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THE EFFECTS OF PROBLEM REPRESENTATION ON THE SURE-THING AND SUBSTITUTION PRINCIPLES*

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This paper reports an experimental investigation of the effects of three forms of problem representation on compliance with the Sure-Thing and Substitution Principles. The most common form of representation, written problem statements, was compared with two visual representations: decision matrices with each column proportional in size to the probability of the corresponding event and tubes containing 100 labeled balls. The proportional matrices led to fewer violations of both principles. Moreover, when subjects were trained to construct proportional matrices from written problem statements, they exhibited even fewer violations. (UTILITY/PREFERENCE; DECISION ANALYSIS; PROBLEM REPRESENTATION)

1. Introduction

This paper reports an experimental investigation of the effects of three forms of problem representation on compliance with the Sure-Thing and Substitution Principles. In addition, the effects of training subjects to transform a written problem statement into a visual representation are described.

Von Neumann and Morgenstern's (1947) expected utility theory provides a normative model for decision making under risk. A number of axiom sets have been proposed as alternative bases of expected utility. The Sure-Thing Principle serves as an axiom in Savage (1954) and the Substitution Principle is an axiom in Marschak (1950). Experimental studies of conformance with expected utility theory have relied heavily on investigations of conformance with these two principles. Subjects have customarily been presented with choice problems formatted only in written problem statements. In this paper, two visual problem representations are compared with written problem statements. Investigations of alternative problem representations, such as the study reported here, should contribute to the development of a theory of the way people frame decision problems. Tversky and Kahneman (1981) contains an insightful discussion of how the framing of decisions can influence choice.

First, the Sure-Thing and Substitution Principles are briefly described in this section. Next, the three forms of problem representation (written statements, proportional matrices, and balls in tubes) are presented in §2. §3 contains a description of the investigation of the effects of problem representation. The investigation of the effects of training subjects to construct proportional matrices is reported in §4. §5 contains a summary and discussion.

The Sure-Thing Principle

The principle that *choices* between alternatives should not depend on an event for which each alternative has the same "sure-thing" outcome was termed the Sure-Thing Principle by Savage (1954, Postulate 2). The classical pair of problems testing adher-

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ence to the Sure-Thing Principle has become known as the Allais (1953) Paradox:

Original Problem: Choose one of the following:
Option A: 10% chance of receiving 500 million (old) French francs
1% chance of receiving nothing
89% chance of receiving 100 million francs
Option B: Receive 100 million francs for sure

Transformed Problem: Choose one of the following:
Option A′: 10% chance of receiving 500 million francs
90% chance of receiving nothing
Option B′: 11% chance of receiving 100 million francs
89% chance of receiving nothing

These problems can be represented in decision matrices with a row for each option and a column, labeled with the appropriate probability, corresponding to each probabilistic event.

		Probabilities			
		0.89	0.10	0.01	
Original Problem	A	100M	500M	0	
	B	100M	100M	100M	*
Transformed Problem	A′	0	500M	0	*
	B′	0	100M	100M	

The most common choice in the original problem is B (as indicated by the asterisk), guaranteeing the receipt of 100 million francs. Notice that there is an 89% chance that the outcome will be 100 million francs no matter whether alternative A or B is chosen. The transformed problem is constructed by changing this "sure-thing" payoff amount from 100 million to 0 francs. The Sure-Thing Principle states that this transformation should not alter the order of preference. Thus, if a person prefers B in the original problem, B′ should be preferred in the transformed problem. However, most people prefer A′, perhaps because they prefer the chance of the very high payoff of 500 million francs to a just slightly better chance of a 100 million franc payoff.

MacCrimmon (1965), Slovic and Tversky (1974), Kahneman and Tversky (1979), and MacCrimmon and Larsson (1979) have shown that when subjects are presented with written versions of the Allais Paradox, their choices often violate the Sure-Thing Principle. One previous study investigated the effects of alternative visual problem representations on conformance with the Sure-Thing Principle. Moskowitz (1974) presented subjects with decision trees, matrices, and written statements on three Allais Paradox-type tests, and found that decision trees led to more violations than matrices or written statements.

The Substitution Principle

Kahneman and Tversky (1979) presented the pair of choices below to illustrate violations of the Substitution Principle.

