OMEGA Int. J of Mgmt Sci., Vol. 13, No. 4, pp. 349–358, 1985
Printed in Great Britain.

0305-0483/85 $3.00 + 0.00

Testing of the 'Reduction of Compound Alternatives' Principle

L ROBIN KELLER

University of California, Irvine, USA

(*Received in October 1984; in revised form December 1984*)

This paper reports an empirical investigation of the effects of three pictorial forms of problem representation on conformance with the Reduction of Compound Alternatives Principle of expected utility theory. The most common form of representation, written problem statements, was compared with three pictorial representations: tubes containing one hundred labeled balls, decision matrices with each column proportional in size to the probability of the corresponding event, and bar graphs. The tubes representation led to fewer violations of the Principle. In addition, when subjects were trained to construct proportional matrices from written problem statements, they exhibited fewer violations than those who received the same problems already formatted in proportional matrices. The results reported here should contribute to the development of a theory of the way people frame decision problems.

1. INTRODUCTION

THE REDUCTION of Compound Alternatives Principle serves as an axiom in von Neumann and Morgenstern's [10] axiomatic development of the expected utility model for guiding rational decision making. Though the Principle is normatively appealing, Kahneman and Tversky [4, 9] have shown that subjects often violate the Principle when given choice problems presented in written problem statements. This paper presents an empirical investigation of the effects of alternative pictorial problem representations on conformity with the Reduction of Compound Alternatives Principle. In addition, the effects of training subjects to transform a written problem statement into a visual representation are described. The effects of pictorial problem representations on the Sure-Thing and Substitution Principles of expected utility theory have been reported previously in Keller [6]. Studies such as these should contribute to the development of a theory of the way people frame decision problems. See Tversky and Kahneman [9] and Fischhoff [2] for insightful discussions of how the framing of decisions can influence choice. Berkeley and Humphreys [1] present an in-depth discussion of the process of structuring decision problems.

First, the Reduction of Compound Alternatives Principle is briefly described in this section. Section 2 contains an investigation of the effects of the four forms of problem representation on conformance with the Principle. Section 3 contains a second investigation of conformance with the Principle in which colored and non-colored tubes representations were compared with written statements of problems containing automobile payoffs. The investigation of the effects of training subjects to construct proportional matrices is in Section 4. Section 5 contains a summary and discussion.

The Reduction of Compound Alternatives Principle serves as an expected utility axiom in von Neumann and Morgenstern [10, Axiom C] and Luce and Raiffa [7, Assumption 2]. Before stating the Principle formally, we look at a simple example. Consider alternative A: [+\$100, 0.25; +\$0, 0.75] which has a 25% chance of gaining \$100 and a 75% chance of nothing. In conformance with the Principle, one should be indifferent between alternative A and compound alternative C, which consists of two stages:

Stage 1: 50% chance of going to Stage 2
50% chance of getting nothing (and not advancing to Stage 2)

Stage 2: 50% chance of gaining \$100
50% chance of getting nothing

Note that the compound alternative C can be 'reduced' to the simple alternative A in which there is a (50%)(50%) = 25% chance of getting \$100 and a 50% + (50%)(50%) = 75% chance of getting nothing.

Reduction of compound alternatives principle

Alternative A: $[a_1, p_1; \ldots; a_r, p_r]$

is indifferent to Compound Alternative C:

$$[A^1, q_1; \ldots; A^n, q_n];$$

where

Alternative A^i is

$$[a_1, p_1^i; \ldots; a_r, p_r^i] \quad \text{for} \quad i = 1, \ldots n;$$

and

$$p_i = q_1 p_i^1 + q_2 p_i^2 + \ldots + q_n p_i^n.$$

The Reduction of Compound Alternatives Principle has also been called the 'No Fun in Gambling' Principle, since a compound alternative which has two or more sequential probabilistic events is not supposed to be more 'fun' (due to heightened suspense) than the corresponding alternative with only one stage of probabilistic events. Fishburn [3] explicitly introduces a utility of gambling term to the expected utility of a risky alternative as a means of allowing 'joy in gambling'. In this paper, we focus on examining means for enhancing conformance with the Principle rather than modifying the expected utility model by relaxing the requirement that the Principle hold.

