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Decision Technologies

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Abstract

This chapter covers the use and limitations of decision technologies, beginning with the construction and use of decision trees for decisions whose outcomes depend on probabilistic events. Next, we describe methods for determining probabilities of events and for valuing decision outcomes, which are decision components that are entered onto the decision tree. Then, a method for valuing decision alternatives measured by performance on multiple objectives is covered. Throughout the chapter the prescriptive decision technology is presented, then behavioral biases in using these technologies are briefly discussed.

Keywords: chance nodes, decision nodes, decision tree, expected utility, hierarchy of objectives, multiple objectives, probability, probability assessment, swing weights, utility function, value function

Introduction

For researchers, practitioners, and students interested in judgment and decision making, there can be sizable benefits in better understanding the use and limitations of decision technologies. This chapter covers decision analysis technologies that are widely used in practice (Keefer et al. 2004) and taught in management and engineering schools (see Clemen and Reilly's (2004) textbook).

Just as knowing how your vision deviates from optimal can help it be corrected with glasses or by changing your environment (e.g. with a car's rear view mirror's warning that objects are closer than they appear), comparing the prescriptions of a decision technology with how unaided people make judgments or decisions can be helpful in identifying ways of improving decisions. Many of the chapters in this handbook emanate from comparisons of differences between normatively desired vs. actual judgments and choices that have led to new technologies, new prescriptive theories, and new descriptive theories.

Knowing the steps in a decision technology can suggest places where it may fail or be subject to manipulation. Prior research has examined the assumptions (called axioms) underlying expected utility theory or probability theory to see if they are descriptively accurate ways to describe people's choices in decisions involving uncertainty or their probability judgments. For example, when people were found to not be very good at providing accurate probabilities, a stream of research focused on improving probability assessments. One method is to use a scoring rule to grade a probability assessor's judgments (Johnstone et al. 2011, Bickel 2010). So, understanding a technology and its vulnerabilities can lead to improvements in the technology.

In the next section, we introduce the construction of decision trees. Influence diagrams, which provide another way of portraying and analyzing the information in a decision tree, are not covered here (see Howard and Matheson, 2005). Then, we describe methods for determining probabilities of events and for valuing decision outcomes, which are decision components that are entered onto the decision tree. Valuing decision alternatives measured by performance on multiple objectives is covered next. Throughout the chapter we present the prescriptive decision technologies, and then discuss behavioral biases in using these technologies that have been discovered.

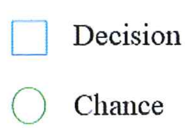
Decision Trees

The purpose of a decision tree is to assist decision makers in the process of mapping out each decision with the possible chance events and potential outcomes using a visual display that supports subsequent analysis of the decision. A primary advantage of a decision tree is its ability to incorporate the uncertainty associated with each decision alternative. Several examples of this tool can be found in recent literature. Lippman and McCardle (2004) analyze a high profile, high-stakes, high risk lawsuit case involving an heir-claimant to the estate of the deceased Larry Hillblom. They analyze the young heir-claimant's decision to proceed with the lawsuit using decision trees with utility functions. Brandao, Dyer, and Hahn (2005) use a decision tree to calculate real-option valuation problems with complex payoffs and uncertainty associated with the changes in the value of a project over time. Bakir (2008) presents a detailed decision tree to evaluate how different security measures could reduce vulnerabilities to terrorism in cargo truck crossings to the United States from Mexico. Different types of industries can benefit from using decision trees in assessing their available options. For example, oil exploration firms may utilize decision trees to decide which area is most likely to yield the highest expected profits, taking into account the uncertainty of how much oil may be extractable in such a location (see the oil wildcatter problem in Raiffa (1968)).

In this section, we present how decision trees could assist decision makers in choosing the best option from those available to them. For simplicity, we will assume here that the decision maker is risk neutral, and thus the person should choose the decision alternative that maximizes expected monetary value (when the outcomes are monetary). The next section covers how to value outcomes with utility functions for decisions involving uncertainty and value functions for decisions under certainty. Interested readers can explore the area in further detail by studying expected utility theory (von Neumann and Morgenstern 1947; Arrow 1971; Friedman and Savage 1948). Expected utility theory states that when individuals are faced with decisions under uncertainty, they should compare the expected utilities of each decision alternative.

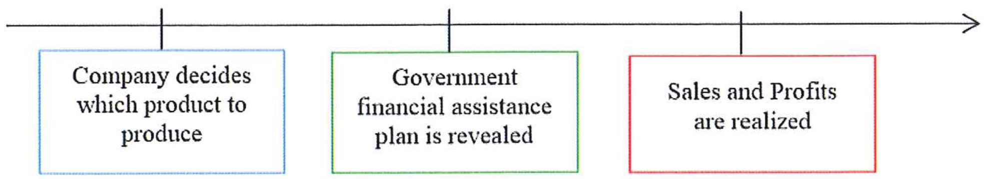
Decisions are challenging in part because the future is uncertain. If the only objective is to maximize wealth and the decisions being made have known monetary outcomes, then it is relatively simple to choose the decision that would yield the highest amount. Unfortunately, we cannot foresee the outcome of future uncertain events that would affect our decisions. For example, let us consider a small company that is faced with making either Product A or Product B. Product B will only sell well if the government provides financial assistance to its citizens that year. Therefore, a decision tree could assist substantially in making this type of decision by mapping out the sequence of events and the consequences of each decision.

Decision trees are drawn from left to right in sequential order to map out the chronological order of how the events unfold. The following symbols are used:



Terminal

The timeline of the company's decision is:



Then the decision of which product to produce is depicted as:

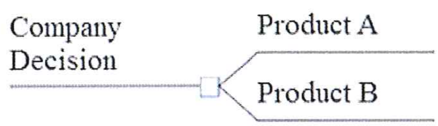


Figure 1: Two Alternatives in Decision Tree

After the company makes the decision of which product to produce, a chance event occurs. This chance event is whether or not the government will provide financial assistance for the calendar year. For this example, let us suppose that the probability of the government giving financial assistance this year is 20%. This number is indicated below the tree branch of the corresponding chance event. Notice that at every chance node, the sum of the all probabilities emanating from that chance node must be equal to 1.

