Kalman Filtering, Sensor Fusion, and Eye Tracking

ECCV OpenEyes Workshop
August 23, 2020

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Outline

- Introduction to Kalman Filtering
- Multi-sensor, multirate fusion
- Distributed/decentralized Kalman filtering
- Thoughts on potential for eye tracking

R. E. Kalman 1930-2016

A Tribute to Rudolf Kalman

His Research, Life, and Influence

ATHANASIOS ANTOULAS, TRYPHON T. GEORGIOU, PRAMOD P. KHARGONEKAR,
A. BÜLENT ÖZGÜLER, EDUARDO D. SONTAG, and YUTAKA YAMAMOTO

Digital Object Identifier 10.1109/MCS.2016.2643261
Date of publication: 17 March 2017

Rudolf Kalman produced a wealth of groundbreaking contributions that revolutionized control engineering and founded the modern field of mathematical system theory.

Prof. R.E. Kalman—A Personal Tribute

My Debt to an Intellectual Giant

PRAMOD P. KHARGONEKAR

Digital Object Identifier 10.1109/MCS.2016.2643339
Date of publication: 17 March 2017

OBITUARY «

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Editor's note: A special issue commemorating Rudolf Kalman's work and life is under preparation and will be published in the April 2017 issue of *IEEE Control Systems Magazine.*

Digital Object Identifier 10.1109/MCS.2016.2621578
Date of publication: 19 January 2017

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Professor R.E. Kalman–Reflections on his way of thinking

Pramod P. Khargonekar

Annual Reviews in Control 45 (2018) 207–210

Kalman Filtering: Linear System + Gaussian Noise

$$x((k+1)T) = A(k)x(kT) + B(k)u(kT) + w(kT), x(0) = x_0$$
$$y(kT) = C(k)x(kT) + D(k)u(kT) + v(kT)$$

Dynamic System Model

$$E(x_0 x_0^T) = P(0)$$
$$E(w(kT)w^T(kT)) = Q(k)$$
$$E(v(kT)v^T(kT)) = R(k)$$

- Linear time-varying discrete time system, T sampling period
- State variable x
- Known input u (=0 for this talk)
- Process noise w
- Measurement noise v
- We will simplify by assuming that the known input is 0

Kalman Filter

Optimal estimate of the state consists of time-update and measurement update:

$$\begin{aligned} \hat{x}((k+1)T|k) &= A(k)\hat{x}(kT|k) \\ \hat{x}(kT|k) &= \hat{x}(kT|(k-1)) + L(k)(y(kT) - C(k)\hat{x}(kT|(k-1))) \end{aligned}$$

Kalman gain L and estimation error covariances evolve according to:

$$\begin{aligned} P^{-1}(k|k) &= P^{-1}(k|(k-1)) + C^T(k)R^{-1}(k)C(k) \\ L(k) &= P(k|k)C^T(k)R^{-1}(k) \\ P((k+1)|k) &= A(k)P(k|k)A^T(k) + Q(k) \end{aligned}$$

Rich Heritage from Kalman

- The most important and impactful result from control theory and one of the most important from engineering
- Equivalent to Bayes theorem (under the specified conditions)
- Extensions to continuous-time, nonlinear, non-Gaussian, distributed, networked, …
- Real-time, on-line, recursive algorithms and implementations
- Computationally efficient implementations
- Applications in many fields of engineering and sciences (including machine vision)
- Huge literature

Multi-Sensor Data Fusion

- Combine data from multiple, disparate sensors to arrive at a unified estimate of the unknown system/signal

- Wide variety of techniques to address disparate challenges related to the system, the sensors, and the data characteristics: probabilistic, Dempster-Schafer, fuzzy, …

- Many application domains:

    - Aerospace, naval, and defense applications
    - Sensor networks, IoT
    - Digital twins, manufacturing, monitoring, …
    - Robotics
    - Medical
    - Physical sciences: weather, environment, air, water, …

Kalman Filtering and Multi-Sensor Data Fusion

“Nonetheless, the Kalman filter is one of the most popular fusion methods mainly due to its simplicity, ease of implementation, and optimality in a mean-squared error sense. It is a very well-established data fusion method whose properties are deeply studied and examined both theoretically and in practical applications. On the other hand, similar to other least-square estimators, the Kalman filter is very sensitive to data corrupted with outliers. Furthermore, the Kalman filter is inappropriate for applications whose error characteristics are not readily parameterized.”

