type-II deviation to an exclusive group. The most a competing entrepreneur can do to attract L types is to set $s = \tilde{s}_L$ (defined by (9)), which yields

$$\max_{s\in[0,1]} \; \pi\big(1 - s\big) + \theta_L\big((1-p)s\big)^{1/2} = \pi\left[1 + (1-p)\left(\frac{\theta_L}{2\pi}\right)^2\right].$$

The most a competing entrepreneur can do to attract H types is to set $s = \tilde{s}_H$ (defined by (10)), which yields

$$\max_{s\in[0,1]} \; \pi\big(1 - s\big) + \theta_H\big(ps\big)^{1/2} = \pi\left[1 + p\left(\frac{\theta_H}{2\pi}\right)^2\right].$$

Hence the following conditions rule out a profitable type-II deviation:

$$\pi\big(1-s^*\big)+\theta_L\big(s^*\big)^{1/2} \geq \pi\left[1+(1-p)\left(\frac{\theta_L}{2\pi}\right)^2\right], \tag{42}$$

$$\pi\big(1-s^*\big)+\theta_H\big(s^*\big)^{1/2} \geq \pi\left[1+p\left(\frac{\theta_H}{2\pi}\right)^2\right]. \tag{43}$$

The following two participation constraints rule out a profitable type-III deviation:

$$\pi\big(1-s^*\big)+\theta_L\big(s^*\big)^{1/2} \geq \Delta\pi \tag{44}$$

$$\pi\big(1-s^*\big)+\theta_H\big(s^*\big)^{1/2} \geq \Delta\pi. \tag{45}$$

Case 1: $\Delta \leq \underline{\Delta}$ and $p \leq \left(\theta_L/\theta_H\right)^2$. $\underline{\Delta}\pi$ is an upper bound on the RHS of conditions (42)-(45). The LHS of (42) is a lower bound on the LHS of conditions (42)-(45). Therefore, it is sufficient to show there exists $s^* \in [s_L, s_H]$ such that

$$\begin{aligned}\pi\big(1-s^*\big)+\theta_L\big(s^*\big)^{1/2} &\geq \underline{\Delta}\pi \\ &= \max_{s\in[0,1]}\ \pi\big(1-s\big)+\theta_L s^{1/2}.\end{aligned} \tag{46}$$

Hence (46) is satisfied for $s^* = s_L$ and there exists a cohesive RE.

Case 2: $\Delta \leq \underline{\Delta}$ and $p > \left(\theta_L/\theta_H\right)^2$. First, note that (45) is satisfied strictly whenever (44) is satisfied. Hence it is sufficient to show there exists $s^* \in [s_L, s_H]$ that satisfies (42)-(44).

Case 2a: $\Delta \leq 1+(1-p)\left[\theta_L/(2\pi)\right]^2$. In this case, the RHS of (42) is no less than the RHS of (44), so the relevant constraints are (42) and (43). Denote the smallest s^* that satisfies (43) by z_H. We have:

$$z_H = \left(\frac{\theta_H}{2\pi}(1-\sqrt{1-p})\right)^2.$$

Denote the largest s^* that satisfies (42) by z_L. We have:

$$z_L = \left(\frac{\theta_L}{2\pi}(1+\sqrt{p})\right)^2.$$

Notice from (11) that $z_L \geq s_L$ and $z_H \leq s_H$.

For case 2a then, it suffices to show that $z_L > z_H$. In this case, there exists an $s^* \in [z_H, z_L]$ which satisfies both constraints and also lies in $[s_L, s_H]$. Comparing:

$$\begin{aligned} z_L &\geq z_H \\ \left(\frac{\theta_L}{2\pi}[1+\sqrt{p}]\right)^2 &\geq \left(\frac{\theta_H}{2\pi}[1-\sqrt{1-p}]\right)^2 \\ \frac{\theta_L}{\theta_H} &\geq \frac{1-\sqrt{1-p}}{1+\sqrt{p}}. \end{aligned} \tag{47}$$

The RHS of (47) increases monotonically from 0 to 1/2 as p goes from 0 to 1. Therefore, (47) is satisfied for all p if $\theta_L \geq (1/2)\theta_H$.