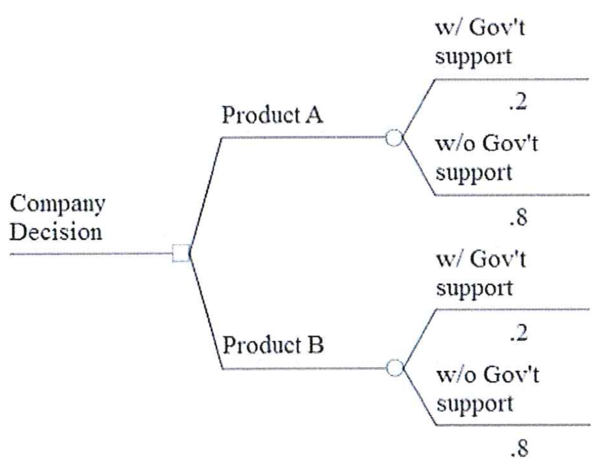


Figure 2: Chance Nodes added to Decision Tree

The company's profits are realized at the end, denoted by the terminal node. Notice that the profitability of each product is dependent on whether the government gives financial support. While both products are affected by the government giving support, Product B is more affected.

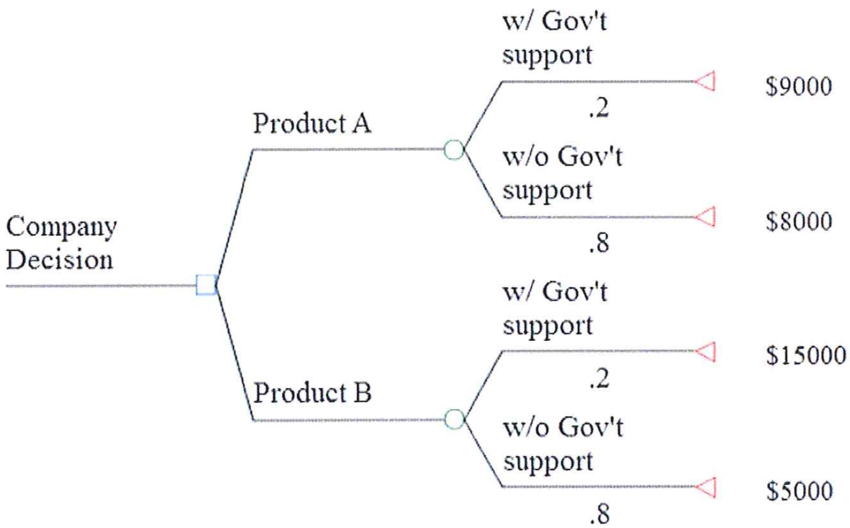


Figure 3: Monetary Outcomes added to Decision Tree

The decision tree is now complete. All of the events are properly drawn out in sequential order with corresponding outcomes. Recall that a primary purpose of drawing a decision tree is to provide a good visual aid to illustrate how the sequences of events unfold, which will assist in the decision making process. To make use of this decision tree, we must calculate the expected monetary value of each outcome.

Let □ represent the monetary gain (or loss) incurred with a probability □ of occurring. The subscript n refers to the total number of possible events occurring, in this case $n = 2$. The formula for computing the expected value of each decision is:

□□ □ □□□□□□ □ □□□□□□ □□□□□ □ □ □□

The expected monetary value is calculated at each chance node using the corresponding terminal outcome value and probability for each tree branch following the chance node (Figure 4). Based on the calculated expected values, producing Product A would yield higher monetary profits for the company, on average. Therefore, given the assumption of risk-neutrality, it would be in the company's best interest to produce Product A. Since the process of calculating the expected value of each decision is done starting from the far right side (terminal node) back to the left side, it is often called "rolling back" the decision tree. Once a decision is made, we "prune" any tree branches that represent alternatives which will not be chosen, as an indication of that alternative being "cut off" from further consideration.

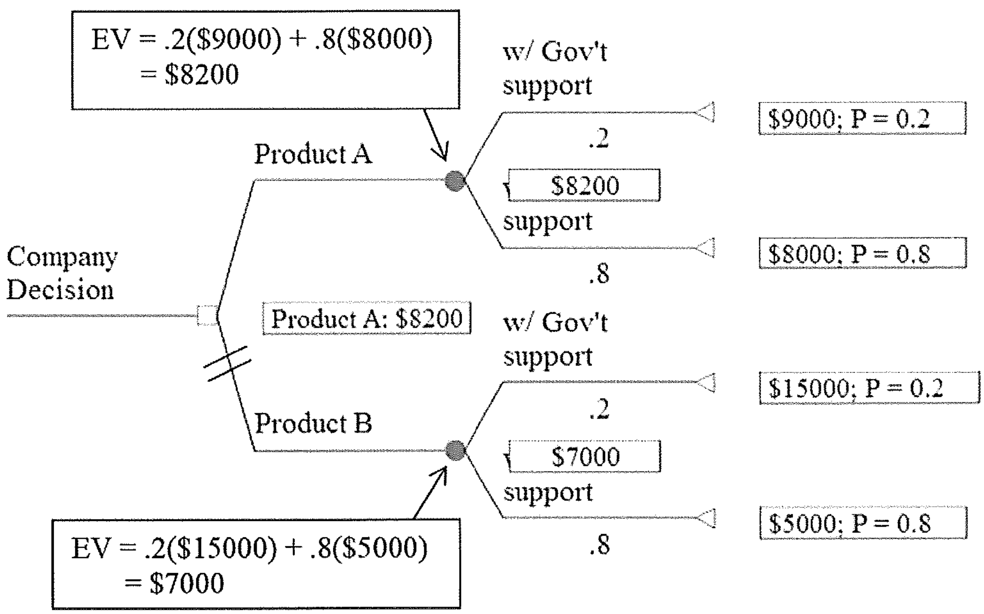


Figure 4: Expected Monetary Value Calculation with Lower Branch Pruned off on Decision Tree

Various software packages (e.g., TreeAge 2011), have made the task of calculating the expected value of each decision in a tree as easy as the touch of a mouse button. See Chapter 10 in this Handbook for more details on decisions under risk.

