String GUTs

Michael Ratz

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Based on collaborations with:
M. Blaszczyk, F. Brümmer, W. Buchmüller, M.–C. Chen, M. Fallbacher, M. Fischer, S. Groot Nibbelink, K. Hamaguchi, R. Kappl, T. Kobayashi, O. Lebedev, H.M. Lee, R. Mohapatra, A. Mütter, H.P. Nilles, B. Petersen, F. Plöger, S. Raby, S. Ramos–Sánchez, G. Ross, F. Ruehle, R. Schieren, K. Schmidt–Hoberg, C. Staudt,V. Takhistov, A. Trautner, M. Trapletti, P. Vaudrevange & A. Wingerter

Why SUSY

and

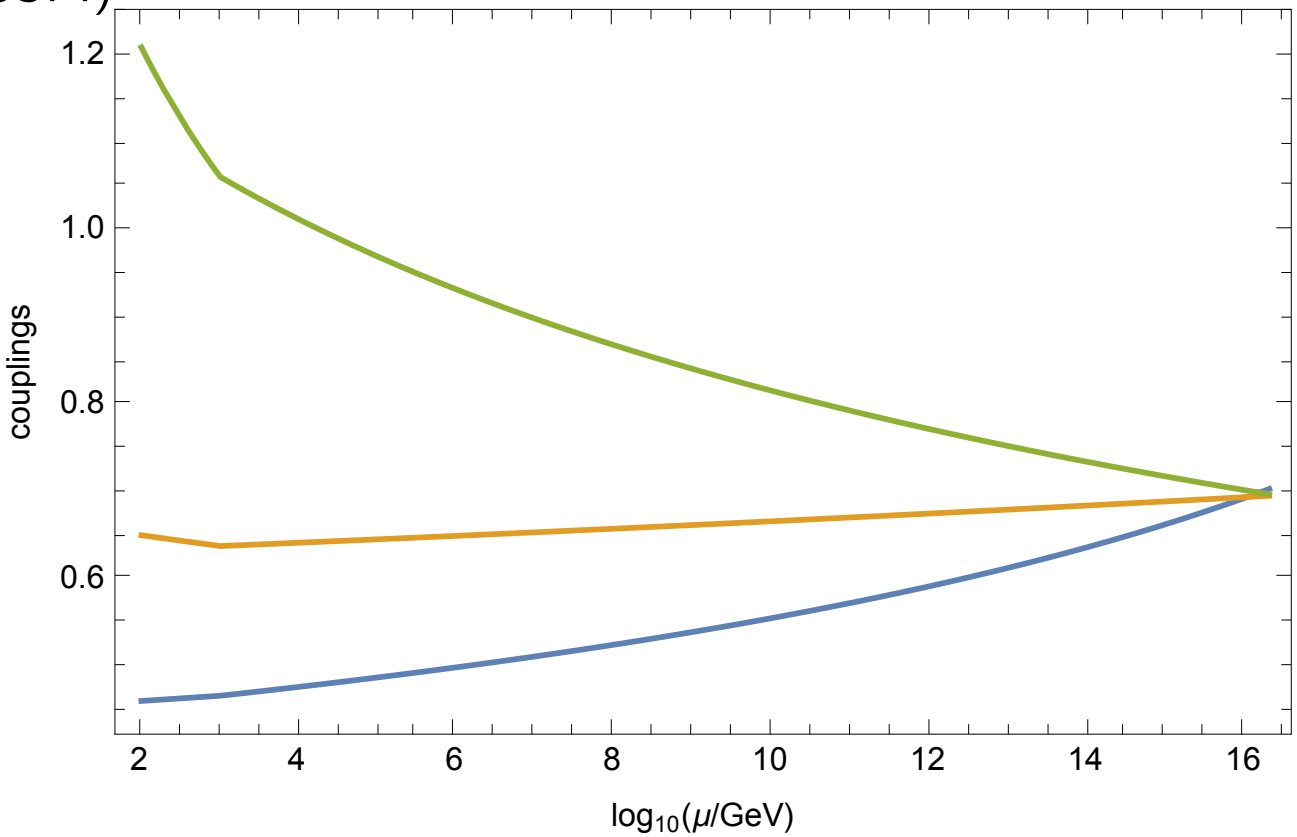
Grand Unification

?

Gauge coupling unification in the MSSM

- Running couplings in the (minimal) supersymmetric standard model (MSSM)

Dimopoulos, Raby & Wilczek [1981]



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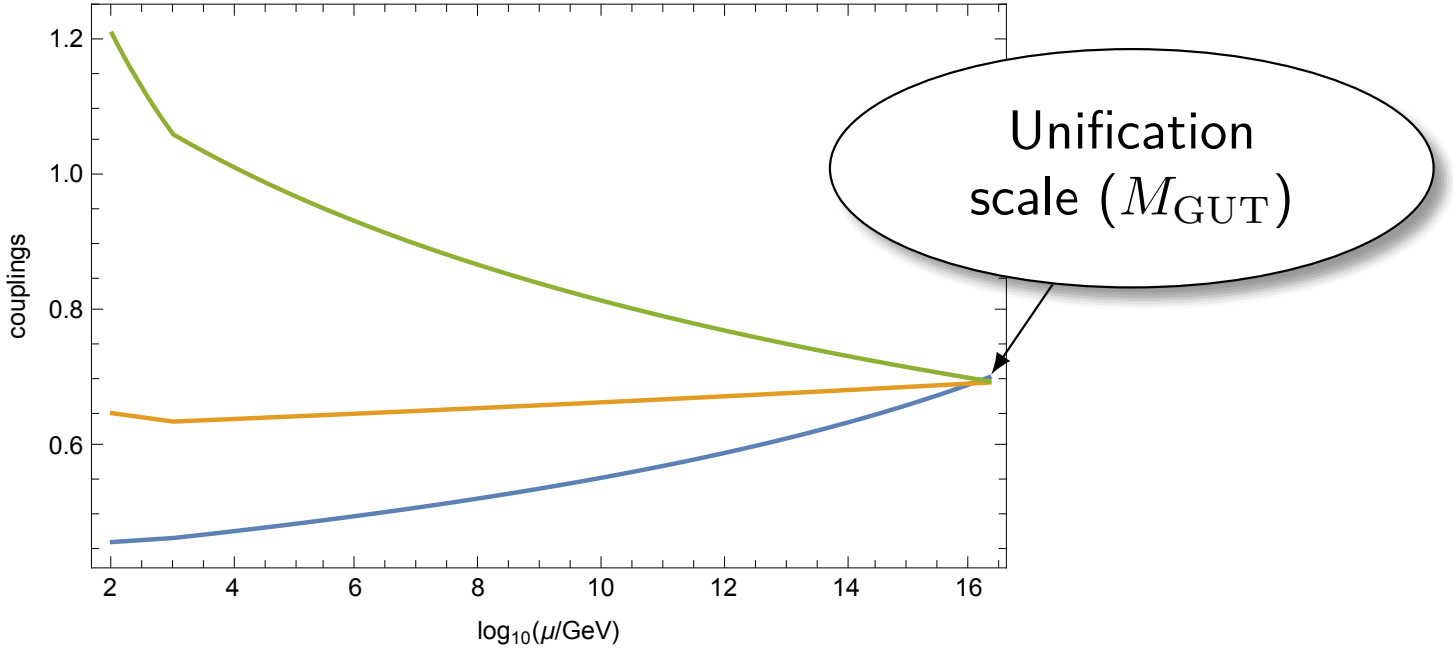

- Gauge coupling unification might be a consequence of $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \subset \mathrm{SU}(5)$

Where is SUSY?

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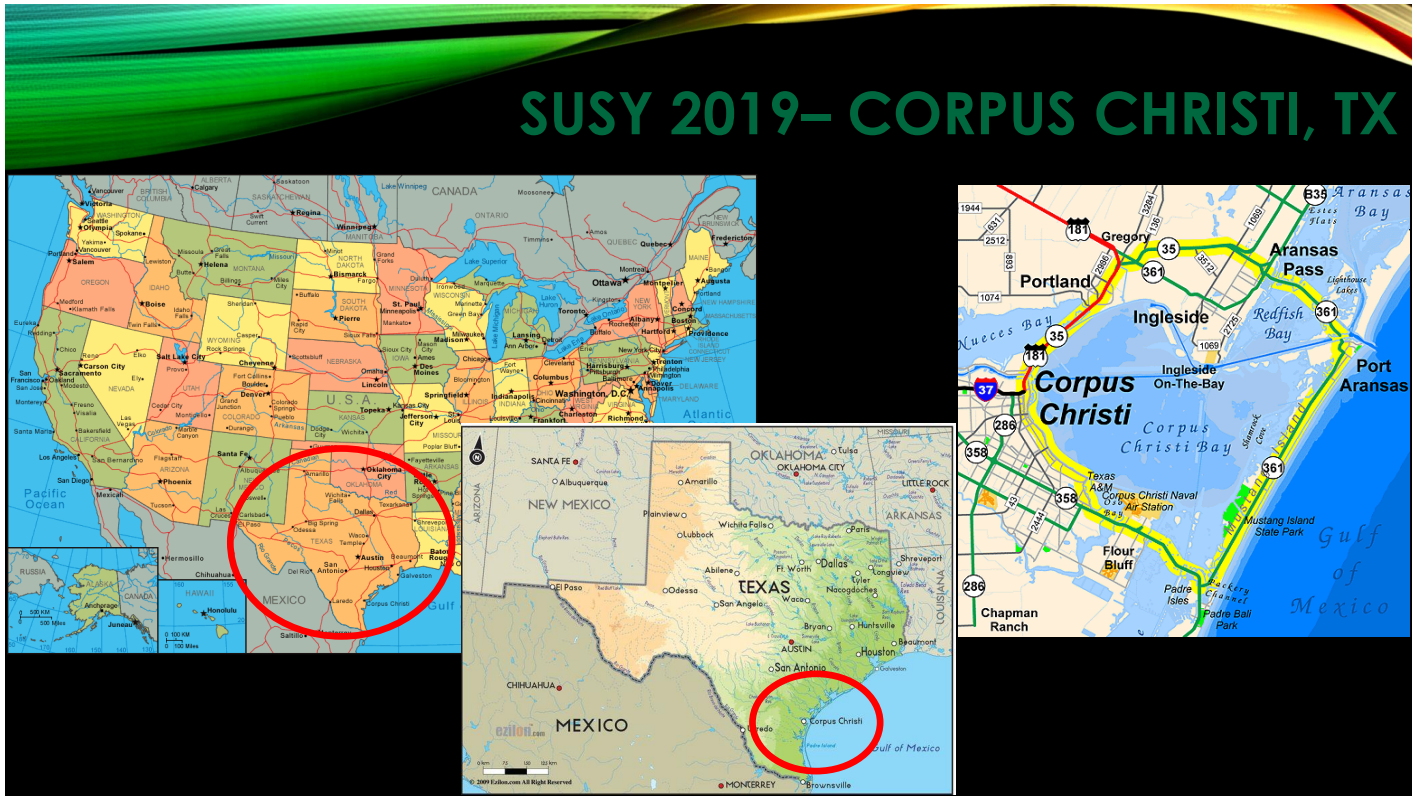
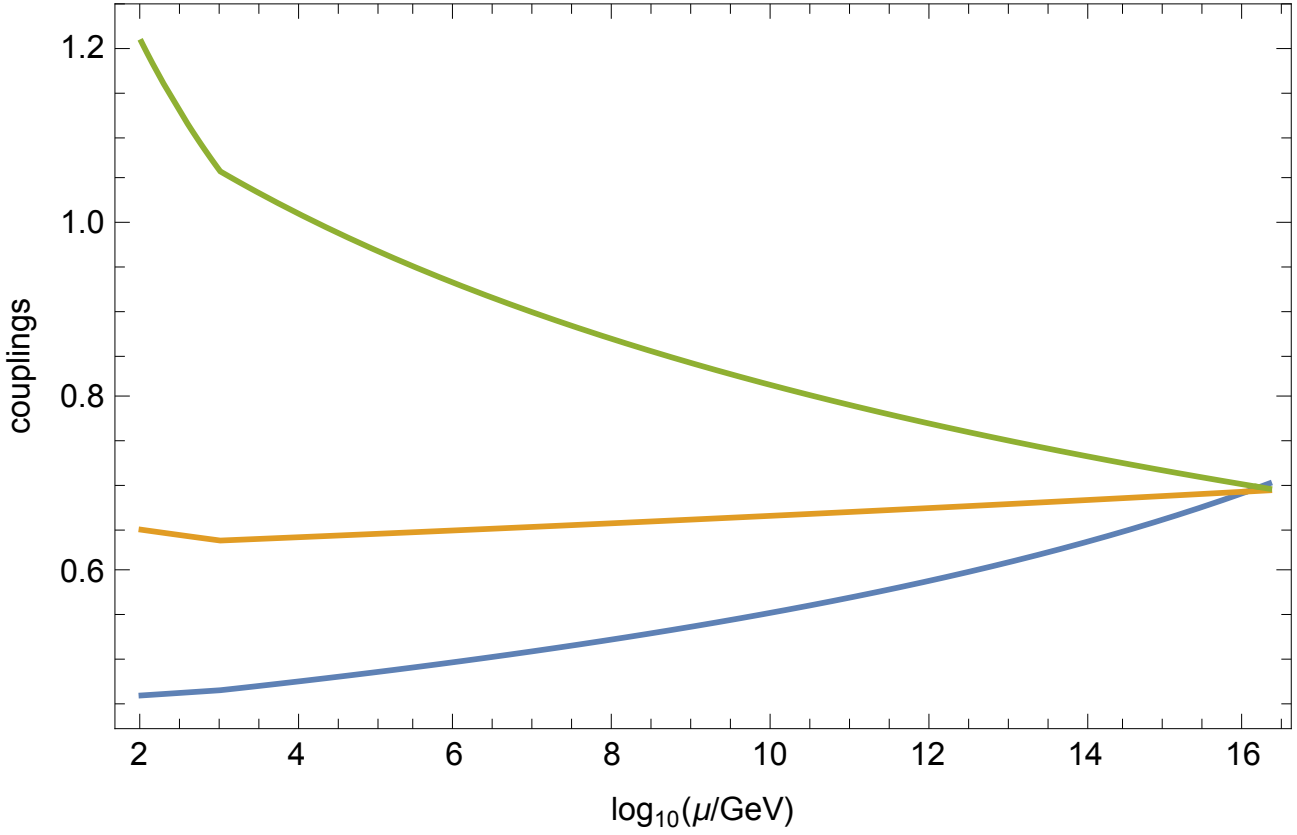
- Answer: in 2019 in Corpus Christi (TX)



