

An Empirical Investigation of Relative Risk Aversion

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Abstract—A single-attribute utility function models a decisionmaker's preferences as revealed by choices among risky alternatives. A measurable value function models a decisionmaker's strength of preference for various levels of the attribute. An experimental investigation of the relationship between the utility function and measurable value function for twenty-nine cases is described. The utility function was the same as the measurable value function in only three cases. Further, the recently developed concept of a relative risk attitude was used to categorize the observed preferences. Although relative risk aversion occurred in ten cases, it was not found to be the universal relative risk attitude. In addition, a person's relative risk attitude differed across attributes.

I. INTRODUCTION

A SINGLE-ATTRIBUTE utility function $u(x)$ models a decisionmaker's preferences as revealed by his choices among risky alternatives. Traditionally, the shape of the utility curve is examined to reveal the "risk attitude" of the decisionmaker [22]. Similarly, a measurable value function $v(x)$ models a decisionmaker's strength of preference for various levels of the attribute. Thus, a measurable value function will indicate whether a decisionmaker has decreasing marginal value for additional equal-sized increments of the attribute. This paper reports an investigation of the relationship between $u(x)$ and $v(x)$ for 29 actual cases.

Expected utility is often used as a criterion for ranking risky alternatives. Satisfaction of the von Neumann-Morgenstern [25] axioms or equivalent conditions (e.g., see Marschak [20]) provides a theoretical basis for this use of the expected utility criterion. The computation of the expected utility of a risky alternative requires specification of the probability distribution for the alternative's outcomes and the decisionmaker's $u(x)$ over various levels of the attribute. A utility function can be assessed by arbitrarily setting the utility values of the best and the worst levels of the attribute to be $u(x_1) = 1$ and $u(x_0) = 0$. The decisionmaker is then asked to provide the attribute level $x_{0.5}$ for which he would be indifferent between receiving $x_{0.5}$ for sure and a risky alternative with a half chance of receiving x_1 and a half chance of receiving x_0 . The utility of $x_{0.5}$ is then equal to the expected utility of the risky alternative, which is 0.5. In a similar manner the $x_{0.25}$ and $x_{0.75}$ levels, representing $u(x_{0.25}) = 0.25$ and $u(x_{0.75}) =$

0.75, are obtained. Details of this assessment procedure are provided in Keeney and Raiffa [16].

A measurable value function is used to measure the differences in the decisionmaker's strength of preference among various attribute levels. Suppose, for attribute levels x_i , x_j , and x_k , that $v(x_j) - v(x_i)$ is equal to $v(x_k) - v(x_j)$. Then the decisionmaker perceives the same difference in his strength of preference between levels x_j and x_i as he does between x_k and x_j . Axioms for the existence of $v(x)$ are presented by Krantz, *et al.* [18], and Dyer and Sarin [4] have developed a multiattribute measurable value theory. One method for assessing measurable value functions is to directly rate the preference differences among pairs of attribute levels. Several other methods of eliciting measurable value functions are given in Fishburn [11] and Dyer and Sarin [4]. While the assessment of a utility function requires the use of probabilistic question formats, measurable value function assessment requires only deterministic interrogation procedures.

The first research question addressed here is whether the utility function is the same as the measurable value function for most decisionmakers. Harsanyi [14] claims that $u(x)$ should equal $v(x)$; and some psychological studies (e.g., [1]) have made this implicit assumption. Sarin [23] presents conditions under which $u(x)$ and $v(x)$ are logically equivalent. On the contrary, arguments by Fishburn [12], Ellsberg [9], and others claim that no identity relationship between $u(x)$ and $v(x)$ needs to exist in general. Since a utility function is assessed solely on choices among risky alternatives, no information about the decisionmaker's perception of the relative preference differences between the various attribute levels is collected. Thus, $u(x)$ does not, by construction, measure preference differences, which are modeled by $v(x)$. Empirical evidence on this question was collected by assessing the single-attribute $u(x)$ and $v(x)$ functions for 12 subjects on from one to three attributes.

Dyer and Sarin [5] define and discuss the concept of *relative risk aversion* and provide further insight into the relationship between $u(x)$ and $v(x)$. In utility assessment applications, decisionmakers have indicated at least two factors that affect their decisions in risky situations: 1) varying preference differences for incremental changes in the amount of the attribute and 2) attitude toward risk taking. The first factor can be singled out by assessing the measurable value function. Each equal increment in the

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level of the value function will represent an equal preference difference. In an attempt to isolate risk attitude, define a utility function $u_v(v(x)) = u(x)$. The $u_v(v(x))$ function should reflect only the risk factor, since it is assessed over $v(x)$. The pairs of points $(v(x), u(x))$ can be plotted to determine the shape of the $u_v(v(x))$ utility curve. An examination of the shape of the curve will provide additional insight into the decisionmaker's risky choice behavior. For example, if a decisionmaker's $u_v(v(x))$ curve is concave, he will be termed *relatively risk averse*. (The terms concave and convex are used here in the strict sense, excluding the case of a linear function.)