How to Assess Probabilities

Perhaps the most challenging aspect of decision making under uncertainty is assessing the probability of each uncertain event. Recall the example presented previously of a decision tree involving the decision of which product to select. We assumed that the probability that the government would provide support was 20%. Often, however, the probabilities of uncertain events are not known. Decision makers frequently resort to examining uncertain events based on prior experiences and/or expert opinions, and regularly make systematic errors in assessing probabilities. Since the future is unpredictable, decisions under uncertainty require thorough assessment of the probabilities of possible desirable and undesirable events occurring.

The process of assessing probabilities is often challenging; proper techniques are required to elicit subjective probabilities from individuals. Spetzler and Staël von Holstein (1975) described practical methods for eliciting subjective probabilities that reduce biases and aid in the process of quantifying judgments. For a detailed explanation of how to assess subjective probabilities rigorously, see Morgan and Henrion (1990) and Clemen and Reilly (2004). These methods of eliciting subjective probabilities from experts are widely accepted and applied in many research papers. See Keeney and von Winterfeldt (1991) for an application in nuclear safety and Merkhofer (1987) for a detailed review on the probability encoding process and examples and further insights regarding assessment. See Chapter 16 in this Handbook for more detailed coverage.

A common approach is to ask directly: "What is your belief regarding the probability that [insert event] will occur?" Although this approach is the most straightforward way to assess probabilities, the responses received may be inaccurate due to biases (that will be further discussed later in this section). Another method involves asking the decision maker to construct a bet such that she is indifferent between taking either side of the bet. For example, suppose the probability to be assessed is the probability of winning the next game for the San Diego Chargers professional football team, and the two bets are:

Bet 1: Win \$X if the Chargers win	Bet 2: Lose \$X if the Chargers win
Lose \$Y if the Chargers lose	Win \$Y if the Chargers lose

The decision maker would select the values of X and Y, such that she is indifferent between choosing Bet 1 or Bet 2. Once the numerical values of X and Y are obtained, the subjective probability with which the decision maker believes the Chargers will win can be computed by:

$$P(\text{Chargers win}) = \frac{Y}{X+Y}$$

This method is not without limitations, since it assumes the person is risk neutral, and calculates the value of a bet by computing the expected monetary value. Many people may be uncomfortable with the idea of betting, and may also be risk averse. One possible way to overcome this is by considering only small amounts of money. There are also people who are loss averse, meaning they consider losses to be more significant than equal-sized gains.

A method that avoids direct assessment, loss aversion, and risk aversion is to conduct a thought experiment that involves gains (prizes) which are equivalent between lotteries, and no losses. Set up a lottery such that Prize A is significantly better than Prize B; for example, Prize A can be a brand new 60" television, and Prize B can be a movie ticket. The decision maker is asked to compare two lotteries:

1: Win Prize A if the Chargers win	2: Win Prize A with a specified probability p
Win Prize B if the Chargers lose	Win Prize B with a specified probability $1-p$

Adjustments are made to the value of p until the decision maker is indifferent between choosing lotteries 1 and 2. The value of p that makes the decision maker indifferent between the two lotteries is also the subjective probability with which the decision maker believes the Chargers will win. Finding this value is often challenging, and is not error free. There is a phenomenon known as preference reversal, in which a person's preference between lotteries may change depending on how preferences are assessed. A more detailed discussion on preference reversal can be found later in this section.

People tend to rely on a number of heuristic principles to reduce the complexity of the task of assigning probabilities and judging frequencies. They try to estimate the availability or representativeness of the uncertain event. If they are able to simulate it more easily in their mind, then they are likely to assign higher probabilities to the event. One such principle is representativeness, illustrated by the observation that a large majority of respondents believe a sequence of six coin tosses is more likely to follow the pattern: HTTHTH than: HHHHTH or: HHHTTT. The first sequence appears to be more representative of a typical "random" pattern than the second or third sequence. In actuality, all three of these sequences are equally likely to occur. Another such principle is availability, illustrated by the observation that the majority of people believe there are more words in the English language that start with the letter R than there are words with the letter R being in the third position. This is because people can more easily generate words that start with the letter R, than words that have the letter R in the third position. For more details on these and other heuristics, see Kahneman and Tversky (1972), and Tversky and Kahneman (1973, 1974).

The way people assess probabilities and frequency of events is often affected by a cognitive bias known as anchoring. People who are given an initial value tend to make insufficient adjustments to that value; that is, people tend to be "anchored" by the initial value. Tversky and Kahneman (1974) showed the anchoring effect by providing test subjects with a random number, then asking the subjects to assess the likelihood or frequency of an event. Subjects were first given a random number (which was determined by spinning a wheel in the subject's presence). The subjects were then asked whether the percentage of African countries in the United Nations was higher or lower than the given number, and to adjust their estimate by decreasing or increasing the number. The starting value had a significant effect on the final estimate. Subjects tend to remain close to the initial value; a subject who was given the number 10 percent would, on average, estimate 25 percent, while a subject who was given 65 percent would, on average, estimate 45 percent. For more details on the anchoring bias, see Cohen, Chesnick, and Haran (1972), Slovic (1972), and Bar-Hillel (1973).

The human brain has limited capacity to process information that involves uncertainty, which often leads to poor subjective probability estimation due to poor calibration. This has also been found to happen to experts; see Kahneman, Slovic, and Tversky (1982) and Gilovich, Griffin and Kahneman (2002) for details. Fox and Clemen (2005) studied one such bias in probability assessment that arises from the initial structure of elicitation, where subjects typically assign equal probabilities to all specified events then adjust those probabilities insufficiently to reflect his or her beliefs.