Now suppose that $\theta_L < (1/2)\theta_H$. Evaluating (47) at $p = (\theta_L/\theta_H)^2$ yields

$$\frac{\theta_L}{\theta_H} \geq \frac{1 - \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2}}{1 + \frac{\theta_L}{\theta_H}}. \tag{48}$$

Note:

$$\begin{aligned}
\left(\frac{\theta_L}{\theta_H}\right)^2 + 1 - \left(\frac{\theta_L}{\theta_H}\right)^2 &= 1 \\
\left(\frac{\theta_L}{\theta_H}\right)^2 + \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} &> 1 \\
\left(\frac{\theta_L}{\theta_H}\right)^2 &> 1 - \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} \\
\frac{\theta_L}{\theta_H} &> \frac{1 - \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2}}{\frac{\theta_L}{\theta_H}},
\end{aligned}$$

which implies that (48) holds. Thus (47) is satisfied at $p = (\theta_L/\theta_H)^2$. But not at $p = 1$. Therefore, there exists a threshold $\tilde{p}$ such that (47) is satisfied if and only if $p \leq \tilde{p}$.

Case 2b: $\Delta \in \left(1 + (1-p)\left[\theta_L/(2\pi)\right]^2, \underline{\Delta}\right]$. Now the RHS of (42) is less than the RHS of (44), so the relevant constraints are (44) and (43). (The community now fragments through assimilation not schism.)

Denote the largest s^* that satisfies (44) by $\tilde{z}_L$. We have:

$$\tilde{z}_L = \left(\frac{\theta_L}{2\pi} + \frac{1}{2}\sqrt{\left(\frac{\theta_L}{\pi}\right)^2 - 4(\Delta - 1)}\right)^2.$$

Notice from (11) that $\tilde{z}_L \geq s_L$ since $\Delta \leq \underline{\Delta}$.

Hence it suffices to show that $\tilde{z}_L > z_H$. In this case, there exists an $s^* \in [z_H, \tilde{z}_L]$ which satisfies both constraints and also lies in $[s_L, s_H]$. Comparing:

$$\begin{aligned}
\tilde{z}_L &\geq z_H \\
\left(\frac{\theta_L}{2\pi} + \frac{1}{2}\sqrt{\left(\frac{\theta_L}{\pi}\right)^2 - 4(\Delta - 1)}\right)^2 &\geq \left(\frac{\theta_H}{2\pi}[1 - \sqrt{1-p}]\right)^2 \\
\theta_H\sqrt{1-p} + \pi\sqrt{\left(\frac{\theta_L}{\pi}\right)^2 - 4(\Delta - 1)} &\geq \theta_H - \theta_L. \qquad (49)
\end{aligned}$$

Note that the LHS is strictly decreasing in both Δ and p.

Evaluating (49) at $p = (\theta_L/\theta_H)^2$ and $\Delta = \underline{\Delta}$:

$$\theta_H \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} \geq \theta_H - \theta_L$$
$$\sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} \geq 1 - \frac{\theta_L}{\theta_H}$$
$$1 - \left(\frac{\theta_L}{\theta_H}\right)^2 \geq 1 - 2\frac{\theta_L}{\theta_H} + \left(\frac{\theta_L}{\theta_H}\right)^2$$
$$\frac{\theta_L}{\theta_H} \geq \left(\frac{\theta_L}{\theta_H}\right)^2,$$

which is true since $\theta_H > \theta_L$. Evaluating (49) at $\Delta = \underline{\Delta}$ and $p = 1$ yields $0 \geq \theta_H - \theta_L$, a contradiction. Thus when Δ is at its maximum point, there exists a value $\tilde{p} \in \left([\theta_L/\theta_H]^2, 1\right)$ such that a cohesive RE exists if and only if $p \leq \tilde{p}$.

Where it exists, define $\tilde{p}(\Delta)$ as the value of p that equates the two sides of (49) for a given value of Δ. As the LHS of (49) is strictly decreasing in both Δ and p, $\tilde{p}(\Delta)$ is strictly decreasing in Δ.

At $\Delta = 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$, (49) is the same as (47). By continuity of the LHS of (49), if $\theta_L > (1/2)\theta_H$, there exists a value $\hat{\Delta} \in \left(1 + (1-p)\left[\theta_L/(2\pi)\right]^2, \underline{\Delta}\right)$ such that (i) for $\Delta \leq \hat{\Delta}$ a cohesive RE exists for all p and (ii) for $\Delta > \hat{\Delta}$ a cohesive RE exists if and only if $p \leq \tilde{p}(\Delta)$. If $\theta_L < (1/2)\theta_H$, for all $\Delta \in \left(1 + (1-p)\left[\theta_L/(2\pi)\right]^2, \underline{\Delta}\right]$, a cohesive RE exists if and only if $p \leq \tilde{p}(\Delta)$.

Schismatic RE. The following conditions are necessary and sufficient. To rule out emergence of an inclusive club, there must not exist an $s^* \in [0,1]$ such that (42) and (43) hold.