Definition: Suppose that for any i, j , and k satisfying $v(x_j) - v(x_i) = v(x_k) - v(x_j)$, some decisionmaker would chose to take attribute level x_j rather than a risky alternative with a half chance of getting either x_i or x_k . Then this decisionmaker will be defined as *relatively risk averse* since he or she would choose the riskless alternative x_j rather than taking a risk in which the expected measurable value of the risky alternative $0.50(v(x_i)) + 0.50(v(x_k))$ is equal to the measurable value $v(x_j)$ of the riskless alternative. A *relatively risk neutral* decisionmaker would be indifferent between the riskless alternative x_j and the risky alternative, and a *relatively risk prone* decisionmaker would prefer the risky alternative over the riskless alternative x_j .

For example, suppose a person's preferences among different annual salary amounts can be modeled by the utility function $u(x) = x^{0.5}$, where x ranges from 0 to 1.0 and represents fractional amounts of a \$50000 total. Note that this utility function is concave, so the person is risk averse [22]. The same person's measurable value function may be $v(x) = x^2$, where annual salary x is on the same scale as above. This value function is convex, and thus the person feels there is increasing marginal value for equal-sized increments in the annual salary. We represent the person's preferences under risk with the utility function $u_v(v(x))$. This utility function is modeled over values of $v(x)$, where equal-sized increments in $v(x)$ represent equal preference differences. In this case $u_v(v(x)) = v(x)^{0.25}$. [Since $v(x) = x^2$, we know $v(x)^{0.5} = x$. Substituting for x in $u_v(v(x)) = u(x) = x^{0.5}$, we get $u_v(v(x)) = (v(x)^{0.5})^{0.5} = v(x)^{0.25}$.] Thus $u_v(v(x))$ is concave and this person is relatively risk averse. To illustrate the definition of relative risk aversion, consider $x_i = 0$, $x_j = 0.71$, and $x_k = 1.0$. Note that $v(x_j) - v(x_i) = 0.5 - 0$ is the same as $v(x_k) - v(x_j) = 1.0 - 0.5$. This person would prefer the riskless alternative x_j over the risky alternative with a half chance of either x_i or x_k since $u_v(v(x_j)) = 0.84 > 0.5u_v(v(x_i)) + 0.5u_v(v(x_k)) = 0.5(0^{0.25}) + 0.5(1.0^{0.25}) = 0.50$.

Notice that a decisionmaker may be risk prone [22] and be relatively risk averse. This would occur, for example, if both $u(x)$ and $v(x)$ were convex, but $v(x)$ were more convex than $u(x)$, so that $v(x)$ lay completely below $u(x)$. Bell and Raiffa [2] propose that $u_v(v(x))$ is concave (corresponding to relative risk aversion) and suggest specifically that $u_v(v(x))$ is strategically equivalent to $-e^{-c(v(x))}$. The second research question of this study is whether $u_v(v(x))$ is generally concave, and further, whether it has the exponential form.

Few empirical studies have elicited both $u(x)$ and $v(x)$ functions. In an experimental study with 24 cases and an examination of ten previously reported cases, Krzysztofowicz [19] found support for a constant relative risk attitude (i.e., $u_v(v(x))$ is either linear or it is concave or convex exponential). His study did not support Dyer and Sarin's [5] conjecture that a decisionmaker may have an intrinsic relative risk attitude, which is invariant across attributes. Currim and Sarin [4] experimentally evaluated the use of utility and value functions in consumer preference models and discussed the application of the relative risk attitude in consumer research. McCord and de Neufville [21] elicited single-attribute utility and value functions. Their results can be shown to illustrate relative risk aversion for those who were risk averse (in the sense of Pratt [22]) and relative risk proneness for those who were risk prone. Fischer [10] and Beach [1] also conducted empirical studies eliciting and comparing utility and measurable value functions. Finally, Eliashberg [7] assessed single-attribute $u(x)$ and $v(x)$ functions, but didn't compare the two types of preference functions.

A measure of the merit of decision analysis research is its potential impact on actual preference assessment applications. If a functional relationship between a decisionmaker's $v(x)$ and $u(x)$ can be established, a broader range of assessment techniques may be appropriate. For example, a decisionmaker faced with a decision among risky alternatives might refuse to answer lottery-type assessment questions, but be willing to answer riskless questions, thus yielding $v(x)$. Then the utility function $u(x)$ could be derived using the established functional relationship. The application of multiattribute utility theory to risky large-scale societal decisions (such as siting power plants) may benefit from explicit consideration of relative risk attitudes. Preferences for levels of different attributes are often modeled by assessing the single-attribute utility functions of experts in the different areas. But the experts' utility functions confound their strength of preference with their risk attitudes. Different experts may have very different relative risk attitudes, either because of personal traits or academic training. It may be possible to use the experts' opinions about their strength of preference over levels of the attributes (by assessing measurable value functions) without having to accept their relative risk attitudes. Taken to an extreme, the relative risk attitude might become a decision variable. For example, one might choose to exhibit constant relative risk aversion. For more discussion of the benefits of investigating relative risk attitudes, see Dyer and Sarin [5].