Khaleghi et al., Multisensor Data Fusion, Information Fusion (2011)

Related Techniques and Extensions

- Extended Kalman filter (EKF)
- Unscented Kalman filter (UKF)
- Particle filters (Monte-Carlo algorithms)
- Other related methods

Multi-Rate Kalman Filtering

- Setting: Multiple sensors operating at different sampling rates
- Sensor 1 sampling time $T_1 = NT$
- Sensor 2 sampling time $T_2 = MT$
- …
- Sensor N sampling time …
- Kalman filter equations generalize to this case quite easily although the notation can get quite complicated

Lifting Approach

Start with a discrete-time sequence z(0), z(1), … z(N), …

$$Z^N = Z^N(0) = \begin{Bmatrix} z(0) \\ z(1) \\ . \\ . \\ z(N-1) \end{Bmatrix}, Z^N(1) = \begin{Bmatrix} z(N) \\ z(N+1) \\ . \\ . \\ z(2N-1) \end{Bmatrix}, \ldots$$

A multi-rate system can be rewritten as a standard discrete-system using lifted representation for various signals

Many papers in the control systems literature on this lifting-based approaches

Contents lists available at ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

Full Length Article

An overview of multirate multisensor systems: Modelling and estimation

Honglei Lin, Shuli Sun*

School of Electronics Engineering, Heilongjiang University, Harbin 150080, China

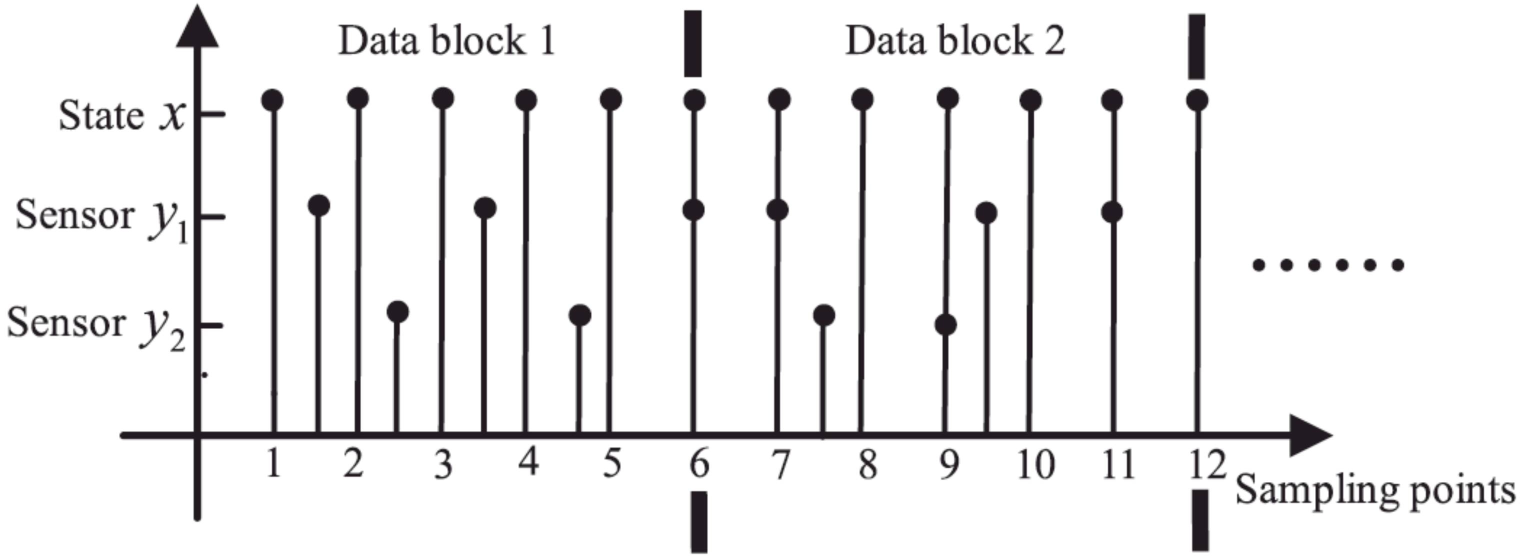

Fig. 3. Illustration of nonuniform sampling scheme with data blocks.