Original Problem: Choose one of the following:

Option A: 80% chance of $4,000
20% chance of $0

Option B: $3,000 for sure*

Transformed Problem: Choose one of the following:

Option A′: 20% chance of $4,000*
80% chance of $0

Option B′: 25% chance of $3,000
75% chance of $0

Observe that alternative A′ is just the risky alternative [$0,0.75; A, 0.25], in which there is a 75% chance of getting nothing and a 25% chance of getting alternative A. Likewise, B′ is [$0,0.75; B, 0.25]. As Kahneman and Tversky (1979) showed, many people prefer the certain alternative B over A, as indicated by the asterisk. However, the Substitution Principle then requires a preference of B′ over A′ in the transformed problem. *The principle states that if alternative B is preferred to A then the compound alternative* $[C, p; B, 1-p]$ *should also be preferred to* $[C, p; A, 1-p]$, where C is some common outcome (see Marschak 1950, Axiom 4). In violation of the principle, people often choose alternative A′ over alternative B′.

Although Allais (1953) presented his test of the Substitution Principle 30 years ago, tests of this principle have received much less empirical attention than the Allais Paradox (see MacCrimmon and Larsson 1979 and Kahneman and Tversky 1979). Moreover, the effects of different problem formulations on conformance with the Substitution Principle have not been reported. Conformance with other utility principles was examined in Keller (1982) and MacCrimmon and Larsson (1979). Schoemaker (1982) provides a comprehensive review of research on the expected utility model.

2. Forms of Problem Representation

Three forms of problem representation were examined in this study. The first was a written problem statement such as:

I: 100% chance of $3000 versus J: 75% chance of $3000
5% chance of $0
20% chance of $4000

The second form was a picture of balls in tubes. Here problems were represented as a choice between two tubes, each containing 100 labeled marbles as in Figure 1A. Notice that each tube is portrayed as partly sunk into the ground, with the below-ground portion representing the "sure-thing" payoff of $3000.

The third form was a proportional decision matrix where the width of a column is proportional to the probability of the corresponding event as illustrated in Figure 1B.

Tubes and proportional matrices share some important features. In both representations, alternatives were visually partitioned so that identical "sure-thing" outcomes were lined up opposite each other and not colored. This might induce a *cancellation operation* (Kahneman and Tversky 1979) leading a person to focus attention on distinguishing features of events, thereby reducing violations of the Sure-Thing Principle by deemphasizing the amount of the sure-thing payoff. Also, the *certainty effect* (Kahneman and Tversky 1979) could be partially counteracted since a certain alternative would be partitioned and only partly colored, making it appear less certain. Another feature of these representations is that the events are portrayed in proportion to their probabilities. This should reduce Substitution Principle violations by emphasiz-

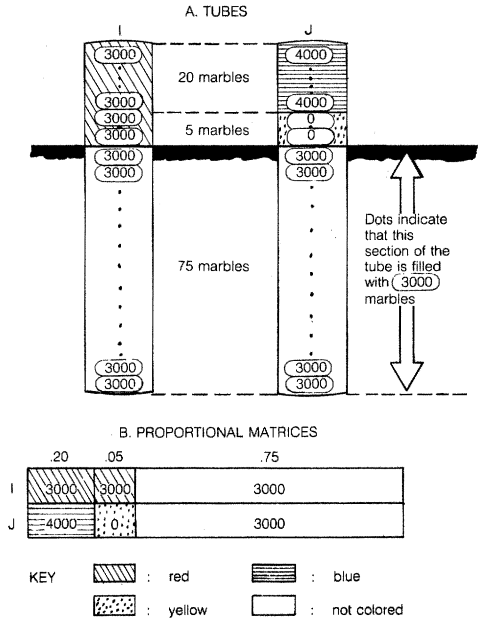

FIGURE 1. Forms of Problem Representation.

ing the constant probability ratio in the original and transformed problems. Indeed, this aspect is highlighted by the noncoloring of the identical outcomes, and coloring of all other outcomes, as illustrated in Figure 1.

3. Effects of Problem Representation

Two separate experimental investigations of utility violations were conducted. The first examined the effects of problem representation and is reported in this section. The investigation of the effects of training in problem structuring is reported in the next section.