The Reduction of Compound Alternatives Principle can be tested by the following pair of choice problems:

Original problem \$0/0.75
LP: 75% chance of \$0
25% chance of \$3000
HP: 80% chance of \$0
20% chance of \$4000

Game version of problem \$0/0.75 (g)
Consider the following two-stage game. In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice between:
LP′: 100% chance of \$3000
HP′: 80% chance of \$4000
20% chance of \$0
Your choice must be made before the game starts.

A decision matrix representation of the two problems displays their identical structures. Label the original problem \$0/0.75 since there is a $p = 0.75$ chance of ending up in the first column of the matrix and receiving the common 'sure-thing' outcome of C = \$0. Label the game version \$0/0.75 (g) since its structure is identical except the problem is framed as a two-stage game.

		Probabilities			
		$p = 0.75$	0.20	0.05	
Original problem \$0/0.75	HP	C = \$0	\$4000	\$0	*
	LP	C = \$0	\$3000	\$3000	
Game version of problem \$0/0.75 (g)	HP′	C = \$0	\$4000	\$0	
	LP′	C = \$0	\$3000	\$3000	*
		(end in 1st stage)	(continue to 2nd stage)		

Through examination of the decision matrix above, it can be seen that HP (HP stands for the Higher Payoff option with a chance of \$4000) and HP′ are isomorphic, as are LP (for the Lower Payoff of \$3000) and LP′. Thus, if HP is preferred to LP, HP′ must be preferred to LP′ and vice versa. However, in violation of the Reduction of Compound Alternatives Principle, subjects often prefer HP over LP in the original problem (as indicated by the asterisk) but prefer LP′ over HP′ in the game version. Kahneman and Tversky [4, 9] found violations of the Principle when subjects were presented with written statements of similar problems. Additional tests of the Principle can be constructed by varying the sure-thing outcome *C* and the probability *p* of this sure-thing outcome. For example, a test comparing choices on a pair of problems with a 0.75 chance of the sure-thing payoff \$3000 is represented as \$3000/0.75–\$3000/0.75 (g).

Two similar cognitive processes have been proposed as possible reasons for violations of the Reduction of Compound Alternatives Principle: the *isolation effect* and the *pseudo-certainty effect*. Kahneman and Tversky [4] described the *isolation effect* as occurring when people disregard components that are shared by alternatives, isolating their focus on components which are different. In a sequential game formulation, people may isolate the first stage and focus on the second stage of the game only. Tversky and Kahneman [9] suggested that a decision problem will be framed in this way when (a) there is an event which has identical outcomes for all alternatives, and (b) the probabilities of other outcomes are presented as being conditional on the non-occurrence of this event. This heuristic reasoning process is used to simplify the task of choosing among options. For example, the *isolation effect* can successfully lead to a satisfaction of the Substitution Principle of expected utility. The pattern is termed heuristic because, though it may often lead to a satisfactory choice, it can sometimes be dysfunctional. The decision frame resulting from the *isolation effect* may serve as a catalyst for a violation of the Reduction of Compound Alternatives Principle since focusing on the second stage of the game tends to hide the fact that the original problem and game version of the problem contain isomorphic alternatives.

In the event that there is an alternative in the second stage of the game containing a 100% chance of some outcome, that alternative may be perceived as being certain, even though its occurrence is conditional upon making it to the second stage. In this case, the pseudo-certain alternative may appear relatively more attractive due to the perceived certainty. Tversky and Kahneman [9] defined the *pseudo-certainty effect* to describe this situation, and discussed its presence in decisions about insurance coverage.

2. EXPERIMENT 1: ALTERNATIVE FORMS OF PROBLEM REPRESENTATION

The purpose of this experiment was to evaluate the use of four forms of problem representation on two pairs of problems testing conformance with the Reduction of Compound Alternatives Principle. The two pairs of problems testing the Principle were described in Section 2. The first pair of problems was $0/0.75 and the associated game version, $0/0.75 (g). The second pair of problems was $3000/0.75 and $3000/0.75 (g).