In addition, there are the participation constraints

$$\pi\left[1 + (1-p)\left(\frac{\theta_L}{2\pi}\right)^2\right] \geq \Delta\pi, \tag{50}$$

$$\pi\left[1 + p\left(\frac{\theta_H}{2\pi}\right)^2\right] \geq \Delta\pi. \tag{51}$$

For $\Delta \leq \underline{\Delta}$ and $p < (\theta_L/\theta_H)^2$, we have established that a club leader can break up a schismatic state and form an inclusive club at $s^* = s_L$. For $\Delta > 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$, clearly (50) is violated. That leaves $\Delta \leq 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$ and $p \geq (\theta_L/\theta_H)^2$. For such Δ, (50) is satisfied. For such p, the LHS of (50) is less than the LHS of (51). Hence (51) is also satisfied. We established in case 2a above that either (42) or (43) are violated for all $s^* \in [0,1]$ if and only if $p \geq \tilde{p}$, where $\tilde{p}$ equates (47). Therefore, a schismatic RE exists for $\Delta \leq 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$ and $p \geq \tilde{p}$.

Exclusive RE. The conditions are the same as for a schismatic RE except that the weak inequality in (50) is reversed. Hence an exclusive RE exists for $\Delta \leq \underline{\Delta}$ wherever neither a cohesive or schismatic RE exists. □

Proof of Proposition 6.

Proof. The proof proceeds in five cases.

Case 1: $\Delta \leq \theta_L/\pi$. In this case $\underline{s} = 1$, while $s^* \in [s_L, s_H] \subset (0,1)$. Therefore $\Psi^{\mathcal{M}} = 0 < 1 - s^* = \Psi^{\mathcal{C}}$.

Case 2: $\Delta \in \left(\theta_L/\pi, \tilde{\Delta}\right]$. Recall that, under monopoly, the RE is cohesive if $p \leq \hat{p}$ and exclusive otherwise. Under competition, the RE is either cohesive or schismatic if $p \leq \tilde{p}(\Delta)$ and exclusive otherwise.

Case 2a: $p \leq \min\{\hat{p}, \tilde{p}\}$. By Proposition 3, a monopolist leader chooses strictness $\underline{s}$ while a leader who faces competition chooses some $s^* \in [s_L, s_H]$. Note $s^* \leq \underline{s}$ by construction of $\underline{s}$ [see (5)], so $\Psi^{\mathcal{M}} = 1 - \underline{s} \leq 1 - s^* = \Psi^{\mathcal{C}}$.

Case 2b: $\hat{p} < p \leq \tilde{p}$. Under monopoly the RE is exclusive with strictness $\overline{s}$; under competition the RE is cohesive with strictness s^*. Therefore assimilation is weakly greater under competition if

$$\begin{aligned}
\Psi^{\mathcal{M}} &\leq \Psi^{\mathcal{C}} \\
1 - p\overline{s} &\leq 1 - s^* \\
p\overline{s} &\geq s^*.
\end{aligned}$$

As $p > \hat{p} \equiv \underline{s}/\overline{s}$, $p\overline{s} > \underline{s} \geq s^*$. Thus the above inequality holds.

Case 2c: $p > \max\{\hat{p}, \tilde{p}\}$. In this case, the monopolist leader always chooses strictness $\overline{s}(p)$, while under competition there will be two active leaders with strictness levels $\tilde{s}_L$ and $\tilde{s}_H$, respectively (schism). Social integration under monopoly is no greater than under competition if and only if

$$\begin{aligned}
1 - p\overline{s}(p) &\leq 1 - (1-p)\tilde{s}_L - p\tilde{s}_H \\
(1-p)\tilde{s}_L + p\tilde{s}_H &\leq p\overline{s}(p).
\end{aligned}$$

Note that the RHS is at its minimum when $\Delta = \tilde{\Delta}$, whereby $\overline{s}(p) = \left(\sqrt{\tilde{s}_H} + \sqrt{\tilde{s}_H - \tilde{s}_L}\right)^2$. Thus it is sufficient to verify

$$\begin{aligned}
(1-p)\tilde{s}_L + p\tilde{s}_H &\leq p\left(\sqrt{\tilde{s}_H} + \sqrt{\tilde{s}_H - \tilde{s}_L}\right)^2 \\
(1-p)\tilde{s}_L + p\tilde{s}_H &\leq p\left(\tilde{s}_H + \tilde{s}_H - \tilde{s}_L + 2\sqrt{\tilde{s}_H(\tilde{s}_H - \tilde{s}_L)}\right) \\
\tilde{s}_L &\leq p\left(\tilde{s}_H + 2\sqrt{\tilde{s}_H(\tilde{s}_H - \tilde{s}_L)}\right). \qquad (52)
\end{aligned}$$