The next section contains a description of the assessment of the subjects' $u(x)$ and $v(x)$ functions. Section III contains the analysis of the results. The research findings are in Section IV. The final section contains the summary and suggestions for future research.

II. ASSESSMENT OF PREFERENCE JUDGMENTS

Subjects' $v(x)$ and $u(x)$ functions were assessed with self-administered questionnaires over one attribute in a few

unrelated decision problems that were framed as both riskless and risky hypothetical situations. Twelve graduate and upper-division engineering students in a UCLA decision analysis seminar participated in the study. Though the subjects had a background in basic probability and were learning about single-attribute utility functions, they were naive about the underlying research questions.

The usual limitations in using students as subjects were of special concern in this study. In an attempt to ensure that the subjects would provide meaningful survey responses based on well-formed preferences, a preliminary survey was conducted to establish the subjects' familiarity with various attributes. Four attributes were selected for the study based on the results of the preliminary survey: minutes waiting in a gas line, minutes spent on a bus trip, grade earned in a class, and annual salary level. Further, though no direct reward was offered for participation in the study, the decision analysis students were provided with relevant academic knowledge in the debriefing.

In each questionnaire, preliminary questions were designed to screen out cases in which the subject apparently misunderstood the instructions. A bracketing procedure [21] was introduced as a means for arriving at an indifference judgment by making a series of strict preference judgments from both sides of the indifference point. Also, pairs of questions eliciting the same response were used to check the consistency of the subjects' answers. Subjects were instructed to answer surveys in pencil and to feel free to change answers if, upon more thought, they felt their answers were inconsistent with their preferences. The twelve subjects answered questions on from one to three attributes, depending on their familiarity with decisions involving the various attributes. In 29 cases, the subjects understood the questioning procedure and consistently provided the $u(x)$ and $v(x)$ functions. The assessment procedures for each attribute are described next.

Gasoline Line

Riskless Judgments: This experiment was conducted at a time when waiting times at gasoline stations sometimes were greater than one hour. The scenario was that the subject's car was almost out of gas, the car held ten gallons, and the subject had three hours free before school. The subject was asked to specify a gas price which would yield indifference between two gasoline stations. The first question was

You are now planning to go to the gas station at which gas costs \$0.90/gallon and there is a 60-minute wait. Fill in the blank: If I could go to the alternative station below, I'd be willing to pay $\$x_1$ = _____/gallon with a 50-minute wait.

The ensuing questions elicited prices x_2, x_3, x_4, x_5 such that $(x_1, 50 \text{ min}) \sim (x_2, 40 \text{ min}) \sim (x_3, 30 \text{ min}) \sim (x_4, 20 \text{ min}) \sim (x_5, 10 \text{ min})$. Note that to realistically model the gasoline-line problem, the second attribute of price per gallon was introduced. For the relatively small dollar amounts in this problem, it was verified that subjects' preferences were linear in money. (See [16, p. 125] for a

discussion of necessary assumptions in willingness-to-pay assessment procedures.)

Risky Judgments: Subjects adjusted waiting times to achieve indifference between gasoline stations. With some stations, the waiting time was known and at other stations the exact waiting time was unknown, but was specified probabilistically. The first task was to fill in the blank portion of the following:

I am indifferent between a station at which for sure I'll wait ten minutes versus a station at which I have a half chance of waiting five minutes and a half chance of waiting y_1 = _____ minutes.

Ensuing questions sequentially elicited y_2, y_3, \dots, y_n such that

(y_i min; 100-percent chance)

$\sim (y_{i-1} \text{ min, 50 percent, } y_{i+1} \text{ min, 50 percent}).$

Bus Trip Time

Riskless Judgments: The scenario was that subjects commuted to school daily on a public bus, choosing among routes with varying fares and trip times. Subjects stated the fares x_1, x_2, \dots, x_6 to achieve indifference between the alternative routes: $(\$0.20, 60 \text{ min}) \sim (x_1, 50 \text{ min}) \sim (x_2, 40 \text{ min}) \sim \dots \sim (x_5, 10 \text{ min}) \sim (x_6, 5 \text{ min})$. As in the gasoline problem, the fare price has been introduced as a second attribute. It was verified that subjects' preferences were linear in money for the relatively small dollar amounts in this problem.

Risky Judgments: Subjects adjusted bus trip time lengths to achieve indifference between routes. With some routes the length was known and on others the length was specified probabilistically. A suggested reason for a varying route time length was the uncertainty of whether the subject would be able to make transfer connections quickly. The question format was similar to that described earlier for the assessment of risky judgments about lengths of time in gasoline lines.

