Evolution & Learning in Games

Econ 243B

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Lecture 9.

Cultural Transmission

Motivation

- Prior to the 1960s, it was conventional wisdom among social scientists that immigrants from various ethnic and religious backgrounds would assimilate into American culture.
- It became obvious that this was not occurring not only in America, but elsewhere.
- High rates of homogamy persisted along with distinctive cultural traits:
    - Basque and Catalan culture in Spain,
    - Ultra-Orthodox Judaism in New York,
    - Conservatives and liberals in the United States.

The Bisin-Verdier Model

Bisin & Verdier (2000 QJE, 2001 JET)

- Consider a simple baseline model of cultural transmission.
- Agents form a continuum and can have either cultural trait a or b.
- Each parent (asexually) produces one child, socializes them and then dies.
- With probability τ_i a parent with trait $i \in \{a, b\}$ successfully passes on her trait to her child (**vertical transmission**).
- With probability $1 - \tau_i$ the child is matched at random with someone from her parent's generation and acquires their trait (**oblique transmission**).

Dynamics: Exogenous Socialization

- Let q equal the proportion of type a individuals in the population.
- The probability that a type b individual has a type a child is $P_{ba} = (1 - \tau_b)q$.
- The probability that a type a individual has a type b child is $P_{ab} = (1 - \tau_a)(1 - q)$.
- In continuous time the dynamic is:

$$\begin{aligned} \dot{q} &= \underbrace{(1-q)\,P_{ba}}_{\textit{inflow}} - \underbrace{q\,P_{ab}}_{\textit{outflow}} \\ &= (1-q)(1-\tau_b)q - q(1-\tau_a)(1-q) \\ &= (\tau_a - \tau_b)q(1-q). \end{aligned} \tag{1}$$

The Melting Pot

- We have a melting pot, i.e. a monomorphic cultural equilibrium:
  - $q = 1$ is asymptotically stable if $\tau_a > \tau_b$.
  - $q = 0$ is asymptotically stable if $\tau_b > \tau_a$.
- How can we get cultural diversity, i.e. a polymorphic cultural equilibrium?

Endogenous Socialization

- Bisin and Verdier's contribution is to introduce a choice of socialization effort. For example:
  - teaching,
  - school choice,
  - residential choice,
  - homogamy.

Imperfect Empathy

- To model socialization choice, parents need to have preferences over the traits that their children can acquire.
- **Imperfect empathy**: parents evaluate their children's behavior based on their own preferences.
- Formally, a parent with trait i gets a payoff of V_{ij} if their child acquires trait j, where $V_{ii} > V_{ij}$ whenever $i \neq j$.

Objective Functions

- A parent with trait a in state q has payoff function:

$$U^a(q) = [\underbrace{\tau_a + (1-\tau_a)q}_{P_{aa}}]V_{aa} + \underbrace{(1-\tau_a)(1-q)}_{P_{ab}}V_{ab} - c(\tau_a).$$

  They choose socialization effort τ_a at cost $c(\tau_a)$ to maximize this function.

- A parent with trait b in state q has payoff function:

$$U^b(q) = [\underbrace{\tau_b + (1-\tau_b)(1-q)}_{P_{bb}}]V_{bb} + \underbrace{(1-\tau_b)q}_{P_{ba}}V_{ba} - c(\tau_b).$$

First-Order Conditions

Define 'cultural intolerances'

$$\Delta_a = V_{aa} - V_{ab} \text{ and } \Delta_b = V_{bb} - V_{ba}.$$

- The FOC for an a type is:

$$(1-q)\Delta_a = c'(\tau_a).$$

- The FOC for a b type is:

$$q\Delta_b = c'(\tau_b).$$

Optimal Socialization Effort

Proposition 1. Optimal socialization effort varies as follows:

(i) τ_i is strictly increasing in 'cultural intolerance' Δ_i,

(ii) τ_a is strictly decreasing in q,

(iii) τ_b is strictly increasing in q,

(iv) $\tau_a > \tau_b$ if and only if $q < \frac{\Delta_a}{\Delta_a + \Delta_b}$.

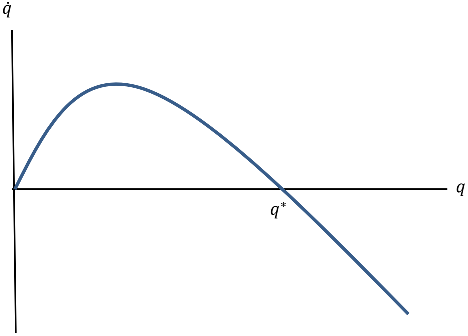
Hence 'minorities' expend more effort on socialization

- See evidence on religious minorities by Bisin, Topa and Verdier 2004.