There is a growing interest in finding an effective method to combine probability estimates of individuals (often assumed to be experts). For example, a corporation may be interested in estimating the probability that a new product will increase its company revenue by at least x dollars. They could hire multiple experts to provide a set of estimates. Suppose the number of experts is n, which is greater than or equal to 2. Then $p_i(A)$ is the estimate by individual i of the probability of event A happening, where $i = 1, \ldots, n$. There are a number of aggregating methods, including Morris' approach (1974, 1977) applying Bayes' Theorem. The group probability, $G(A)$, is the conditional probability of A happening given the individual $p_i(A)$, $i = 1, \ldots, n$. Bates and Granger (1969) proposed the weighted average method, where each individual i is assigned a weight . The group probability is defined by the weighted average of all the individual estimates of the probability,

For a more detailed description of this method, including the weight assignment process, see Dickinson (1973, 1975). A common approach to assigning weights is by taking into account individuals' previous performance on similar tasks as a guideline. Another popular method is the Delphi method (Dalkey and Helmer 1963), which utilizes an iterative process that allows individuals to know other individuals' probability estimates and then adjust their own estimates.

Since the human brain has limited capacity to process information about a gamble involving uncertainty, people may express inconsistent preferences. One pattern of inconsistent preferences was called a "preference reversal" by Slovic and Lichtenstein in their research work during the 1970s. Let us consider a gamble that considers four basic risk dimensions: probability of winning (P_W), amount to win ($\$_W$), probability of losing (P_L), and amount to lose ($\$_L$). People in general will focus more on a particular dimension of a gamble, and consequentially will give less emphasis to other dimensions. For example, a person with very little money might focus more on the losing amount ($\$_L$) than the other dimensions.

When asked to select or provide attractiveness ratings between two choices, people tend to place more emphasis on the probabilities of the gambles. But when asked to place a bid price on the gamble, people tend to place more emphasis on the amount to win or lose. Preference reversal occurs when an individual prefers one gamble over another when presented with a choice, but places a higher price on the bet that they prefer less when making a direct choice. Consider the following gambles:

P bet: 28/36 chance of winning $10	$ bet: 3/36 chance of winning $100
8/36 chance of winning nothing	33/36 chance of winning nothing

Notice that the bet with the higher chance of winning is referred to as the P bet (for its higher probability of winning), and the bet with the higher dollar amount of winning is the $ bet. Typically, when an individual is presented with the above gambles, (s)he is more likely to <u>choose</u> the P bet rather than the $ bet. However, when asked to give the lowest selling price at which they would be willing to sell each bet, most individuals give a higher amount for the $ bet than for the P bet. This phenomenon was first studied by Slovic and Lichtenstein (1968). They conducted more studies including subjects who were experienced gamblers in casino settings (Lichtenstein and Slovic 1971, 1973) and found results that supported their previous finding. Grether and Plott (1979) found through a series of experiments that the preference reversal phenomenon exists in realistic settings involving monetary payouts, and the phenomenon cannot be explained by standard economic theory. For further analysis and experimental designs on preference reversal, see Pommerehne, Schneider, and Zweifel (1982).

In summary, there are many challenges to assessing probabilities because the human mind has limited capacity to process multiple dimensions of a decision problem involving uncertainty. It requires appropriate elicitation methods to collect probability estimates from individuals. Various biases may affect an individual's stated subjective probability estimate, such as anchoring and insufficient adjustment. Individuals may also exhibit a lack of internal consistency, and may be influenced by the structure of the elicitation method. There are still challenges to assessing subjective probabilities even with help from experts who are trained to avoid such biases. Often there are several experts estimating the probability of an event happening. The remaining obstacle in these situations is finding a suitable method to aggregate the probability estimates provided by different individuals.

How to Value Outcomes

In some decision problems, the preferences of the decision maker will be simple and transparent enough such that outcomes can be evaluated without additional elicitation or analysis. In the Decision Trees section, for example, we described each outcome in terms of a monetary amount, which was sufficient for comparing the possible alternatives for decision makers who are risk neutral over monetary outcomes. However, some decision problems will involve potential outcomes which cannot be described easily by a single existing number (such as a monetary amount) capturing the value of the outcome to the decision maker. In such decisions in an environment of certainty, we typically use a value function to determine the desirability of an outcome. A value function is a function whose domain is the set of possible outcomes, and whose output is a number on an ordinal or interval scale which captures the preferences of the decision maker. For details on value functions, see Debreu (1960), Fishburn (1970), or Keeney and Raiffa (1976).

An ordinal value function yields a greater value for outcome 1 than for outcome 2 if and only if the decision maker prefers outcome 1 over outcome 2. An interval value function captures strength of preferences between outcomes as well; the difference in value between outcomes 1 and 2 is greater than the difference in value between outcomes 3 and 4 if and only if the decision maker's strength of preference between outcomes 1 and 2 is greater than that between outcomes 3 and 4. Such an interval value function carrying strength of preference information can also be called a measurable value function. See Dyer and Sarin (1979) for details.

When the set of possible outcomes is a range of a single variable, there are several possible approaches for assessing an interval value function. Midvalue splitting is a common technique: the decision maker is given two outcomes 1 and 2, and asked to identify an intermediate outcome 3 such that the strength of preference between 1 and 3 is equal to the strength of preference between 3 and 2. The value of outcome 3 must therefore be halfway between the values of outcome 1 and outcome 2. Marginal assessment is often used as well: the decision maker is asked to assign a score to each marginal improvement in the outcome, and the marginal increases in value are proportional to these elicited marginal scores. A third approach is to assume a particular functional form for the value function, and then assess one or more parameters from the decision maker. The functional form would be selected based on properties deemed reasonable for the particular decision context. Keller (1985) provides details of how she assessed such interval (i.e. measurable) value functions from experiment participants and fitted them to a number of functional forms.

Assessing and using value functions as described previously is a prescriptively compelling approach to decision making under certainty. However, as discussed throughout this handbook, many challenges arise when trying to apply the tools and techniques for judgment and decision making. For example, the way in which an outcome is presented may influence the value that the decision maker places on it, and the decision maker's current situation or emotional state may have an unduly large effect on the value assigned to various outcomes.

