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Discrete public goods under threshold uncertainty

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Abstract

A discrete public good is provided when total contributions exceed the contribution threshold, yet the threshold is often not known with certainty. I show that the relationship between the degree of threshold uncertainty and equilibrium contributions and welfare is not monotonic. For a large class of threshold probability distributions, equilibrium contributions will be higher under increased uncertainty (e.g., a mean-preserving spread) if the public good's value is sufficiently high. Otherwise, and if another condition on the distribution's mode is met, contributions will be lower. The same result also obtains if a single-crossing condition of the pdfs is met.

JEL classification: C70; D80; H41
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1. Introduction

An important class of public goods is so-called *discrete public goods*. A discrete public good is provided if contributions exceed the required threshold level of contributions; otherwise, no good is provided. Examples of such goods abound: multiple plaintiffs raising funds to achieve a commonly desired judicial ruling, neighborhood residents petitioning a local government to build a public project, and, more dramatically, plotters planning the size of their attempted coup. Because in many of these settings we observe voluntary contributions or participation, it is not surprising that research on discrete public goods focuses on the voluntary contributions

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mechanism. In fact, as shown by Palfrey and Rosenthal (1984) and Bagnoli and Lipman (1989, 1992), the voluntary contributions mechanism also has important efficiency properties: it can implement the first-best outcome when individuals have certain knowledge of the threshold level of contributions needed for provision.

However, there is often not full certainty about the threshold. It might not be known how much money will be needed to complete the project, or coup plotters might not know if their faction will be large enough to successfully take power. Recognizing that threshold uncertainty affects an individual's contribution decision, Nitzan and Romano (1990) and Suleiman (1997) examined the voluntary contributions mechanism when individuals are uncertain about the threshold. They find that an uncertain threshold often results in an inefficient voluntary contributions equilibrium. This may occur because ex post excess contributions will be discarded, or because equilibrium contributions fall short of the threshold.

This paper extends the research in a new direction by examining how the inefficiency caused by the uncertainty about the threshold relates to the degree of uncertainty. This question is of interest for various reasons. From a theoretical perspective, an answer will help us understand the manner in which uncertainty leads to inefficient outcomes, but there are also potential normative implications. If the level of uncertainty can be controlled or influenced, then my results may lend insights into the strategic use of uncertainty by planners or mechanism designers.

While a preliminary guess may posit that the inefficiency worsens as the uncertainty increases (thought of as a mean-preserving spread), I show that this is not the case. Voluntary contributions do not relate monotonically to uncertainty but instead depend on the value of the public good itself. Moreover, by making assumptions about the threshold distribution, we can make more specific claims about the relationship between uncertainty and contributions. The first key result of this paper is that for strictly unimodal and totally feasible threshold distributions, a mean-preserving spread will lead to an increase in voluntary contributions if the value of the public good is sufficiently high. Moreover, with an additional assumption it is also true that an increase in uncertainty will lead to a decrease in contributions if the public good value is sufficiently low. This result follows from the underlying strategic nature of the contribution decision. Given others' contributions, an individual only contributes if her probability of being a pivotal contributor is sufficiently high. With a high public good value, the increase in uncertainty increases the equilibrium probability of being pivotal, while it decreases the equilibrium probability of being pivotal when the value is low. Because the pivot probabilities are related directly to the threshold distribution, in a second key result I show that the claim made above about contribution level changes as uncertainty increases also holds for two threshold distributions if a certain single-crossing property is met; namely, if their pdfs cross only once in the range of contribution levels between the feasible mode with higher mass and the total feasible contributions.

My findings complement earlier research on threshold uncertainty. My analysis differs in three ways from Nitzan and Romano (1990). First, I focus on changes in the threshold distributions under various public good values. Second, I assume each player makes a binary contribution choice, which better represents collective action scenarios that involve participation, in–out, or yes–no decisions, while they assume individuals choose contribution levels from a continuous set. Third, I assume no refunding of contributions that fall short of the threshold. That said, I consider continuous contributions and refunded contributions later in the paper and explain how my findings are not substantively affected. My work also differs from Suleiman

(1997), who assumes a uniform threshold distribution and considers various types of preferences. My analysis applies to a wider range of threshold distributions, and, as I discuss later in the paper, it even can be applied to general threshold distributions.

My work also fits into the broader literature on the effects of various types of uncertainty on collective action. For example, Palfrey and Rosenthal (1988) consider a setting where individuals are uncertain about others' degree of altruism; Palfrey and Rosenthal (1991) examine uncertainty about others' contribution costs; and Menezes et al. (2001) study uncertainty about others' valuations of the public good. These examples of threshold certainty and incomplete information are to be contrasted with my case of threshold uncertainty and complete information. However, in both cases, the lack of certainty leads to inefficient outcomes, and the degree of the inefficiency can depend on the value of the public good. Finally, my work is also closely related to the research on common pool resources with unknown pool size (e.g., Budescu et al., 1995).

This paper proceeds next in Section 2 by presenting the basic model of the public good game. Section 3 examines the game's equilibria and presents the main results. In Section 4, I relate my findings on contribution levels from Section 3 to individuals' welfare. In Section 5, I briefly discuss general threshold distributions, changes in the contribution refund policy, continuous contribution choices, risk aversion, and sequential contributions. Section 6 concludes.

2. Model

Consider a set of expected payoff maximizing players $N=\{1,\ldots,n\}$, $2<n<\infty$. Players have identical strategy sets $S_i=\{0,1\}$. Choosing strategy $s_i=0$ is to be interpreted as not contributing, while choosing $s_i=1$ implies contributing. The cost of contributing one unit is $c>0$, the value of a provided public good is $v>0$, and both are the same for all individuals. The contribution threshold t to provide the public good is chosen from a publicly known distribution cdf F with pdf f s.t. $F(0)=0$. Thus, the probability of providing the public good is $F(\sum_{j=1}^{n} s_j)$, and i's expected payoff given some profile s of contribution choices is $u_i(s)=F(\sum_{j=1}^{n} s_j)v-s_ic$.[1] With n, v, c and F and all of the above commonly known, and assuming the players make their contribution choices simultaneously, we have a well-defined normal form game.

My focus on binary contributions implies that the total number of any contributions in any strategy profile must be an integer. As such, my analysis in Section 3 assumes a discrete threshold distribution. There are two ways to think of this. First, the threshold distribution F is itself a discrete distribution. Such would be the case, for example, if the contribution takes the form of participation and a certain number of participants are required for successful provision. Second, the threshold distribution F is actually continuous, but contributions will only be evaluated at integer values. An example of this would be where contributions are monetary amounts and the threshold can take any value (including decimals), but where each individual can contribute only \$0 or \$1.