Dynamics: Endogenous Socialization

Population dynamics are given by (1) except that now τ is endogenous.

Proposition 2. The process converges to the interior steady state $q^* = \frac{\Delta_a}{\Delta_a + \Delta_b}$ from any $q \in (0, 1)$.



Therefore, a polymorphic cultural distribution emerges from almost every initial state.

Generalizing the Analysis

- How can we extend the Bisin and Verdier framework to n traits?
- What is the relationship between Bisin-Verdier style cultural evolution and standard dynamics in evolutionary game theory?

The Montgomery Analysis

- Agents form a continuum and possess one of n cultural traits, $i \in \{1, \dots n\}$.
- Each parent (asexually) produces one child, socializes them and then dies.
- A parent with trait i will have a child with trait $j \neq i$ with probability:

  $$P_{ij} = (1 - \tau_i)q_j \tag{2}$$

  and a child with trait i with probability:

  $$P_{ii} = \tau_i + (1 - \tau_i)q_i. \tag{3}$$

The Cultural Evolutionary Dynamic

- In discrete time:

$$q_i(t+1) = \sum_j q_j(t) P_{ji}. \tag{4}$$

- Substituting (2) and (3) into (4):

$$\begin{aligned} q_i(t+1) &= q_i(t)\left[\tau_i + (1-\tau_i)q_i(t)\right] + \sum_{j\neq i} q_j(t)(1-\tau_j)q_i(t) \\ &= q_i(t)\tau_i + (1-\tau_i)q_i(t)^2 + q_i(t)\sum_{j\neq i} q_j(t)(1-\tau_j) \\ &= q_i(t)\tau_i + q_i(t)\sum_j q_j(t)(1-\tau_j) \\ &= q_i(t) + q_i(t)\left[\tau_i - \sum_j q_j(t)\tau_j\right]. \end{aligned} \tag{5}$$

The Cultural Evolutionary Dynamic

- Taking the continuous-time limit, we have:

$$\dot{q}_i = q_i \left[\tau_i - \sum_j q_j \tau_j \right] \tag{6}$$

for all $i = 1, \ldots n$.

- Clearly, when the τs are exogenous, the dynamic converges from every interior state to a monomorphic distribution centered on trait $\arg\max_i \{\tau_i\}_{i=1}^n$.

Endogenous Socialization

- Let us proceed along the lines of Bisin and Verdier (2000) except with n traits and a quadratic socialization cost:

$$\max_{\tau_i} \sum_j P_{ij} V_{ij} - \frac{1}{2}(\tau_i)^2, \tag{7}$$

where V_{ij} is an i type's payoff from having a child with trait j.

- The FOC is:

$$\begin{aligned} \tau_i^* &= (1 - q_i) V_{ii} - \sum_{j \neq i} q_j V_{ij} \\ &= V_{ii} - \sum_j q_j V_{ij} \\ &= \sum_j q_j \left[V_{ii} - V_{ij} \right] \\ &\equiv \sum_j q_j \Delta_{ij}, \end{aligned} \tag{8}$$

where Δ_{ij} is an i type's intolerance toward j.

The Replicator Dynamic

- Substituting into the dynamic (6), we have:

$$\dot{q}_i = q_i \left[\sum_j q_j \Delta_{ij} - \sum_j q_j \sum_j q_k \Delta_{jk} \right] \tag{9}$$

  for all $i = 1, \ldots n$.

- Interpreting Δ_{ij} as the payoff from playing strategy i against j, this becomes a well-known evolutionary dynamic:
  - The **replicator dynamic** operating on a particular population game, i.e. random matching to play an $n \times n$ coordination game.

Convergence Results

- Hence we can exploit standard results from evolutionary game theory on the replicator dynamic to study cultural evolution.
- Suppose that $\Delta_{ij} = \Delta_i$ for all $j \neq i$ (and $\Delta_{ii} = 0$), i.e. each group is intolerant of all other traits to an equal degree.
- Then this is a **strictly stable** game:
  - There is a unique Nash equilibrium (distribution of traits), which is globally asymptotically stable.
  - Every trajectory of the replicator dynamic in the interior of the n-dimensional simplex converges to this state.
- More generally, we can cast this as a potential game and exploit the corresponding results on such games.