Grades

Riskless Judgments: Subjects were asked to consider how much better one grade is than another in a calculus class. They were told to think of an exchange of a higher grade for a lower grade as a measure of how much academic improvement has occurred. Represent the exchange of a B+ grade in place of a B by $B \rightarrow B+$. Subjects received 10 cards containing exchanges of adjacent pairs of grades from $D \rightarrow D+$ to $A \rightarrow A+$. Subjects directly rated the degree of improvement (a surrogate for preference difference) on each card by giving ten points to the card showing the least improvement and assigning points to the other cards relative to this benchmark. For example, if a card represented twice as much improvement as the ten-point card, it was to be awarded 20 points.

Risky Judgments: Subjects specified probability values to achieve indifference between alternative sections of the

TABLE I
PAIRED $u(x)$ AND $v(x)$ VALUES FOR THE ATTRIBUTE: GRADE IN A CLASS

Case Number	1	2	3	4	5	6	7	8	9	10
Subject	A	B	D	E	G	H	I	J	K	L
Attribute										
Level x_i	u	v	u	v	u	v	u	v	u	v
D	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D +	0.13	0.04	0.46	0.07	0.39	0.05	0.33	0.04	0.61	0.18
C -	0.44	0.11	0.91	0.15	0.77	0.13	0.54	0.08	0.81	0.34
C	0.57	0.19	0.96	0.22	0.94	0.20	0.69	0.14	0.87	0.48
C +	0.66	0.28	0.98	0.32	0.95	0.24	0.78	0.21	0.94	0.60
B -	0.75	0.40	0.99	0.42	0.98	0.36	0.85	0.29	0.96	0.71
B	0.80	0.49	0.99	0.52	0.98	0.47	0.89	0.38	0.98	0.80
B +	0.86	0.62	1.0	0.62	0.99	0.52	0.93	0.49	0.99	0.87
A -	0.92	0.79	1.0	0.74	1.0	0.73	0.95	0.62	0.99	0.92
A	0.96	0.87	1.0	0.85	1.0	0.87	0.98	0.77	1.0	0.97
A +	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

calculus class. The first problem was

I am indifferent between a course in which I'm sure to receive an A versus a course in which there is a p chance of an A+ and a $1 - p$ chance of an A-.

Annual Salary Level

Riskless Judgements: Subjects were asked to consider how much better a job with a specified annual salary was than another job with a different annual salary that did not differ by any other important attributes. First, subjects made the bisection judgment

$$\$6000 \rightarrow x_1 \sim^* x_1 \rightarrow \$12,000$$

which they were told would be interpreted as meaning "the improvement in getting annual salary level x_1 rather than \$6000 is the same as the improvement of getting \$12,000 rather than x_1 ." Then, subjects sequentially specified x_2, x_3, \dots, x_n constructing a uniform sequence of equal preference difference intervals

$$x_1 \rightarrow \$12,000 \sim^* \$12,000 \rightarrow x_2$$

$$\sim^* x_2 \rightarrow x_3 \sim^* \dots \sim^* x_{n-1} \rightarrow x_n.$$

Risky Judgments: Subjects adjusted salary levels to achieve indifference between jobs differing only by salary levels. The first task was to specify the salary level y_1 such that a job with a fixed annual salary of \$8000 was indifferent to a job with a starting salary of \$6000 and a 50-percent chance of a raise to y_1 dollars within a short time. The second judgment was to specify the salary level y_2 to achieve indifference between a fixed salary of y_1 dollars and a job with a starting salary of \$8000 and a 50-percent chance of a raise to y_2 dollars. Ensuing judgments followed this pattern, generating a sequence of y_i values. Note that in order to realistically introduce risk to the salary decision, some jobs contain the possibility of a raise, which occurs after a time delay. In this analysis, the utility of an annual salary of y_1 dollars occurring due to a raise is interpreted as equal to the utility of a fixed annual salary of y_1 dollars. Thus the time delay before the raise is not explicitly considered.

III. ANALYSIS

Among the 29 cases of assessed $u(x)$ and $v(x)$ functions, eight cases represented subjects' preferences for time on a bus trip, three were for time in a gas line, eight were for annual salary, and ten were for calculus class grades. In each of the cases, once $u(x)$ and $v(x)$ were assessed, the two functions were transformed to a 0-1 scale. Pairs of $u(x)$ and $v(x)$ values for the different attribute levels are in Tables I-IV. For the annual salary attribute, both functions were plotted versus x , and curves were faired through the points by visual inspection. Pairs of $u(x)$ and $v(x)$ values were read from the graph for a number of attribute levels to be used in the analysis.

This section contains a description of the method of analysis for each of the research questions. The initial research question, "Does $u(x) = v(x)$ in general?", can also be stated as "Is the linear form $u_v(v(x)) = v(x)$ appropriate in most cases?". First, various functions of $v(x)$ were considered as possible models of $u_v(v(x))$. Distinguish the model $\overline{u_v(v(x))} = f(v(x))$ from the actual data $u_v(v(x))$ by an overbar. If a model provided an approximate fit to the data, then the actual $u_v(v(x))$ curve was assumed to share the curvature properties of the model. The initial research question was then addressed by examining whether the linear model $\overline{u_v(v(x))} = v(x)$ provided the best acceptable fit among all the models considered in each of the cases.