If the underlying distribution, call it G, is continuous, then the discrete version of the marginal increase in provision probability as contributions increased by one from $x-1$ to x, denoted $f(x)$, would be defined as $f(x)=\int_{x-1}^{x} G(s)s$. My analysis in Section 3 is then with

[1] This can be thought of as transferable utility, where the contributions equal $c\sum_{j=1}^{n} s_j$ with provision probability is $F(c\sum_{j=1}^{n} s_j)$, and where the units for t are chosen such that we can ignore the c in front of the summation.

regard to the discrete transformation of the underlying continuous distribution, so that the comparison of distributions is a comparison of their discrete transformations.[2]

3. Equilibrium analysis

3.1. Pure equilibria

By the definition of Nash Equilibrium, for any pure equilibrium profile s^* it must be that

$$u_i(1, s^*_{-i}) \geq u_i(0, s^*_{-i}) \quad \text{for any } i \text{ with } s^*_i = 1$$

$$u_i(0, s^*_{-i}) \geq u_i(1, s^*_{-i}) \quad \text{for any } i \text{ with } s^*_i = 0.$$

The first condition implies the following for those that contribute:

$$F\left(\sum_{j\neq i} s^*_j + 1\right)v - c \geq F\left(\sum_{j\neq i} s^*_j\right)v \Rightarrow F\left(\sum_{j\neq i} s^*_j + 1\right) - F\left(\sum_{j\neq i} s^*_j\right) \geq \frac{c}{v}.$$

Notice that the left hand side is the marginal increase in the probability of provision due to i's contribution, and, as such, we can write it the condition as

$$f\left(\sum_{j\neq i} s^*_j + 1\right) \geq \frac{c}{v}. \tag{1}$$

The condition for a non-contributor is

$$F\left(\sum_{j\neq i} s^*_j\right)v \geq F\left(\sum_{j\neq i} s^*_j + 1\right)v - c \Rightarrow \frac{c}{v} \geq F\left(\sum_{j\neq i} s^*_j + 1\right) - F\left(\sum_{j\neq i} s^*_j\right) \Rightarrow \frac{c}{v} \geq f\left(\sum_{j\neq i} s^*_j + 1\right).$$

The number of contributions in equilibrium is of greater interest than which players contribute in equilibrium, so I will treat two equilibria with the same number of contributions as one equilibrium. Denote C^* to be the number of contributors in equilibrium s^*. Notice that in equilibrium s^*, a contributing player believes with probability one that exactly $C^* - 1$ others are contributing, so that the contributing player is pivotal with probability $f\left(\sum_{ji} s^*_j + 1\right)$, which equals $f(C^*)$. A non-contributing player is pivotal with probability $f(C^*+1)$.

Assuming that a player in a pure equilibrium who is indifferent between contributing and not contributing will contribute, the conditions for existence of an equilibrium s^* are:

$$C^* = \begin{cases} 0 & \text{if} \quad f(1) < \frac{c}{v} \\ x \in \{1, \ldots, n-1\} & \text{if} \quad f(x) \geq \frac{c}{v} \text{ and } f(x+1) < \frac{c}{v}. \\ n & \text{if} \quad f(n) \geq \frac{c}{v} \end{cases} \tag{2}$$

[2] This is not too fine a point. I will compare one distribution with one of its mean-preserving spreads, and a spread of G does not imply that the discrete transformation of the spread is a mean-preserving spread the original distribution's discrete transformation. Hence, my discrete analysis only corresponds to discrete transformations.

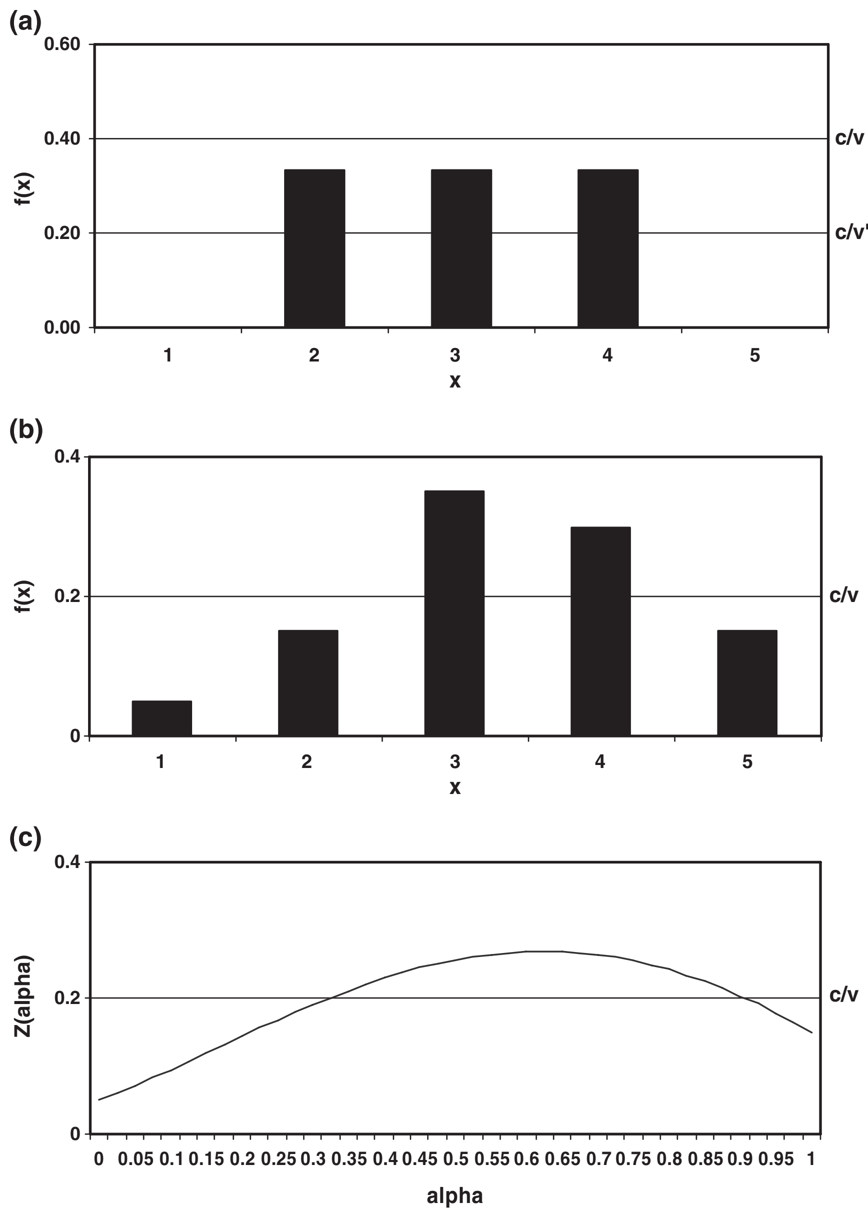

Fig. 1. Finding equilibria graphically. (a) A uniform pdf. (b) A strictly unimodal pdf. (c) A $Z(\alpha)$-curve.

In words, a player contributes in equilibrium if her probability of being pivotal is sufficiently high, i.e., greater than c/v.

Fig. 1(a) illustrates these conditions with the uniform threshold distribution

$$f(x) = \begin{cases} \frac{1}{3}, & x = 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}.$$

Suppose $n=5$. With $c/v=0.4$, as illustrated by the horizontal line at 0.4, we have $f(x)<c/v$ for all x. Thus, the only equilibrium has 0 contributions. However, with $c/v'=0.2$, as illustrated by the lower horizontal line, we have $f(0)<0.2$ and $f(4)>0.2>f(5)$. In this case, there are two equilibria: a trivial one with 0 contributions and a non-trivial one with 4 contributions.

In Sections 3 and 4, I will focus on strictly unimodal (i.e., strictly quasi-concave) threshold distributions, but will consider general threshold distributions in Section 5. Formally, denote F *strictly unimodal* if its pdf is single-peaked and if $f(x)\neq f(x+1)$ for any $f(x)>0$, while the pdf can be flat at any x where $f(x)=0$. Whereas a discrete normal distribution is strictly unimodal, a uniform distribution is not because it is flat. The substantive results in this paper can be obtained without this condition (see Section 5), but this condition is not unrealistic and greatly simplifies the preliminary analysis.

Proposition 1 contains a preliminary characterization of pure equilibria. In it and throughout the rest of the paper, the *feasible mode* m is the mode of the distribution over $x\in\{1,\ldots,N\}$. More formally, $x\in N$ is the feasible mode where $f(x)>f(x')$ for all $x'\in N$, $x'\neq x$ ("$>$" by strict unimodality).