A set of nine choice problems was designed to provide tests of the Sure-Thing and Substitution Principles. The problems concerned choices between different kinds of automobiles. For example, one problem was:

HP: 60% chance of Porsche versus LP: 80% chance of new convertible Volkswagen
40% chance of nothing 20% chance of nothing

This sample problem can be represented in a proportional matrix as:

	0.20	0.60	0.20
HP	Nothing: 0	Porsche: PO	Nothing: 0
LP	Nothing: 0	New Convertible VW: NVW	NVW

The general form of this problem is:

	p	$0.75(1-p)$	$0.25(1-p)$
HP	Common Outcome: C	Porsche: PO	Nothing: 0
LP	Common Outcome: C	New Convertible VW: NVW	NVW

TABLE 1

Choice Problems

Problem Label: Common Outcome C / Prob. p of Common Outcome	Choice Problems: Higher-payoff alternative								Lower-payoff alternative			
	C,	p;	Porsche (PO),	Prob. of Porsche;	Nothing (0),	Prob. of Nothing	vs.	C,	p;	New conv. VW (NVW),	Prob. of NVW
0/0.96	[0,	0.96;	PO,	0.03;	0,	0.01]	vs.	[0,	0.96;	NVW,	0.04]
RVW/0.96	[RVW,	0.96;	PO,	0.03;	0,	0.01]	vs.	[RVW,	0.96;	NVW,	0.04]
NVW/0.96	[NVW,	0.96;	PO,	0.03;	0,	0.01]	vs.	[NVW,	0.96;	NVW,	0.04]
0/0.20	[0,	0.20;	PO,	0.60;	0,	0.20]	vs.	[0,	0.20;	NVW,	0.80]
RVW/0.20	[RVW,	0.20;	PO,	0.60;	0,	0.20]	vs.	[RVW,	0.20;	NVW,	0.80]
Ø/0	[Ø,	0;	PO,	0.75;	0,	0.25]	vs.	[Ø,	0;	NVW,	1.00]
0/0.66	[0,	0.66;	PO,	0.33;	0,	0.01]	vs.	[0,	0.66;	NVW,	0.34]
RVW/0.66	[RVW,	0.66;	PO,	0.33;	0,	0.01]	vs.	[RVW,	0.66;	NVW,	0.34]
NVW/0.66	[NVW,	0.66;	PO,	0.33;	0,	0.01]	vs.	[NVW,	0.66;	NVW,	0.34]

KEY: PO: Porsche
NVW: new convertible Volkswagen
RVW: regular 1965 Volkswagen
0: nothing
Ø: no common outcome, i.e., probability of common outcome is zero

The higher-payoff alternative, HP, is the one with a chance of getting a Porsche; the lower-payoff alternative, LP, has a $1 - p$ chance of a "lower-payoff" new convertible Volkswagen. Each problem can be characterized by specifying the probability p of receiving the common outcome C. The sample problem is labeled 0/0.20 since the "sure-thing" payoff C is "nothing" and the probability of this outcome is 0.20. In the nine choice problems, the "sure-thing" payoff C was either nothing, a 1965 regular Volkswagen, or a new convertible VW. The complete set of problems is shown in Table 1.

Undergraduate students enrolled in different sections of lower division UCLA problem solving classes served as subjects. Subjects were not paid, though extra credit was earned in some classes. The nine problems were presented with other choice problems in a questionnaire administered during class sessions. The subjects were divided into three groups, by class sections (thus due to differing enrollments, group sizes varied). One group of 30 subjects received written problem statements. A group of 43 subjects received problems formatted in the tubes representation. Another group of 42 subjects received proportional matrix representations. When the visual (tube or matrix) forms of problem representation were used, the questionnaire also contained a written version of each choice problem. The complete questionnaires are contained in Keller (1983).

Each subject received the same written instructions. Subjects were instructed to work entirely on their own. They were told to indicate a preference for each problem; in the unlikely event that they were indifferent between alternatives, they could choose both options. The instructions included an operational interpretation of probability, namely drawing a ball at random from a hat full of 100 colored balls. Finally, subjects were instructed to consider each problem separately, and not to assume their assets were augmented upon completion of each choice problem. Subjects also received detailed written explanations of a sample problem formatted in the representation used in their questionnaire.

Results

Table 2 summarizes the choices on the nine problems by the three groups. For example, 80.0% of the subjects who received only written problem statements chose the higher-payoff option in problem 0/0.96 as listed in the first row of the table. (Each subject's choices are in Keller 1983.) Over all nine problems, the mean percentage of choices of the higher-payoff option was 57.4% for those receiving only written problem statements. In contrast, those receiving the tubes representation had a mean of 63.3% and those receiving proportional matrices had a mean of 67.5%. There is a significant difference in the mean percentage of higher-payoff choices among these three groups ($\chi^2 = 6.85$, 2 d.f., $\alpha < 0.05$). Thus the alternative forms of problem representation led to different patterns of choices. These differences are discussed in relation to utility principle violations in the next paragraph and in the ensuing analysis.