Doublet–triplet splitting vs. full generations

☺ Gauge coupling unification: $M_{\rm GUT} \sim 10^{16}\,{\rm GeV}$ with SUSY



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☺ One generation of observed matter fits into $\mathbf{16}$ of $\mathrm{SO}(10)$

$$\begin{aligned} \mathrm{SO}(10) &\to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}} = G_{\mathrm{SM}} \\ \mathbf{16} &\to (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\ &\quad \oplus (\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{0} \end{aligned}$$

☹ However: Higgs only as doublet(s):

$$\mathbf{10} \to \underbrace{(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2}}_{\text{doublets: needed}} \oplus \underbrace{(\mathbf{3},\mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3}}_{\text{triplets: excluded}}$$

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☞ The GUT–breaking Higgs representations are hard to get in string theory

Dienes & March-Russell [1996]

Purpose of this talk

- ☞ Discrete R symmetries to solve some of the most stringent problems of the MSSM
  - μ problem
  - proton decay operators

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- ☞ Discrete R symmetries to solve some of the most stringent problems of the MSSM
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- ☞ Discrete flavor symmetries as the origin of $\mathcal{CP}$ violation
- ☞ Stringy origin of these discrete symmetries

Outline

1. Introduction & Motivation ✓
2. Anomaly–free discrete symmetries & unification
   - anomaly cancellation
   - consistency with unification
   - unique $\mathbb{Z}_4^R$ symmetry
   - no–go theorems in 4D
   - stringy realization
3. $\mathcal{CP}$ violation from strings
   - $\mathcal{CP}$ violation from finite groups
   - discrete (family) symmetries from strings
   - stringy origin
4. Summary

Anomaly–free

discrete symmetries

and

Grand Unification

- anomaly cancellation
- consistency with unification
- unique $\mathbb{Z}_4^R$ symmetry
- no–go theorems in 4D

Superpotential of the MSSM

Yukawa couplings

$$\mathscr{W}_{\text{gauge invariant}} = \mu\, \boldsymbol{h}_d \boldsymbol{h}_u + \kappa_i\, \boldsymbol{\ell}_i \boldsymbol{h}_u$$
$$+ Y_e^{gf}\, \boldsymbol{\ell}_g \boldsymbol{h}_d \boldsymbol{e}^{\mathcal{C}}{}_f + Y_d^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_d \boldsymbol{d}^{\mathcal{C}}{}_f + Y_u^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_u \boldsymbol{u}^{\mathcal{C}}{}_f$$
$$+ \lambda_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{\ell}_f \boldsymbol{e}^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{q}_f \boldsymbol{d}^{\mathcal{C}}{}_k + \lambda''_{gfk}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{d}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k$$
$$+ \kappa_{gf}\, \boldsymbol{h}_u \boldsymbol{\ell}_g\, \boldsymbol{h}_u \boldsymbol{\ell}_f + \kappa^{(1)}_{gfk\ell}\, \boldsymbol{q}_g \boldsymbol{q}_f \boldsymbol{q}_k \boldsymbol{\ell}_\ell + \kappa^{(2)}_{gfk\ell}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{u}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \boldsymbol{e}^{\mathcal{C}}{}_\ell$$

effective neutrino mass operator

☞ Want: Yukawa couplings and Weinberg operator

Superpotential of the MSSM

$$
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\end{aligned}
$$

R–parity violation

- ☞ Want: Yukawa couplings and Weinberg operator
- ☞ Do not want/need R–parity violation

Superpotential of the MSSM

$\overset{!}{\sim} \mathcal{O}(m_{3/2})$

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$\overset{!}{\lesssim} \frac{10^{-18}}{M_{\mathrm{P}}}$

- ☞ Want: Yukawa couplings and Weinberg operator
- ☞ Do not want/need R–parity violation
- ☞ Want to tie μ term to supersymmetry breaking and suppress proton decay operators

Prejudices, assumptions & goals

Assumptions:

- ☞ $\mathrm{SO}(10)$ unification of matter is not an accident
- ☞ μ term is forbidden by a symmetry but appears after SUSY breaking
- ☞ Want to preserve gauge coupling unification

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

if violated, gauge coupling unification will be spoiled

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} \rho \mod \eta \quad \text{for all } G$$

$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

$\mathbb{Z}_N$ charge

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden (before ~~SUSY~~)

need to forbid the μ term to be able to appreciate the Kim–Nilles and/or Giudice–Masiero mechanisms Kim & Nilles [1984] ; Giudice & Masiero [1988]

Anomaly–free symmetries, μ and unification

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(iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT

$$\begin{aligned} \mathrm{SO}(10) \quad &\rightarrow \quad \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}} = G_{\mathrm{SM}} \\ \mathbf{16} \quad &\rightarrow \quad (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\ &\qquad \oplus (\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{0} \end{aligned}$$

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- Can prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term

Hall, Nomura & Pierce [2002] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

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  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry

	$\boldsymbol{q}$	$\boldsymbol{u}^{\mathcal{C}}$	$\boldsymbol{d}^{\mathcal{C}}$	$\boldsymbol{\ell}$	$\boldsymbol{e}^{\mathcal{C}}$	$\boldsymbol{h}_u$	$\boldsymbol{h}_d$	$\boldsymbol{\nu}^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

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  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry
  3. R symmetries are not available in 4D GUTs

     uneaten parts of the Higgs that breaks the GUT symmetry cannot be paired up

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

☞ Powerful tool: anomaly matching

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- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

extra

matter

gauginos

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SM gauginos

universal

extra gauginos from X, Y bosons

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- Assume now that some mechanism eliminates the extra gauginos

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- Assume now that some mechanism eliminates the extra gauginos
- ➥ Extra stuff must be non–universal (split multiplets)

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bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ;Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ;Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

Yukawa couplings ✓

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\end{aligned}
$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

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forbidden by exact $\mathbb{Z}_2^R \subset \mathbb{Z}_4^{\mathrm{R}}$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ;Lee, Raby, M.R., Schieren, et al. [2011a] ;Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

$\mathcal{O}(m_{3/2})$

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$\mathcal{O}\left(\frac{m_{3/2}}{M_{\rm P}^2}\right)$

- ☞ R parity violating couplings forbidden
- ☞ μ term of the right size

  order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

  Planck units

- ☞ proton decay under control

String theory realization

and

String models

- evading the no–go theorem
- origin of $\mathbb{Z}_4^R$
- higher–dimensional operators (effective μ term etc.)

Grand unification in higher dimensions

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- Even more, R symmetries have a clear geometric interpretation in terms of the Lorentz symmetry of compact dimensions

String compactifications

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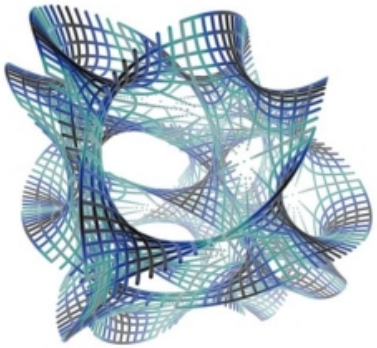

- String compactification: twist the string in such a way that the excitations carry the quantum numbers of the standard model particles

From strings to the real world?

☞ Many popular attempts to connect strings with observation:

- heterotic orbifolds
- intersecting D–branes
- Calabi–Yau compactifications
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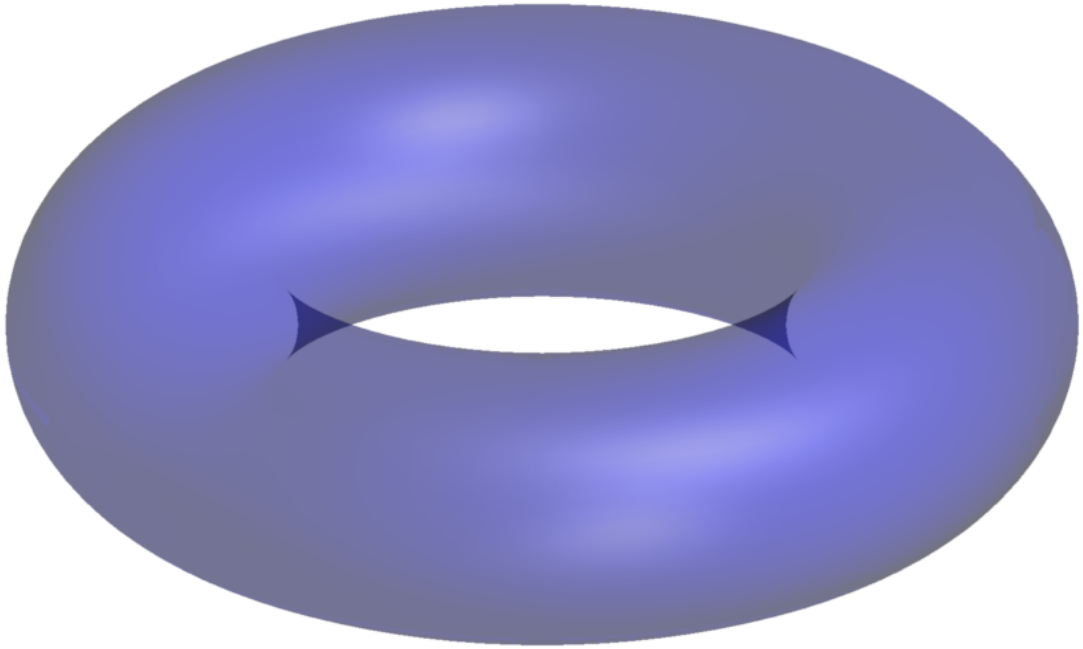
main theme of the rest of this talk:

orbifold compactifications of the heterotic string

$\mathbb{Z}_2$ orbifold pillow

☞ Starting point: torus



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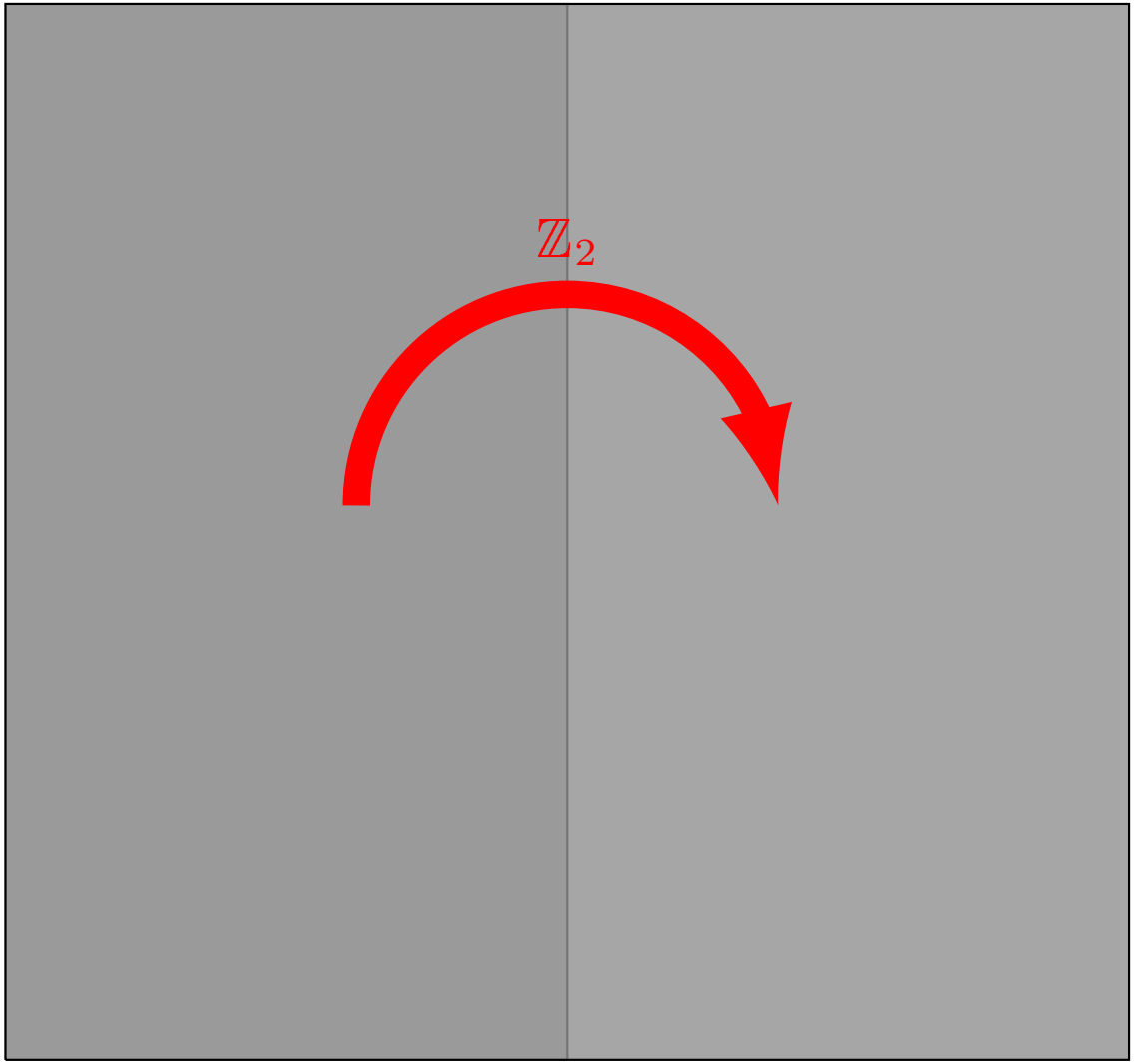