In addition to the linear form, three other model forms were considered. The exponential form

$$\overline{u_v(v(x))} = \frac{1 - e^{-c(v(x))}}{1 - e^{-c}}, \quad c \neq 0$$

is concave when the parameter c is positive and convex for $c < 0$. For c very close to zero, the exponential model is essentially linear. The logarithmic form

$$\overline{u_v(v(x))} = \frac{\log(v(x) + c) - \log c}{\log\left(\frac{1+c}{c}\right)}, \quad c > 0$$

TABLE II
 PAIRED $u(x)$ AND $v(x)$ VALUES FOR THE ATTRIBUTE: ANNUAL SALARY LEVEL¹

Case Number Subject	11 B			12 C			13 F			14 H			15 J			16 K			17 L			18 M		
	\$K	u	v	\$K	u	v	\$K	u	v	\$K	u	v	\$K	u	v	\$K	u	v	\$K	u	v	\$K	u	v
6.0	0.0	0.0		6.0	0.0	0.0	6.0	0.0	0.0	6.0	0.0	0.0	6.0	0.0	0.0	6.0	0.0	0.0	6.0	0.0	0.0	6.0	0.0	0.0
8.8	0.13	0.09		9.5	0.29	0.08	9.5	0.14	0.20	9.0	0.12	0.20	9.5	0.16	0.16	8.5	0.41	0.50	9.0	0.38	0.09	8.7	0.12	0.08
12.0	0.19	0.17		12.0	0.42	0.17	12.0	0.33	0.40	12.0	0.25	0.40	12.0	0.27	0.29	9.4	0.50	0.67	11.0	0.50	0.17	12.0	0.32	0.17
16.0	0.29	0.26		15.0	0.50	0.23	15.0	0.50	0.59	13.0	0.50	0.52	15.0	0.35	0.43	12.0	1.0	1.0	12.0	0.53	0.19	16.0	0.42	0.25
20.0	0.38	0.33		16.0	0.56	0.25	15.0	0.51	0.60	14.0	0.66	0.60	25.0	0.50	0.66				15.0	0.59	0.28	20.0	0.50	0.31
24.0	0.42	0.42		20.0	0.63	0.33	18.0	0.75	0.80	16.0	0.80	0.80	27.0	0.53	0.71				18.0	0.63	0.38	21.0	0.53	0.34
29.0	0.50	0.50		25.0	0.70	0.42	21.0	1.0	1.0	18.0	1.0	1.0	35.0	0.62	0.86				22.0	0.72	0.47	27.0	0.63	0.42
35.0	0.56	0.59		30.0	0.77	0.50							45.0	1.0	1.0				24.0	0.75	0.53	33.0	0.72	0.51
42.0	0.63	0.67		35.0	0.83	0.58													28.0	0.81	0.63	40.0	0.75	0.58
50.0	0.72	0.76		40.0	0.87	0.67													29.0	0.84	0.72	48.0	0.81	0.66
60.0	0.81	0.83		45.0	0.88	0.75													33.0	0.88	0.81	57.0	0.85	0.75
71.0	0.98	0.92		50.0	0.92	0.83													41.0	0.97	0.91	68.0	0.89	0.83
84.0	1.0	1.0		55.0	0.96	0.92													46.0	1.0	1.0	81.0	0.95	0.92
				60.0	1.0	1.0																96.0	1.0	1.0

¹ \$K Annual salary level in thousands of dollars.

TABLE III
 PAIRED $u(x)$ AND $v(x)$ VALUES FOR THE ATTRIBUTE:
 MINUTES WAITING IN A GAS LINE

Case Number Subject Attribute Level	19 A		20 B		21 I	
	u	v	u	v	u	v
45 min	0.0	0.0	0.0	0.0	0.0	0.0
40 min	0.14	0.14	0.15	0.47	0.08	0.42
30 min	0.43	0.43	0.49	1.0	0.32	0.72
20 min	0.71	0.71	0.66	1.0	0.44	0.78
10 min	1.0	1.0	1.0	1.0	1.0	1.0

is concave for all values of the parameter c , but for $c \geq 100$ the curve becomes essentially linear. Finally, a power function model was considered

$$\overline{u_v(v(x))} = v(x)^c, \quad c > 0.$$

This model is concave for $c < 1$, linear for $c = 1$, and convex for $c > 1$.

The closeness of fit of the estimated $\overline{u_v(v(x))}$ to the actual $u(x) = u_v(v(x))$ was measured by the root mean-squared error (RMSE), which is the square root of the mean-squared error (MSE)

$$MSE = \frac{1}{N} \sum_{i=1}^N [\overline{u_v(v(x_i))} - u(x_i)]^2.$$

The best-fitting exponential, logarithmic, and power model for each case was found by a computer search for the parameter value which minimized the RMSE. Fishburn and Kochenberger [13] fit exponential and power functions, and Spetzler [24] describes a method for estimating the logarithmic parameter. Finally, among all models considered for a specific case, the model with the minimum RMSE was chosen as the best-fitting model.