Proposition 1 *(Characterization of pure equilibria). Fix N, c, v, and F, and suppose F is strictly unimodal.*

(a) The unique equilibrium has $C^*=0$ iff $c/v>f(m)$, while the unique equilibrium has $C^*=n$ iff $c/v<f(x)$ for all $x\in N$.(b) Any non-zero equilibrium has $C^*\geq m$.(c) There is at most one non-zero equilibrium with $C^*>0$. Furthermore, if there is more than one equilibrium then there are exactly two equilibria: the trivial equilibrium $C^*=0$ and a non-zero equilibrium $C^*>0$.

Proof. (a) (Necessity) Suppose the unique equilibrium is $C^*=0$. From Eq. (2), it must be that $c/v>f(1)$. From Eq. (2), it also must be the case that there is no $x=1,\ldots,n-1$ such that $f(x)\geq c/v\geq f(x+1)$ or that $f(n)$. Thus $c/v>f(x)$ for all $x=1,\ldots,n$ which implies $c/v>f(m)$. (Sufficiency) $c/v>f(m)$ implies $c/v>f(1)$, which, from Eq. (2), implies an equilibrium $C^*=0$. $c/v>f(m)$ also implies there is no x such that $f(x)\geq c/v$, so from Eq. (2), there cannot be any equilibrium with non-zero contributions.

(Necessity) Suppose $C^*=n$ is the unique equilibrium. From Eq. (2), it must be that $f(n)\geq c/v$. From Eq. (2), it must also be true that $c/v>f(1)$, and that there is no x such that $f(x)<c/v$. Thus, $c/v>f(x)$ for all $x=1,\ldots,n$ which implies $c/v<f(m)$. (Sufficiency) $c/v<f(x)$ for all $x\in N$ implies, from Eq. (2), that there is no x for which $c/v>f(x)$, so the conditions for a non-zero equilibrium or an equilibrium with $C^*=n$ cannot be met. The conditions for $C^*=n$ are met, so that is the unique equilibrium.

(b) $m\leq n$ by the definition of feasible mode m. If $C^*=n$ in an equilibrium, then $C^*=n\geq m$, so $C^*\geq m$.

Now suppose an equilibrium with $0<C^*<n$. By Eq. (2), it must be that $f(C^*)\geq c/v\geq f(C^*+1)$, which means C^* must be in the "downward sloping" region of the pdf. With the pdf's peak at m, it follows that to be in the downward sloping region, it must be that $C^*\geq m$.

(c) Suppose two non-zero equilibria with C^* and $C^{*\prime}$ s.t. $C^{*\prime}>C^*>0$. From Eq. (2), it must be true that (i) $f(C^*)\geq c/v>f(C^*+1)$ and (ii) $f(C^{*\prime})\geq c/v$. Moreover, from (b) it must also be true that $C^*\geq m$ and $C^{*\prime}\geq m$, which implies both C^* and $C^{*\prime}$ are in the downward sloping region of the pdf. Combining (i) with the fact about the downward sloping pdf we have $f(C^*)\geq c/v>f(x)$ for all $x=C^*+1,\ldots,C^{*\prime},\ldots,n$. However, this contradicts (ii). □

Proposition 1 can be illustrated in Fig. 1(b), which displays a strictly unimodal threshold distribution. There is a trivial equilibrium $C^*=0$ since $f(1)<c/v$. That is, if no one else

contributes, then a single contribution is very unlikely to lead to provision, so no individual will contribute. The non-trivial equilibrium is $C^*=4$. Each of the four contributors is willing to make her contribution given that three others are contributing since her probability of being pivotal is higher than c/v. The non-contributor is not willing to contribute given that four others contribute since her probability of being pivotal is less than c/v. Also, notice that for a strictly quasi-concave distribution, a non-zero equilibrium must be to the right of the feasible mode $m=3$ (on the downward-sloping side of the pdf).

Hereafter, I focus on the *non-trivial equilibrium* C^*. From Proposition 1(a), a zero-contribution equilibrium is non-trivial only if c/v is higher than the feasible mode. Otherwise, the non-trivial equilibrium has contributions. As shown later, this non-trivial equilibrium is the Pareto-undominated equilibrium, although it can be inefficient.

I now state the two main theoretical propositions of this paper. The first of these will consider a threshold distribution that is totally feasible. Say that F is *totally feasible* when the public good can be provided with probability 1, i.e., $F(n)=1$.

Proposition 2 *(2nd-order stochastic dominance). Fix N, c, and v, and consider two strictly unimodal threshold distributions F and $\hat{F}$, $F\neq\hat{F}$. Denote C^* and $\hat{C}^*$ the respective non-trivial equilibria. If (i) $\hat{F}$ is a mean-preserving spread of F and (ii) F and $\hat{F}$ are both totally feasible, then there exists a scalar k, $0<k<1$, such that $\hat{C}^*\geq C^*$ if the cost–value ratio $c/v\leq k$. Furthermore, if it is also true that the feasible mode of F is strictly greater than the feasible mode of $\hat{F}$, then there exists a second scalar $k'\geq k$, $0<k'<1$, such that $C^*\geq\hat{C}^*$ if the cost–value ratio $c/v>k'$.*

In words, Proposition 2 claims that if $\hat{F}$ is a mean-preserving spread of F and both are totally feasible, then equilibrium contributions are higher under the spread if the c/v-ratio is sufficiently small, and, with an additional assumption about the feasible modes, contributions are lower under the spread if the c/v-ratio is sufficiently large. Proposition 3 claims that second-order stochastic dominance is not necessary for the claim in Proposition 2. Instead, a sufficient condition is that their pdfs cross only once on their downward sloping sides.

Proposition 3 *(Single-crossing condition). Fix N, c, and v, and consider two strictly unimodal threshold distributions F and $\hat{F}$, $F\neq\hat{F}$ with feasible modes m and m, respectively. Denote C^* and $\hat{C}^*$ the respective non-trivial equilibria. If $f(m)>\hat{f}(\hat{m})$ and f and $\hat{f}$ cross exactly once over $\{m,\ldots,n\}$, then there exists a scalar k, $0<k<1$, such that $\hat{C}^*\geq C^*$ if $c/v\leq k$, and $C^*\geq\hat{C}^*$ if $c/v>k$.*

The proofs of Propositions 2 and 3 will follow directly from a more general result, Lemma 1, about games that differ only in their threshold distributions. In the rest of Section 3.1, I establish Lemma 1 and then prove Propositions 2 and 3.

Because $C^*\geq m$ by Proposition 1(b), we can restrict our attention to that part of the threshold pdf that is between m and n. And we can go one step further when comparing the non-trivial equilibria of otherwise identical games with different threshold distributions. The following corollary to Proposition 1 states that when looking for the equilibrium with higher contributions of the two games, we can restrict our attention to that part of the two distributions that is between the feasible mode with higher mass, denoted by $\tilde{m}$, and n. Specifically, define $\tilde{m}$ as follows. Let m and m be the modes of f and f, respectively. Then,

$$\tilde{m}=\begin{cases} m & \text{if} \quad f(m)\geq\hat{f}(\hat{m}) \\ \hat{m} & \text{if} \quad f(m)<\hat{f}(\hat{m}) \end{cases}.$$

Corollary 1 *(Comparing non-trivial equilibria). Fix N, c, and v, and consider two strictly unimodal threshold distributions F and $\hat{F}$, $F\neq\hat{F}$. Denote C^* and $\hat{C}^*$ the respective non-trivial* *equilibria. (a)* $\hat{C}^* > C^*$ *iff there exists some level of contributions* $x \in \{C^* + 1, \ldots, n\}$*, such that* $\hat{f}(x) \geq c/v$*. (b) If* $\hat{C}^* > C^*$ *then* $\hat{C}^* \geq \tilde{m}$*.*

With our attention now restricted to the right of the feasible mode with higher mass, we look more closely at the behavior of the two distributions from $\tilde{m}$ to n. I will refer to this specific range of feasible contribution levels as the *interior* $I = \{\tilde{m}, \ldots, n\}$. One key condition of interest is when one of the pdfs has a higher interior-right tail, that is, one pdf is greater than the other pdf for all contribution levels from some number x to n. The phrase "interior-right" comes from the fact that the interior-right tail is a subset of I containing the highest values in I. Another key condition is as analog for the interior-left, but this condition will also be defined by the height of the pdfs to the right of the interior-left. After formally defining these conditions, I will illustrate them graphically.