Forms of problem representation

The first form of problem representation was a written problem statement such as:

J: 80% chance of $4000
20% chance of $0

K: 100% chance of $3000

The second form was a picture of balls in tubes. Here problems were represented as a choice between two tubes, each containing 100 labeled marbles as in Fig. 1A. The third form of problem representation was a proportional decision matrix with the width of a column proportional to the probability of the corresponding event as illustrated in Fig. 1B. The last form was a bar graph as illustrated in Fig. 1C, with the height of the bar representing the numerical amount of the payoff.

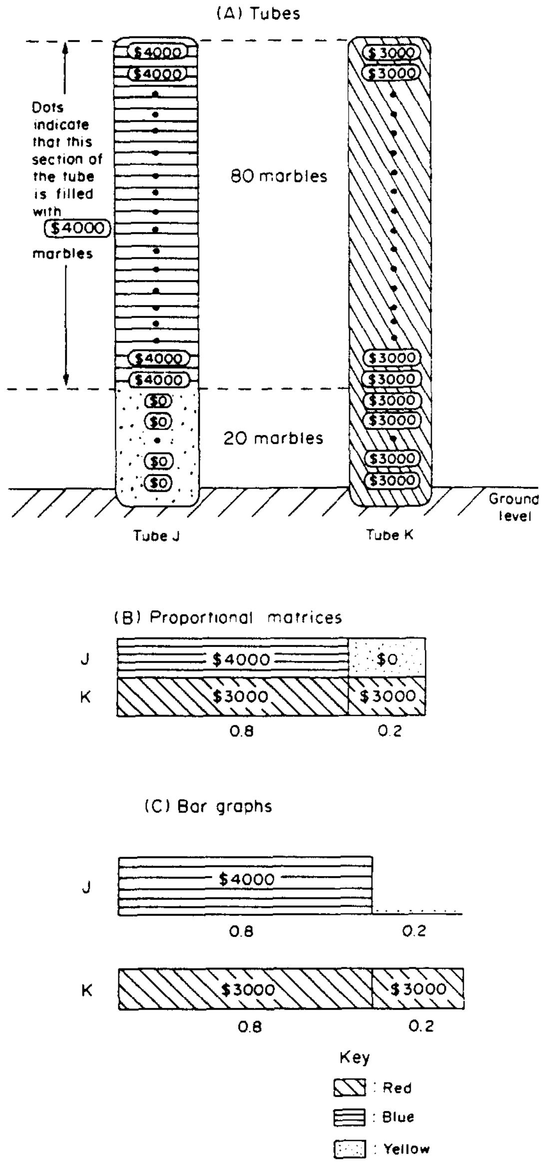

Fig. 1. Forms of problem representation.

Table 1. Use of pictorial problem representations. Percentage of subjects violating principle

Principle Tests	Written statements	Proportional matrices	Graphs	Tubes
C p–C p(game)	$N = 44$	$N = 32$	$N = 26$	$N = 43$
S0 0.75–S0 0.75(g)	45.5%	46.9%	46.2%	45.5%
S3000 0.75–S3000/0.75(game)	43.2%	43.8%	34.6%	18.2%
Aggregate violation percentage	44.4%	45.4%	40.4%	31.9%

Tubes, proportional matrices, and graphs share some important features. In these pictorial representations payoffs were color-coded, thus highlighting the relative probabilities of the outcomes. Two-stage game outcomes were represented in a single tube, matrix row, or graph in an attempt to overcome the *pseudo-certainty* and *isolation effects*. Though two-stage games are commonly represented with decision trees, they were not used in this study because they do not display these desired features.

Method of administration

Undergraduate students enrolled in different sections of lower division UCLA problem solving classes served as subjects. Subjects were not paid, though extra credit was earned in some classes. The four problems were presented with other choice problems in a questionnaire administered during class sessions. The two problems in each Principle test appeared on separate pages of the questionnaire, interspersed with the other problems. Subjects were able to go back to previous pages if they wished as they worked through the questionnaire. (The other choice problems and results are in Keller [6].) The subjects were divided into four groups by class sections, thus due to differing enrolments the group sizes varied. One group of 44 subjects received written problem statements. A group of 32 subjects received proportional matrix representations. Twenty-six subjects received bar graphs and 43 subjects received problems formatted in the tubes representation. When the pictorial forms of problem representation were used, the questionnaire also contained a written version of each choice problem. The questionnaires are in Keller [5].