Distributed/Decentralized Kalman Filtering

Decentralized Structures for Parallel Kalman Filtering

HAMID R. HASHEMIPOUR, SUMIT ROY, AND ALAN J. LAUB

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 33, NO. 1, JANUARY 1988

KALMAN FILTER ALGORITHMS FOR A MULTI-SENSOR SYSTEM*

D. Willner, C. B. Chang, and K. P. Dunn
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1976 IEEE Conference on Decision and Control

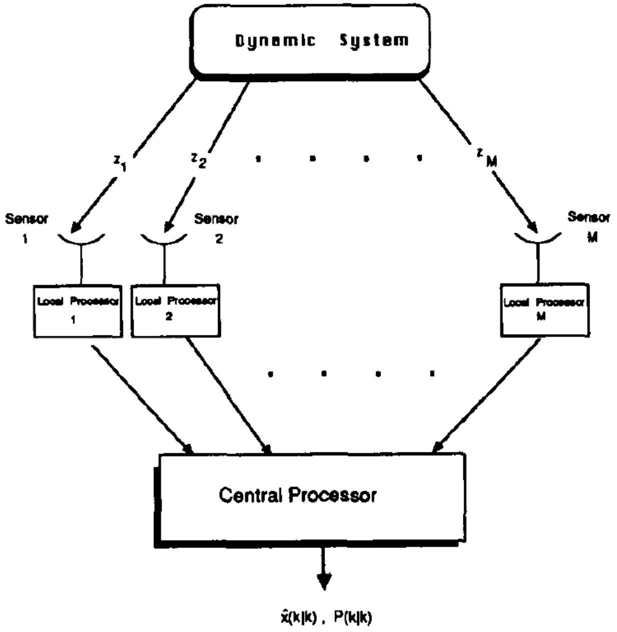

Fig. 1. Collocated sensors.

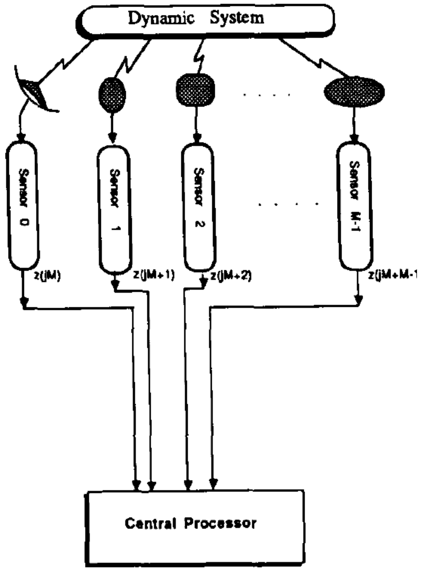

Fig. 2. Time sequential measurements for dispersed :

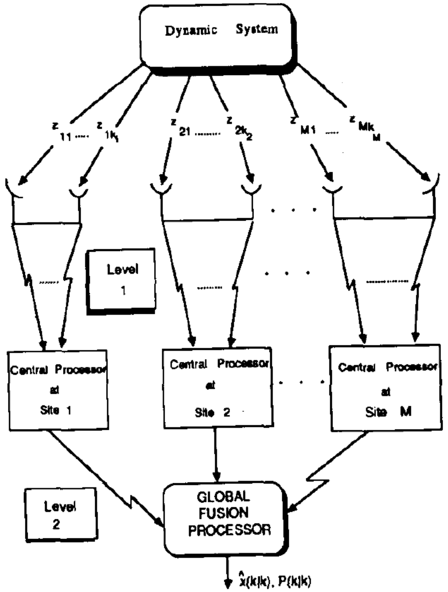

Fig. 3. Two-tier Kalman filter structure.