Interior tails conditions. *Consider two strictly quasi-concave distributions* F *and* $\hat{F}$*,* $F \neq \hat{F}$*, and the resulting interior* $I = \{\tilde{m}, \ldots, n\}$*. (a) Say that* $\hat{f}$ *has a fatter interior-right tail than* f *if there exists an* $x \in I$*, such that* $\hat{f}(x') \geq f(x')$ *for all* $x' \in \{x, \ldots, n\}$*. The fatter interior-right tail is the range* $I_R = \{x', \ldots, n\}$*. (b) Say that* f *has a fatter interior-left tail than* $\hat{f}$ *if there exists an* $x \in I$*, such that (i)* $f(x') \geq \hat{f}(x')$ *for all* $x' \in \{\tilde{m}, \ldots, x\}$*, and (ii) if* $\hat{m} > m$ *with* $\hat{f}(\hat{m}) > f(\hat{m})$*, then* $mf(x') > \hat{f}(\hat{m})$ *for all* $x' \in \{\tilde{m}, \ldots, x\}$*. The fatter interior-left tail is* $I_L = \{\tilde{m}, \ldots, x\}$*.*

Fig. 2(a) illustrates these conditions with smooth pdfs drawn for clarity. It shows the case where both a fatter interior-right tail I_R and a fatter interior-left tail I_L exist. Notice that these tails do not necessarily meet. Fig. 2(b) shows that we cannot distinguish I_R from I_L when one pdf is always above the other in I. Fig. 2(c)–(d) show that the tails meet when the pdfs cross once in the interior. The reason for condition (ii) in a fatter interior-left tail is that we want to know when the non-trivial equilibrium C^* will be in that interior-left tail *and* when $C^* \geq \hat{C}^*$. This idea is illustrated on Fig. 2(c). Notice that if $c/v = k_1$, then $\hat{C}^*$ is higher than $C^* = x_1$ even though $f(x_1) > \hat{f}(x_1)$. This is because x_1 is to the left of the feasible mode of F.

We can use these figures to demonstrate the two main propositions of the paper and bring us closer to Lemma 1. Notice that the k and k' in Fig. 2(a) satisfy the k and k' in Proposition 2. F 2nd-order stochastically dominates F, and F has a higher feasible mode. We see that if $c/v \leq k$ then $\hat{C}^* \geq C^*$, whereas if $c/v > k'$ then $C^* \geq \hat{C}^*$.

Lemma 1 *(Fatter interior tails and pure equilibria). Consider two games that are identical except for their strictly unimodal threshold distributions* F *and* $\hat{F}$*,* $F \neq \hat{F}$*. Denote* C^* *and* $\hat{C}^*$ *the respective non-trivial equilibria.*

(a) If $\hat{f}$ *has a fatter interior-right tail than* f*, then there exists a scalar* k*,* $0 < k < 1$ *such that* $\hat{C}^* \geq C^*$ *if the cost–value ratio* $c/v \leq k$ *.*

(b) If f *has a fatter interior-left tail than* $\hat{f}$*, then there exists a scalar* k'*,* $0 < k' < 1$*, such that* $C^* \geq \hat{C}^*$ *if the cost–value ratio* $c/v > k'$*.*

Proof. (a) Suppose F and $\hat{F}$ are strictly unimodal such that $\hat{f}$ has a fatter interior-right tail $I_R = \{x, \ldots, n\}$ than f. Choose k such that $k = f(x)$, and pick any $c/v \leq k$. Because f is strictly unimodal and downward sloping over I_R, the highest contribution level $x' \in I_R$ such that $f(x') \geq c/v$ must also have $f(x'+1) < c/v$, which implies $C^* = x'$ by Eq. (2). Since $\hat{f}$ has higher mass than f for each contribution level in I_R by the definition of the fatter interior-right tail, it must be true that $\hat{f}(C^*) \geq f(C^*)$. Using Eq. (2) again implies $\hat{C}^* \geq fC^*$.

(b) Suppose F and $\hat{F}$ are strictly unimodal such that f has a fatter interior-left tail $I_L = \{\tilde{m}, \ldots, x\}$ than $\hat{f}$. Choose $k' = f(x)$, and pick any c/v, $f(\tilde{m}) < c/v < k'$. Because $\hat{f}$ is strictly unimodal and

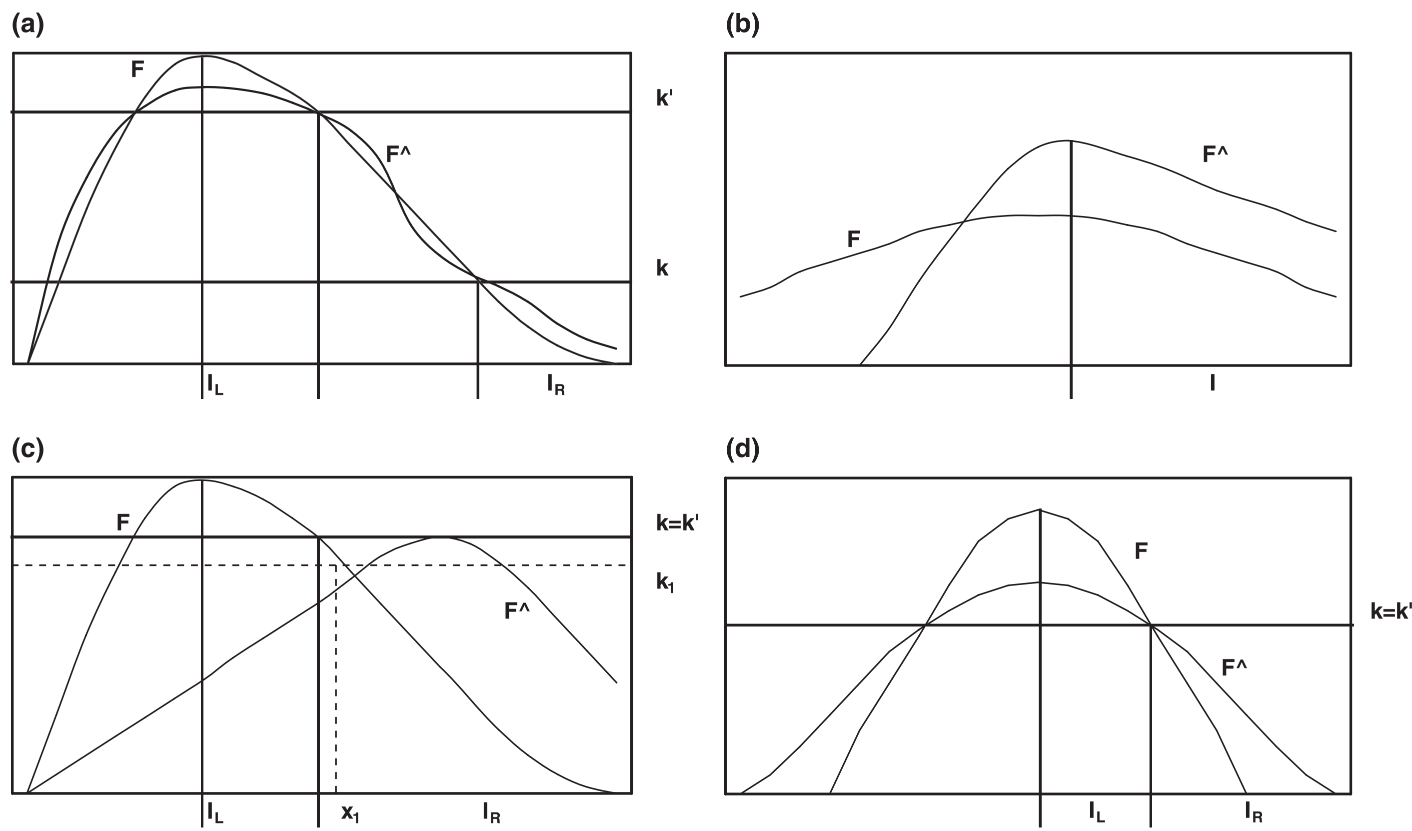

Fig. 2. Illustrations of interior tails. (a) Interior tails exist but do not meet. (b) One pdf above the other. (c) Interior tails meet under single-crossing. (d) A simple mean-preserving spread.