Each subject received the same written instructions. Subjects were instructed to work entirely on their own. They were told to indicate a preference for each problem; in the unlikely event that they were exactly indifferent between alternatives, they could choose both options. The instructions included an operational interpretation of probability, namely drawing a ball at random from a hat full of 100 colored balls. Finally, subjects were instructed to consider each problem separately, and not to assume their assets were augmented upon completion of each choice problem. Subjects also received detailed written explanations of a sample problem formatted in the representation used in their questionnaire.

Results

As shown in Table 1, among those receiving the tubes representation, 31.9% of the responses violated the Reduction of Compound Alternatives Principle. In contrast, those receiving graphs and matrices had higher violations of 40.4% and 45.4%, respectively. Those receiving

Table 2. Choices on problems by those receiving written statements, tubes, matrices, or graphs

Problem Label: Common outcome C / Probability p of common outcome	Written $N = 44$ % chose higher-payoff option	Proportional matrices $N = 32$ % chose higher-payoff option	Bar graphs $N = 26$ % chose higher-payoff option	Tubes $N = 43$ % chose higher-payoff option
S0/0.75	43.2	50.0	50.0	45.5
S0/0.75(g)	6.8	28.1	26.9	24.2
S3000/0.75	34.1	46.9	50.0	24.2
S3000/0.75(g)	22.7	18.8	34.6	30.3
All four problems	mean = 26.7	mean = 35.9	mean = 40.4	mean = 31.1

Reject hypothesis of independence of mean % choosing HP option from type of problem representation, $\chi^2 = 6.44$, 3 d.f., $\alpha < 0.10$.

only the written problem statements had 44.4% violations. Thus an additional 8.5 to 13.5% of the subjects in the graph, matrix, and written statement groups violated the Principle in comparison with those in the tube group.

The percentage of subjects choosing the higher payoff option on each of the four problems was found to differ significantly among the four problems representation groups, as displayed in Table 2.

Analysis

The First Principle test was:

Original problem	LP:	75% chance of \$0 25% chance of \$3,000
	HP:	80% chance of \$0 20% chance of \$4000

Game version of problem \$0/0.75 (g)

Consider the following two-stage game. In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice between:

LP′: 100% chance of \$3000

HP′: 80% chance of \$4000
20% chance of \$0

Your choice must be made before the game starts.

In the game version, the LP′ option is 'pseudo-certain' since once you've gotten to the second stage, there is a 100% chance of receiving \$3000 if LP′ is chosen. The *pseudo-certainty effect* was expected to lead a greater percentage of the written statements group to choose the LP′ option in the \$0/0.75 (g) problem than the percentages of the pictorial problem representation groups which chose that option. As expected, only 6.8% of the written statements group chose the HP option in the game version, while between 24.2 and 28.1% of the pictorial representation groups chose the HP option, as shown in Table 2. It was also expected that more subjects would choose the higher payoff option in the \$0/0.75 original problem than in the \$0/0.75 (g) game version since the original problem only differs from the game version by having a frame which does not lead to a *pseudo-certainty effect*. In all cases, more subjects chose the HP option in the original problem than in the game version, as expected. It was further expected that the difference in the percentage of subjects choosing the higher payoff option on the \$0/0.75 and \$0/0.75 (g) problems would be greatest for those receiving only written problem statements since the pictorial representations might counteract the *pseudo-certainty effect*. As can be calculated from the data in Table 2, the additional percentage of subjects choosing the HP option in the original problem was greatest (43.2% − 6.8% = 36.4%) with written statements and ranged from 21.3 to 23.1% for the pictorial problem representations.

Though the pictorial problem representations led to the expected shifts in responses, they did not lead to large differences in violations of the Reduction of Compound Alternatives Principle for the first Principle test. The greatest differences in violation percentages occurred for the second Principle test, \$3000/0.75–\$3000/0.75 (g):

Original problem \$30000/0.75	LP:	100% chance of \$3000
	HP:	75% chance of \$3000 5% chance of \$0 20% chance of \$4000

Game version \$3000/0.75 (g)

Consider the following two-stage game. In the first stage, there is a 75% chance to end the game immediately and win \$3000. There is a 25% chance to move into the second stage without winning anything in the first stage. If you reach the second stage, you have a choice between:

LP′: 100% chance of \$3000

HP′: 80% chance of \$4000
20% chance of \$0

Your choice must be made before the game starts.