Multiplying both sides by $[p/(1-p)](1/\tilde{s}_H)$ yields

$$\left(\frac{\theta_L}{\theta_H}\right)^2 \leq \frac{p^2}{1-p} + \frac{2p}{1-p}\sqrt{\frac{\tilde{s}_H - \tilde{s}_L}{\tilde{s}_H}}.$$

As $\tilde{s}_H > \tilde{s}_L$ for all $p > (\theta_L/\theta_H)^2$, the second term on the right-hand side is nonnegative. Therefore it suffices that

$$\left(\frac{\theta_L}{\theta_H}\right)^2 \leq \frac{p^2}{1-p}. \qquad (53)$$

The right-hand side is strictly increasing in p. Therefore, if the inequality holds at $\tilde{p}$, it holds for all $p > \tilde{p}$. Given that schism is occurring, $2\theta_L < \theta_H$, so by the proof of Proposition 5, $\tilde{p}$ is defined implicitly by

$$\frac{\theta_L}{\theta_H} = \frac{1-\sqrt{1-\tilde{p}}}{1+\sqrt{\tilde{p}}}. \tag{54}$$

Substituting (54) into (53), it remains to show that

$$\begin{aligned}\left(\frac{1-\sqrt{1-\tilde{p}}}{1+\sqrt{\tilde{p}}}\right)^2 &\leq \frac{\tilde{p}^2}{1-\tilde{p}} \\ \frac{1-\sqrt{1-\tilde{p}}}{1+\sqrt{\tilde{p}}} &\leq \frac{\tilde{p}}{\sqrt{1-\tilde{p}}} \\ \sqrt{1-\tilde{p}}-(1-\tilde{p}) &\leq \tilde{p}+\tilde{p}^{\frac{3}{2}} \\ \sqrt{1-\tilde{p}} &\leq 1+\tilde{p}^{\frac{3}{2}},\end{aligned}$$

which is true for all $\tilde{p} \in [0,1]$.

Case 2d: $\tilde{p} < p \leq \hat{p}$. In this case, the RE is cohesive under monopoly and schismatic under competition. Therefore, social integration is weakly greater under competition if

$$\begin{aligned}\Psi^M &\leq \Psi^C \\ 1-\underline{s} &\leq 1-p\tilde{s}_H-(1-p)\tilde{s}_L \\ \underline{s} &\geq (1-p)\tilde{s}_L+p\tilde{s}_H.\end{aligned}$$

In *case 2c*, we showed for $p > \tilde{p}$ that

$$p\overline{s} \geq (1-p)\tilde{s}_L + p\tilde{s}_H.$$

By the proof of Proposition 2, $\underline{s} \geq p\overline{s}$ if and only if $p \leq \hat{p}$. Therefore,

$$\underline{s} \geq p\overline{s} \geq (1-p)\tilde{s}_L + p\tilde{s}_H.$$

Case 3: $\Delta \in \left(\tilde{\Delta}, \underline{\Delta}\right]$. In this case, schism is impossible. Under both monopoly and competition, there is either cohesion or exclusivity, with the possibility of cohesion under one and exclusivity under the other.

Case 3a: $p \leq \min\{\tilde{p}, \hat{p}\}$. In this case, there is cohesion under both monopoly and competition, so the argument is identical to that of *case 2a*.

Case 3b: $\hat{p} < p \leq \tilde{p}$. In this case, the club is exclusive under monopoly and there is a single cohesive club under competition. Hence we need to show

$$\begin{aligned}\Psi^{\mathcal{M}} = 1-p\overline{s} &\leq 1-s^* = \Psi^{\mathcal{C}} \\ p\overline{s} &\geq s^*.\end{aligned}$$

As $p > \hat{p}$, $p\overline{s} > \underline{s} > s^*$. Thus the above inequality holds.

Case 3c: $p > \max\{\tilde{p}, \hat{p}\}$. In this case, both are exclusive. The monopolist leader sets strictness $\overline{s}(p)$ while the only active leader under competition sets strictness $\tilde{s}_H$. Social integration is no greater under monopoly than competition if and only if

$$\begin{aligned} 1 - p\overline{s} &\leq 1 - p\tilde{s}_H \\ \tilde{s}_H &\leq \overline{s}, \end{aligned}$$

which holds by construction of $\overline{s}$ (so that IR_H binds).