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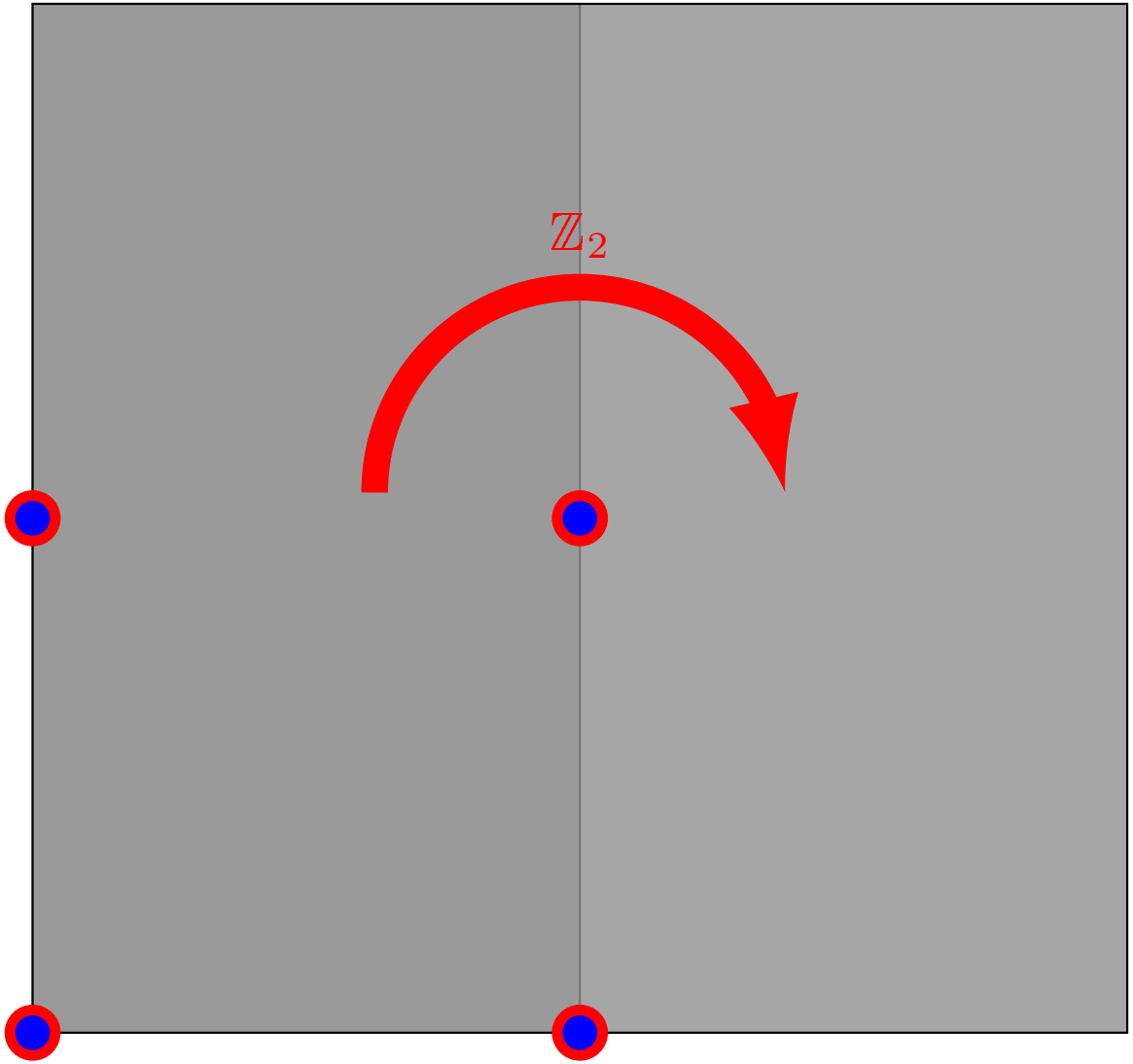

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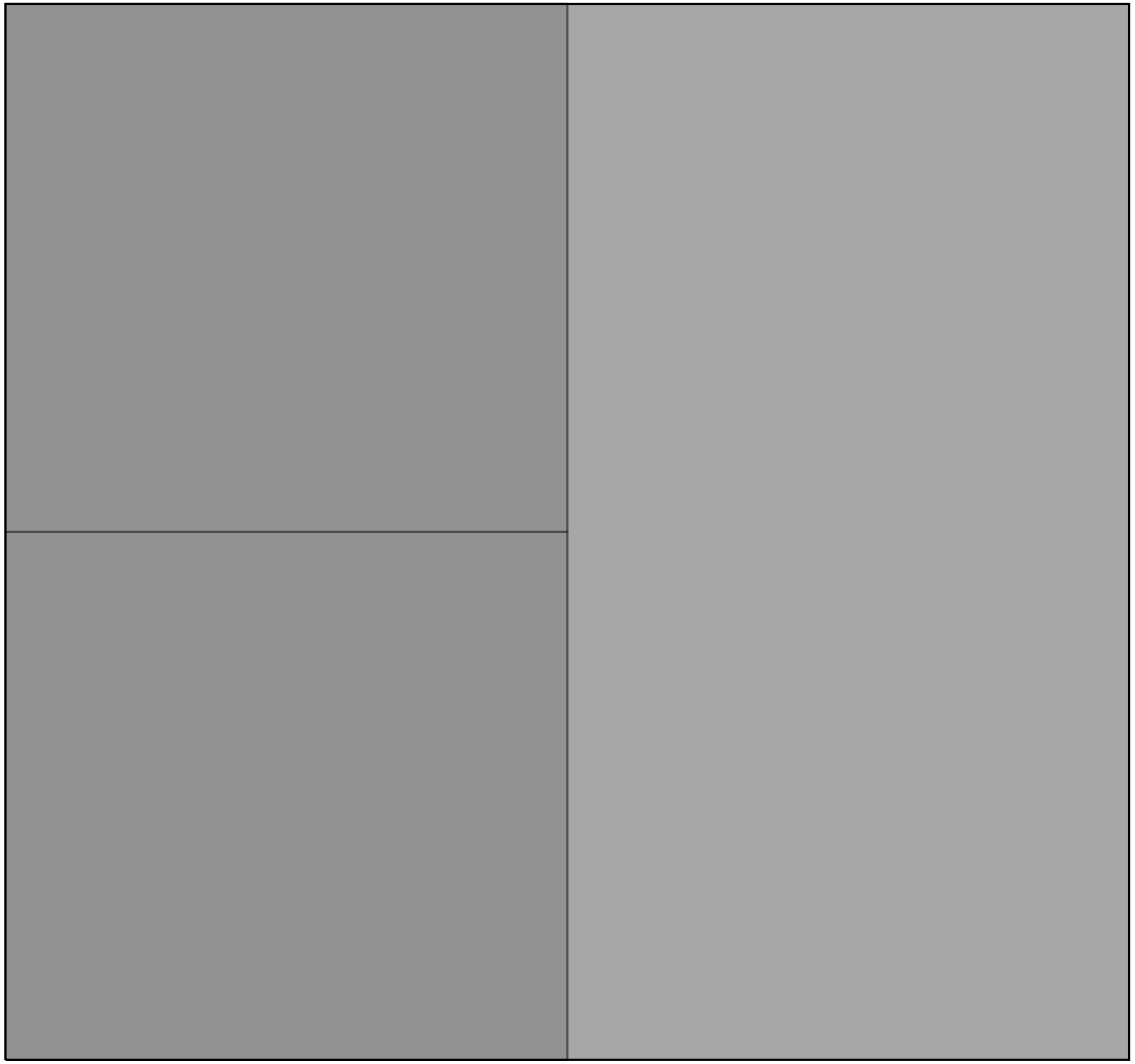

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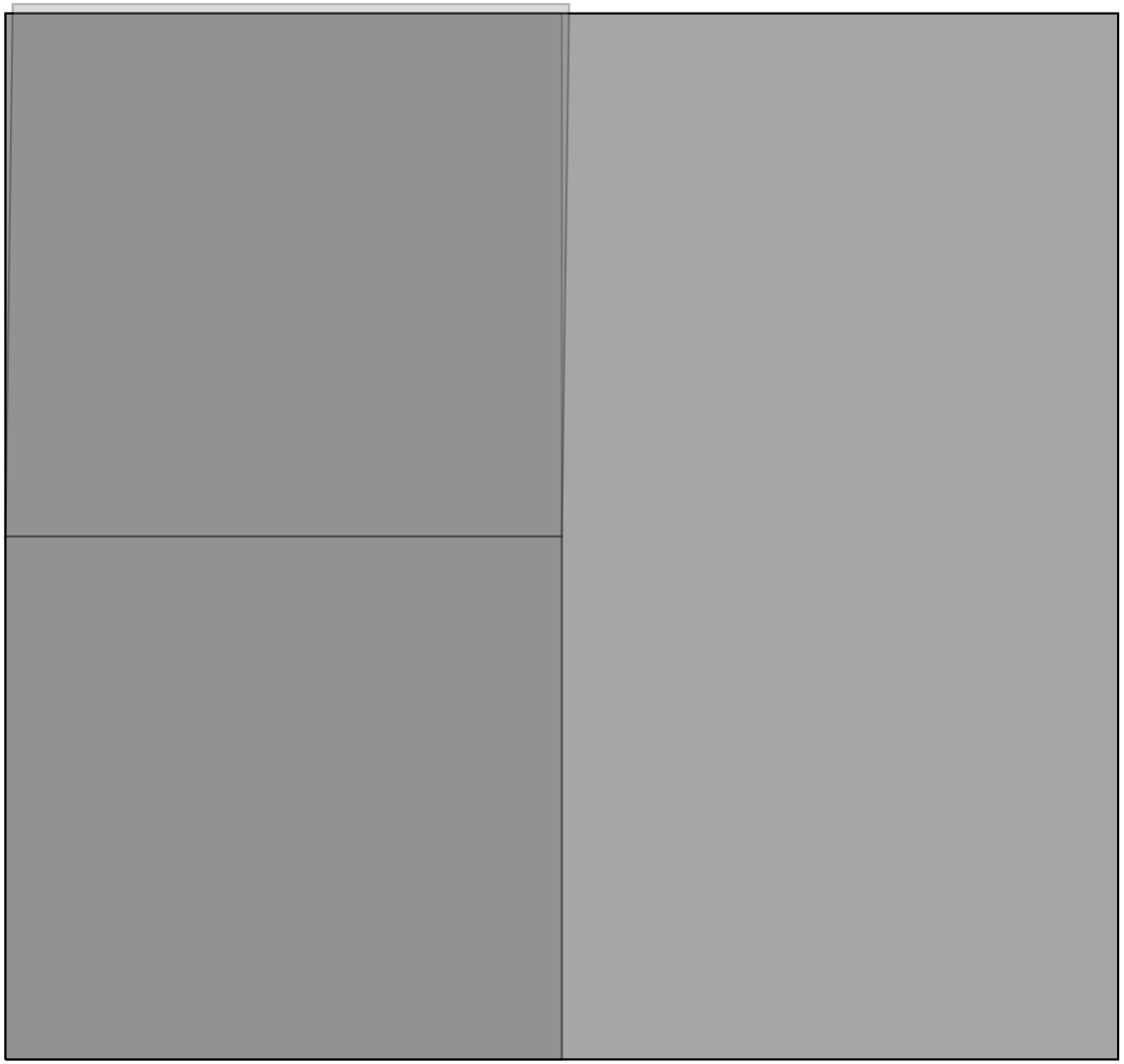

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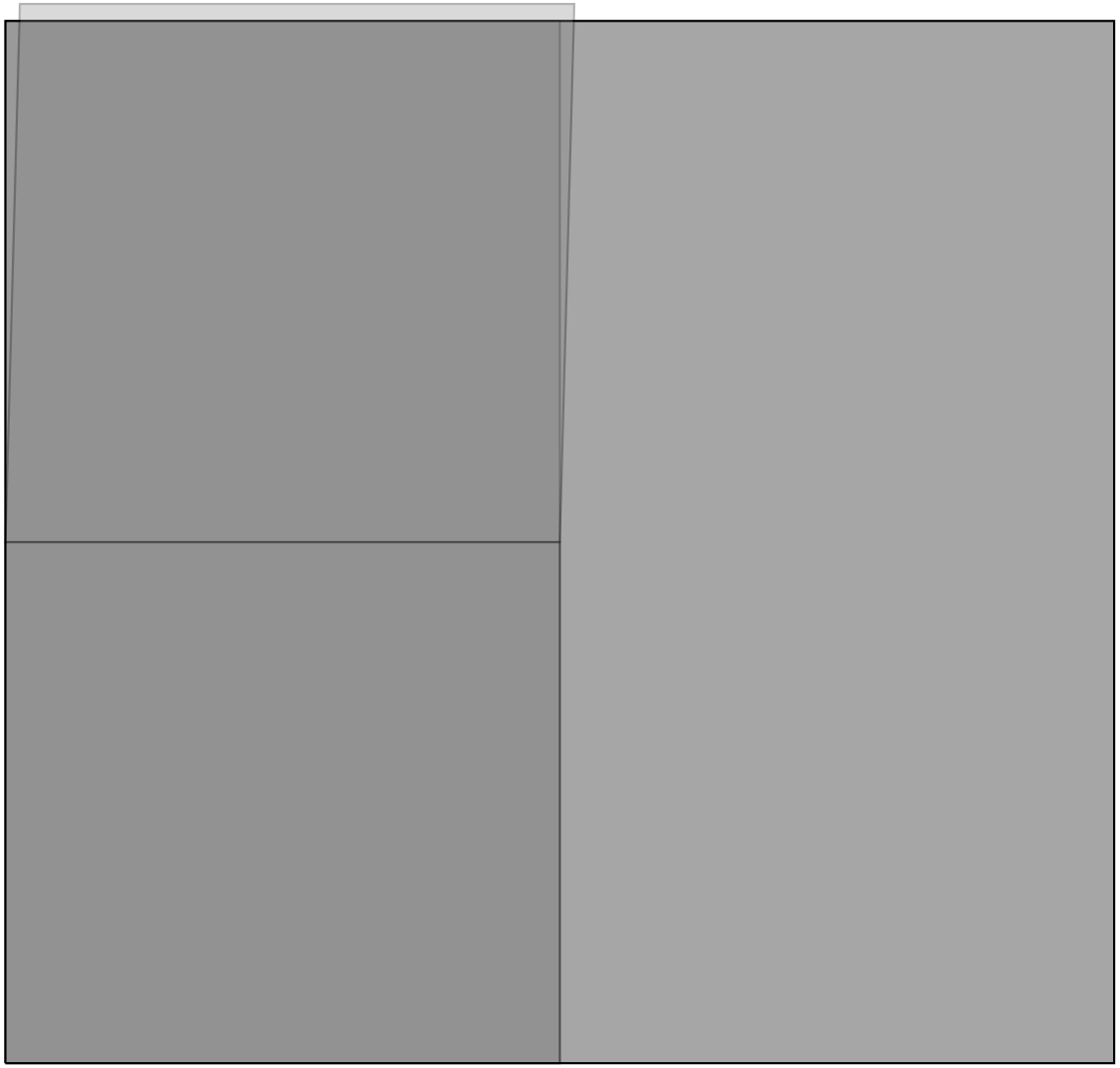