In the original problem, the *certainty effect* [4] would lead more of the written statement group than those receiving pictures to choose the certain LP option of \$3000. Those receiving the pictorial problem representations had the certain alternative of \$3000 partitioned into three parts to correspond to the HP option (with 75, 5 and 20% chances of occurrence), and the sure-thing payoff of a 75% chance of \$3000 was not colored [6]. This visual partition might counteract the *certainty effect*. Thus, fewer would be expected to choose the LP option than in the written problem statement group. As seen in Table 2 this occurred for matrices and bar graphs, which had an extra 12.6% (i.e. 46.9% of the matrix subjects chose the HP option compared with only 34.1% of the written statement subjects who chose HP) and 15.9% of the subjects choosing the HP option. But surprisingly, for the tubes representation, 9.9% fewer of the subjects' responses were choices of the HP option compared with the written problem statements.

In the game version of the problem, more of those receiving written problem statements were expected to choose the LP′ option than those receiving pictures due to *pseudo-certainty effect*. As expected, tubes and bar graphs led to an extra 7.6 and 11.9% of subjects choosing the HP option. Matrices led to slightly fewer (3.9% of the subjects) choosing the HP option.

Next we examine the pattern of responses for the different groups on both of the problems, \$3000/0.75 and \$3000/0.75 (g). More subjects chose the higher payoff option in the original problem than in the game version for the written, matrix, and graph groups. A tentative explanation for this might be that for these problem representations and this Principle test, the *certainty effect* in the original problem was weaker than the *pseudo-certainty effect* in the game version (or that the certain alternative in the game version was somehow perceived to be certain). The tubes representation exhibited the opposite pattern: fewer subjects chose the higher payoff option in the original problem than in the game version. Perhaps in the tubes representation the *pseudo-certainty effect* was weaker, for this problem.

It is interesting that the tubes representation was best, at least for this problem, in enhancing conformity with the Reduction of Compound Alternatives Principle. As reported in Keller [6], the proportional matrix representation led to substantially fewer violations of the Sure-Thing Principle than did the tubes representation on six principle tests involving problems with automobile payoffs. The Sure-Thing Principle, as described by Savage [8], requires a person to make the same choice in an original problem C/p as he/she does in a new problem constructed by replacing the sure-thing payoff C with C'. The four problems in the current experiment can be combined to form two tests of the Sure-Thing Principle. The first test compares choices in the problem \$0/0.75 with choices in the problem \$3000/0.75. Only 31.8% of the written subject group violated the Sure-Thing Principle, compared with 38.5, 42.4, and 46.9% of the graph, tubes, and matrix groups. It is likely that the heuristic reasoning process behind the *certainty effect* led to greater conformity with the Sure-Thing Principle on this test among the written statements group by increasing the choices of the LP option in the \$3000/0.75 problem. The second principle test compares responses on the game versions of the problems: \$0/0.75 (g) vs \$3000/0.75 (g). In this test, the tubes led to the least violation of 18.2%, followed by matrices and written statements at 25.0% and graphs at 30.8%. In this case, the LP′ option was probably not perceived as certain, so the *certainty effect* heuristic would not have had an impact. A future study could take problems in which matrices have been shown to lead to fewer violations of the Sure-Thing or other principles and construct Reduction of Compound Alternatives tests from the problems.

3. EXPERIMENT 2: TUBES REPRESENTATION WITH AUTOMOBILE PROBLEMS

The results of Experiment 1 suggested that the tubes representation might be effective in leading to conformance with the Reduction of Compound Alternatives Principle in other situations. The purpose of the second experiment was to examine conformance with the Principle when using the tubes representation with problems containing automobile payoffs rather than the monetary payoffs in Experiment 1. A second purpose was to examine the effect of color-coding the payoffs in the tubes representation.

Method of administration

The method of administration was similar to that in the first experiment. Three Reduction of Compound Alternatives Principle tests were constructed from the six choice problems containing automobile payoffs which are displayed in Table 3. Subjects were divided into three groups. Thirty subjects received written problem representations. Forty-three subjects received color-coded tubes representations, like those in Experiment 1. Fifteen subjects received tubes representations which were not color-coded, but were otherwise identical to the colored version. Those receiving pictorial problem representations also received the problems in a written format.