The first six rows in Table 2 correspond to problems in which the probability of a Porsche is equal to 0.75 times the probability of a new convertible Volkswagen, while the last three rows correspond to problems with the probability of a Porsche equal to 0.97 times the probability of a new VW. The first six problems differ only by the common outcome C and the probability of the common outcome. Conformance with the Sure-Thing Principle requires the same choice (always choose the higher-payoff option, always choose the lower-payoff option, or always be indifferent) on all problems varying only by the common outcome C. Likewise, conformance with the Substitution Principle requires the same choice on all problems varying only by the probability of the common outcome. Thus, conformance with both principles requires the same choice on the first six problems. Combining each subject's pattern of choices

TABLE 2

Choices on Problems by Those Receiving Written Statements, Tubes, or Matrices

Problem Label: Common Outcome C / Probability p of Common Outcome	Written $N = 30$ % chose Higher-Payoff Option	Tubes $N = 43$ % chose Higher-Payoff Option	Proportional Matrices $N = 42$ % chose Higher-Payoff Option
0/0.96	80.0	79.1	88.1
RVW/0.96	73.3	83.7	73.8
NVW/0.96	63.3	83.7	66.7
0/0.20	23.3	23.3	42.9
RVW/0.20	26.7	32.6	54.8
Ø/0	13.3	20.9	35.7
First 6 problems have probability of Porsche = 0.75 × probability of new VW	MEAN = 46.7 Standard deviation = 26.3	MEAN = 53.9 Standard deviation = 28.5	MEAN = 60.3 Standard deviation = 18.0
0/0.66	93.3	88.4	92.9
RVW/0.66	90.0	81.4	83.3
NVW/0.66	53.3	76.7	69.0
Last 3 problems have probability of Porsche = 0.97 × prob. of new VW	MEAN = 78.9 Standard deviation = 18.1	MEAN = 82.2 Standard deviation = 4.8	MEAN = 81.7 Standard deviation = 9.8
All nine problems	MEAN = 57.4	MEAN = 63.3	MEAN = 67.5*

*Reject hypothesis of independence of mean % choosing HP option from type of problem representation, $\chi^2 = 6.85$, 2 d.f., $\alpha < 0.05$.

on the six problems, we can look at the aggregate results for indications of differences between the three groups. If every subject were to make the same choice on each of the first six problems in conformity with the utility principles, then the percentage choosing the HP option should be the same for each problem. Thus, for example, among those receiving written problem statements, every entry in the first column would be the same percentage X%, the mean would be X%, and the standard deviation would be 0. However, the percentage of the written statement group choosing the HP option actually varied from 80.0% to 13.3% with a standard deviation of 26.3 and mean of 46.7%. Among those receiving the tubes representation, the percentage choosing the HP option varied from 83.7% to 20.9% with a standard deviation of 28.5 and mean of 53.9%. There was a noticeably smaller standard deviation of 18.0 and higher mean of 60.3% among those receiving the proportional matrices. This lower variability in the percentage choosing the higher-payoff option suggests the potential of greater conformity with the utility principles when the proportional matrix representation is used.

Analysis

The effects of the three types of problem representation on violations of the two utility principles were examined by checking whether the subjects' choices on pairs of problems were inconsistent with the expected utility principles. Six pairs of problems provided tests of the Sure-Thing Principle; each pair of problems differed only in the amount of the "sure-thing" payoff C. Six other pairs of problems tested the Substitution Principle; each pair of problems differed only in the probability of the common "sure-thing" outcome. Aggregating the results of all twelve utility principle tests, the group receiving the written problem representations exhibited the highest degree of violations (40.3% of all pairs of choices were violations). The group receiving the tubes

representation displayed 33.4% violations, and those receiving proportional matrix representations exhibited the least violations (26.2%). The results for the two utility principles are presented separately below.

Sure-Thing Principle Effects

The effects of tube, proportional matrix, and written problem representations were evaluated on six Sure-Thing Principle tests. For three of the tests, the value of the probability of the "sure-thing" outcome was 0.66. Recall that the Sure-Thing Principle is tested by pairs of problems varying only by the "sure-thing" payoff C; the test can be generally represented as a pair of problems of the form C/p and C'/p. The first three tests are labeled: RVW/0.66-NVW/0.66, 0/0.66-NVW/0.66, and 0/0.66-RVW/0.66. The remaining three tests (with the value of $p = 0.96$), are labeled: RVW/0.96-NVW/0.96, 0/0.96-NVW/0.96, and 0/0.96-RVW/0.96.