downward sloping over I_L, the highest contribution level $x' \in I_L$ such that $\hat{f}(x') \geq c/v$ must also have $\hat{f}(x'+1) < c/v$, which implies $C^* = x'$ by Eq. (2). Since f has higher mass than $\hat{f}$ for each contribution level in I_L by the definition of the fatter interior-left tail, it must be true that $f(C^*) \geq \hat{f}(C^*)$. Using Eq. (2) again implies $\hat{C}^* \geq C^*$.

Now pick any $c/v > f(\tilde{m})$. Since $\tilde{m}$ is the feasible mode with highest mass, if $c/v > f(\tilde{m})$, then by Eq. (2), the non-trivial equilibrium under f is $C^* = 0$, and under $\hat{f}$ it is $\hat{C}^* = 0$. Thus, $C^* = \hat{C}^*$. □

Lemma 1(a) says that contributions will be higher under the distribution with the fatter interior-right tail if c/v is sufficiently small, and Lemma 1(b) says that contributions will be higher under the distribution with fatter interior-left tail if c/v is sufficiently large. The reason is that the fatter tail implies a higher probability of being pivotal at the contribution levels in the tail. We can now prove Propositions 2 and 3.

Proof of Proposition 2. F 2nd-order stochastically dominates $\hat{F}$ implies that $\sum_{x=0}^{x'} \hat{F}(x) \geq \sum_{x=0}^{x'} F(x)$ for all $x' \in N$. Total feasibility and same means together imply that $\sum_{x=0}^{n} \hat{F}(x) = \sum_{x=0}^{n} F(x)$ (see Laffont (1989)). Subtracting the first condition from the second condition yields

$$\sum_{x=0}^{n} \hat{F}(x) - \sum_{x=0}^{x'} \hat{F}(x) \leq \sum_{x=0}^{n} F(x) - \sum_{x=0}^{x'} F(x) \Rightarrow \sum_{x=x'+1}^{n} \hat{F}(x) \leq \sum_{x=x'+1}^{n} F(x).$$

This last equation says that, starting from n and moving to the left on the graph of the cdfs, when F and $\hat{F}$ first separate, $\hat{F}$ must be below F. This implies that $\hat{f}$ must have a fatter interior-right tail than f. (Notice that if F and $\hat{F}$ do not separate in interior I, then $\hat{f}$ and f have identical interiors, which means that $\hat{f}$ has a fatter interior-right tail by the weakness.) Since $\hat{f}$ has a fatter interior-right tail, invoking Lemma 1 establishes that there exists a k that satisfies the first claim in Proposition 2.

If the feasible mode of f has mass strictly greater than the feasible mode of $\hat{f}$ then it follows that f has a fatter interior-left tail. Invoke Lemma 1 to establish that there exists a k' as in the second claim in Proposition 2. □

The intuition for Proposition 2 is straightforward. Spreading the distribution pushes probability mass to the right part of the tail, and the total feasibility restriction means that this mass will stay in the feasible region. With more mass in the interior-right tail, the probability of being pivotal is higher at the high levels of x. Alone, this is not enough to ensure that contributions will be higher in the game with the spread probability. If the cost–value ratio c/v is too high, then the mass increase on the right will not be enough, and there might be a drop in contributions. This is seen in Fig. 2(a) when $c/v > k'$. The role of the k or k' is to push us sufficiently far enough into the tail of the interior.

Notice that total feasibility is sufficient but not necessary. What is necessary in this case of a mean-preserving spread is that enough mass is spread to the interior-right. In other words, all we need is a fatter interior-right tail (Lemma 1). Proposition 3 demonstrates this point (because it does not assume total feasibility) while making another claim about an implication of the single-crossing property.

Proof of Proposition 3. The claim assumes that $I = \{m, \ldots, n\}$ (i.e., $\tilde{m} = m$). Suppose the pdfs cross at $x \in \{m, \ldots, n\}$, so that $f(x') \geq \hat{f}(x')$ for all $m \leq x' < x$, and $f(x'') \leq f(x'')$ for all $x \leq x'' \leq n$. It follows then that $\hat{f}$ has a fatter interior-right tail and f has a fatter interior-left tail. It remains to show that $k = k'$ in Lemma 1.

If $f(\hat{m}) \geq \hat{f}(\hat{m})$ then by strict quasi-concavity, f has a fatter interior-left tail from m to $x+1$ and $\hat{f}$ has a fatter interior-right tail from x to n. These two tails meet each other, so $k=k'$ in Lemma 1, thus satisfying the claim. Now suppose that $f(\hat{m})<\hat{f}(\hat{m})$ (akin to Fig. 2(b)). Set $k=\hat{f}(\hat{m})$, and then find where f crosses k. For any $c/v \leq k$ we satisfy Lemma 1(a), and for any $c/v>k$, we satisfy Lemma 1(b). Thus $k=k'$ in Lemma 1. □

Proposition 3 applies for a wide variety of threshold distributions. For example, many monotone mean-preserving spreads will meet this single-crossing condition, such as shown in Fig. 2(d). The class of uniform threshold distributions also meets this single-crossing condition.