Table 3. Automobile problems

Test number	Problem label	LP option	vs	HP option
1	0/0.66	66% chance of NONE 34% chance of VW		67% chance of NONE 33% chance of PORSCHE
	0/0.66(g)	Consider the following two-stage game. In the first stage, there is a 66% chance to end the game without winning anything, and a 34% chance to move into the second stage. If you reach the second stage you have a choice between: 100% chance of VW		97% chance of PORSCHE 3% chance of NONE
2	VW/0.66	100% chance of VW		66% chance of VW 1% chance of NONE 33% chance of PORSCHE
	VW/0.66(g)	Consider a different two-stage game. In the first stage, there is a 66% chance to end the game immediately and win a VW. There is a 34% chance to move into the second stage (without having won anything in the first stage). If you reach the second stage, you have a choice between: 100% chance of VW		3% chance of NONE 97% chance of PORSCHE
3	0/0.20	80% chance of VW 20% chance of NONE		60% chance of PORSCHE 40% chance of NONE
	0/0.20(g)	Consider the following two-stage game. In the first stage, there is a 20% chance to end the game without winning anything, and an 80% chance to move into the second stage. If you reach the second stage you have a choice between: 100% chance of VW		75% chance of PORSCHE 25% chance of NONE

VW = new convertible Volkswagen.

Results

The results of this experiment are shown in Table 4. Tubes with color-coded payoffs led to 21.7% of the responses violating the Reduction of Compound Alternatives Principle while tubes without added coloring led to violations in an additional 7.2% of the responses. In comparison with the colored tubes group, those receiving written problem statements had an added 9.4% of the responses violating the Principle.

Analysis

Table 5 contains the choices on the six problems by the subjects in the written, colored tube, and non-colored tube groups. In the first Principle test, the results agree with a *pseudo-certainty effect* interpretation. For all three groups more subjects chose the HP option in the original problem 0/0.66 than in the game version in which the second stage contained the lower-payoff choice of a 100% chance of a new Volkswagen.

In the second Principle test, the *certainty effect* was illustrated for all three groups in the original problem VW/0.66 since fewer subjects chose the HP option in the written group than in either of the tube groups. For the game version of the problem, the colored tubes repre-

Table 4. Use of tubes representation. Percentage of subjects violating principle

	Principle Tests	Written Statements	Colored Tubes	Non-Colored Tubes
	C/p–C/p(game)	$N = 30$	$N = 43$	$N = 15$
Set 1	0/0.66–0/0.66(g)	36.7%	27.9%	26.7%
	VW/0.66–VW/0.66(g)	36.7%	25.6%	33.3%
Set 2	0/0.20–0/0.20(g)	20.0%	11.6%	26.7%
Aggregate violation percentage		31.1%	21.7%	28.9%

Set 1: Probability of Porsche = 0.97 × Probability of new Volkswagen.
Set 2: Probability of Porsche = 0.75 × Probability of new Volkswagen.

Table 5. Choices on problems by those receiving written statements, colored tubes, or non-colored tubes

Problem Label: Common outcome C / Probability p of common outcome		Written $N = 30$ % chose higher-payoff option	Colored Tubes $N = 43$ % chose higher-payoff option	Non-Colored Tubes $N = 15$ % chose higher-payoff option
Set 1	0/0.66	93.3	88.4	93.3
	0/0.66	63.3	60.5	66.7
	VW/0.66	53.3	76.7	80.0
	VW/0.66(g)	66.7	65.1	86.7
Set 2	0/0.20	23.3	23.3	13.3
	0/0.20(g)	30.0	20.9	40.0
All 6 problems		Mean = 55.0	Mean = 55.8	Mean = 63.3

Set 1: Probability of Porsche = 0.97 × Probability of new Volkswagen.
Set 2: Probability of Porsche = 0.75 × Probability of new Volkswagen.

sentation resulted in slightly fewer (1.6% of the subjects) choosing the HP option than did the written statements. Thus, it appears that the colored tubes representation either made the *pseudo-certainty effect* stronger or led to subjects noticing there was a certain alternative in the game version.