Dyer, Farrell, and Bradley [6] and Fishburn and Kochenberger [13] used mean-squared error for measuring the closeness of fit of a model to an assessed function, but they did not propose a criterion for determining whether a model is "close enough" to reality. An *ad hoc* rule was

developed here: A model is acceptable if the root mean-squared error of the model's predicted $\overline{u_v(v(x))}$ from the actual $u_v(v(x))$ is less than or equal to 0.05. Recall that the range of the normalized $u_v(v(x))$ curve is one, and note that if the model and the actual data varied by no more than 0.05 on each pair of values, then the RMSE would be less than or equal to 0.05.

Seventeen of the 29 cases were acceptably modeled by one of the four functional forms. The best-fitting models and their properties are listed for each case in Table V. A linear model provided the best acceptable fit in one case. In two additional cases, the linear model was close to the best, as measured by an $RMSE \leq 0.05$. The exponential model provided the best fit in nine cases, the logarithmic model was best in three cases, and the power model was best in four cases. In one case, $u_v(v(x))$ was convex, but none of the four models yielded an acceptable fit. In eleven cases no uniformly concave, linear, or convex model was found to be acceptable.

Before responding to the first research question, it was necessary to develop a measure of the difference between the best-fitting model and the linear model in each case. If both models provide approximately the same degree of fit to the data, then both models are equally appropriate. Define G , a measure of how much better the best-fitting model fits the actual data than the linear model, as

$$G = 1 - \frac{MSE(\text{best-fitting model})}{MSE(\text{linear model})},$$

for $MSE(\text{linear model}) \neq 0$.

If the mean-squared error for the linear model is zero, the data are linear since $u(x) = v(x)$. In this case define G to be zero. Similar to R^2 , which is the coefficient of determination, G can be interpreted as a measure of the additional explanation of the variance (of predicted data points from actual points) by the best-fitting model over that explained by the linear model. (Fishburn and Kochenberger [13] introduce a similar measure for comparing a model with the linear model.) A value of G which is

TABLE IV
PAIRED $u(x)$ AND $v(x)$ VALUES FOR THE ATTRIBUTE: MINUTES IN A BUS TRIP

Case Number	22	23	24	25	26	27	28	29
Subject	A	F	H	J	L	I	K	M
Attribute Level	u	v	u	v	u	v	u	v
60 min	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50 min	0.15	0.29	0.25	0.0	0.01	0.03	0.25	0.13
40 min	0.49	0.43	0.49	0.21	0.11	0.17	0.53	0.33
30 min	0.63	0.57	0.74	0.40	0.22	0.33	0.63	0.47
20 min	0.88	0.86	0.88	0.80	0.57	0.50	0.88	0.67
10 min	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

TABLE V
BEST-FITTING MODELS¹

Case Number	Logarithmic		Exponential		Power		Linear	Best Acceptable		Other Acceptable	Curve Shape
	c	RMSE	c	RMSE	C	RMSE	RMSE	Fit	G	Fits	
1	0.038	0.2995	3.701	0.04708	0.384	0.05600	0.25352	Log	0.986	Exp	Concave
2	0.0004	0.10346	11.572	0.04051	0.113	0.10791	0.47608	Exp	0.993	None	Concave
3	0.0004	0.10313	11.279	0.01967	0.135	0.11136	0.47978	Exp	0.998	None	Concave
4	0.0036	0.05287	8.097	0.04303	0.215	0.07258	0.40937	Exp	0.989	None	Concave
5	0.007	0.2529	4.77	0.01528	0.226	0.03155	0.26775	Exp	0.997	Log, Pow	Concave
6	100.0	0.13799	-1.365	0.06754	1.604	0.05487	0.13721	None	0.840	None	Near Convex
7	0.443	0.03435	1.199	0.03518	0.693	0.04242	0.10833	Log	0.899	Exp, Pow	Concave
8	0.029	0.17496	5.689	0.15782	0.375	0.19481	0.30872	None	0.739	None	Near Concave
9	100.0	0.09318	-0.400	0.08573	1.196	0.08166	0.09283	None	0.226	None	S-Shaped
10	100.0	0.15703	-1.675	0.06084	1.757	0.04200	0.15620	Pow	0.923	None	Convex
11	100.0	0.03222	0.006	0.03222	0.981	0.03184	0.03223	Pow	0.024	Lin, Log, Exp	Near Linear
12	0.079	0.01405	2.697	0.03311	0.441	0.02445	0.21481	Log	0.996	Pow, Exp	Concave
13	100.0	0.06322	-0.693	0.00682	1.269	0.01021	0.06234	Exp	0.988	Pow	Convex
14	100.0	0.06897	-0.328	0.06147	1.145	0.05791	0.06856	None	0.287	None	S-Shaped
15	100.0	0.12433	-1.265	0.7439	1.465	0.08949	0.12369	None	0.638	None	Convex
16	100.0	0.09695	-1.099	0.03105	1.470	0.03685	0.09618	Exp	0.896	Pow	Convex
17	0.062	0.03996	2.789	0.07302	0.418	0.02151	0.22073	Pow	0.991	Log	Concave
18	0.257	0.01821	1.625	0.01731	0.603	0.03458	0.14124	Exp	0.985	Log, Pow	Concave
19	100.0	0.00076	-0.001	0.00008	1.0	0.0	0.0	Lin	0.0	None	Linear
20	100.0	0.30948	-3.180	0.27412	2.513	0.27412	0.30922	None	0.214	None	Convex & Jump
21	100.0	0.28050	-3.778	0.01073	3.336	0.01282	0.27971	Exp	0.999	Pow	Convex
22	32.201	0.06727	0.032	0.06727	1.016	0.06719	0.06733	None	0.004	None	S-Shaped
23	0.0688	0.10529	2.891	0.10665	0.413	0.10628	0.20932	None	0.747	None	Near Concave
24	100.0	0.05939	-0.23	0.05656	1.101	0.05410	0.05916	None	0.164	None	S-Shaped
25	0.1665	0.04450	1.85	0.05201	0.538	0.03490	0.15389	Pow	0.949	Log	Concave
26	10.66	0.02758	0.091	0.02757	0.975	0.02796	0.02858	Exp	0.069	Log, Pow, Lin	Near Linear
27	0.165	0.10950	1.98	0.10862	0.512	0.11063	0.18632	None	0.660	None	Jump
28	100.0	0.15280	-1.469	0.12060	1.642	0.12878	0.15242	None	0.374	None	Jump
29	4.0012	0.10089	0.192	0.10101	0.793	0.09277	0.10182	None	0.170	None	Jump