Distributed Kalman Filtering and Sensor Fusion

Distributed Kalman Filtering and Sensor Fusion in Sensor Networks

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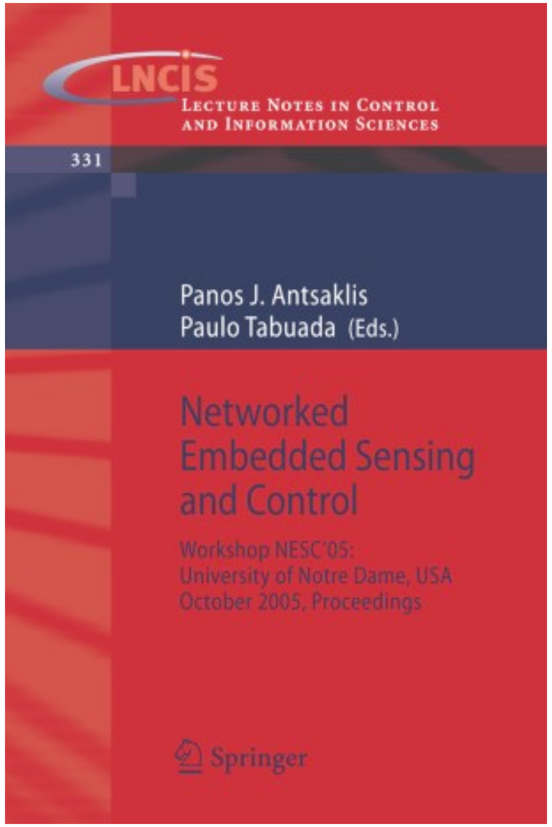

Key Ideas

Suppose we have J different sensors described by:

$$
\begin{aligned}
y_1(kT) &= C_1(k)x(kT) + v_1(kT) \\
y_2(kT) &= C_2(k)x(kT) + v_2(kT) \\
&\dots \\
y_J(kT) &= C_J(k)x(kT) + v_J(kT)
\end{aligned}
$$

Centralized optimal solution: combine these measurements and solve the resulting KF problem:

$$
y(kT) = \begin{bmatrix} y_1(kT) \\ y_2(kT) \\ \dots \\ y_J(kT) \end{bmatrix} \qquad v(kT) = \begin{bmatrix} v_1(kT) \\ v_2(kT) \\ \dots \\ v_J(kT) \end{bmatrix}
$$

Local Kalman Filters and Fusion of Results

- Create a Kalman Filter at each sensor and generate state estimates and covariance matrices
- Variety of algorithms for exchanging information between sensors with or without a central processor
- Analytical results on the performance of the resulting estimates and convergence to the centralized estimator
- Voluminous literature on these themes

Example: Fusion without Feedback

$$P^{-1}(k|k) = P^{-1}(k|(k-1)) + \sum_{i=1}^{J} \left(P_i^{-1}(k|k) - P_i^{-1}(k|(k-1)k)\right)$$

$$\begin{aligned} P^{-1}(k|k)\hat{x}(kT|k) &= P^{-1}(k|(k-1))\hat{x}(kT|(k-1)) \\ &+ \sum_{i=1}^{J} \left[P_i^{-1}(k|k)\hat{x}_i(kT|k) - P_i^{-1}(k|(k-1)k)\hat{x}_i(kT|(k-1))\right] \end{aligned}$$