For each of the six Sure-Thing Principle tests, Table 3 displays the percentage of subjects whose pair of choices violated the principle. The table contains the results for the three groups of subjects who received tube, matrix, or written problem representations. For example, 43.3% of the 30 subjects (i.e., 13 subjects) who received written statements made a pair of choices violating the Sure-Thing Principle on the RVW/0.66-NVW/0.66 test. In contrast, on the same test, 18.6% of the subjects receiving the tubes representation violated the principle, while 14.3% of those receiving the matrices did. The group receiving written statements had the highest aggregate violation percentage (on all six tests) of 32.8%. Noticeably less aggregate violation was observed for those receiving the tubes representation (21.7%). Even less violation was observed among those receiving matrices; in the aggregate these subjects violated the principle in only 19.1% of the choice pairs.

An in-depth examination of the results shown in Table 3 provides greater insight. Within each set of three tests, the test listed in the middle row has the maximum violation, as was expected. In all cases, the maximum violation percentage occurred when there was the widest disparity between the values of the "sure-thing" payoff parameters (note that there were two cases in which there was a tie for the maximum). This pattern occurred for all three problem representation groups, but was most pronounced for those receiving written statements.

Next, the results for the group receiving written problem statements will be examined. Note that there were substantially fewer violations of the Sure-Thing Principle on the two tests in which C was "nothing" and C′ was RVW, "a 1965 Regular VW," compared to the other tests. On one test, 0/0.66-RVW/0.66, 10% of the pairs of

TABLE 3

Percentage of Subjects Violating Sure-Thing Principle

Sure-Thing Principle Tests Original Problem—Transformed Problem C/p—C'/p		Written Statements $N = 30$ subjects	Tubes $N = 43$	Proportional Matrices $N = 42$
Set 1 $p = 0.66$	RVW/0.66—NVW/0.66	43.3	18.6	14.3
	0/0.66—NVW/0.66	46.7	20.9	23.8
	0/0.66—RVW/0.66	10.0	20.9	14.3
Set 2 $p = 0.96$	RVW/0.96—NVW/0.96	40.0	23.3	16.7
	0/0.96—NVW/0.96	40.0	27.9	26.2
	0/0.96—RVW/0.96	16.7	18.6	19.0
Aggregate Violation Percentage on all 6 tests		32.8	21.7	19.1
Standard Deviation of 6 violation percentages		14.1	3.2	4.6

responses by the written statement group violated the principle. On the other test, 0/0.96-RVW/0.96, 16.7% of the pairs of responses were violations. In contrast, there were violations of 40.0% to 46.7% for the other four tests. The fewer violations might have resulted from subjects believing that the difference between receiving nothing and a 1965 regular VW was relatively small in terms of strength-of-preference among various outcomes. Since the original and the transformed problems in a Sure-Thing Principle test differ only by the "sure-thing" payoff amounts, it is possible the pairs of problems in the two tests were perceived as being similar. For example, in the 0/0.96 "original" problem a subject might choose the higher-payoff option with a 0.03 chance of receiving a Porsche and a 0.97 chance of nothing. In conformance with the principle, the subject might also choose the higher-payoff alternative in the RVW/0.96 "transformed" problem, choosing a 0.03 chance of a Porsche, a 0.01 chance of nothing, and a 0.96 chance of a 1965 regular VW. In the Sure-Thing Principle tests which resulted in higher degrees of violation, one "sure-thing" payoff was a brand-new convertible VW and the other was either a 1965 VW or nothing. It is possible that many subjects believed there was a substantial difference between a new VW and nothing; and that this difference was larger than, but fairly close to, the difference between a new VW and a 1965 VW.

It is important to note that in the two tests in which C was "nothing" and C′ was a "1965 Regular VW," written problem statements led to less violation than did matrices or tubes, though violations never exceeded 21%. It appears that subjects' choice processes applied to written problem statements may encourage conformity when the original and transformed problems are perceived to be similar. In the other four tests, when the "sure-thing" payoffs were wider apart, the written statement group exhibited substantially more Sure-Thing and Substitution Principle violations than the matrix or tube groups. In this case, it is possible the choice process induced by the matrix or tube representation differs from that used when presented with written statements. It can be seen from the table that those receiving written statements displayed much greater variability in the degree of principle violation than those with matrices or tubes, as measured by the standard deviations of the aggregate violation percentages.