Case 3d: $\tilde{p} < p \leq \hat{p}$. In this case, the club is cohesive under monopoly and there is a single exclusive club under competition. Thus

$$\begin{aligned} \Psi^{\mathcal{M}} = 1 - \underline{s} &\leq 1 - p\tilde{s}_H = \Psi^{\mathcal{C}} \\ \underline{s} &\geq p\tilde{s}_H. \end{aligned}$$

The RHS is increasing in p while the LHS is constant. Therefore it is sufficient to show this holds for $p = \hat{p}$:

$$\begin{aligned} \underline{s} &\geq \hat{p}\tilde{s}_H \\ \underline{s} &\geq \frac{\underline{s}}{\overline{s}}\tilde{s}_H \\ \overline{s} &\geq \tilde{s}_H, \end{aligned}$$

which is true. Hence $\Psi^{\mathcal{M}} \leq \Psi^{\mathcal{C}}$.

<u>Case 4</u>: $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\right]$. Under both monopoly and competition, the RE is exclusive. Therefore $\Psi^{\mathcal{M}} < \Psi^{\mathcal{C}}$ if

$$\begin{aligned} 1 - p\overline{s}\left(p\right)) &< 1 - p\tilde{s}_H \\ \tilde{s}_H &< \overline{s}\left(p\right) \\ \sqrt{p}\frac{\theta_H}{2\pi} &< \sqrt{p}\frac{\theta_H}{2\pi} + \sqrt{p\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}, \end{aligned}$$

which holds for $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\right]$.

Case 5: $\Delta > \overline{\Delta}$. In this case, there is complete assimilation under both monopoly and competition, so $|M(s)| = 0$ for all s and $\Psi^{\mathcal{M}} = \Psi^{\mathcal{C}} = 0$.

□

Proof of Proposition 7.

Proof. Observe the program (15). By Proposition 3, $x_j^*(s, p)$ is nondecreasing in p. Therefore, if there exists an s^t that maximizes both $X^t = \sum_{j \in N} \omega_j x_j^*(s^t, p^t)$ and p^{t+1}, then s^t maximizes $V(p^t)$.

In addition, $s^t \in \{\underline{s}, \overline{s}(p^t)\}$ in equilibrium. Otherwise, the leader could increase strictness (and participation) without a decline in membership in period t. We rely on these arguments below.

(i) $\Delta \leq \delta\theta_L$. By Proposition 3, $\underline{s} = 1$. Hence the leader can form an inclusive club at $s^t(p^t) = 1$ regardless of p^t. This maximizes both X^t and p^{t+1} for all α and t. Therefore, $p^{t+1} = X^t = 1$ for all $t \geq 0$. This establishes part (i).

(ii) By Proposition 2, there always exists a positive strictness level s^t such that $M^*(s^t) \neq \emptyset$. We know $s^t \in \{\underline{s}, \overline{s}(p^t)\}$. By hypothesis, $\alpha = 0$. Hence

$$p^{t+1} = \begin{cases} \underline{s} & \text{if } s^t = \underline{s} \\ p^t\overline{s}(p^t) & \text{if } s^t = \overline{s}(p^t). \end{cases}$$

Note that $p^{t+1} = X^t$ (because $\alpha = 0$). Hence both p^{t+1} and X^t are maximized by the same s^t. Thus, as in the static case, $s^t = \overline{s}(p^t)$ if and only if

$$\begin{aligned} p^t\overline{s}\left(p^t\right) &> \underline{s} \\ p^t &> \frac{\underline{s}}{\overline{s}(p^t)} \equiv \hat{p}. \end{aligned}$$

Case 1: $p^0 \leq \hat{p}$. Hence an inclusive club forms and $p^{t+1} = \underline{s}$. Observe that $\underline{s} \leq \hat{p}$, because $\overline{s}(p^t) \leq 1$. Thus, $p^{t+1} \leq \hat{p}$. By induction, an inclusive club forms in every period $p^t = \underline{s}$ for all $t > 0$.

Case 2: $p^0 > \hat{p}$ and $p^0 \geq \Delta\left(\frac{\pi}{\theta_H}\right)^2$. In this case, $s^0 = \overline{s}(p^0) = 1$ so that $p^1 = p^0\overline{s}(p^0) = p^0$. By induction, an exclusive club forms at $s^t = 1$ and $p^t = p^0$ for all $t > 0$.