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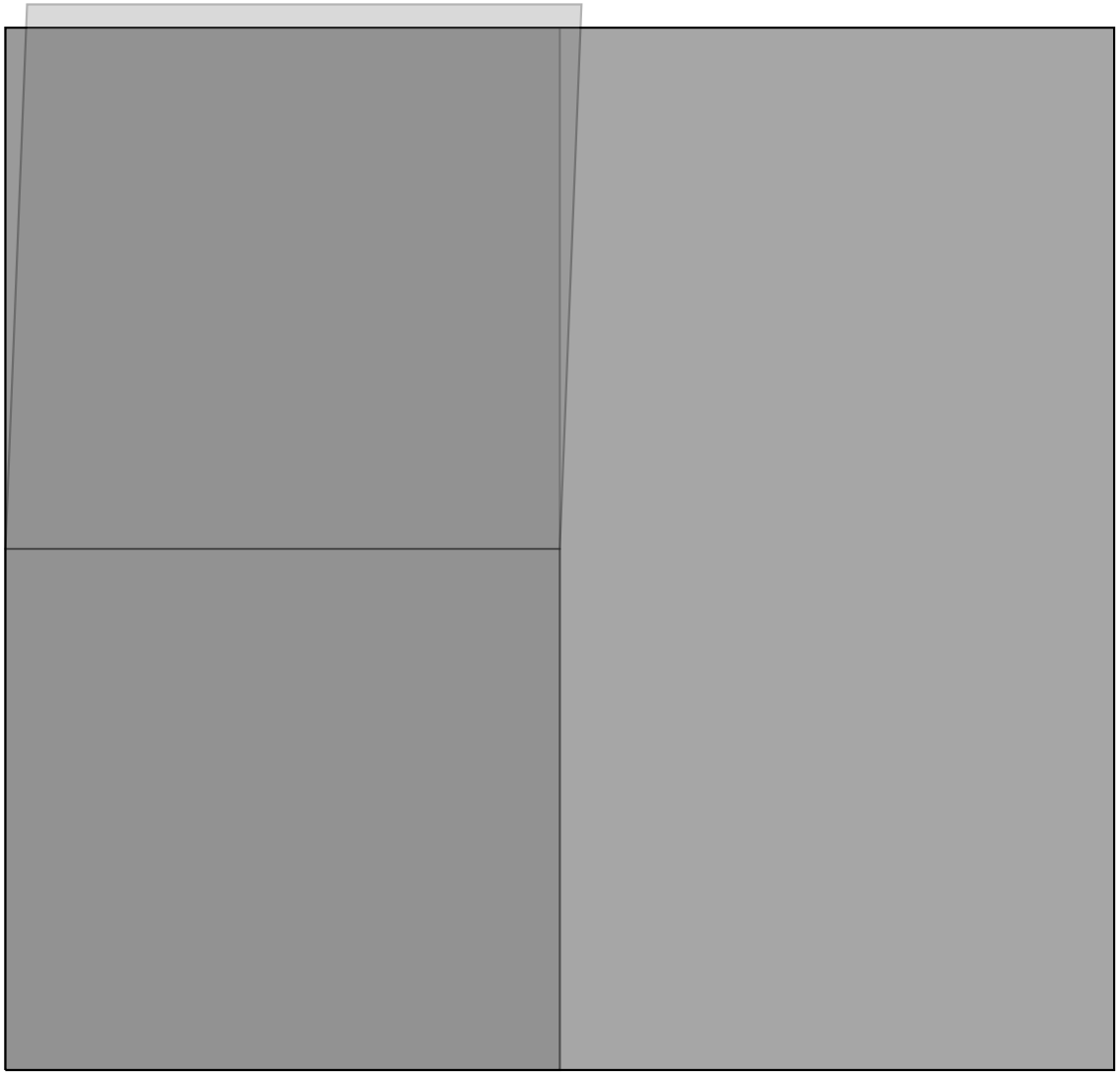

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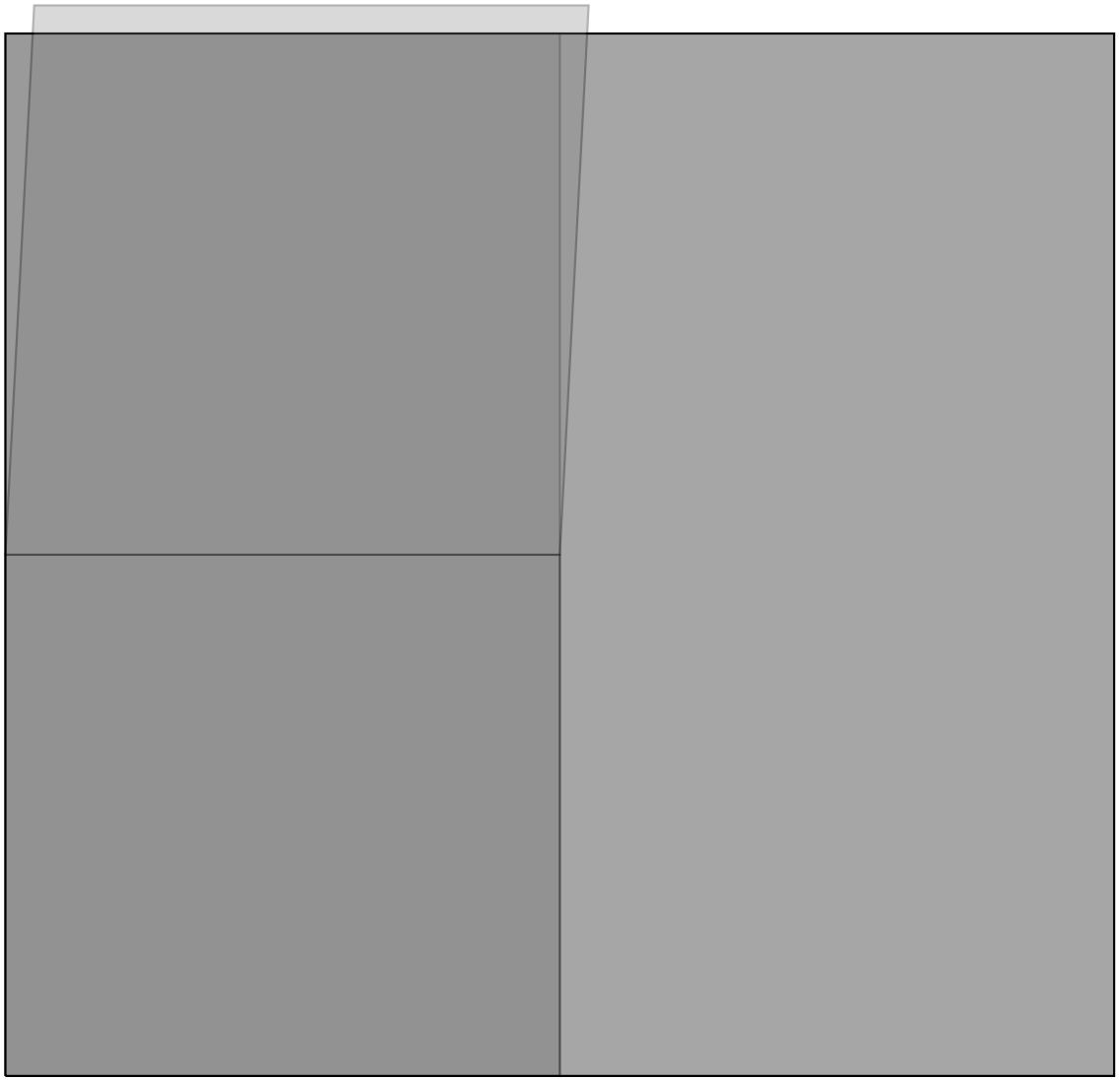

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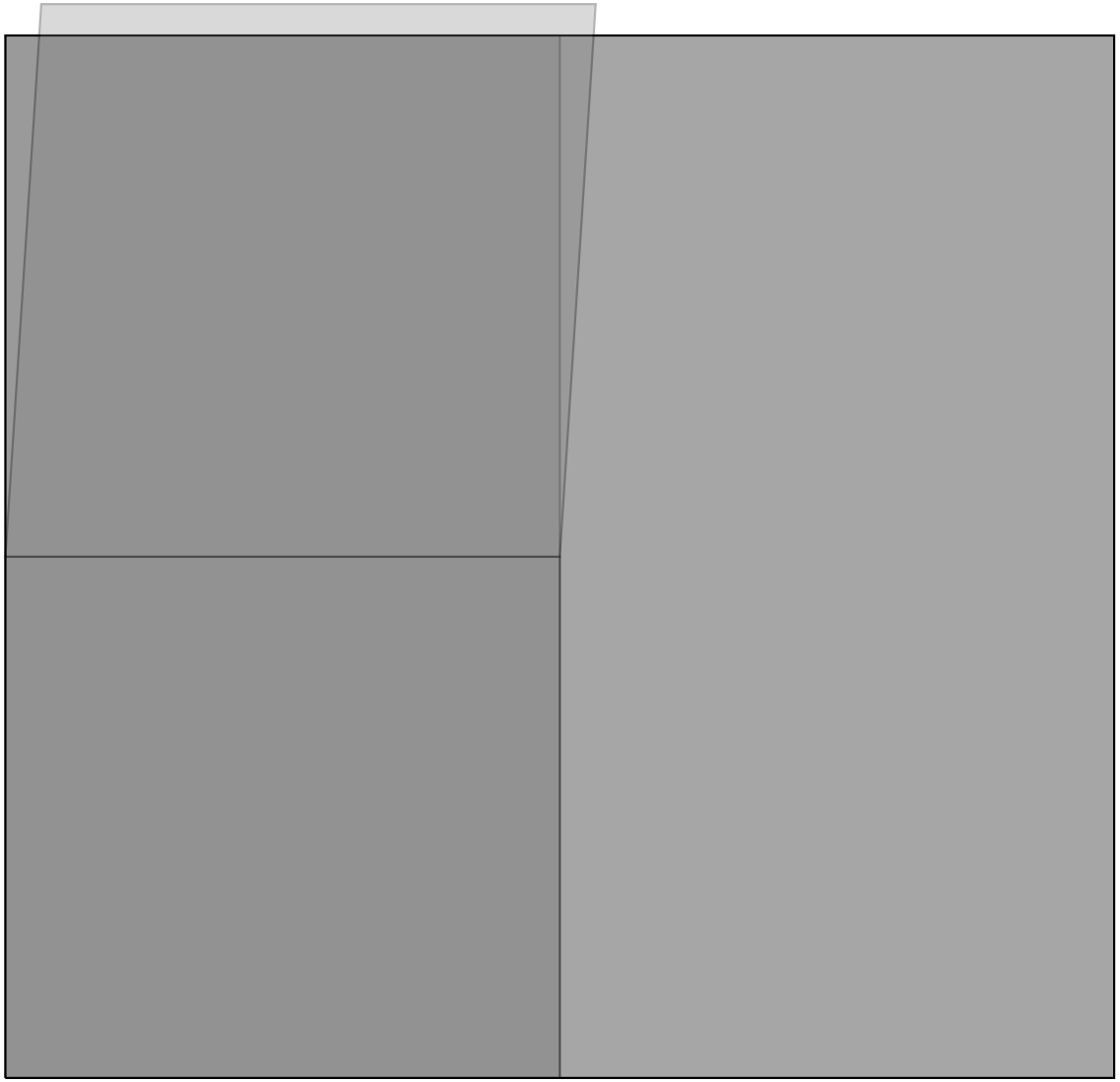

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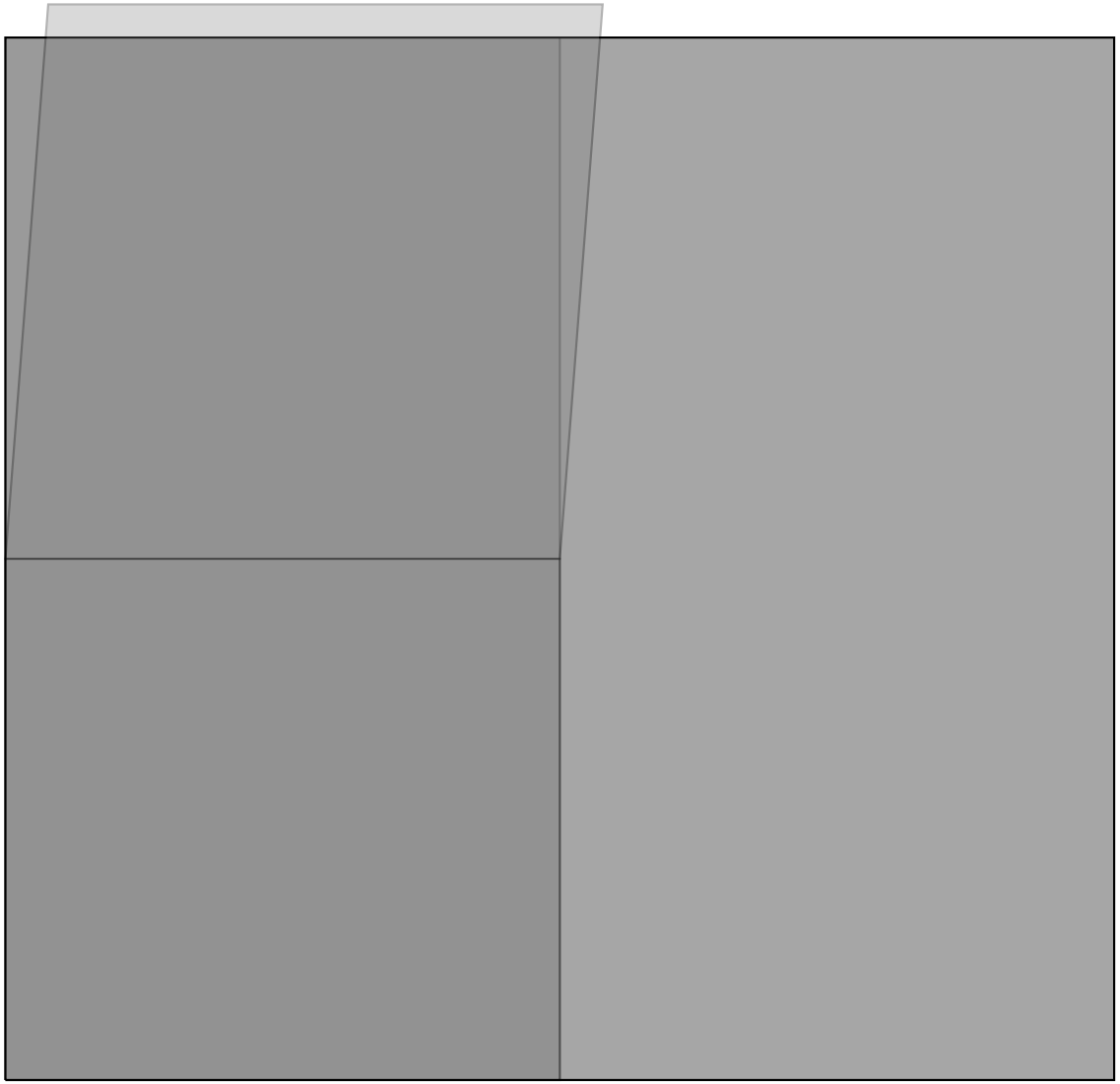