Substitution Principle Effects

The effects of tube, proportional matrix, and written problem representations were evaluated on six Substitution Principle tests. The Substitution Principle is tested by pairs of problems varying only by the probability p of receiving the "sure-thing" outcome C. The first three tests are labeled: RVW/0.96-RVW/0.20, Ø/0-RVW/0.96, and Ø/0-RVW/0.20. Note that in problems with $p = 0$, there is no "sure-thing" payoff C, so in the problem labeling the payoff is represented by the empty set symbol Ø. The other three Substitution tests are labeled: 0/0.96-0/0.20, Ø/0-0/0.96, and Ø/0-0/0.20.

The group receiving written problem statements had the highest aggregate violation percentage (on all six tests) of 47.8%. The aggregate violation percentage for those receiving tubes was 45.0%, close to that for the written statements. Substantially less violation was observed among those receiving proportional matrices, in the aggregate these subjects violated the Principle in only 34.1% of the choice pairs.

A deeper examination of these results provides added insight. The problems in a Substitution Principle test vary only by the values of p and p', the probability of the common outcome. It is expected that pairs of problems in which there is a large increase in the probability from p to p' will lead to more violations of the Substitution Principle. First, we consider a person's likely choice pattern on a Substitution Principle test as the probability of the common outcome increases from p to p'. Next, the results of the six Substitution Principle tests are shown to exhibit the expected pattern.

Suppose an individual prefers the lower-payoff alternative, Option LP, in the

following problem which has no common outcome, so $p = 0$:

		0.75	0.25	
Original Problem	HP	Porsche	0	
	LP	NVW	NVW	*

We construct the transformed problem by increasing the probability of the common outcome to p':

		p'	$0.75(1 - p')$	$0.25(1 - p')$	
Transformed Problem	HP′	C = 0	Porsche	0	*
	LP′	C = 0	NVW	NVW	

A person conforming with the Substitution Principle would either always choose the higher-payoff option or always choose the lower-payoff option. Here we describe the expected pattern of choices among those who violate the principle. When there is a minor increase in probability from p to p', the person probably perceives little difference between the original and transformed problems and still chooses option LP′ in the transformed problem, in conformance with the Substitution Principle. At some point, as p' gets bigger, the higher-payoff option HP′ in the transformed problem may start to appear equally attractive. At this point, the added value of a Porsche exactly offsets the accompanying $0.25(1 - p')$ lower chance of receiving a car in comparison with the lower-payoff option. The choice of option LP over HP in the original problem, and indifference between Options LP′ and HP′ in the transformed problem jointly violate the Substitution Principle. For even larger values of p', it is expected that the person would choose option HP′ over option LP′, also violating the principle. Note that when $p = 0$, Option LP in the original problem is a certain alternative. In such a case, the violation pattern described above illustrates the *certainty effect* (i.e., overweighting of certain outcomes) discussed in Kahneman and Tversky (1979).

Table 4 contains a display of the Substitution Principle test results. Within each set of three tests, the test in the middle row has the greatest difference between p and p'. Based on the above reasoning, it was expected that the greatest violation would be observed in these tests; as seen in the table this pattern was observed, regardless of the form of problem representation.

TABLE 4

Percentage of Subjects Violating Substitution Principle

Substitution Principle Tests	Original Problem—Transformed Problem C/p—C/p'	Written Statements N = 30 subjects	Tubes N = 43	Proportional Matrices N = 42
Set 1 C = RVW	RVW/0.96—RVW/0.20	56.7	60.5	28.6
	Ø/0—RVW/0.96	63.3	67.4	47.6
	Ø/0—RVW/0.20	16.7	11.6	19.0
Set 2 C = 0	0/0.96—0/0.20	66.7	60.5	45.2
	Ø/0—0/0.96	70.0	62.8	52.4
	Ø/0—0/0.20	13.3	7.0	11.9
Aggregate Violation Percentage on all 6 tests		47.8	45.0	34.1
Standard Deviation of % choosing HP		23.5	25.4	15.2