Case 3: $p^0 > \hat{p}$ and $p^0 < \Delta\left(\frac{\pi}{\theta_H}\right)^2$. In this case, $s^0 = \overline{s}(p^0) < 1$ so that $p^1 = p^0\overline{s}(p^0) < p^0$. By induction, $p^{t+1} < p^t$ as long as an exclusive club is formed. Suppose this continued forever. Then by recursion,

$$p^t = p^0\prod_{\tau=0}^{t-1}\overline{s}(p^\tau). \tag{55}$$

Because $\overline{s}(p)$ is strictly increasing in p and p^t is strictly decreasing in t,

$$p^t = p^0\prod_{\tau=0}^{t-1}\overline{s}(p^\tau) < p^0\left[\overline{s}\left(p^0\right)\right]^t. \tag{56}$$

By hypothesis, $\overline{s}(p^0) < 1$. Hence if an exclusive club formed in every period, $\lim_{t\to\infty} p^t = 0$.

Therefore, there exists a finite time T such that $p^{T-1} > \hat{p}$ and $p^T \leq \hat{p}$. Therefore, a cohesive club forms at time T. From there we enter case 1: $s^t(p^t) = \underline{s}$ and $p^t = \underline{s}$ for all $t \geq T$.

(iii) $\Delta \in \left(\underline{\Delta}, \overline{\Delta}(p^0)\right]$. As $\underline{\Delta}$ is time invariant, $\Delta > \underline{\Delta}$ for all time. By Proposition 2(i) then, $M^*(s^t) \neq N$ for all t. In addition, $M^*(s^t)$ is nonempty for some s^t, only if $\Delta \leq \overline{\Delta}(p^t)$. In this case, we know the strictness s^t that maximizes both X^t and p^{t+1} is $\overline{s}(p^t)$.

By (6), $\overline{s}(p^0) = 1$ for $p^0 \geq \Delta \left(\frac{\pi}{\theta_H}\right)^2$. Hence $p^1 = p^0 \overline{s}\left(p^0\right) = p^0$. By induction, $p^t = p^0$ for all t.

By (6), $\overline{s}(p^0) < 1$ otherwise. Hence $p^1 = p^0 \overline{s}(p^0) < p^0$. If $p^0 < \Delta \left(\frac{\pi}{\theta_H}\right)^2$ then, $p^1 < \Delta \left(\frac{\pi}{\theta_H}\right)^2$, so $\overline{s}(p^1) < 1$. By induction, $p^{t+1} < p^t$ for all $t > 0$.

From there, the argument in part (ii) case 3 establishes $\lim_{t\to\infty} p^t = 0$.

Recall $\overline{\Delta}(0) = 1$. By hypothesis, $\Delta > \underline{\Delta} > 1$. Therefore, there exists a finite time T such that $\Delta \leq \overline{\Delta}\left(p^{T-1}\right)$ and $\Delta > \overline{\Delta}\left(p^T\right)$, so that complete assimilation occurs at time T. This establishes part (iii) of the proposition.

(iv) $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}(p^0)\right\}$. By Proposition 2(ii), $M^*(s^t) = \emptyset$ for all s^t. Hence for any p^0, $p^1 = 0$. Recall that $\overline{\Delta}(p)$ is strictly increasing in p. Hence $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}(p^1)\right\}$. By induction, there is complete assimilation $p^{t+1} = 0$ for all $t \geq 1$.

□

Proof of Proposition 8.

Proof. First, note that by inspection of (6), $\Delta < \theta_H/\pi$ is sufficient for $\overline{s}(1) = 1$ and a fixed point at $p^* = 1$. In this case, there is a steady state with maximal religious participation of $X^* = n$. In addition, for μ sufficiently large, the leader maximizes the steady-state level of religious participation—the stream of payoffs along the transition path is irrelevant. Hence, if a transition path from p^0 to $p^* = 1$ can be achieved through feasible choices of s^t, then the leader will choose such a path. We shall now characterize conditions under which this occurs.

Recall that $p^{t+1} = [\alpha + (1-\alpha)p^t]\overline{s}(p^t)$. Because $\Delta < \theta_H/\pi$ by hypothesis, there exists a $\overline{p} < 1$ such that $\overline{s}(p) = 1$ for all $p \in [\overline{p}, 1]$. In this interval, $p^{t+1} = \alpha + (1-\alpha)p^t$. Thus $p^{t+1} - p^t = \alpha(1-p^t) > 0$, and $p^{t+1} > p^t$ for all $p \in [\overline{p}, 1)$. Hence there exists a feasible transition path from any $p \in [\overline{p}, 1]$ to $p^* = 1$.