Outcomes may occur over time, in which case a single assigned value must aggregate experiences at different points in the future. The standard normative approach is to elicit a discount rate from the decision maker indicating the degree to which the decision maker is willing to sacrifice a portion of potential overall gains to receive those gains sooner. This discount rate is then used to compute a single net present value for an entire stream of outcomes (Samuelson 1937, Koopmans 1960, Fishburn and Rubenstein 1982). However, experimental results indicate that decision makers deviate from this approach, often in predictable ways. For example, gains tend to be discounted more heavily than losses (Thaler 1981, Loewenstein and Prelec 1991, Shelley 1993, Ahlbrecht and Weber 1997). See Frederick, Loewenstein, and O'Donoghue (2002) for a discussion of this and several other observed deviations from the normative approach. Further analysis of anomalies in time preferences is provided by Guyse, Keller, and Eppel (2002), Frederick and Loewenstein (2008), and Guyse and Simon (2011).

Deviations from the normative approach to valuing outcomes are likely to occur when some possible outcomes can be characterized as gains, and others as losses. This distinction was observed by Kahneman and Tversky (1979) in simple gambles, and subsequently in a wide range of contexts. For example, Kahneman, Knetsch, and Thaler (1990) distributed an item to some (but not all) of the subjects in their experiment at random, and found that the subjects who were given an item placed a much higher value on it.

Chapters 11-16 and 21-25 provide detailed discussion on these issues and many others related to the valuation of outcomes, including time pressure, construal theory, morals, and fairness. All of these challenges can make it more difficult for decision makers to implement effectively the approaches presented in this chapter.

Further challenges arise in the realm of decisions under uncertainty. Consider the decision tree approach presented earlier. At the end of each path through the tree is an outcome, on which the decision maker must assign some level of utility. We assumed that the decision maker was risk-neutral, which allowed us to bypass considerations of utility functions or risk attitude. If the outcomes can be expressed as cardinal numbers, e.g. amounts of money, then we can choose the alternative(s) yielding the highest expected value. (Of course, the same potential pitfalls in valuing certain outcomes will apply here as well.)

This process becomes more challenging when there is no obvious way to represent each outcome with a single number. There are many reasons this might be difficult. Outcomes might consist of multiple dimensions, in which case the decision maker will have to consider multiple objectives (see the next section of this chapter). Outcomes might be expressed in terms of a nominal (e.g., blue, red, or yellow car) or ordinal (e.g., 1st, 2nd, or 3rd) measure, on which an expected value has no meaning. Outcomes may occur over time, in which case discounting or other methods of aggregation would be necessary, with the potential hazards discussed previously. In all of these situations (and many more), it will be necessary for the decision maker to provide some information about preferences to measure the outcomes with numbers that can be rolled back through the decision tree.

When the decision maker was risk-neutral, issues of risk and utility were not germane to the decision problem. However, as mentioned, there are many situations in which decision makers are not risk-neutral, and in such situations in an environment of uncertainty, we must use utility functions rather than value functions. While a value function expresses the desirability of an outcome, a utility function expresses the desirability of a lottery or gamble of multiple possible outcomes, with associated probabilities. The foundations of this approach are presented by Savage (1954) and von Neumann and Morgenstern (1947).

Utility functions can be assessed using the concept of a *certainty equivalent*. A certainty equivalent is obtained by presenting the decision maker with a gamble between outcome 1 and outcome 2 with associated probabilities p and $(1-p)$, and asking the decision maker to specify an outcome 3 such that (s)he is indifferent between outcome 3 and the gamble. This imposes a relationship between the utilities of the three outcomes: U(outcome 3) = pU(outcome 1) + $(1-p)U$(outcome 2). An alternate approach uses a related concept known as a *probability equivalent*. With that approach, outcome 3 is already specified, but the decision maker must choose a value of p such that outcome 3 and the gamble are equally desired. Normatively speaking, the two approaches should provide the same utility function. Repeated application of either approach leads to an estimate of an interval scaled function, with which relative differences between utilities can be used in making decisions between decision alternatives. Keller (1985) provides details of how she assessed utility functions from experiment participants and fitted them to a number of functional forms.

Utility functions provide the numbers to be assigned to each outcome at the end of a path through a decision tree. They are constructed such that the certainty equivalent of a gamble achieves the same expected utility as the gamble itself, thus, normatively speaking, they can be rolled back through a decision tree without any loss of preference information which could affect the decision. Just as decision makers often deviate from the normative procedure for providing and applying value functions, they often do so for utility functions as well.

First, empirical evidence shows that assessing utility functions with certainty equivalents and probability equivalents does *not* necessarily yield the same utility functions (Hershey and

Schoemaker 1985). In general, assessments using probability equivalents tend to yield greater aversion to risk. There are multiple behavioral causes for the incongruence of the results, one of which is that decision makers tend to treat gains and losses differently. Hershey, Kunreuther, and Schoemaker (1982) examine other possible sources of bias in eliciting utility functions, including some discussed earlier in the context of value functions.

Gneezy, List, and Wu (2006) find that people will sometimes value a gamble less than its worst possible outcome. This is inconsistent with approaches based on expected utility; as such approaches suggest that the desirability of a gamble should be a weighted average of the desirability of each of the possible outcomes. This phenomenon is not likely to arise in monetary gambles, but can occur when the value of each individual outcome is less transparent. Wang, Feng, and Keller (2013) explore boundary conditions under which the phenomenon disappears: when the decision maker is under high cognitive load or when the value of the lowest sure outcome is judged prior to judging the gamble.

Loewenstein et al. (2001) propose that decision makers do not make choices under uncertainty based on an approach grounded in expected utility; instead they are heavily influenced by their emotional state at the time the decision is made. This is called the *risk-as-feelings* hypothesis. They argue that the vividness with which outcomes are described plays a significant role in the decision process, because while it does not affect the analytical interpretation of the gamble, it may have a drastic effect on the emotional response of the decision maker.