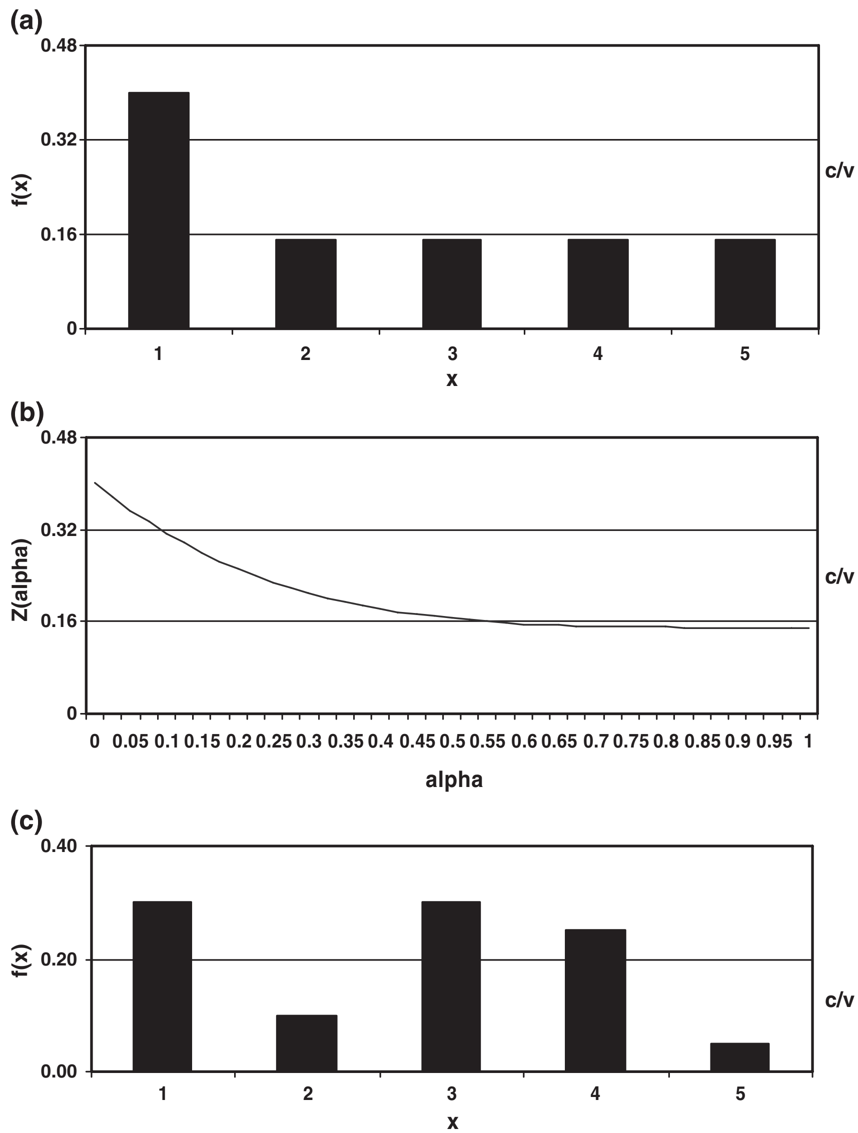

Fig. 3. Other distributions. (a) $f(x)$ for Proof of Proposition 4(b). (b) $Z(\alpha)$-curve for (a). (c) A multi-modal distribution.

3.2. Mixed equilibria

To examine mixed strategies, let α_i be the probability that i chooses $a_i=1$, and no longer assume a player contributes if indifferent between contributing and not contributing. Similar logic is used to examine mixed equilibria as was used for pure equilibria, but there is one important difference. While the results for pure equilibria come from looking at fatter interior tails of the probability distributions, the results for the mixed equilibria come from looking at fatter interior tails of *transformations* of the probability distributions.

I restrict my attention to symmetric equilibria in which $\alpha=\alpha_i=\alpha_j$, for all $i,j\in N$. As before, player i's best response is to contribute only if her probability of being pivotal is higher than c/v. Given that all $j\neq i$ choose $\alpha_j=\alpha$, i's probability of being pivotal, denoted $Z(\alpha)$, is

$$Z(\alpha)\equiv \sum_{x=1}^{n}\binom{n-1}{x-1}\alpha^{x-1}(1-\alpha)^{n-x}f(x).$$

Since a player must be indifferent between contributing and not contributing to be willing to mix, any mixed equilibrium must have α such that $Z(\alpha)=c/v$.

The transformation of the probability distribution of interest is the $Z(\alpha)$-curve, which maps the probability player i is pivotal given that all others are mixing at rate $\alpha\in[0,1]$. Fig. 1(c) illustrates this curve for the pdf in Fig. 1(b). There are three symmetric equilibria: $\alpha^*=0$, $\alpha^*=0.32$ and $\alpha^*=0.91$. Symmetric equilibria can only occur at three places on the figure: at the origin if the $Z(\alpha)$-curve is less than c/v at $\alpha=0$, at a place where the $Z(\alpha)$-curve intersects the c/v-line, and at $\alpha=1$ if $Z(1)>c/v$. This last possibility would happen in Fig. 1(b) if $c/v<0.15$.

Equilibria at 0 and 1 have a nice stability property: an ε increase in α from 0 would drive contributions back down to zero, and an ε decrease in α from 1 would drive contributions back to one. Strictly mixing equilibria only share this property if the slope of the $Z(\alpha)$-curve is downward sloping where it crosses the c/v-line. In Fig. 1(c), the equilibrium at 0.32 is not stable, but the one at 0.91 is stable. The stable symmetric equilibria have qualitative properties similar to the pure equilibria: they occur where the distribution (or its $Z(\alpha)$-curve transformation) crosses the c/v-line from above. We take advantage of this fact in the propositions and corollaries for symmetric equilibria.

This stability notion coincides with the concept of evolutionarily stable strategies (ESS) (see Samuelson (1997)): i is at least as better off playing α than playing the perturbed strategy given that the others play α, and if i is indifferent to playing the perturbed strategy given the other play α, then i is strictly better off playing α than playing the perturbed strategy when all others play the perturbed strategy.

I now use this stability concept to state Proposition 1A, which is the symmetric, mixed equilibrium analog to Proposition 1. In so doing, I refer to strictly unimodal $Z(\alpha)$-curves. Although these curves are not threshold distributions per se, I use the term to refer to the single-humped shape of the curve. However, this is not without justification since a distribution that is strictly unimodal over the feasible range $\{1,\ldots,n\}$ will often produce a strictly unimodal $Z(\alpha)$-curve.

Proposition 1A *(Characterization of symmetric, mixed equilibria). Fix N, c, v, and F, and suppose Z(α) is strictly unimodal.*

(a) The unique equilibrium is α= 0 iff c/v is strictly greater than the maximum of the Z(α)-curve. The unique equilibrium is α= 1 iff c/v is strictly less than Z(α)-curve for all α∈[0, 1].*

(b) Any stable equilibrium with $\alpha^*>0$ has α^* (weakly) to the right of the mode of the $Z(\alpha)$-curve.

(c) There is at most one stable equilibrium with $\alpha^*>0$. Furthermore, if there is more than one stable equilibrium then there are exactly two stable equilibria: the trivial equilibrium $\alpha^*=0$ and a non-trivial equilibrium $\alpha^*>0$.

I omit the proof since it follows by using exactly the same logic as that used for Proposition 1 except the $Z(\alpha)$-curve is used instead of the pdf.

I now focus now on non-trivial symmetric equilibria that are stable, and there are good reasons for this focus. First, the ESS concept's stability properties suggest that such strategies are more likely to be observed. Second, ESS can arise out of many dynamic processes which again suggests they are more likely to be observed. Third, symmetric mixed ESS will exhibit comparative static properties that are qualitatively similar to the asymmetric pure equilibria thereby giving added justification to the comparative static predictions of these equilibria.

The analog to the non-trivial pure equilibrium C^* is the non-trivial stable and symmetric equilibrium α^*. Lemma 1 can be restated as Lemma 1A in terms of the fatter interior tails of the $Z(\alpha)$-curves.

Lemma 1A *(Fatter interior tails and symmetric equilibria). Consider two games identical except for their threshold distributions F and $\hat{F}$, $F\neq\hat{F}$. Denote α^* and $\hat{\alpha}^*$ the respective symmetric and stable non-trivial equilibria, and denote their pivotalness curves as $Z(\alpha)$ and $\hat{Z}(\alpha)$, respectively.*

(a) If the $\hat{Z}(\alpha)$-curve has a fatter interior-right tail than the $Z(\alpha)$-curve, then there exists a scalar k, $0<k<1$, such that $\hat{\alpha}^*\geq\alpha^*$ if the cost–value ratio $c/v\leq k$.

(b) If the $Z(\alpha)$-curve has a fatter interior-left tail than the $\hat{Z}(\alpha)$-curve, then there exists a scalar k', $0<k'<1$, such that $\alpha^*\geq\hat{\alpha}^*$ if the cost–value ratio $c/v>k'$.

Again, the proof follows using the exact same logic as for Lemma 1 but instead using the $Z(\alpha)$-curve transformation instead of the pdf.