Next the pattern of responses for the different groups on the two problems VW/0.66 and VW/0.66(g) is examined. (The same pattern occurred for the third Principle test, 0/0.20–0/0.20(g).) Fewer subjects chose the higher payoff option in the original problem than in the game version for the written and non-colored tubes groups. A tentative explanation for this might be that for these problem representations and this Principle test, the *certainty effect* in the original problem was stronger than the *pseudo-certainty effect* in the game version (and/or that the certain alternative in the game version was not perceived to be certain). The tubes representation exhibited the opposite pattern: more subjects chose the higher payoff option in the original problem than in the game version. Perhaps in the tubes representation the *pseudo-certainty effect* was stronger, for this problem. In the tubes representation the identical sure-thing outcomes of a 66% chance of a new convertible Volkswagen were represented as 66 marbles inside a tube and sunk below ground level. These subjects may have isolated their attention to the non-identical 'above-ground' outcomes more fully than those in the other groups.

Two tests of the Sure-Thing Principle can be constructed from choices on four of the problems in this experiment. The first test compares the 0/0.66 problem with VW/0.66. This test was one of six tests of the Sure-Thing Principle reported in Keller [6]. Over all six tests, proportional matrices were found to lead to fewer violations of the Sure-Thing Principle than tubes or written problem statements. However, on this specific 0/0.66–VW/0.66 test, tubes led to only 20.9% violations, compared with 23.8% violations with matrices and the much worse 46.7% with written statements. The second test of the Sure-Thing Principle which can be constructed here compares the responses on the game versions 0/0.66(g) and VW/0.66(g). The average violation percentage on the two tests for the colored tubes group was only 15.1%, while written statements and non-colored tubes led to 31.7% and 30.0% violations.

4. EXPERIMENT 3: EFFECTS OF TRAINING IN PROBLEM STRUCTURING

The purpose of this experiment was to investigate whether proportional matrix structuring training would lead to greater conformity with the Reduction of Compound Alternatives Principle than would passive receipt of pre-structured proportional matrices.

Method of administration

The effects of training in problem structuring were examined for three Principle tests with problems containing monetary payoffs. The first two tests were the same as in Experiment 1,

\$0/0.75–\$0/0.75 (g) and \$3000/0.75–\$3000/0.75 (g). One other test was added, ϕ/0–ϕ/0(g):

Original problem ϕ/0	LP:	100% chance of \$3000
	HP:	20% chance of \$0
	HP:	80% chance of \$4000

Game version ϕ/0(g)

Imagine you must choose between receiving \$3000 or entering two-stage game R. In the first stage of the game, there is a 50% chance of ending the game immediately and winning \$4000. There is a 50% chance of moving into the second stage without having won anything in the first stage. If you reach the second stage, you have a 60% chance of winning \$4000 and a 40% chance of getting nothing.

LP′: take the \$3000 and don't play the game

HP′: play game R

The original problem was displayed in Fig. 1 and is labeled ϕ/0 since there's a 0% chance of getting a sure-thing payoff, and this 'null payoff' is represented by the null set symbol.

There were two groups of subjects. The group of 32 subjects who received questionnaires with the problems formulated in the proportional matrix representation in Experiment 1 served as one group. The other group of 15 subjects received training in matrix-structuring. During the training they were presented with written statements of the automobile-payoff problems and shown how to construct proportional matrices. Later, the trained subjects were given a written version of the questionnaire with the monetary-payoff problems and requested to construct a proportional matrix for each problem prior to making a choice.

Results

The group receiving training in proportional matrix structuring had an average of 28.9% violations of the Reduction of Compound Alternatives Principle compared with 35.4% violations by those receiving pre-structured proportional matrices, as displayed in Table 6. Thus an additional 6.5% of those receiving pre-structured proportional matrices violated the Principle in comparison with those who were trained.

Analysis

At the same time these subjects gave responses to test the Reduction of Compound Alternatives Principle, they were responding on three tests of the Sure-Thing Principle and six tests of the Substitution Principle, as reported in Keller [6]. The trained group did better on all three Sure-Thing tests and on five of the six Substitution tests. The one Substitution Principle test in which pre-structured matrices led to less violation was ϕ/0–\$0/0.10. For the three Reduction of Compound Alternatives tests here, ϕ/0–ϕ/0(g) was the only test that led to pre-structured matrices having fewer violations. Both cases where the prestructured matrices were superior contained the problem ϕ/0 with the certain alternative of a 100% chance of \$3000. This may be an illustration of how heuristic reasoning processes lead sometimes to principle conformance and other times to violations.