¹ Definitions for the above categories are as follows:

c	value of parameter in model
RMSE	root mean squared error
G	G value for the best fitting model
acceptable fit	model has RMSE ≤ 0.05
none	none of the models provided an acceptable fit
S-shaped	convex over lower range/concave over higher range
jump	the $u_p(v(x))$ curve has a vertical jump.

close to zero will indicate that the best-fitting model is close to the linear model. Conversely, relatively high values of G will indicate that the best-fitting model provides a substantially superior fit. The values of G in the different cases are recorded in Table V.

In one case the linear model provided the best acceptable fit, so G was necessarily zero. In four of the 16 other cases in which an acceptable model was found, the best-fitting model was considerably better than the linear model,

as measured by G values exceeding 0.89. In the two remaining cases, the best-fitting models were only slightly concave and provided fits that were very close to the fit provided by the linear model. This was shown by G values that were less than 0.07. In the twelve cases for which no acceptable model was found, the linear model was, of course, unacceptable. So, the linear model was appropriate in at most three of 29 cases. Thus it can be reasonably concluded that $u(x)$ is not in general equal to $v(x)$.

TABLE VI
RELATIVE RISK ATTITUDES AND SHAPES OF $u_v(x)$ CURVES¹

Attribute	Relatively Risk Averse (concave)	Relatively Risk Neutral (linear)	Relative Risk Prone (convex)	Relative Risk Attitude Varies (other)
Annual salary level (8 cases)	3	1 ²	3	1
Grade in a class (10 cases)	6	0	1	3
Bus trip minutes (8 cases)	1	1 ²	0	6
Gas Line Minutes (3 cases)	0	1	2	0
Total cases in which shape was observed (out of 29 total cases)	10	3	6	10

¹ Entries in table are number of subjects whose preferences fall into each category.

² This case is very near linear, but a slightly concave model has an RMSE that is better by less than or equal to 0.0011.

The second research question was whether, in general, $u_v(v(x))$ was concave. Examination of the shapes of the best-fitting models identified during the analysis of the first research question provided the answer to this question. A summary of the $u_v(v(x))$ curve shapes is in Table VI. Ten curves were concave, three were essentially linear, and six were convex. In ten cases the curve was not uniformly concave, linear, or convex. Thus, $u_v(v(x))$ was not generally concave.

The second research question can be restated as "In most cases are decisionmakers relatively risk averse?" since a concave $u_v(v(x))$ curve indicates relative risk aversion. Categorization of the observed preferences by relative risk attitude is facilitated by also observing that when $u_v(v(x))$ is linear, the decisionmaker is relatively risk neutral and when $u_v(v(x))$ is convex, the decisionmaker is relatively risk prone. (These observations are based on the reasoning in the proofs of Theorems 4.3 and 4.4 in Keeney and Raiffa [16].) When a $u_v(v(x))$ curve cannot be represented by a uniformly concave, linear, or convex model, then the decisionmaker has a relative risk attitude that varies over the levels of the attribute. The subjects were only relatively risk averse in ten of 29 cases so we can conclude that decisionmakers are not, in general, relatively risk averse. In fact, the subjects displayed different relative risk attitudes for different attributes, as shown in Table VII. For example, Subject A was relatively risk averse for the "grades" attribute but relatively risk neutral for the "gas line minutes" attribute. None of the nine subjects who responded for more than one attribute had the same relative risk attitude across these attributes.