Hashemipour et al, 1988

Combining Estimates in a Sensor Network using Consensus Algorithms

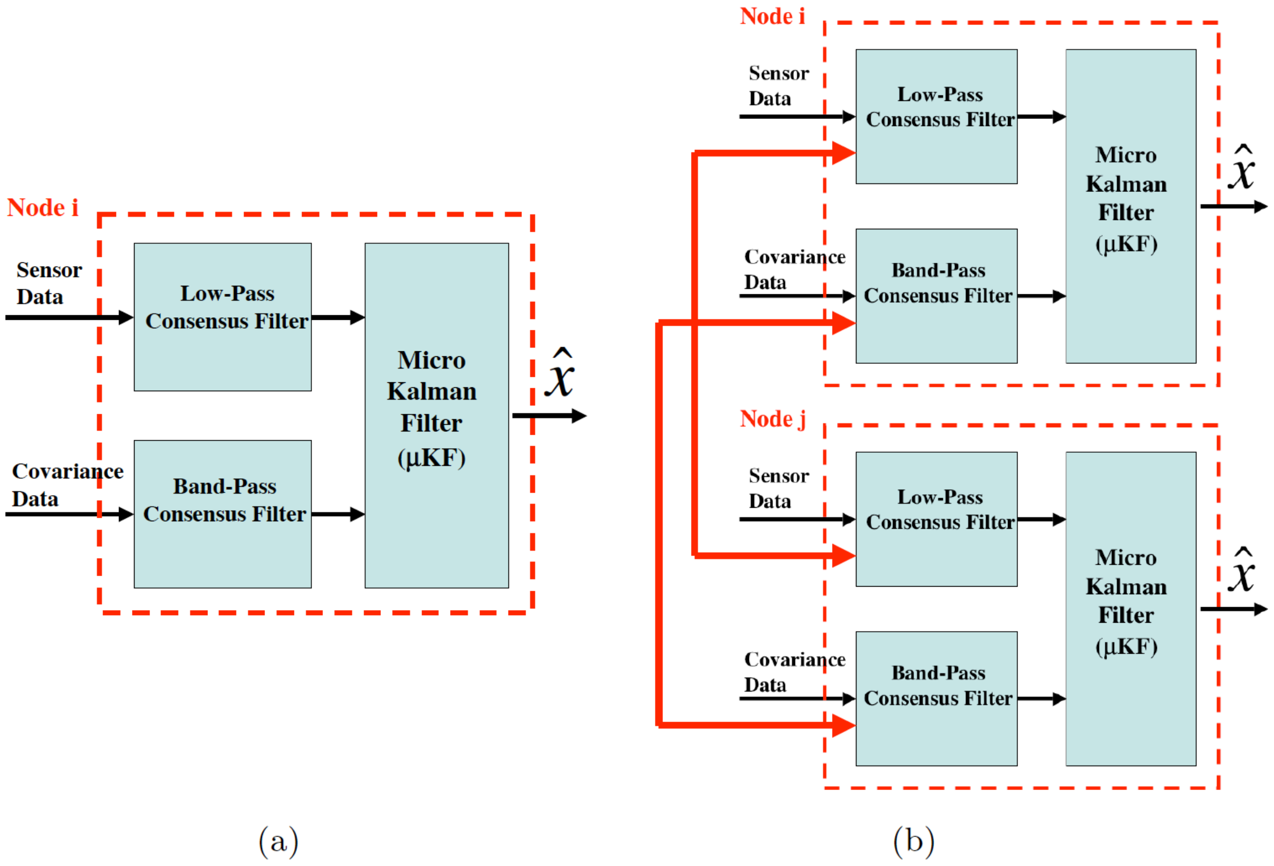

Fig. 2. Node and network architecture for distributed Kalman filtering: (a) architecture of consensus filters and μKF of a node and (b) communication patterns between low-pass/band-pass consensus filters of neighboring nodes.

Olfati-Saber, 2005

FORUM

HISTORY: The Use of the Kalman Filter for Human Motion Tracking in Virtual Reality

Gregory F. Welch
The University of North Carolina at Chapel Hill
Department of Computer Science

"The first published account of the use of a Kalman filter in the context of VR appears to be Rebo's master's thesis (Rebo, 1988)."

"Information and associated databases will be organized by physical location and time, allowing users to both store and retrieve past, present, and future information in the context of physical locality and direction of gaze. The Kalman filter will undoubtedly play a role in this vision, no matter what the underlying sources of signals."

Control theory (Kalman Filtering) is heavily based on mathematical models of the dynamics and observation processes

Newer machine learning avoid such models and rely on data and learning algorithms

How can we combine model-based approaches with machine learning approaches?

Recent Example

2016 IEEE International Symposium on Mixed and Augmented Reality

Learning to Fuse: A Deep Learning Approach to Visual-Inertial Camera Pose Estimation

Jason R. Rambach*
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IEE S.A., Contern, Luxembourg.

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TU Kaiserslautern, Germany.