TABLE 5

Choices on Problems by Trained versus Untrained Groups

Problem label	Higher-payoff option vs. Lower-payoff option	Pre-structured Proportional Matrices $N = 32$ % chose HP	Matrix-Structuring Training $N = 15$ % chose HP
\$0/0.75	[\$0.75; \$4000, 0.20; \$0, 0.05] vs. [\$0, 0.75; \$3000, 0.25]	50.0	33.3
Ø/0	[Ø, 0; \$4000, 0.80; \$0, 0.20] vs. [Ø, 0; \$3000, 1.0]	15.6	6.7
\$1000/0.75	[\$1,000, 0.75; \$4000, 0.20; \$0, 0.05] vs. [\$1000, 0.75; \$3000, 0.25]	50.0	33.3
\$3000/0.75	[\$3,000, 0.75; \$4000, 0.20; \$0, 0.05] vs. [\$3000, 0.75; \$3000, 0.25]	46.9	26.7
\$0/0.95	[\$0, 0.95; \$4000, 0.04; \$0, 0.01] vs. [\$0, 0.95; \$3000, 0.05]	81.3	60.0
\$0/0.10	[\$0, 0.10; \$4000, 0.72; \$0, 0.18] vs. [\$0, 0.10; \$3000, 0.90]	18.8	6.7
Aggregate Violation Percentage on All 6 Tests		43.8	27.8*
Standard Deviation of % choosing HP option		21.8	18.1

*Reject hypothesis of independence of mean % choosing HP option from amount of training, $\chi^2 = 6.59$, 1 d.f., $\alpha < 0.025$

Note that there were substantially fewer violations of the Substitution Principle on the two tests in which $p = 0$ and $p' = 0.20$, compared to the other tests. On these tests, the violations ranged from 7.0% to 19.0%. In contrast, on the rest of the tests, there was one violation percentage of 28.6% and the others ranged from 45.2% to 70.0%. In the two tests with the small and close values of $p = 0$ and $p' = 0.20$ those receiving a tubes representation had slightly fewer violations than those with matrices or written statements. In all other cases (when p and p' are farther apart), matrices led to substantially less violation than tubes, and those receiving tubes exhibited less violation than those with written statements. So, in Substitution Principle tests which are likely to lead many subjects to violation when presented with written statements (i.e., those with p and p' quite different), the use of the matrices led to less violation.

4. Effects of Training in Problem Structuring

An investigation of the effects of training in problem structuring is described in this section. This investigation was conducted in much the same way as that reported in §3. A set of six choice problems was designed to provide tests of the Sure-Thing and Substitution Principles. The problems contained monetary payoffs. The general form of these problems is:

	p	$0.8(1-p)$	$0.2(1-p)$
Higher-payoff alternative	C	\$4000	\$0
Lower-payoff alternative	C	\$3000	\$3000

The problems, labeled by their C/p values, are: Ø/0, \$0/0.10, \$0/0.75, \$0/0.95, \$3000/0.75, and \$1000/0.75. Three Sure-Thing and six Substitution Principle tests are formed by comparing pairs of these problems.

The subjects were divided into two groups. One group of 32 subjects received a questionnaire with the above problems formulated in the proportional matrix representation. The other group of 15 subjects received training in matrix-structuring. During the training they were presented with written statements of the automobile-payoff problems and shown how to construct proportional matrices. Later, the trained subjects were given a written version of the questionnaire with the monetary-payoff

problems and requested to construct a proportional matrix for each problem prior to making a choice. The complete questionnaires and detailed results are in Keller (1983).

Results

Table 5 summarizes the choices on the six problems by the two groups. For example, 33.3% of the subjects who received matrix-structuring training chose the higher-payoff option in problem \$0/0.75 as listed in the first row of the table. The mean percentage choosing the HP option among those receiving pre-structured proportional matrices was 43.8%, which is significantly different from the mean of 27.8% among those receiving matrix-structuring training ($\chi^2 = 6.59$, 1 d.f., $\alpha < 0.025$).

Analysis

The effects of matrix-structuring training on the degree of violation of the Sure-Thing and Substitution Principles were examined by comparing the violation percentages of the trained group with the untrained group. Aggregating the results of the three Sure-Thing and six Substitution Principle tests, the trained group exhibited 28.1% violations, which is significantly less than the 40.6% displayed by those receiving pre-structured matrices (Wilcoxon paired comparison test, $\alpha = 0.006$). As can be seen in Table 6, the trained group (second column) showed less violation in all tests except Ø/0-\$0/0.10, when both p and p' are small. On the Sure-Thing Principle tests, there were only 13.3% violations with matrix-structuring training, versus a much higher 33.3% with pre-structured matrices. On the Substitution Principle Tests, there were 35.5% violations with matrix-structuring training, versus a higher 44.3% with pre-structured matrices. Recall from §3 that receipt of pre-structured proportional matrices was found to lead to fewer violations than receipt of the other problem representations