5. SUMMARY AND DISCUSSION

This paper reports an empirical attempt to examine the effects of pictorial forms of problem representation on conformance with the Reduction of Compound Alternatives Principle. The tubes representation with color-coded outcomes led to the greatest conformance with the Principle on the problems tested. Those receiving tubes displayed patterns of choices differing from the other groups on the problems \$3000/0.75 and \$3000/0.75(g) as well as the problems VW/0.66 and VW/0.66(g). A tentative explanation is that the *pseudo-certainty* and *certainty effects* [4, 9] carry different weight for different forms of problem representation. In addition, for different pairs of problems, the

Table 6. Effects of training in proportional matrix structuring. Percentage of subjects violating principle

Principle tests	Pre-Structured Proportional Matrices	Training in Proportional Matrix Structuring
C/p–C/p(game)	$N = 32$	$N = 15$
\$0/0.75–\$0/0.75(g)	46.9%	20.0%
\$3000/0.75–\$3000/0.75(g)	43.8%	33.3%
ϕ/0 – ϕ/0(g)	15.6%	33.3%
Aggregate Violation Percentage	35.4%	28.9%

relative strengths of the *pseudo-certainty* and the *certainty effects* may shift. Future studies could examine conformance on an expanded list of problems. Tversky and Kahneman [4, 9] have used United States and Israeli currency and "the number of people who die in an epidemic" as problem outcomes, but their problem structures were similar (with a lower payoff and a higher payoff alternative) to the ones tested here.

Subjects who were trained to transform written problem statements into proportional matrices exhibited greater Principle conformance than those receiving pre-structured proportional matrices. Since those receiving the prestructured tubes representation were found to exhibit the greatest Principle conformance, a further experiment could evaluate training in structuring tubes representations. The tubes representation is more complicated for a decision maker to draw than a matrix, but a computer graphics terminal could ease this burden. In this study, each subject received only one form of problem representation, so a with-in subject comparison was not possible. Another experiment might monitor a subject who is allowed to choose from a 'menu' of problem frames available on a graphics display terminal. It may eventually be possible to construct a decision-style profile for an individual which would specify utility-violating tendencies and means to correct them.

Finally, an implicit assumption in this study is that the Reduction of Compound Alternatives Principle is normatively appealing and that examination of the conformance of actual choices with the Principle is appropriate in a study of decision making. Berkeley and Humphreys [1] discuss the pitfalls of interpreting as evidence of bias the disparity between actual choices and those recommended by a normative principle.

REFERENCES

1. Berkeley D and Humphreys P (1982) Structuring decision problems and the 'bias heuristic'. *Acta Psychol.* **50**, 201–252.
2. Fischhoff B (1983) Predicting Frames. *J. exp Psychol. Learning, Memory, Cognition* **9**(1), 103–116.
3. Fishburn PC (1980) A simple model for the utility of gambling. *Psychometrika* **45**(4), 435–448.
4. Kahneman D and Tversky A (1979) Prospect theory: An analysis of decision under risk. *Econometrica* **47**(2), 263–291.
5. Keller LR (1982) The effects of problem representation on conformity with utility properties—An empirical investigation. Unpublished doctoral dissertation, University of California, Los Angeles, Graduate School of Management, (available from University Microfilms).
6. Keller LR (1985) The effects of problem representation on the sure-thing and substitution principles. *Mgmt Sci.* **31**(4).
7. Luce RD and Raiffa H (1957) *Games and Decisions*. pp. 26–27. John Wiley, New York.
8. Savage L (1954) *The Foundations of Statistics*. John Wiley, New York.
9. Tversky A and Kahneman D (1981) The framing of decisions and the psychology of choice. *Science* **211**(Jan. 30), 453–458.
10. Von Neumann J and Morgenstern O (1947) *Theory of Games and Economic Behavior*, 2nd edn. Princeton University Press.

ADDRESS FOR CORRESPONDENCE: *Professor LR Keller, Graduate School of Management, University of California, Irvine, California 92717, USA*