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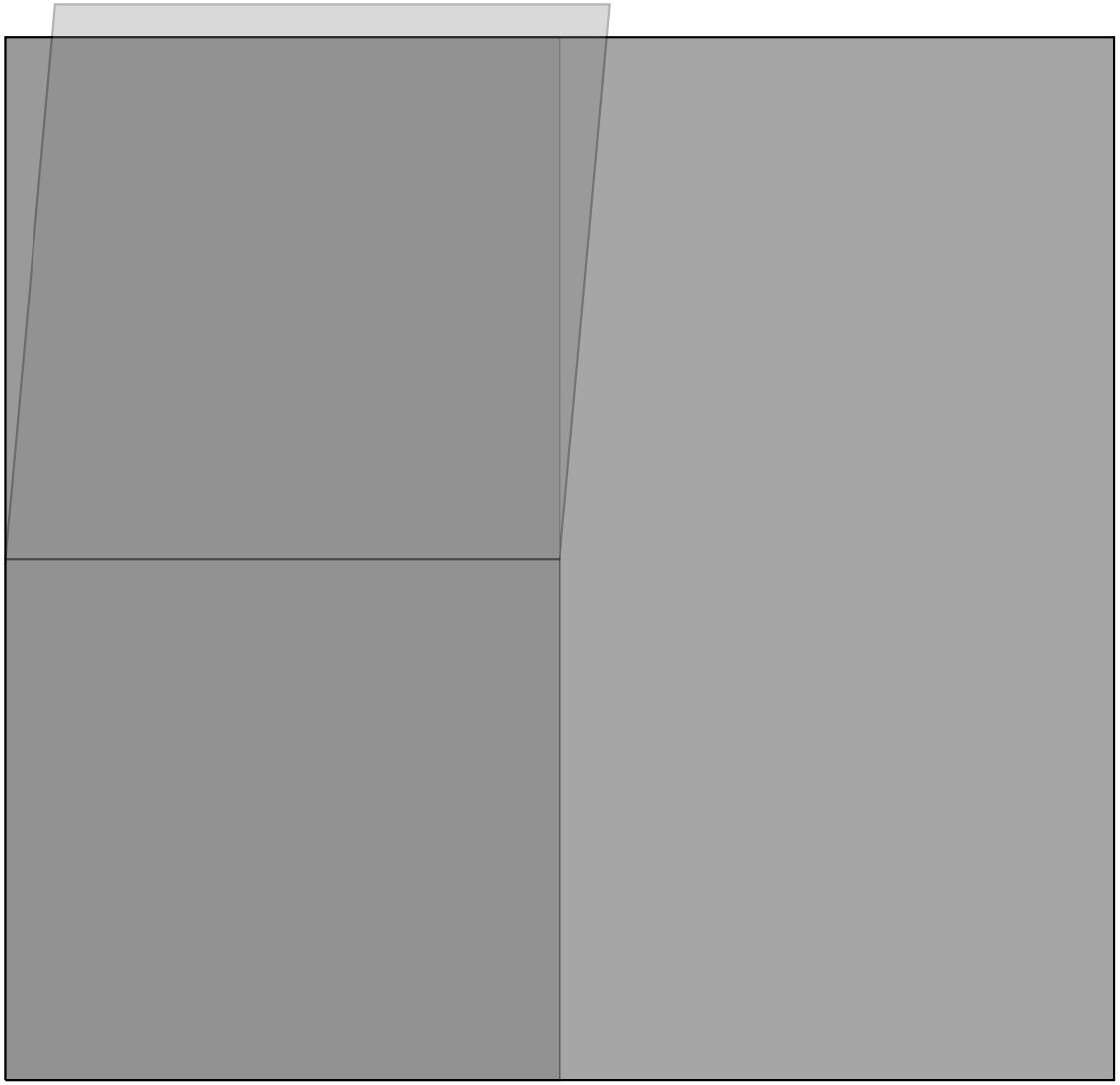

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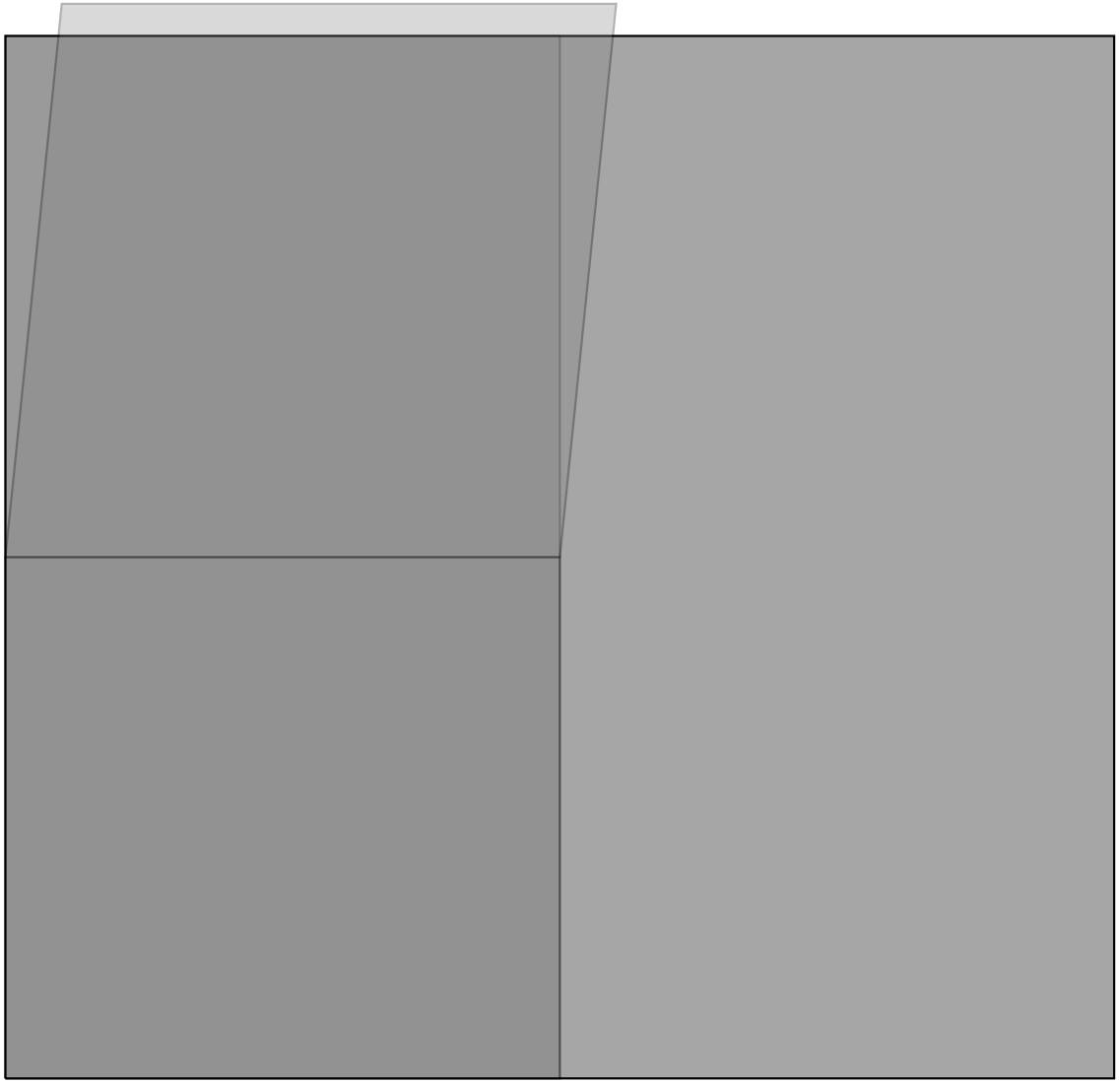

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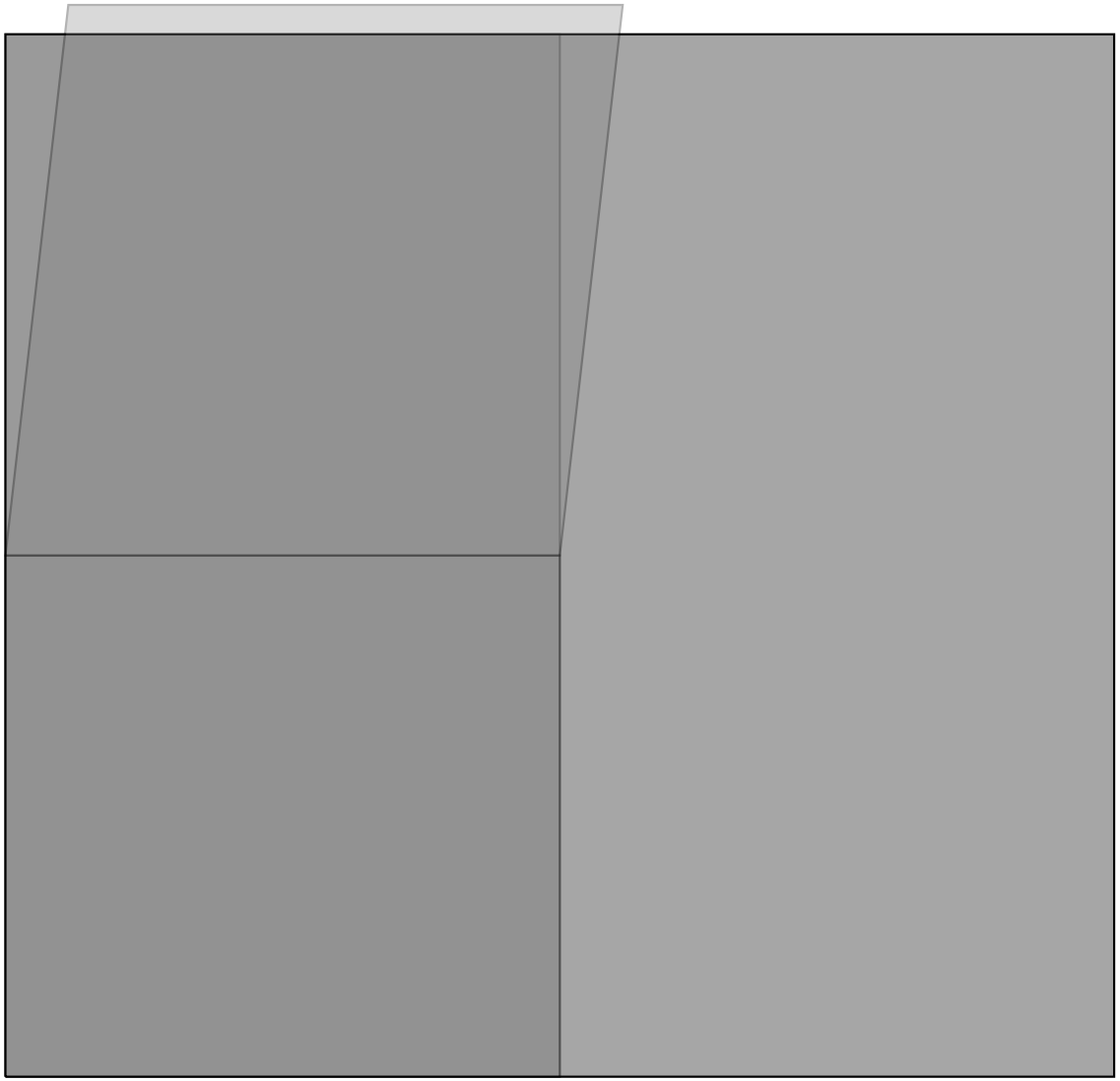

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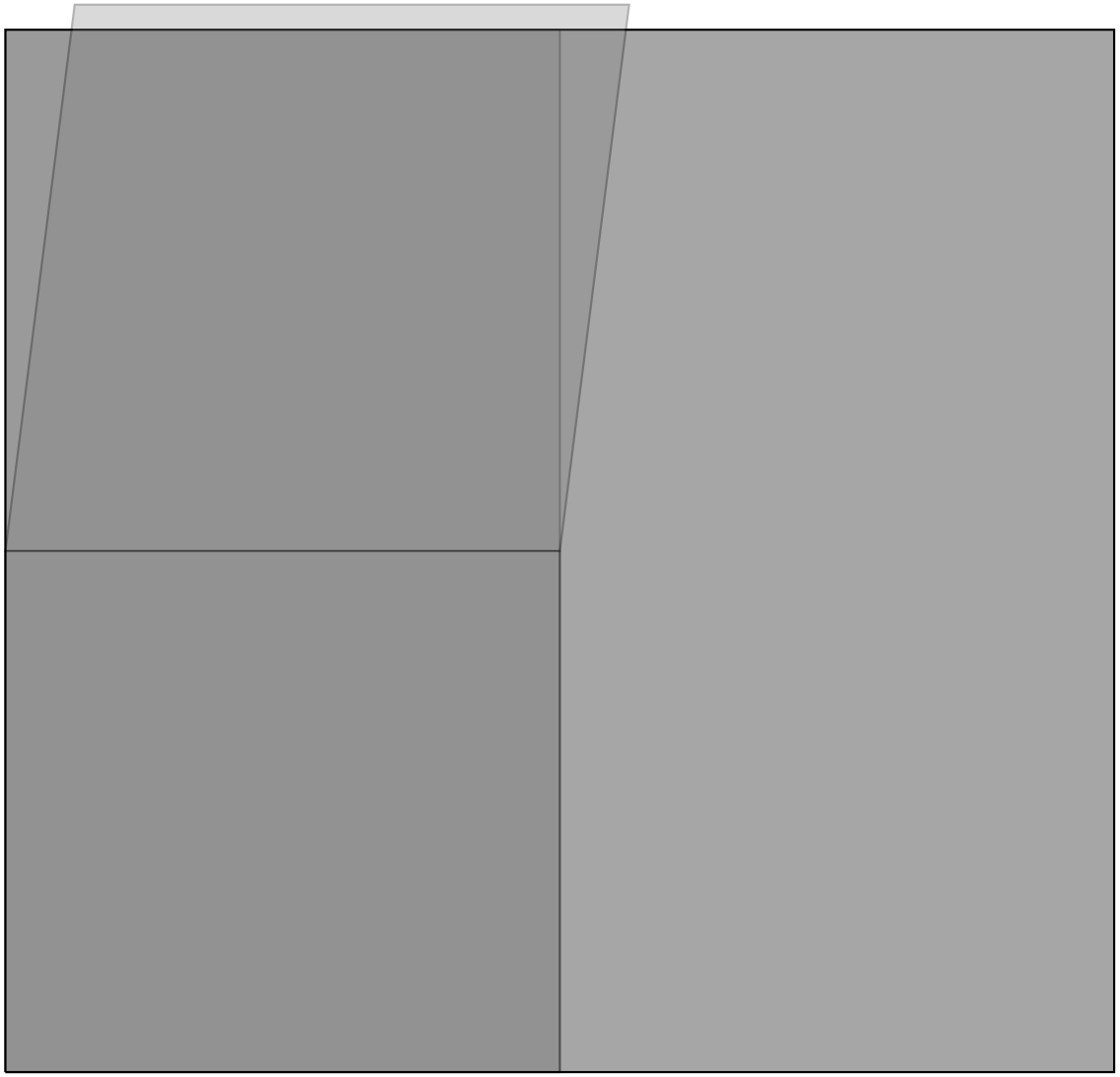