IV. FINDINGS

Two major findings emerge from this analysis. First, the utility function and the measurable value function were not generally the same. In fact, $u(x) = v(x)$ was only an

TABLE VII
RELATIVE RISK ATTITUDES ACROSS ATTRIBUTES¹

Subjects	Attributes			
	Annual Salary Level	Grade in a Class	Bus Trip Minutes	Gas Line Minutes
H	Varies	Varies, near RRP	Varies	
J	RRP	Varies, near RRA	RRA	
K	RRP	Varies	Varies	
L	RRA	RRP	RRN ²	
A		RRA	Varies	RRN
I		RRA	Varies	RRP
F	RRP		Varies, near RRA	
M	RRA		Varies	
B	RRN ²	RRA		RRP

¹ Blank spaces indicate subject did not report preferences on that attribute. Relative risk attitudes are as follows:

RRA relatively risk averse
RRN relatively risk neutral
RRP relatively risk prone
Varies relative risk attitude varies.

² This case is very near linear, but a slightly concave model does marginally better.

acceptable model of the relationship between the functions in three out of 29 cases. Thus decisionmakers will not usually be relatively risk neutral.

Second, $u_v(v(x))$ was not generally concave. The $u_v(v(x))$ models were only concave in ten out of 29 cases. Subjects varied in their relative risk attitudes: ten cases were relatively risk averse, three were relatively risk neutral, and six were relatively risk prone. Ten cases displayed a nonuniform relative risk attitude.

Additionally, a preliminary finding is that decisionmakers appear to have different relative risk attitudes for different attributes. Among the subjects who stated their preferences on more than one attribute, none displayed the same relative risk attitude for all attributes. This finding does not support Dyer and Sarin's [5] conjecture that relative risk attitude may be independent of the attribute on which preferences are assessed. Krzysztofowicz [19] reports a similar result.

Another preliminary finding is that the concave exponential form of $u(v(x))$, which exhibits constant relative risk aversion, appears to be appropriate for some decisionmakers but not for others. This form was the best acceptable model found in five of the 29 cases and was an acceptable model in three additional cases. (In two other cases a very nearly linear concave exponential model provided an acceptable fit.) This finding suggests that the prescriptive theory developed by Bell and Raiffa [2] claiming that constant relative risk aversion is rationally implied by some reasonable assumptions cannot also be used in all cases to model actual preferences descriptively. The results are also in contrast to the finding of a constant relative risk attitude by Krzysztofowicz [19], since only ten of 29 cases were best modeled by the concave or convex exponential form or the linear form. However, in four of the other

cases, the exponential or linear model was close in fit to the best model.

V. SUMMARY AND FUTURE RESEARCH

This study has provided experimental results on the relationship of individuals' single-attribute measurable value functions and their corresponding utility functions. First, the empirical evidence strongly supports the theoretical arguments by Fishburn [12] and Ellsberg [9] that $u(x)$ and $v(x)$ are not generally the same. At the same time the evidence is strongly against the assumption in Beach [1] that $u(x) = v(x)$.

The finding that decisionmakers' relative risk attitudes differ suggests further descriptive and normative research possibilities. The descriptive research possibilities focus on gathering more information on the relative risk attitudes that decisionmakers exhibit. For example, this study was limited to an investigation of unrelated attributes. A logical extension of this work would examine relative risk attitudes for related attributes in a multiattribute setting (see von Winterfeldt *et al.* [26]). In addition, the preferences of a larger number of subjects could be assessed to examine how decisionmakers' relative risk attitudes vary across many attributes. Also, the relationship between the subjects' risk attitudes and relative risk attitudes across many attributes could be examined.

As Bell and Raiffa [2] point out, "our reasoning cannot be said to be 'correct' unless empirical studies suggest close agreement to observed responses or unless many people who wish to behave rationally feel these arguments to be compelling." This study has not empirically supported their concave exponential form of $u_v(v(x))$ (i.e., constant relative risk aversion) as a descriptive model of preferences. However, the form appeared appropriate in some cases, and future studies could present normative arguments which might influence decisionmakers to choose to exhibit constant relative risk aversion.

Finally, there is great potential for research on preference function assessment biases and errors. Hershey, *et al.* [15] and McCord and de Neufville [21] have shown that assessed utility functions depend on the specific assessment procedure used. In this study each preference function was assessed using only one procedure. A further study could use preference functions resulting from multiple assessment procedures. Also, many studies have shown that subjects do not always conform with the principles required by expected utility. In assessing utility functions in the future, choice problems could be formatted in proportional decision matrices which have been shown by Keller [17] to enhance conformity with expected utility. Finally, when a preference model is constructed based upon a subject's actual responses, it is desirable to have the ability to determine statistically if the model provides a close enough fit to the data. For example, we may want to know if the model is sufficiently close to the actual data that any deviation of the model from the data could be attributed to random response error (see Eliashberg and Hauser [8]).

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