It remains to be shown that $p^{t+1} > p^t$ for $p^t < \overline{p}$, which is true if

$$
\begin{aligned}
[\alpha + (1-\alpha)p]\overline{s}(p^t) &> p^t \\
\alpha &> \frac{p^t[1-\overline{s}(p^t)]}{(1-p^t)\overline{s}(p^t)} \equiv \underline{\alpha}^t.
\end{aligned} \tag{57}
$$

Note that $\underline{\alpha}^t < 1$ if and only if $\overline{s}(p^t) > p^t$. Dropping the time subscripts and writing the expression for $\overline{s}(p)$:

$$
\left(\frac{\theta_H}{2\pi}\sqrt{p} + \sqrt{\left(\frac{\theta_H}{2\pi}\right)^2 p - (\Delta - 1)}\right)^2 > p
$$

Simplifying and rearranging the above yields

$$
p\left(\frac{\theta_H}{\pi} - 1\right) > (\Delta - 1). \tag{58}
$$

Suppose (58) is satisfied for all $p \in [0,1]$. Then for all $p^t \in [0,1]$ and all solutions to (57), $\underline{\alpha}^t(p^t) < 1$. Hence for α sufficiently large, $p^{t+1} > p^t$ regardless of p^t. In this case, there exists a feasible transition path from p^0 to $p^* = 1$.

By inspection, (58) is satisfied for all $p \in [0,1]$ if $\Delta < 1$ and $\theta_H \geq \pi$. This establishes part (i).

If $\Delta \leq \underline{\Delta}$ and $\theta_H > \pi$, the leader begins by setting either $\underline{s}$ or $\overline{s}(p^0)$. Clearly, the latter is chosen only if $\alpha\overline{s}(p^0) + (1-\alpha)p^0\overline{s}(p^0) \geq \alpha\underline{s} + (1-\alpha)\underline{s} = \underline{s}$. Hence p^1 is at least $\underline{s}$. Thus if

$$\underline{s}\left(\frac{\theta_H}{\pi} - 1\right) > (\Delta - 1),$$

then (58) is satisfied for all $t > 0$. This establishes part (ii).

Condition (iii) follows from evaluating (58) at $t = 0$. □

Proof of Proposition 9.

Proof. By assumption, agents live for one period and care only about their own payoff. Given coalitional deviations then, the MPRE implements the static outcome (given by Proposition 5) at each stage.

The remainder of the proof proceeds in two steps. First, we show that once a state of cohesion is entered, the community remains cohesive forever. We then show that p^t is strictly decreasing whenever the MPRE implements schism or exclusivity.

To prove that once a community becomes cohesive it remains so, consider $p^T < \tilde{p}$. By Proposition 5, this is the condition under which an inclusive club forms setting strictness s^*. In this case, $p^{T+1} = s^*$. Thus if $s^* < \tilde{p}$, then $p^{T+2} = s^* < \tilde{p}$. By induction, $p^t = s^*$ for all $t > T$.

Because z_H is strictly increasing in p and at $\tilde{p}$, $z_L = z_H = s^*$ (see the proof of proposition 5), we need only show

$$\begin{aligned} z_H &< \tilde{p} \\ \left(\frac{\theta_H}{2\pi}[1 - \sqrt{1-p}]\right)^2 &< \tilde{p} \\ \frac{\theta_H}{2\pi}[1 - \sqrt{1-\tilde{p}}] &< \sqrt{\tilde{p}} \\ \frac{\theta_H}{2\pi} &< \sqrt{\tilde{p}} + \frac{\theta_H}{2\pi}\sqrt{1-\tilde{p}}, \end{aligned}$$

which is true because $\frac{\theta_H}{2\pi} < 1$ by assumption. Thus $s^* < \tilde{p}$ and therefore if $p^t = s^* < \tilde{p}$, $p^{t+1} = s^* < \tilde{p}$.

We now show that p^t is decreasing whenever there is an exclusive club with strictness $\tilde{s}_H$. Given the dynamic defined by (16) and (17), under exclusivity:

$$\begin{aligned} p^{t+1} = \alpha\tilde{s}_H + (1-\alpha)p^t\tilde{s}_H &= [\alpha + (1-\alpha)p^t]\tilde{s}_H \\ &= \underbrace{[\alpha + (1-\alpha)p^t]\left(\frac{\theta_H}{2\pi}\right)^2}_{<1} p^t < p^t. \end{aligned}$$

Thus $p^{t+1} < p^t$ for all t under exclusivity.