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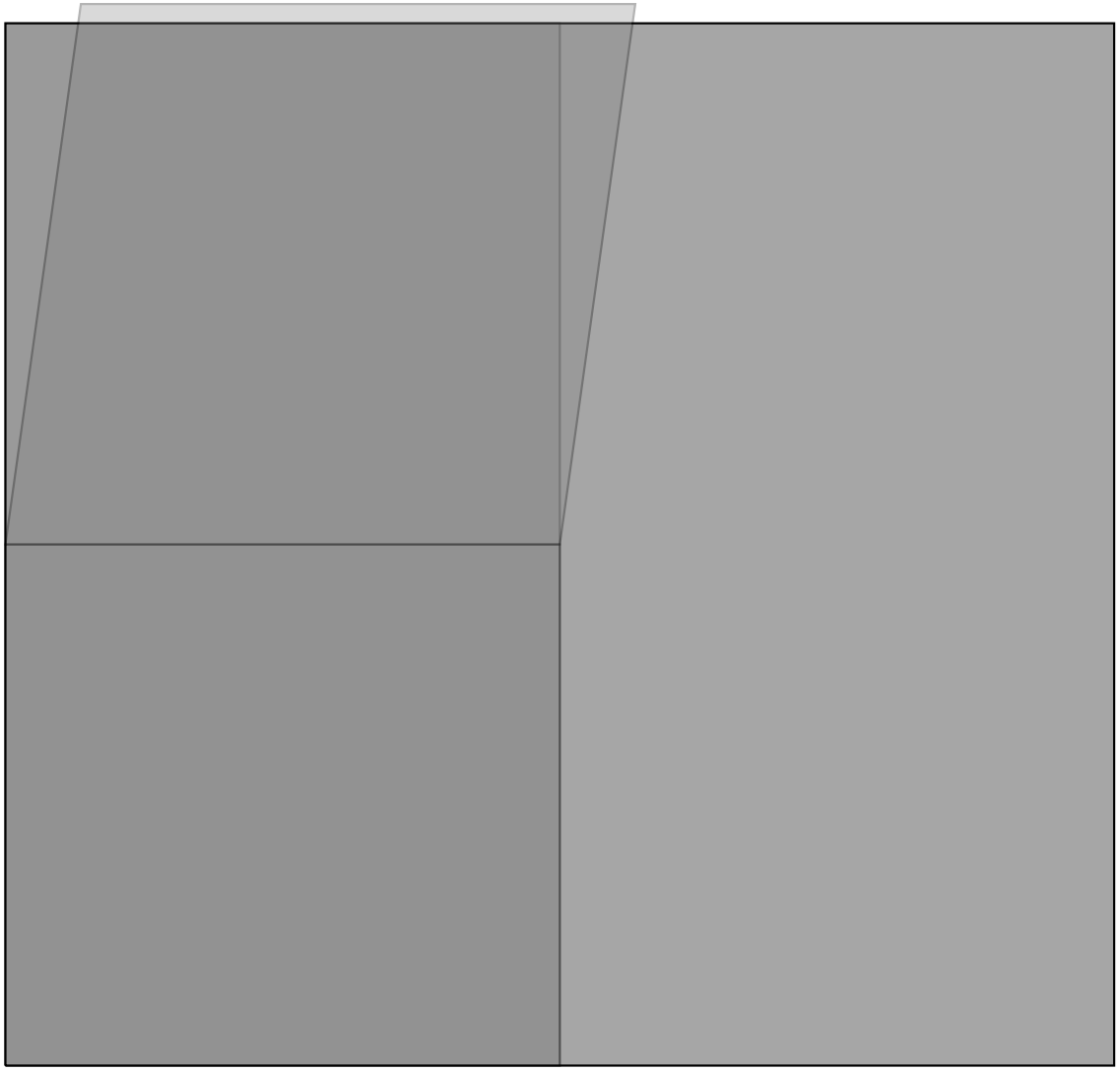

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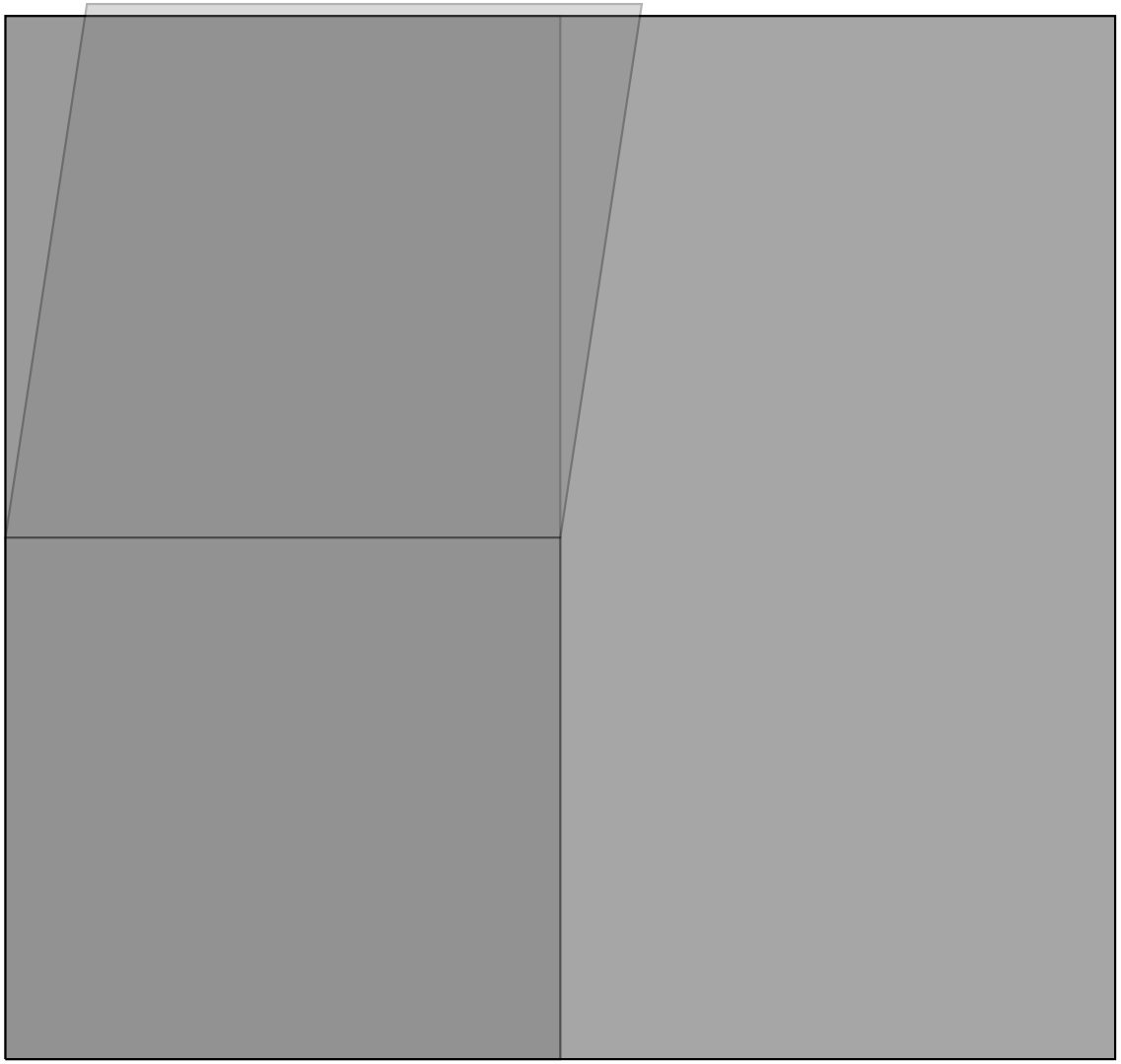

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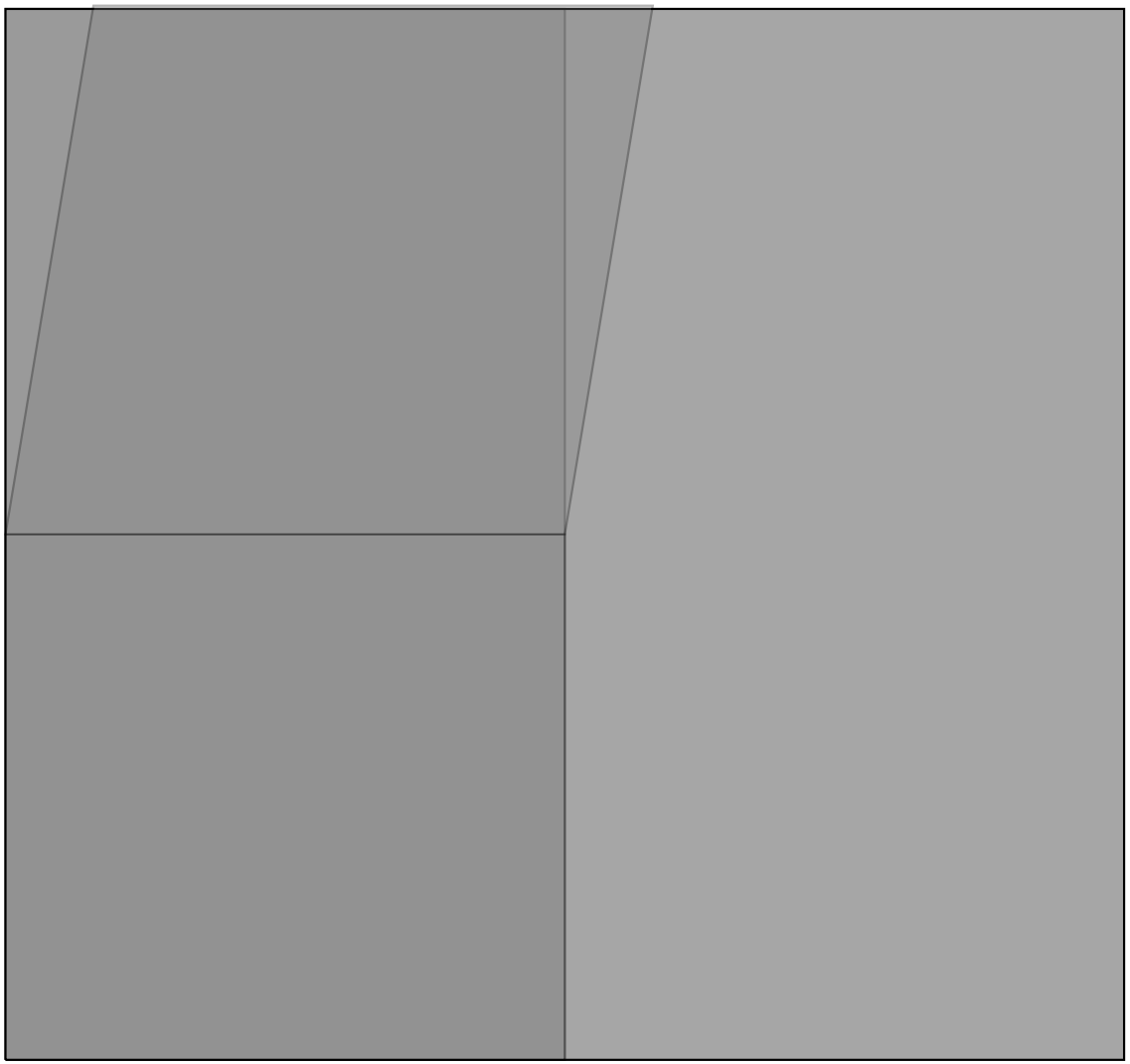