All of these challenges underscore the difficulty in reconciling the normative approaches to decision making under uncertainty with the descriptive approaches observed in reality. Decision trees, expectation, and utility functions satisfy desirable axioms and common-sense properties of "rational" decision making, but they are often violated in practice. Analysts working on prescriptive applications of decision making need to be aware of the ways in which decision makers are likely to deviate from the normative approach, and must be able to exercise judgment regarding whether such deviations reduce the decision quality, and what steps should be taken to resolve them.

Multiple Objective Decisions Under Certainty with Swing Weights

In the previous section, we considered a decision problem involving uncertainty with a single objective, to maximize the expected monetary return. However, we are often faced with decisions that involve many other trade-offs apart from monetary outcomes. For example, when making a decision on which job offer to accept, we not only consider the salary, but also its potential for career advancement, personal interest in the work, the social environment, and perhaps many more factors as well. In this section, we present a method to assist in making a decision that involves more than one objective.

The multiple objective decision under certainty approach has been used by both public organizations and private firms and individuals to assist in choosing the best course of action for their decisions. To further understand decisions with multiple objectives, Keeney and Raiffa's *Decisions with Multiple Objectives* (1976) is a popular presentation of the most commonly used techniques. Several examples can be found in recent literature. Feng and Keller (2006) evaluated different potassium iodide distribution plans to help protect the public from potential thyroid cancer resulting from nuclear incidents. Winn and Keller (2001) examined retrospectively a decision involving multiple stakeholders in a StarKist corporation decision to stop buying tuna caught in association with dolphins, putting dolphins at risk. Some practical examples that appear in Clemen and Reilly (2004) include choosing a location for an additional public library in Eugene, Oregon and an individual choosing a summer internship.

Each of the above examples can be modeled as a decision with multiple objectives under certainty. Although the decisions can be studied without analysis of uncertainties, the complexity of the examples derives from the many trade-offs one must make, such as considering cost versus safety. Therefore, the first step in this decision analysis approach is to construct an objectives hierarchy to help organize the various aspects of the decision that one must consider when choosing an alternative. Then, if we make an assumption known as *preferential independence* (Keeney and Raiffa 1976), we can treat preferences for each objective at the bottom of the hierarchy separately. We assess a value function reflecting the decision maker's preferences for each of these objectives. This provides a value rating, or a score, on each of the objectives for each alternative. Then, for each of the objectives, we will assign weights to reflect how it is traded-off relative to the rest of the objectives. These assigned weights should take into account the trade-offs the decision maker is willing to make between objectives <u>for the ranges that are possible on each objective</u>. The final step is to find the overall value by computing the weighted sum of each alternative to compare which choice would lead to the best outcome.

The objectives hierarchy should include all essential aspects of the decision and so constructing it is often the most challenging task. It is important to note that a list that is too extensive may become overwhelming but a list that lacks pertinent objectives may cause the decision maker to fail to choose the best course of action given the circumstance. Bond, Carlson, and Keeney (2008) found that people often fail to identify up to half of the relevant objectives when making decisions. With practice, compiling a suitable list of objectives should become easier. Bond, Carlson, and Keeney (2010) identified that the two primary reasons why people fail to compile a complete list of objectives that are important to them are because they are either not thinking a) broadly or b) deeply enough about their decision problem. When prompted, individuals can usually add more objectives to their initial lists, so it can be beneficial to revisit a list that was previously compiled.

Objectives must have a clear sense of direction (such as minimizing, maximizing, or improving) on a certain trait to make assigning a rating of performance on the objective possible. We can think of assigning value ratings as grading how well each alternative performs on each objective. The value ratings are the output of a single objective value function, indicating how an alternative performs on an objective. The overall grade of each alternative is calculated based on how well it is rated on each objective. The weight we assign to each objective reflects the trade-offs the decision maker would make between objectives for the ranges that are possible on each objective. We first assess "raw" weights and then normalize the weights to sum to 1.0. The scale we use for raw weights typically ranges from 0 to 100, with 100 specifying the most highly weighted objective. The overall multiple objective value of each alternative is calculated by

Where

The normalized weight is calculated by the following equation, The sum of the normalized weights is 100% or equivalently 1.0 for all lowest-level objectives. The process of assessing the raw weights and value ratings is subjective; it is based on the preferences of the decision maker. The process of assigning raw weights can be challenging. We must take into account how the outcomes vary for each of the alternatives for that objective. For example, suppose we are considering houses where the square footages are very similar, but the ages and quality of the views of the homes vary greatly. The decision maker may consider the square footage of the house to be an important aspect of a home on an absolute scale, but since all of the alternatives score roughly the same on this objective, it should not play a major role in determining the overall score. Assigning a low weight to square footage will account for the fact that the difference in desirability between the highest and lowest available square footages would have a minimal effect on the overall variation in desirability among the alternatives.

An assessment technique called the swing-weight method should be used to assess the raw weights. This approach hypothesizes the existence of a set of alternatives (say houses) that only have one objective met at the highest level. So, one house might perform best on square footage, but worst on everything else. Another house might perform best on price, but worst on everything else, etc. For this thought experiment, the set of houses will be directly rated by the decision maker on a 0-100 scale. Those expressions of the decision maker's preferences are then used to derive the weights that she must have been using, assuming an additive multiple objective measurable value function is appropriate.

A hypothetical "worst case" alternative house with all objectives at their worst level is imagined and hypothetically valued as having a value of 0. The task for the decision maker is: "Assign a value of 100 to the one hypothetical house you'd prefer to choose first." So the decision maker has to think about which house to choose, which will only have one objective satisfied at its highest level (the one with the best age, the best square footage, the best view, etc.). A key concept is that the decision maker should not say what objective is most highly weighted without considering the range of best and worst levels of each objective. Suppose the decision maker chooses the house with the best view. From this judgment, we can derive the raw weight for the objective of having a great view, which turns out to be 100. (Subsequent judgments by the decision maker assigning a rating between 0 and 100 to lesser preferred hypothetical houses will allow the assessment of the raw weights on other objectives.)