Now suppose that there is a schismatic MPRE, with strictness levels $\tilde{s}_L$ and $\tilde{s}_H$, respectively. By (16) and (17),

$$p^{t+1} = p^t \tilde{s}_H + (1 - p^t)\tilde{s}_L.$$

Recall from Proposition 5 that if a schismatic equilibrium exists, it must be that $\tilde{s}_H > \tilde{s}_L$, otherwise an H type could benefit by deviating and joining the all L club. Thus

$$p^{t+1} \;=\; p^t \tilde{s}_H + (1 - p^t)\tilde{s}_L \;<\; \tilde{s}_H \;=\; p^t \left(\frac{\theta_H}{2\pi}\right)^2.$$

As $\frac{\theta_H}{2\pi} < 1$,

$$p^{t+1} < p^t \left(\frac{\theta_H}{2\pi}\right)^2 < p^t.$$

Hence p^t is strictly decreasing under schism.

Therefore, whenever the MPRE specifies schism or exclusivity, p^t is decreasing. Whenever the MPRE specifies cohesion, p^t is constant and below $\tilde{p}$. Under complete assimilation, p^t falls to zero for all subsequent t. Parts (i) and (ii) of the Proposition follow. □

Proof of Proposition 10.

Proof. (i) Suppose that $\Delta > 1$. Then as $\delta \to 0$, $\underline{\Delta}$ and $\overline{\Delta}$ converge monotonically to one. Hence there exists a (non-degenerate) open interval of values $[0, \underline{\delta})$ such that if $\delta^t \in [0, \underline{\delta})$, then $\Delta > \max\{\underline{\Delta}^t, \overline{\Delta}^t\}$. By (IR_H) then, $x_i^* = 0$ for all $i \in N$ at time t regardless of p^t. Hence by (14) and (18), $p^{t+1} = 0$ and $\delta^{t+1} = 0$. Iterating this argument establishes the asymptotic stability of full tolerance at $(\delta, p) = (0, 0)$ when $\Delta > 1$.

To show that $\Delta > 1$ is also necessary, perturb δ^T so that it is small but positive. Recall that for all $\delta^t > 0$, $\underline{\Delta}^t > 1$. Hence for $\delta^T > 0$ and $\Delta \leq 1$, $\Delta < \underline{\Delta}$. By the argument in Proposition 3 then, the religious leader can at least form an inclusive club at $\underline{s} > 0$. Hence there exists some $\tau > T$ such that $p^\tau \geq \underline{s} > 0$ and $\delta^\tau \geq \lambda \underline{s} > 0$ for all $t \geq \tau$. Thus any state in which $\delta = 0$ is not asymptotically stable.

(ii) Suppose $\lambda > \Delta/\theta_H$. For δ^t sufficiently close to λ, $\delta^t > \Delta/\theta_H$. For p close to one, $x_i^* = 1$ for all H types at time t by (IR_H) and the argument in Proposition 3. Hence by (14) and (18), $p^{t+1} = \alpha + (1 - \alpha)p^t > p^t$ (recall $\alpha > 0$ by hypothesis) and $\delta^{t+1} = \lambda p^{t+1} > \lambda p^t = \delta^t$. Iterating this argument establishes the asymptotic stability of zero tolerance $(\delta, p) = (\lambda, 1)$ when $\lambda > \Delta/\theta_H$.

To show that $\lambda > \Delta/\theta_H$ is also necessary, perturb (δ^T, p^T) so that $\delta^T < \lambda$ and $p^T < 1$. If $\lambda \leq \Delta/\theta_H$, then $\delta^T < \lambda$ implies $\delta^T < \Delta/\theta_H$. Thus, $\overline{s}(p) < 1$ for all $p \in [0, 1]$ at time T by (6). Hence, for p^T close to one, $p^{T+1} = \overline{s}(p^T)[\alpha + (1 - \alpha)p^T] \leq \overline{s}(p^T) < \overline{s}(1) < 1$ and $\delta^{T+1} < \lambda \overline{s}(1) < \lambda$. Iterating this argument establishes that $(\delta, p) = (\lambda, 1)$ is not asymptotically stable when $\lambda \leq \Delta/\theta_H$.

(iii) By hypothesis, the initial state exhibits positive religious participation, so that $p^1 > 0$ by (14). By (18), $\delta^1 = \lambda p^1$. Because $p^1 > 0$, for λ sufficiently large, $\delta^1 > \Delta/\theta_L$. By the argument in Proposition 3 then, $x_i^* = 1$ for all i at time 1. Hence $p^2 = 1$ and $\delta^2 = \lambda$. We know from part (ii) that this is an absorbing state when $\lambda \geq \Delta/\theta_L$. This establishes the proposition.

□