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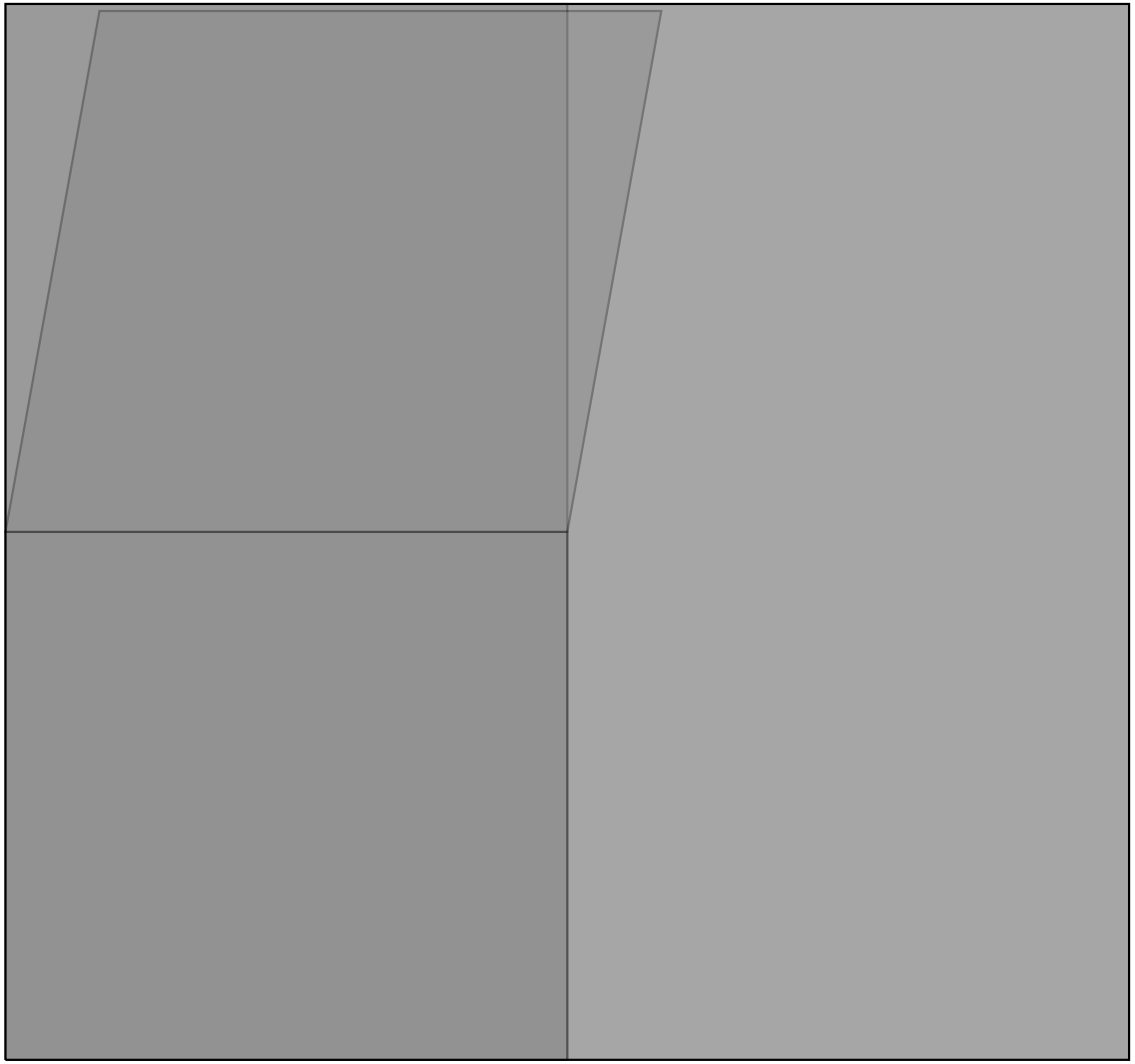

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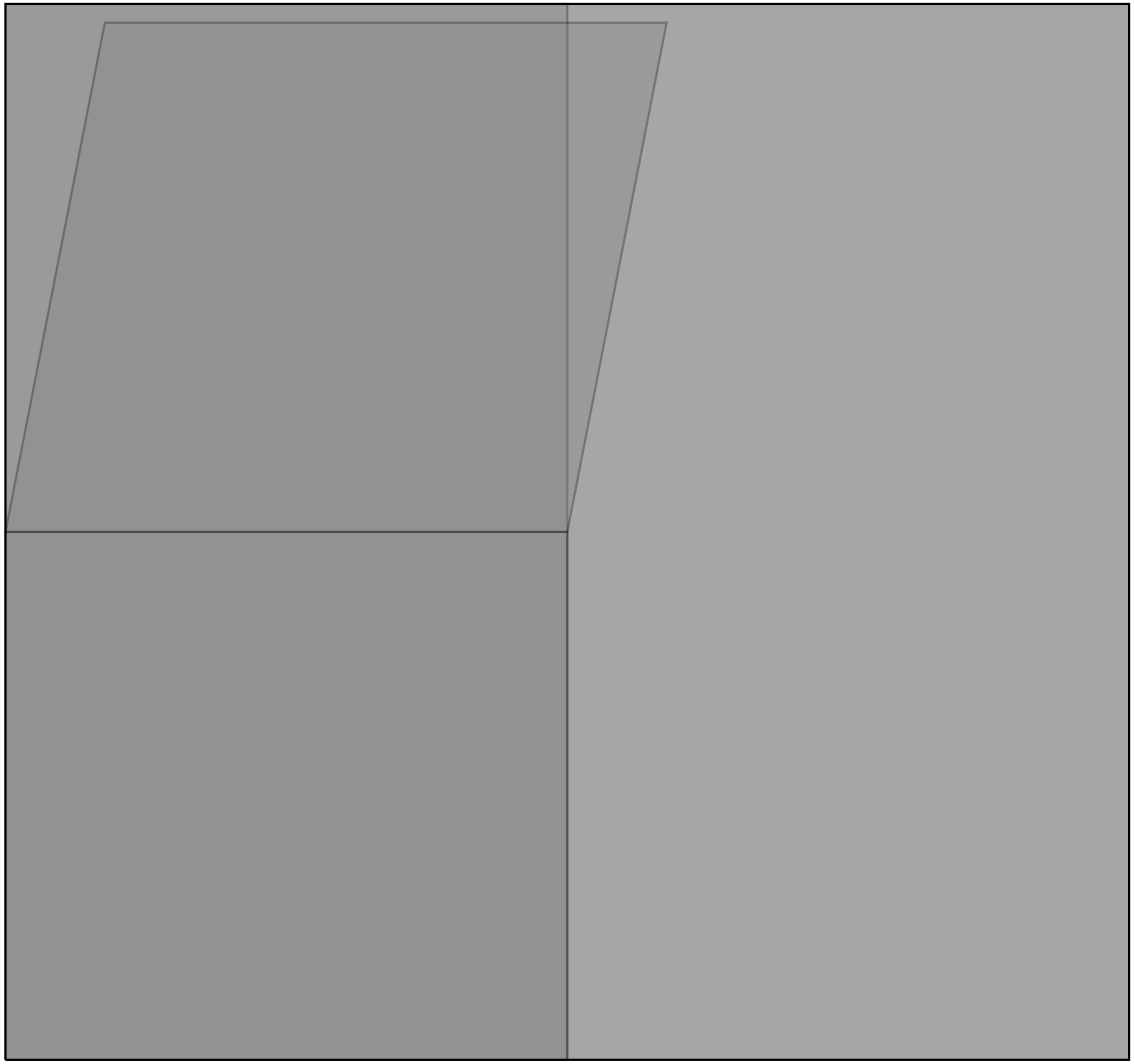

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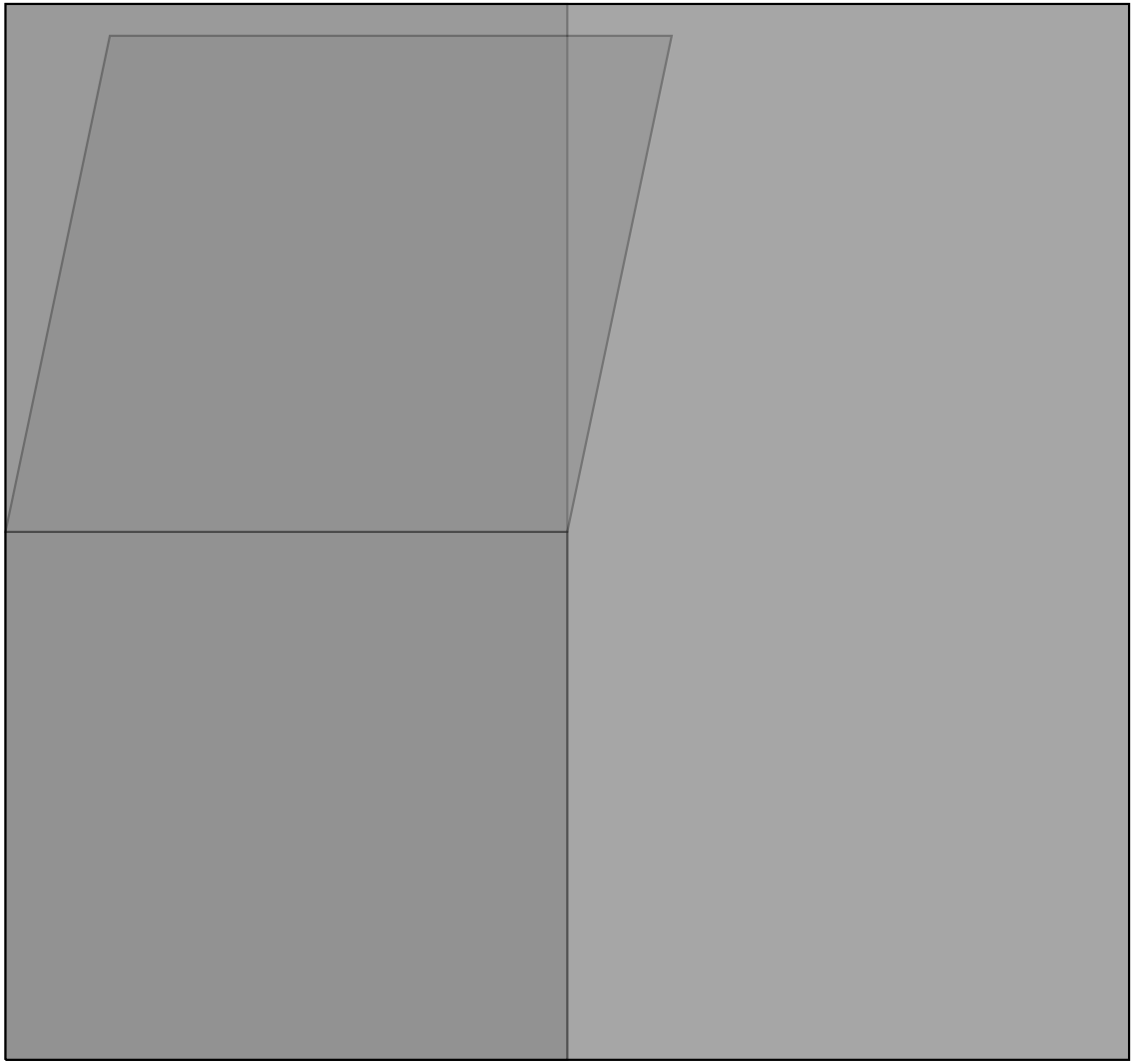

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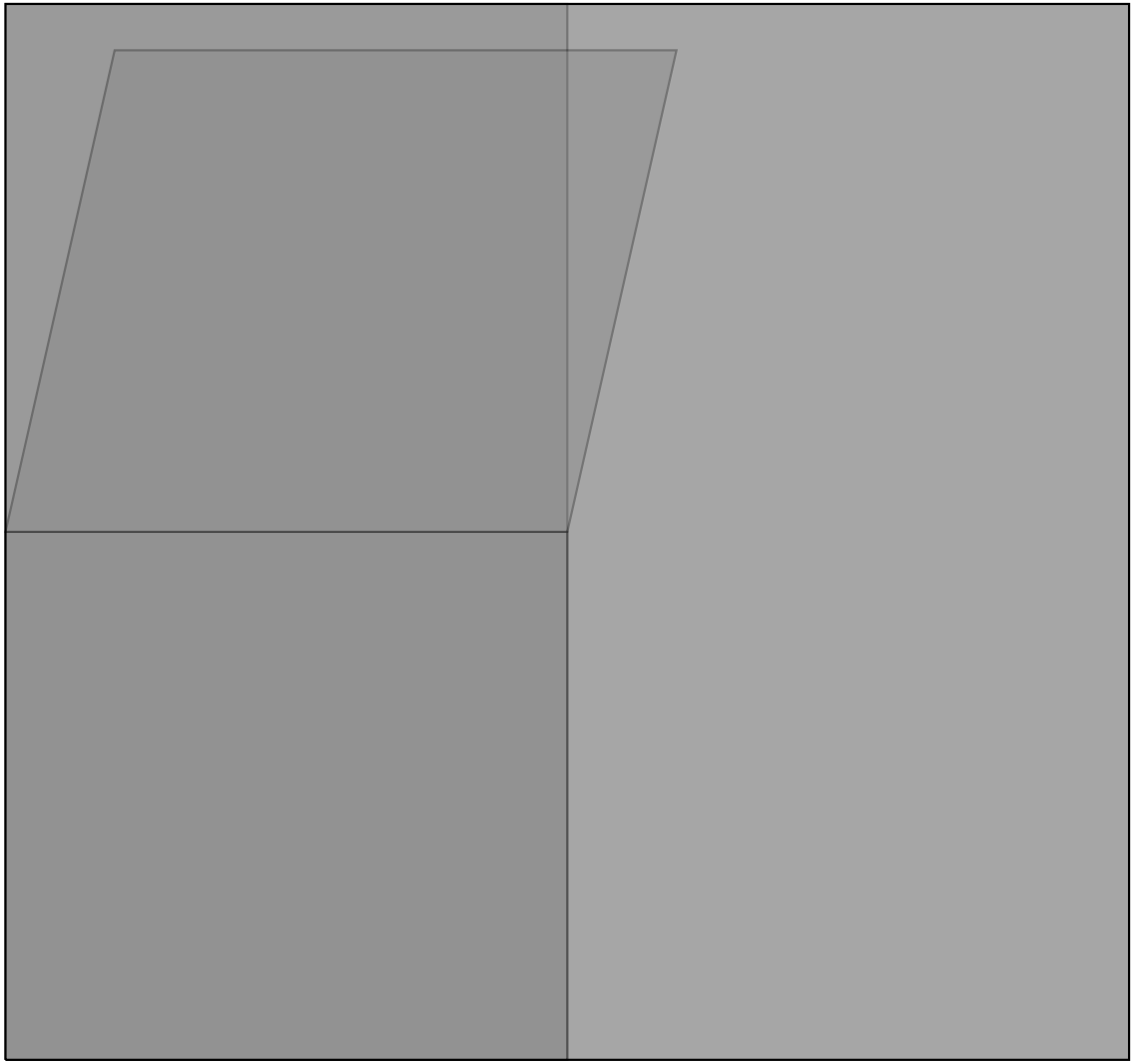