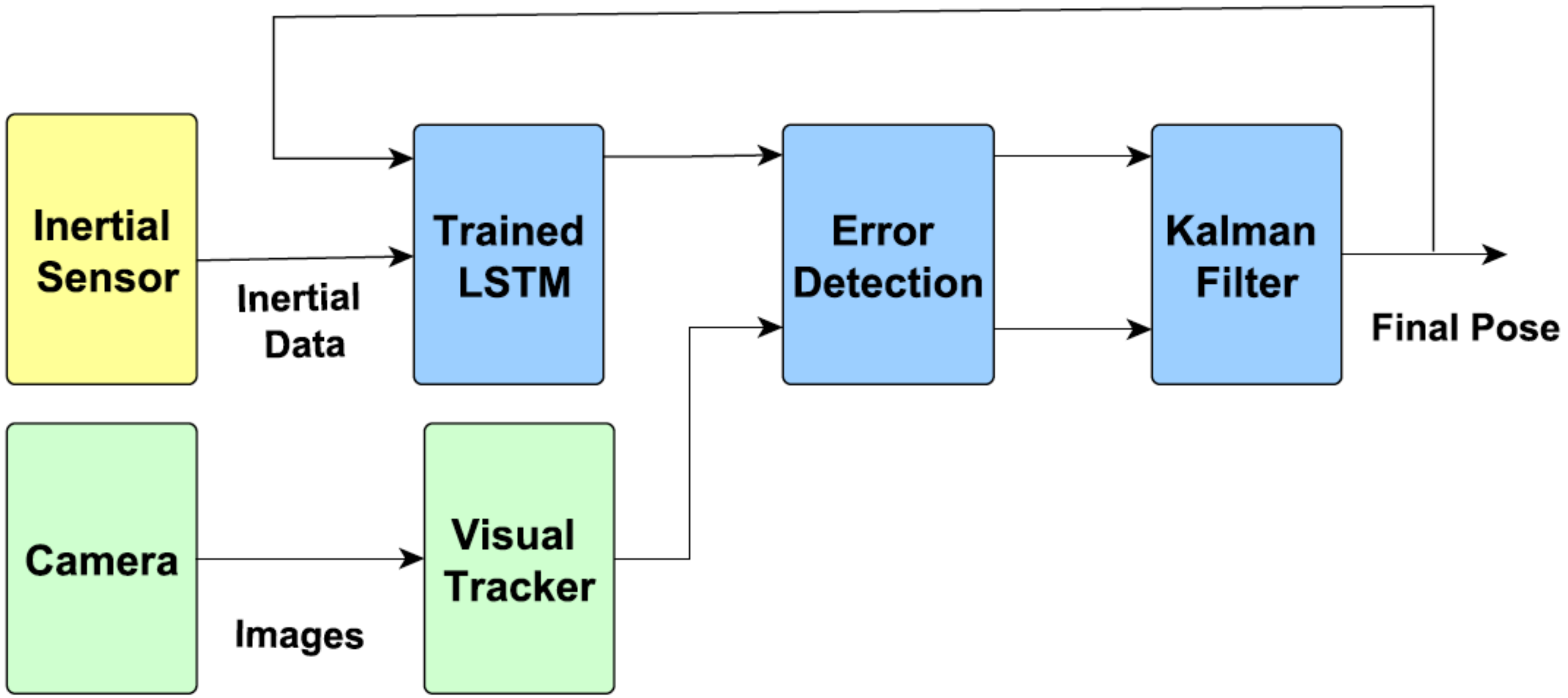

Figure 1: Proposed Fusion System Architecture.

Rambach et al, 2016

Results

Table 2: Tracking accuracy comparison between a pure visual approach and our proposed visual-inertial tracking system. Overlap corresponds to the average overlap between a quadrangle drawn around the $2D$ tracking target (poster) and a quadrangle drawn based on the ground truth (Figure 4). Failed Frames correpsonds to the number of frames where the system could not provide a pose estimate at all.

	Overlap %	**Failed Frames #**
sequence 1(slow) visual	86.3%	249/3389 7.3%
sequence 1(slow) fusion	90.8%	0/3389 0%
sequence 2(fast) visual	77.1%	487/3106 15.7%
sequence 2(fast) fusion	85.6%	0/3106 0%

Rambach et al, 2016

Eye Tracking Issues

- Eye movements
  - Fixations
  - Saccades
  - Dynamic Stimuli: Smooth Pursuits
- Pupil detection
- Tracker calibration
- Slippage or calibration drift

Santini et al 2019 Symposium on Eye Tracking Research and Applications

Physiological measurement

Analysis of eye tracking movements using innovations generated by a Kalman filter

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[2] Service Physiologie Environmentale, Institut National de Recherche et de Sécurité, Vandoeuvre, France

Med. & Biol. Eng. & Comput., 1991, 29, 63–69

Use of KF in eye tracking goes back to Sauter et al, 1991

EYE MOVEMENT ANALYSIS & PREDICTION WITH THE KALMAN FILTER

Thomas Grindinger

07 12, 2006

Advisor: Dr. Andrew Duchowski

Kalman Filtering in the Design of Eye-Gaze-Guided Computer Interfaces

Oleg V. Komogortsev and Javed I. Khan

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A Few Publications in the Control Theory Literature