Suppose the decision maker chose the house with a view, assigning a value of 100. We assume that an additive multiple objective value function is appropriate, which can be calculated by taking the raw weight on view times the single objective value function rating for the best view, assumed to be 1.0, on a scale from 0 to 1, plus a number of terms that come out to be 0 since all other objectives score at their worst value of 0. Thus, the raw weight on the objective of "maximizing the view" is 100. If the decision maker assigns a rating (say 25) to another hypothetical house (say the one with the best square footage), the raw weight on the one objective that performs best in that house will be discovered (so, for example, the raw weight on "maximizing square footage" is 25).

Thus the objective which would be chosen to "swing" from the worst level to the best level, if only one objective could be swung, would be given the highest raw weight of 100. Then, other objectives are compared to this one and given raw weights ranging from 0 to 100. Then, the normalized weights are computed so the weights for use in the actual decision sum to 1. See Clemen and Reilly (2004) for details.

We can use sensitivity analysis to observe how varying the raw weight on an objective could affect the overall values of the alternatives. It enables the decision maker to observe how different raw weights on a particular objective between 0 and 100 affect the overall value of each alternative. See Feng, Keller and Zheng (2008) for examples of how to teach this multiple objective method.

Excel Spreadsheets can assist in making the required calculations and in organizing the objectives and alternatives. The following example involves a newlywed couple who is searching for a perfect home to start a family together. They have already spent months searching for and visiting many potential homes, and have narrowed it down to four houses. There are thus four alternatives: Home 1, Home 2, Home 3, and Home 4.

We approach this problem by setting up an objectives hierarchy for evaluating these four homes. The primary objective for the couple is "Maximize the couple's overall value of their new home." The objectives hierarchy starts with the broad categories of objectives, then branches out to the lower levels. Below is an example of the objectives hierarchy a newlywed couple might consider when purchasing their first home together.

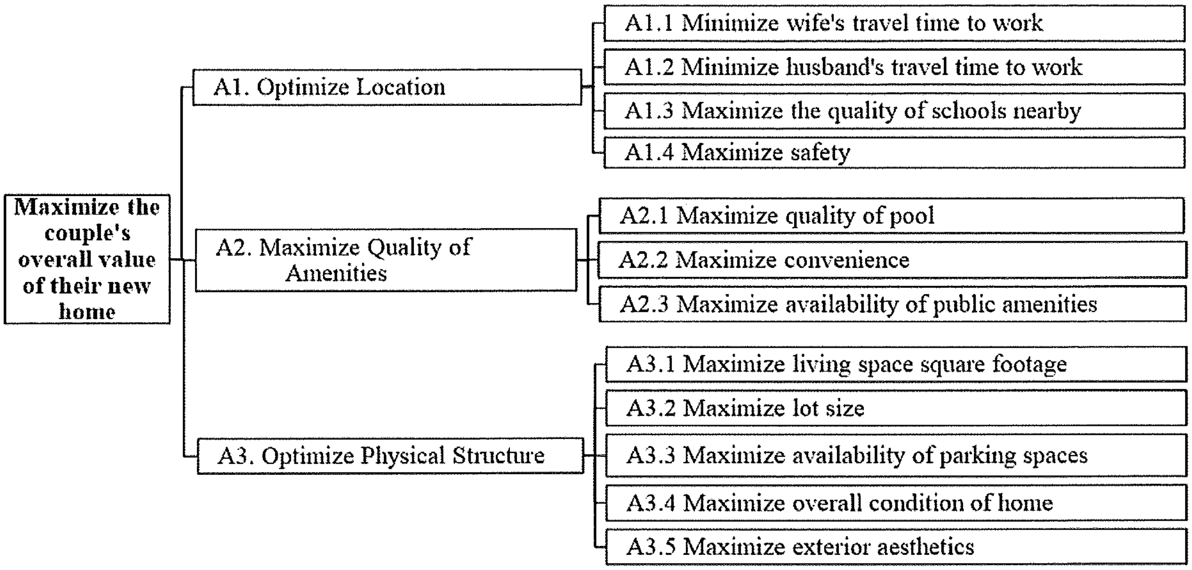


Figure 5. Objectives Hierarchy

In Figure 5, the objectives hierarchy has a total of 12 lowest level objectives. For any given decision, there could be more or fewer objectives depending on the decision maker. Notice that the branches do not need to have the same number of lower level objectives. We assign raw weights to each of the lowest level objectives, on a scale from 0-100, with 0 signifying no weight, and 100 being the most highly weighted, given the existing ranges of performance on each objective.

						Rating on Each Objective 0 - 10 = best			
		Calculated Weights for Major Objectives	Calculated Normalized Weights	Slider	Fill in Raw Swing Weights (0-100)	Home 1	Home 2	Home 3	Home 4
OVERALL OBJECTIVES									
A1. Optimize Location	A1.1 Minimize wife's travel time to work	0.46	0.11		55	0	10	7	4
	A1.2 Minimize husband's travel time to work		0.07		35	10	0	5	8
	A1.3 Maximize the quality of schools nearby		0.19		100	3	10	4	8
	A1.4 Maximize safety		0.10		50	6	8	10	0
A2. Maximize Quality of Amenities	A2.1 Maximize quality of pool	0.12	0.02		10	10	0	0	5
	A2.2 Maximize convenience		0.04		20	10	2	8	4
	A2.3 Maximize availability of public amenities		0.06		30	10	0	2	4
A3. Maximize Living Space	A3.1 Maximize living space square footage	0.42	0.14		75	6	10	8	0
	A3.2 Maximize lot size		0.07		35	4	2	8	10
	A3.3 Maximize availability of parking spaces		0.01		5	6	0	5	10
	A3.4 Maximize overall condition of home		0.13		65	2	6	0	10
	A3.5 Maximize exterior aesthestics		0.08		40	8	4	10	2
	OVERALL VALUE (SUMPRODUCT OF NORMALIZED WEIGHTS TIMES RATINGS)	1.00	1.00		520	5.04	6.46	5.74	5.15

Figure 6. Completed Spreadsheet with Overall Values Calculated

In Figure 6, we filled out the Raw Swing Weights (0-100) and the value rating each alternative received for the corresponding objective on a scale from 0-10, where 10 is best. From the completed raw weights, we can calculate the normalized weights. The calculated weights for each major objective are calculated by summing the lower level objectives for that major objective. The slider position indicates where between 0 and 100 the raw weight resides.