TABLE 6

Percentage of Subjects Violating Principles

Principle Tests	Pre-structured Proportional Matrices $N = 32$	Matrix-Structuring Training $N = 15$
Sure-Thing Principle		
C/p—C'/p		
\$1000/0.75—\$3000/0.75	15.6	6.7
\$0/0.75—\$3000/0.75	46.9	20.0
\$0/0.75—\$1000/0.75	37.5	13.3
Aggregate Violation % on 3 Sure-Thing tests	33.3	13.3
Substitution Principle		
C/p—C/p'		
Ø/0—\$0/0.95	65.6	53.3
\$0/0.10—\$0/0.95	75.0	53.3
Ø/0—\$0/0.75	34.4	26.7
\$0/0.10—\$0/0.75	43.8	40.0
\$0/0.75—\$0/0.95	37.5	26.7
Ø/0—\$0/0.10	9.4	13.3
Aggregate Violation % on 6 Substitution tests	44.3	35.5
Aggregate Violation % Over all 9 Principle tests	40.6	28.1*

*Reject hypothesis of no treatment effect by Wilcoxon paired comparison test; ($\alpha = 0.006$).

on the Sure-Thing and Substitution Principle tests. Thus, on these principle tests, matrix-structuring training is likely to lead to even fewer violations than the already reduced number of violations resulting from using pre-structured proportional matrices.

In Table 6, the Substitution Principle Tests are listed in descending order by the difference between p and p'. The expected pattern of Substitution Principle violations (as described in §3) suggests that the violation percentage should be monotonically decreasing as p'-p decreases. In general, this pattern was observed, though there are two cases which do not adhere to the expected trend. For example, among the group receiving pre-structured matrices, the 34.4% violations of the Ø/0-$0/0.75 test are unexpectedly less than the 43.8% violations of the Ø/0.10-$0/0.75 test. Similarly, the 65.6% violations of the Ø/0-$0/0.95 test are less than the 75.0% violations of the $0/0.10-$0/0.95 test. This illustrates an effect contrary to the *certainty effect* described by Kahneman and Tversky (1979). The results would have been consistent with the certainty effect if the tests with $p = 0$ (thus, having a certain alternative) led to even greater violation than those with very low, but not 0, p values. Future investigation of preference patterns when certain alternatives are involved is needed.

5. Summary and Discussion

This paper reports the first experimental attempt to examine the effects of alternative forms of problem representation on compliance with the Substitution Principle. It is the second attempt to examine the Sure-Thing Principle in a similar way. In addition to written statements, two forms of problem representations were considered here which have not been examined previously in this context. The proportional matrix representation was found to lead to fewer violations of both the Sure-Thing and Substitution Principles than the written or tubes representations. Subjects who were trained to restructure written problem statements into proportional matrices exhibited even fewer violations. The reduction in principle violations with proportional matrices might be due to subjects shifting their focus away from "sure-thing" payoff amounts and gaining a better understanding of the relative probabilities of outcomes. Sure-Thing and Substitution Principle violations with different forms of problem representation were examined here on principle tests exhibiting high degrees of violation, like those in Kahneman and Tversky (1979), as well as on tests with less extreme parameters leading to relatively lower degrees of violation. Thus, the *patterns of violation* as displayed in Tables 3, 4 and 6 are the important results, rather than the violation percentages. As can be seen, the violation percentages vary greatly as the problem parameters are changed. On each principle test, the comparison of the different groups' performances in terms of the violation percentages is of most interest. The aggregate violation percentages across a number of tests allow comparison among groups. Note that if a set of tests with different parameters were used, the aggregate violation percentages would probably be different, but the same pattern of violations among the groups would be expected.

This line of research can be extended in a number of ways. First, in this study each subject received only one form of problem representation, so within subject comparisons were not possible. A study could also be done in which each subject used each form of problem representation (cf. Moskowitz 1974). A further study might attempt to elicit the internal problem frame used by each subject. For example, an experiment could be set up to monitor the amount of attention a subject focuses on different facets of a problem. Another experiment might allow a subject to choose from a "menu" of problem frames on a cathode-ray tube graphic display. Thus the subject could "shop around" among the alternative forms of representation for a choice problem prior to

making a choice. It may ultimately be possible to construct a decision-style profile for an individual which would specify utility-violating tendencies.

This work could also be extended by applying the approach to additional principles of utility theory (see Keller 1982, 1985) or other choice models. Other visual forms of problem representation and alternative training procedures could be evaluated. Finally, unpaid student subjects were presented with hypothetical choices in this study. Since this was the first study of its type, this experimental setting seemed prudent. Either real, consequential payoffs, or paid subjects might lead to less "random error" due to increased subject motivation, see Grether and Plott (1979).[1]

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