A 2nd-order stochastic dominance relationship between threshold distributions F and $\hat{F}$ does not necessarily imply a 2nd-order stochastic dominance between the $Z(\alpha)$- and $\hat{Z}(\alpha)$-curves. However, it will often imply that the $\hat{Z}(\alpha)$-curve has a fatter interior-right tail, and it is the fatter interior-right tail that matters here since it is what yields the existence of k. For example, consider two uniform distributions with identical means but different variances. The smaller variance distribution's pdf and $Z(\alpha)$-curve will have a fatter interior-right tail when compared to the other distribution. Although the k when comparing the pdfs will not necessarily be the same k obtained when comparing $Z(\alpha)$-curves, there will be a k in each instance. The same holds for many other mean-preserving spreads.

Similarly, when m is strictly higher than $\hat{m}$, the mode of the $Z(\alpha)$-curve will often have a higher mass than the mode of the $\hat{Z}(\alpha)$-curve. This will result in a fatter interior-left tail for the $\hat{Z}(\alpha)$-curve, which is sufficient for the existence of the k'.

4. Efficiency

To consider the comparative efficiency of equilibria under different threshold distributions, I use a standard notion of welfare as the sum of expected utilities. That is, let $W(C)=nF(C)v-Cc$ be the welfare generated when C contributions are given under distribution $F(\cdot)$.

Proposition 4 *(Efficiency). Fix N, c, v, and F, and assume F is strictly quasi-concave.*

(a) The non-trivial pure equilibrium C is Pareto-undominated in the class of pure equilibria, and C* is inefficient if* $C^*<n$ *and* $c/vn<f(C^*+1)<c/v$. *(b) The symmetric and stable non-trivial equilibrium* α^* *is generically inefficient, but it can yield higher expected welfare than the non-trivial pure equilibrium C*.*

Proof. (a) From Proposition 1(c), we know that if there is more than one pure equilibrium, then one is $C^{*\prime}=0$ while the other is, say, $C^*>0$. The expected welfare of $C^{*\prime}=0$ is 0. Since it must be true that $f(C^*)\geq c/v$, it must also be true that $F(C^*)\geq c/v$. This implies that the expected welfare of C^* is

$$nF\left(C^*\right)v-C^*c\geq n\frac{c}{v}v-C^*c=\left(n-C^*\right)c,$$

which is weakly greater than 0. Thus, C^* is Pareto-undominated.

Consider the second claim in (a). Let $C^*\in\{1,\ldots,n-1\}$. By Proposition 1, C^* is to the right of the feasible mode. By strict quasi-concavity, $f(C^*)\geq c/v>f(C^*+1)\geq f(C^*+k)$ for all $1<k\leq n-C^*$. This means that the largest marginal welfare gain to be had by an increase in one contribution is from C^* to C^*+1. Welfare is higher under C^*+1 when $W(C^*+1)>W(C^*)$, where $W(C^*+1)$ is the welfare generated if a non-contributor in the equilibrium was to switch to contributing. Doing the algebra shows this to be equivalent to $f(C^*+1)>c/vn$. It follows that the C^* is inefficient when $c/vn<f(C^*+1)<c/v$.

(b) See discussion below. □

As is common in public good games, inefficiencies arise because the marginal gain to an individual from contributing is different from the marginal social gain from that same contribution. This difference comes from the welfare function accounting for all players' marginal gains instead of just one individual's marginal gain. This inefficiency does not arise when $f(C^*)<c/vn$, $C^*=n$, or when $F(C^*)=1$. Notice that this implies that the non-trivial equilibrium C^* is efficient when the threshold is known with certainty—a fact already established by Palfrey and Rosenthal (1984). Their result is thus a special case of the more general result in Proposition 4(a).

That mixed equilibria are generically inefficient, as stated in Proposition 4(b), is trivial. That the symmetric equilibrium can yield higher expected welfare than the pure equilibrium is illustrated by an example. Consider the distribution depicted in Fig. 3(a) and its associated $Z(\alpha)$-curve depicted in Fig. 3(b). With $n=5$, $c=0.16$, and $v=1$, the unique pure equilibrium is $C^*=1$, which yields $W(C^*)=(5)(0.4)(1)-(1)(0.16)=1.84$. The unique symmetric equilibrium is $\alpha^*\simeq 0.55$ and yields expected welfare

$$\begin{aligned}W\left(\alpha^*\right)&=\sum_{x=1}^{5}Pr\left[C=x|\alpha^*\right]W(x)=Pr[C=1]W(1)+\cdots+Pr[C=5]W(5)\\&=\left[\binom{5}{1}\left(\alpha^*\right)^1\left(1-\alpha^*\right)^4\right]W(1)+\cdots+\left[\binom{5}{5}\left(\alpha^*\right)^5\left(1-\alpha^*\right)^0\right]W(5)\simeq 2.76,\end{aligned}$$

which is much higher than the welfare in the pure equilibrium. The symmetric equilibrium has higher welfare because the expected contributions are much higher than under the pure equilibrium. This is seen in Fig. 3(a)–(b), where $f(x)$ crosses the c/v-line at a low contribution level, while the $Z(\alpha)$-curve crosses the c/v-line at about 0.55, which yields an expectation of more than 2.5 contributions. In this example, the higher contributions lead to a higher probability of provision that more than offsets the decline in welfare due to greater total contribution cost, and it occurs because the pdf is lower than the c/v-line for most high contribution levels, but only just below it so that the $Z(\alpha)$-curve is above the c/v-line for many higher symmetric contribution levels. This possibility, that welfare can be higher under the mixed equilibrium than under the pure equilibrium, stands in contrast to case of complete certainty examined by Palfrey and Rosenthal (1984), where mixed equilibria can only yield lower expected welfare than mixed equilibria, and it implies that welfare can actually be higher without formal coordination when there is threshold uncertainty.

Because contributions can increase due to an increase in uncertainty, welfare can be higher under an increase in uncertainty. Again, suppose that the initial distribution has a right tail above C^* that is below c/v but above c/vn from C^*+1 to n or close to n. A widening of uncertainty that drives up the right tail will increase contributions, and if the increase in the probability of provision is sufficient then there will be an increase in expected welfare.

5. Other considerations

5.1. General threshold distributions

I have worked out the analogs to the main claims for when the threshold distributions are not restricted by strict quasi-concavity. The added complication is the non-uniqueness of non-trivial equilibrium. This can be seen in Fig. 3(c), which displays a distribution with multiple modes. Using the conditions from Eq. (2), we can identify two non-zero equilibria: $C^*=1$ and $C^*=4$. In each case, the pdf is downward sloping where it crosses c/v.

How do my earlier results apply to this situation? One approach is to consider the equilibrium with the highest level of expected contributions. Doing so allows us to do the same analysis as before on this high-contribution equilibrium. For pure equilibria, this high-contribution equilibrium is the Pareto-undominated equilibrium, and Lemma 1 and Propositions 2 and 3 can be restated exactly word for word substituting only “high-contribution equilibrium” in place of “non-trivial equilibrium.” The analysis will also be similar for symmetric equilibria.

5.2. Refunds

Palfrey and Rosenthal (1984), Bagnoli and Lipman (1989, 1992), and Nitzan and Romano (1990) consider a version of this game with refunds such that if the total contributions fall short of the (realized) threshold, then contributions are refunded. My primary conclusions still hold in this scenario.

Now, player i contributes given s^*_{-i} if

$$F\left(\sum_{j\neq i} s_j^* + 1\right)(v-c) \geq F\left(\sum_{j\neq i} s_j\right)v \Rightarrow F\left(\sum_{j\neq i} s_j^* + 1\right) - F\left(\sum_{j\neq i} s_j^*\right) \geq F\left(\sum_{j\neq i} s_j^* + 1\right)\frac{c}{v} \Rightarrow f\left(\sum_{j\neq i} s_j^* + 1\right) \geq F\left(\sum_{j\neq i} s_j^* + 1\right)\frac{c}{v}. \tag{3}$$

The difference between Eqs. (3) and (1) is that now the right hand side has decreased. That is, i’s probability of being pivotal necessary to contribute is lower than before. This occurs because there is now no fear of a “lost cause.” Whereas before if i contributes and the threshold is not met, she loses her contribution. Now she only loses her contribution if the threshold is met.