By comparing the four alternatives presented in Figure 6, Home 2 has the highest overall value (6.46 out of 10 possible) to the newlywed couple. Notice that the couple view maximizing the quality of schools nearby as the most highly weighted objective. But perhaps they had not considered sending their children to private schools, in which case the school district that the home is assigned to would become less highly weighted. The couple may be interested in knowing how much the overall score would differ if their objective of maximizing the quality of the schools nearby were to decrease in weight. They can perform sensitivity analysis to observe how susceptible the overall value of each home is to changes in the raw weight of an objective (in this case, maximizing the quality of schools nearby).

						Rating on Each Objective 0 - 10 = best			
		Calculated Weights for Major Objectives	Calculated Normalized Weights	Slider	Fill in Raw Swing Weights (0-100)	Home 1	Home 2	Home 3	Home 4
OVERALL OBJECTIVES									
A1. Optimize Location	A1.1 Minimize wife's travel time to work	0.33	0.13		55	0	10	7	4
	A1.2 Minimize husband's travel time to work		0.08		35	10	0	5	8
	A1.3 Maximize the quality of schools nearby		0.00		0	3	10	4	8
	A1.4 Maximize safety		0.12		50	6	8	10	0
A2. Maximize Quality of Amenities	A2.1 Maximize quality of pool	0.14	0.02		10	10	0	0	5
	A2.2 Maximize convenience		0.05		20	10	2	8	4
	A2.3 Maximize availability of public amenities		0.07		30	10	0	2	4
A3. Maximize Living Space	A3.1 Maximize living space square footage	0.52	0.18		75	6	10	8	0
	A3.2 Maximize lot size		0.08		35	4	2	8	10
	A3.3 Maximize availability of parking spaces		0.01		5	6	0	5	10
	A3.4 Maximize overall condition of home		0.15		65	2	6	0	10
	A3.5 Maximize exterior aesthestics		0.10		40	8	4	10	2
	OVERALL VALUE (SUMPRODUCT OF NORMALIZED WEIGHTS TIMES RATINGS)	1.00	1.00		420	5.52	5.62	6.15	4.48

Figure 7. Raw Weight of "Maximizing the quality of schools nearby" Reduced to 0.

When the raw weight of the quality of schools nearby is reduced to 0, Home 3 becomes the most desirable alternative with an overall value of 6.15 (see Figure 7). We may be interested in knowing the normalized and raw weight that makes the couple indifferent between Home 2 and 3. Sliders in Excel allow us to perform sensitivity analysis to observe how varying the raw weight of an objective changes the overall value of the alternatives. Using the slider, we are able to quickly vary the raw weight from 0 to 100, and find that at the raw weight value of 38 (normalized weight of 0.08), Home 2 and 3 have the same overall values.

		Calculated Weights for Major Objectives	Calculated Normalized Weights	Slider	Fill in Raw Swing Weights (0-100)	Rating on Each Objective 0 - 10 = best Home 1	Home 2	Home 3	Home 4
OVERALL OBJECTIVES									
A1. Optimize Location	A1.1 Minimize wife's travel time to work	0.39	0.12		55	0	10	7	4
	A1.2 Minimize husband's travel time to work		0.08		35	10	0	5	8
	A1.3 Maximize the quality of schools nearby		0.08		38	3	10	4	8
	A1.4 Maximize safety		0.11		50	6	8	10	0
A2. Maximize Quality of Amenities	A2.1 Maximize quality of pool	0.13	0.02		10	10	0	0	5
	A2.2 Maximize convenience		0.04		20	10	2	8	4
	A2.3 Maximize availability of public amenities		0.07		30	10	0	2	4
A3. Maximize Living Space	A3.1 Maximize living space square footage	0.48	0.16		75	6	10	8	0
	A3.2 Maximize lot size		0.08		35	4	2	8	10
	A3.3 Maximize availability of parking spaces		0.01		5	6	0	5	10
	A3.4 Maximize overall condition of home		0.14		65	2	6	0	10
	A3.5 Maximize exterior aesthetics		0.09		40	8	4	10	2
	OVERALL VALUE (SUMPRODUCT OF NORMALIZED WEIGHTS TIMES RATINGS)	1.00	1.00		458	5.31	5.98	5.98	4.77

Figure 8. Swing Weight for Objective A1.3 Maximize the Quality of Schools Nearby

As seen in Figure 8, after performing the sensitivity analysis using the slider, we find that when the raw weight of the "maximizing the quality of schools nearby" objective is between 0 and 38, Home 3 is preferred. When the raw weight is greater than 38, Home 2 is preferred. This information is beneficial to the couple because if they are confident that the raw weight of the objective of maximizing the quality of schools nearby is well above 38, they can comfortably choose Home 3 without reservations. This process can be performed similarly for all other objectives.

In the example we presented, the couple constructed the multiple objective decision under certainty together. Alternatively, the couple could be treated as separate stakeholders of the same decision. They would individually construct their objectives hierarchy, assign scores and weights, and then compute the overall value of each choice separately. The decision of which alternative to select would require further analysis by the stakeholders, but the multiple objective decision under certainty approach with multiple stakeholders would be able to present clearly to the decision makers how each stakeholder views each alternative.

This section has presented the approach for constructing an additive multiple objective measurable value function (Dyer and Sarin 1979) for use when the decision does not involve uncertainty. A similar additive multiple objective utility function could be constructed for decisions under uncertainty and used in a decision tree for valuing the multiple objective outcomes at the ends of the decision tree, before rolling back the tree to compute the expected multiple objective utility. See Keeney and Raiffa (1976) for more details about required independence conditions and assessment procedures.

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