Proceedings of the American Control Conference
Anchorage, AK May 8-10, 2002

A Non-intrusive Kalman Filter-Based Tracker for Pursuit Eye Movement*

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Table 1: Experimental Results

	Noise Variance	Noise-to-Signal %	Estimation Error-to-Signal %	Error Reduction
Experiment 1	1	7.381%	1.619%	78.061%
Experiment 2	4	14.377%	2.637%	81.658%
Experiment 3	1	12.294%	3.22%	73.792%
Experiment 4	4	25.491%	5.439%	78.6631%

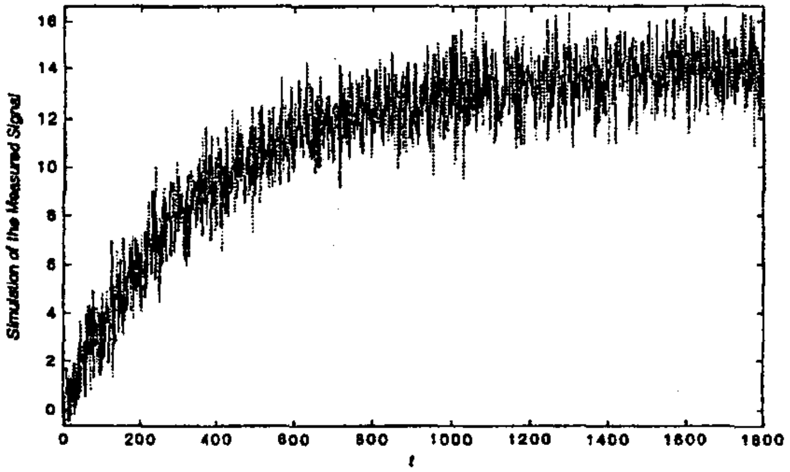

Figure 4.a: Experiment 1: Test Sequence Simulating Measured Eye Position

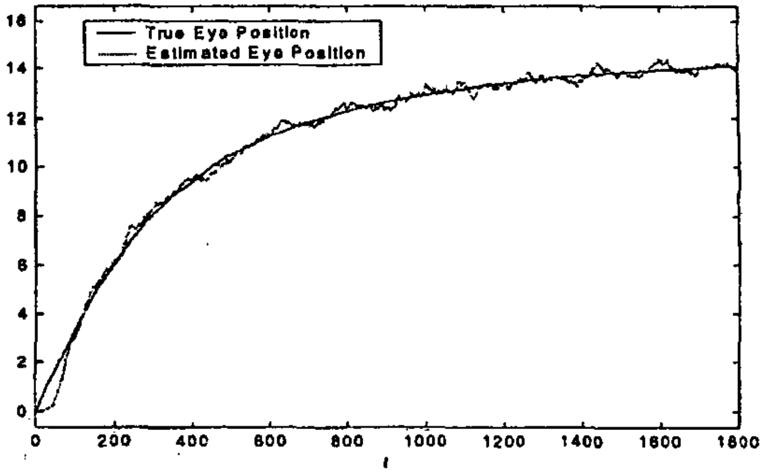

Figure 4.b: Experiment 1: Filtering Results Superimposed on True Eye Position

Control Theory has Potential to Provide Useful Tools

“major remaining challenge hindering a wider adoption of ubiquitous eye-tracking seems to be device slippage.”

Get a Grip: Slippage-Robust and Glint-Free Gaze Estimation for Real-Time Pervasive Head-Mounted Eye Tracking

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ETRA ’19, June 25–28, 2019, Denver , CO, USA

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 44, NO. 3, MARCH 1999 509

A Class of Nonlinear Filtering Problems Arising from Drifting Sensor Gains

Tyrone L. Vincent, *Member, IEEE*, and Pramod P. Khargonekar, *Fellow, IEEE*

Main Idea: *Incorporate sensor drift via a state variable in the system dynamics model and use variants of KF for estimation*

Conclusions

- Rich body of literature on Kalman filtering and myriad extensions
- Rich boy of literature on multirate, multisensory fusion leveraging KF
- Combination with newer ML techniques to leverage their strengths
- Potential for application to eye and gaze tracking problems

Thank you!

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http://faculty.sites.uci.edu/khargonekar/