To identify the equilibrium on the figure, we must now modify the c/v-line by weighting it by the cdf at each contribution level, thus accurately representing the right hand side of Eq. (3). Because the cdf-weight is less than or equal to 1, doing so yields a modified c/v-curve that (weakly) must be below the c/v-line at any contribution level. Since the pdf does not change and since the equilibrium is now identified as where the pdf and modified c/v-curve intersect, the downward shift in the modified c/v-curve can only increase contributions.

Moreover, because distributions with fatter tails, e.g., mean-preserving spreads like $\hat{F}$ in Proposition 2, have lower cdfs at higher contribution levels, their modified c/v-curves shift down more at high contribution levels. This means that the potential for contributions to go up at low c/v-values is larger for distributions like mean-preserving spreads. Thus, when comparing a distribution F with its mean-preserving spread $\hat{F}$, the primary claim of Proposition 2 will still hold.

5.3. Continuous contributions

Bagnoli and Lipman (1989, 1992) and Nitzan and Romano (1990) allow individuals to make continuous contributions, which can be likened to monetary contributions, unlike binary contributions, which can be thought of as participation decisions. To compare the binary contributions equilibrium with the continuous contributions equilibrium, we must consider a continuous threshold distribution $G(x)$, where $x \in [0, \infty)$.

The equilibrium for the continuous contributions case is directly analogous to that of the binary case. Given some profile of strategies s, including her own strategy s_i, player i contributes $\varepsilon > 0$ more if

$$G\left(\sum_{j=1}^{n} s_j + \varepsilon\right)v - (s_i + \varepsilon)c \geq G\left(\sum_{j=1}^{n} s_j\right)v - s_i c \Rightarrow \left(G\left(\sum_{j=1}^{n} s_j + \varepsilon\right) - G\left(\sum_{j=1}^{n} s_j\right)\right)v$$
$$\geq \varepsilon c \Rightarrow \frac{G\left(\sum_{j=1}^{n} s_j + \varepsilon\right) - G\left(\sum_{j=1}^{n} s_j\right)}{\varepsilon} \geq \frac{c}{v}. \tag{4}$$

Letting $\varepsilon \rightarrow 0$, the right hand side of Eq. (4) is exactly the derivative of G at $\sum_{j=1}^{n} s_j$. Thus, the condition for contributing becomes $g\left(\sum_{j=1}^{n} s_j\right) \geq \frac{c}{v}$, where g is the continuous pdf of cdf G. On a graph, the continuous case equilibrium is again found where the pdf and c/v-line intersect on the downward sloping side of the pdf.

The main results of the paper will all hold in this continuous contributions case. Defining the interior tails conditions on the continuous pdf yields version of Lemma 1 for continuous contributions. This, in turn, can be used to establish continuous contributions versions of Propositions 2 and 3. The same logic holds as before because nothing has changed in the strategic aspect of the contribution decision.

5.4. Risk aversion

If players are risk averse then the free-rider effect (the worry about donating a redundant contribution) diminishes while the lost-cause effect (the worry about contributing to a lost-cause) amplifies. A qualitative result similar to Lemma 1 will hold, but there is an important difference. The contribution decision rule will not compare an individual's probability of being pivotal with c/v. Instead of drawing a horizontal c/v-line, there will be a curve that varies by contribution level. On the graph of the pdf, this curve will be decreasing over the domain of contribution levels with $f(x)>0$, and its slope and shape will depend on the size of the risk aversion. Under extreme amounts of risk aversion, the slope becomes more negative and the whole curve shifts up. With a change in uncertainty from F to $\hat{F}$, the curve will also change. For the analog to Lemma 1(a), our definition of the fatter interior-right tail will have to consider not just the comparison of pdfs but also the comparison of these curves.

5.5. Sequential contributions

Since there is no private information in this game, there are not the normal informational issues involved in comparing simultaneous and sequential equilibria. Sequential moves only allow players to condition on observed behavior. This observation will matter when comparing a mixed equilibrium to a sequential equilibrium, but any pure, non-trivial equilibrium with contributions of the simultaneous game is an equilibrium of any sequential move game.[3] Using backward induction, the player who moves at period t will contribute only if she is pivotal, as will the player who moves at period $t-1$, and so on. The subgame perfect equilibrium has the last C^* players contributing and the first $n-C^*$ players not contributing, where $f(C^*)\geq c/v\geq f(C^*+1)$. In short, the sequential order eliminates the trivial equilibrium, but the main results about the non-trivial equilibrium from Section 3.1 will still apply. Hence, the focus on simultaneous contributions in this paper is not missing other important strategic issues (other than timing) that would arise in a sequential move game.

6. Conclusion

For high-valued public goods, a widening of threshold uncertainty will increase individuals' probabilities of being pivotal in providing a discrete public good, thereby driving up contributions. Whether or not overall welfare increases will depend on whether or not the change in expected provision outweighs the change in costs associated with higher contributions. For low-valued public goods, wider threshold uncertainty decreases contributions if the original distribution's feasible mode is higher than that of the distribution with wider uncertainty. Efficiency is always higher when contributions can be made in smaller rather than larger increments.

The main implication of these findings is that whether or not threshold uncertainty hinders collective action will depend on the size of the benefits resulting from successful action. It follows that groups facing threshold uncertainty will sometimes need to undertake costly actions for collective action to succeed. One possibility would be the creation of mechanisms that exclude or punish non-participants. Another possibility, more in the spirit of this paper, would be the costly gathering of information that would reduce the variance in people's beliefs about the threshold, and this in turn raises a number of other strategic issues. For example, a group may actually prefer to not collect more information about the threshold if it is believed doing so will reduce the uncertainty so much that contributions will decrease. Also, a group leader with more

[3] Dekel and Piccione (2000) have similar finding for voting games in symmetric binary elections.

precise information about the threshold may strategically reveal or not reveal her information in an attempt to obtain any surplus that can arise from contributions.

Future research should examine threshold uncertainty in these and other settings. One extension would allow some individuals to have noisier information about the threshold than others. Another setting would have a group leader who must choose whether or not to initiate costly information gathering. By examining these settings we can better understand how individuals' incentives to gather and share information differ across informational environments. Since much collective action occurs within formal groups or in the presence of other institutions, such work will lend insights into the actions taken by these groups to overcome the effects of threshold uncertainty. Another direction is to study collective action in the laboratory.[4] For example, in preliminary laboratory experiments, I find qualitative support for my preliminary result (Proposition 2).[5] Indeed, individuals respond to their perceived pivotalness in a manner similar to that which drives my theoretical results. However, as is common in many public good experiments, the observed behavior is not statistically consistent with expected payoff maximization. Future work should incorporate such findings into theoretical models. These avenues of research will ultimately lead us to a more complete understanding of collective action.

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[4] Ledyard (1995) surveys the public good experiments. Also see Offerman (1996) for a more specific examination of discrete public goods. A few experimental studies have examined threshold uncertainty. Wit and Wilke's (1998) and Au's (2004) conducted experiments with sequential contributions, and they find that contribution levels are lower under higher threshold uncertainty. Gustafsson et al. (1999) report a similar finding in an analogous experiment with simultaneous contributions. Suleiman et al. (2001) find in a simultaneous contributions experiment that the effect of threshold uncertainty can depend on the mean of the threshold distribution. None of these experiments consider how the effect of the uncertainty depends on the value of the public good.

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