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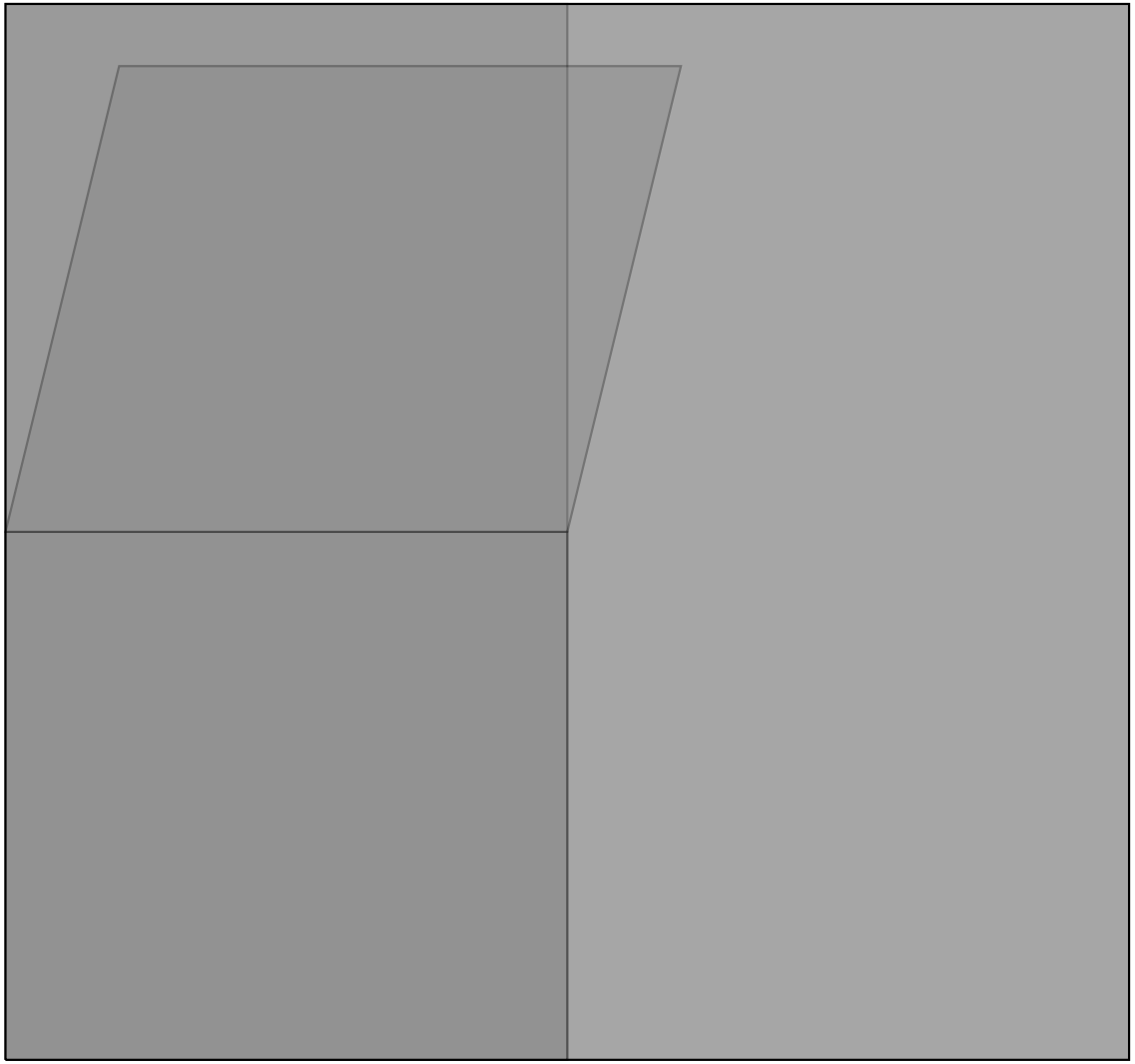

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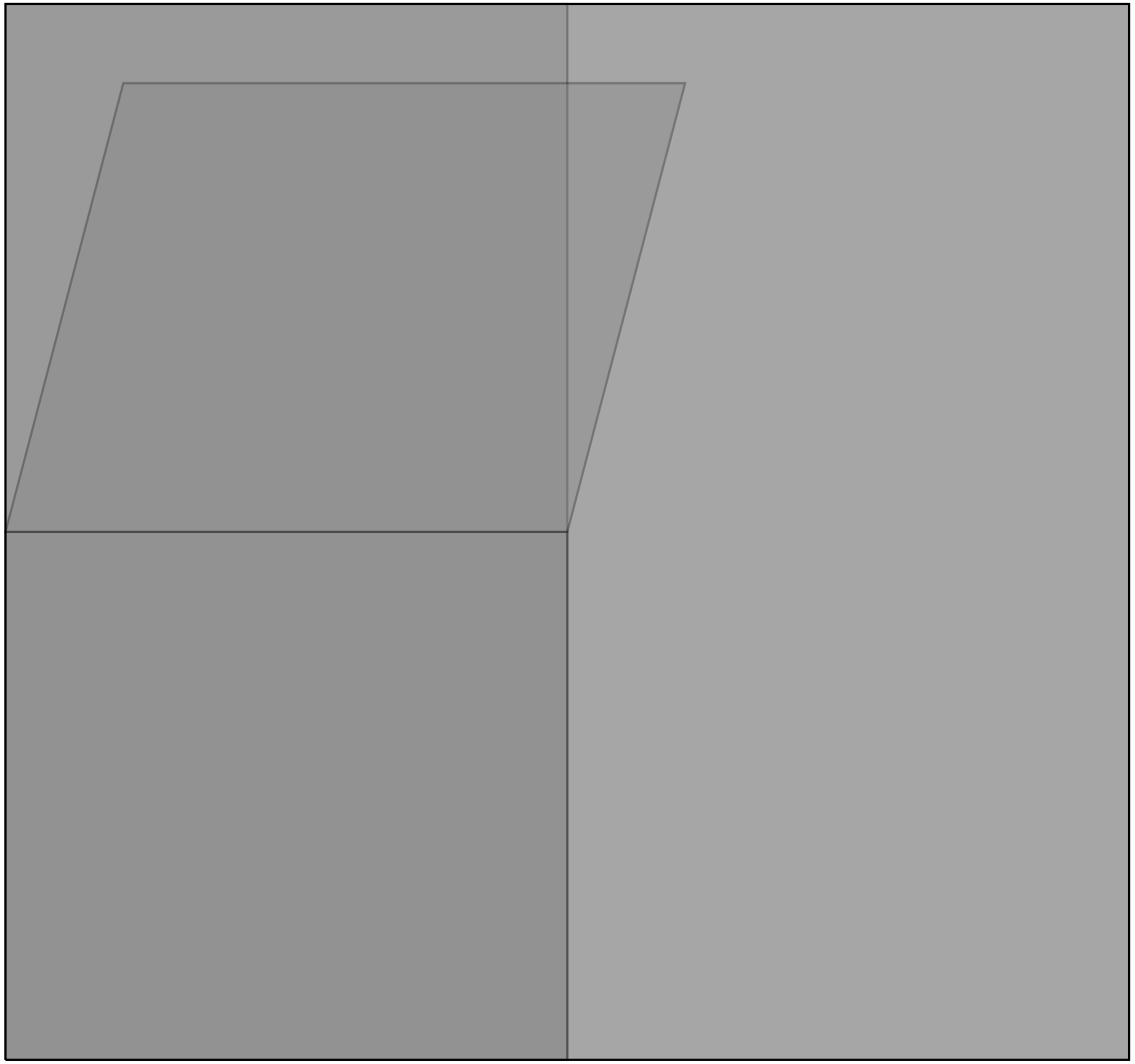

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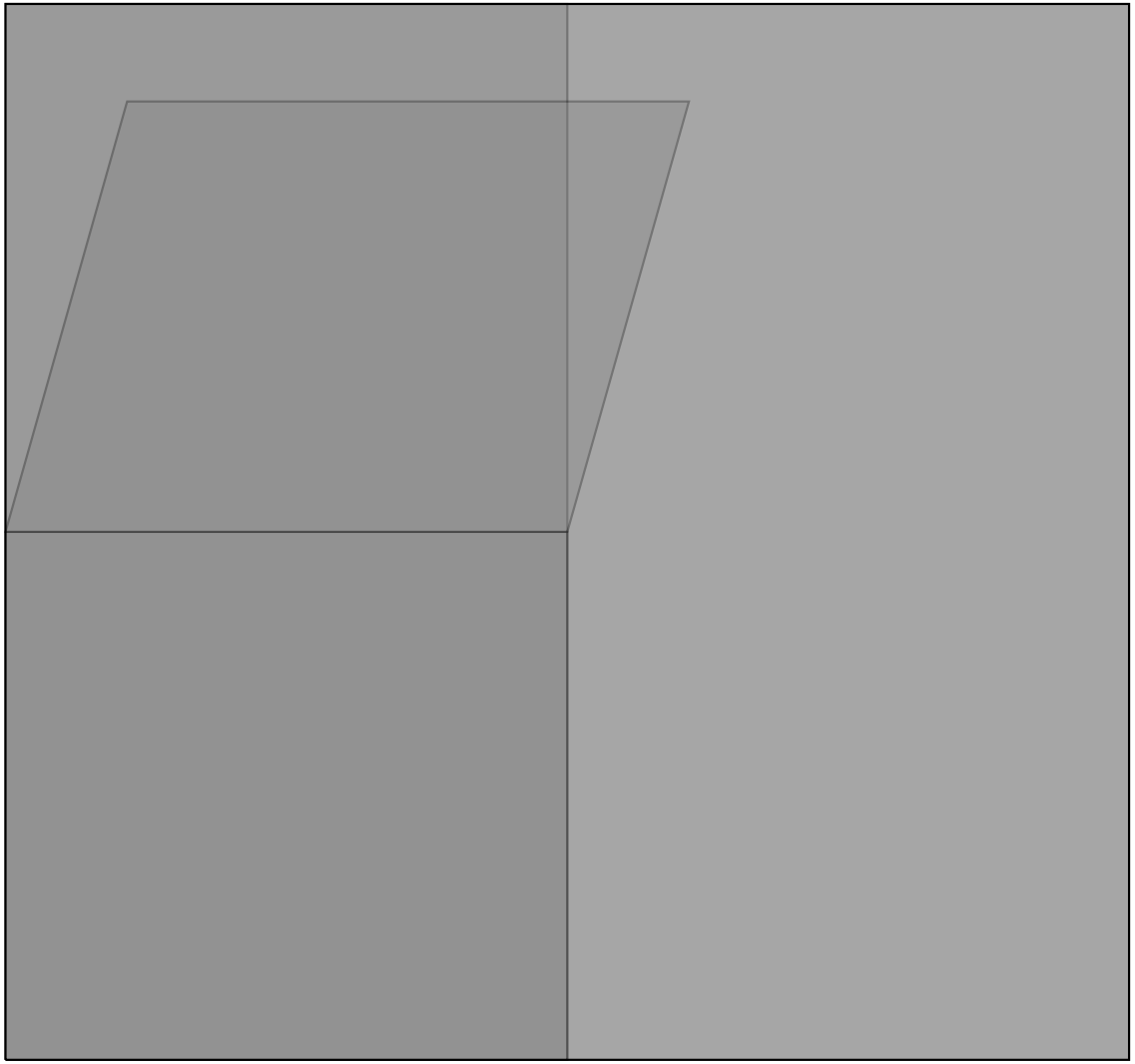

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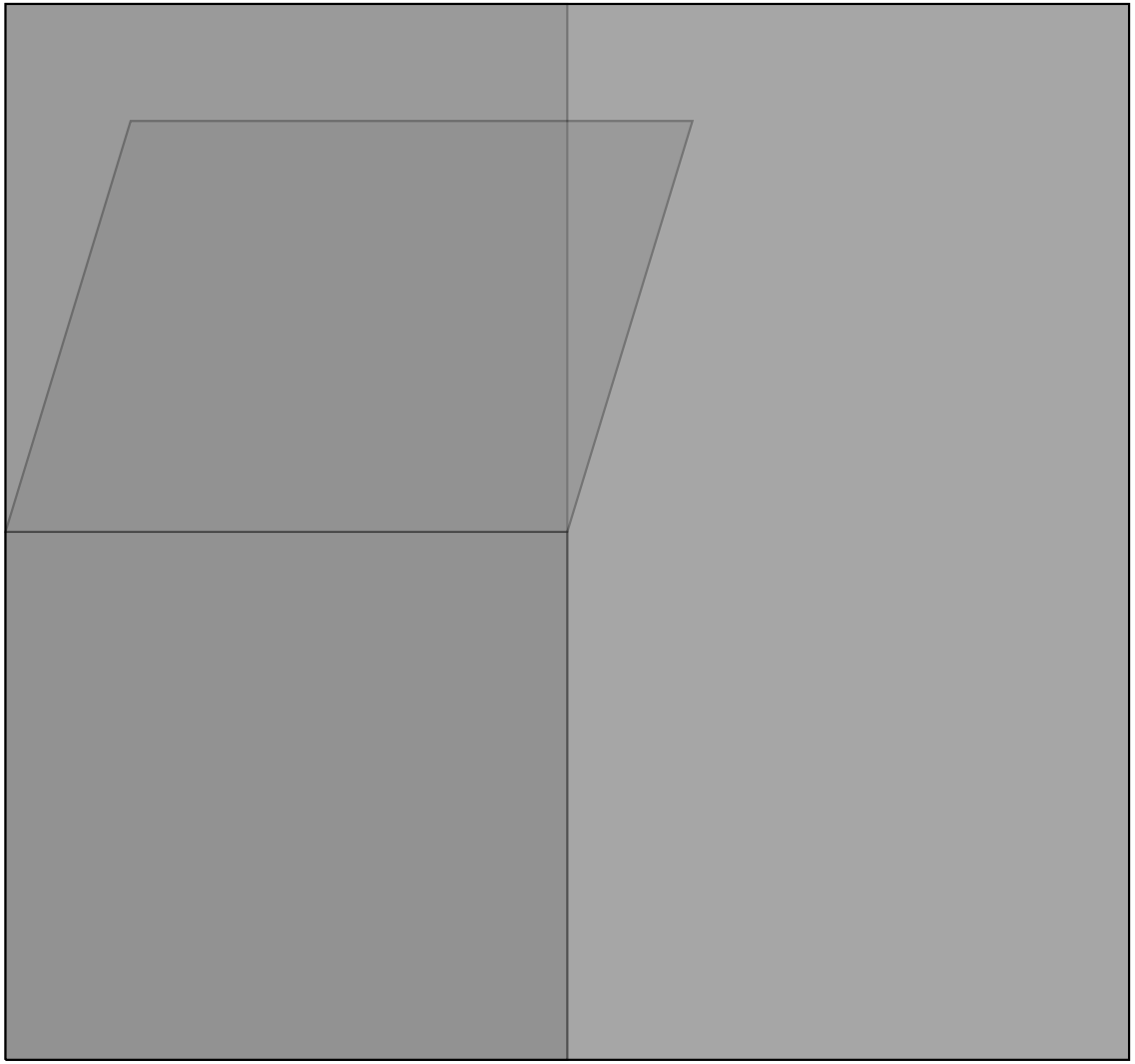

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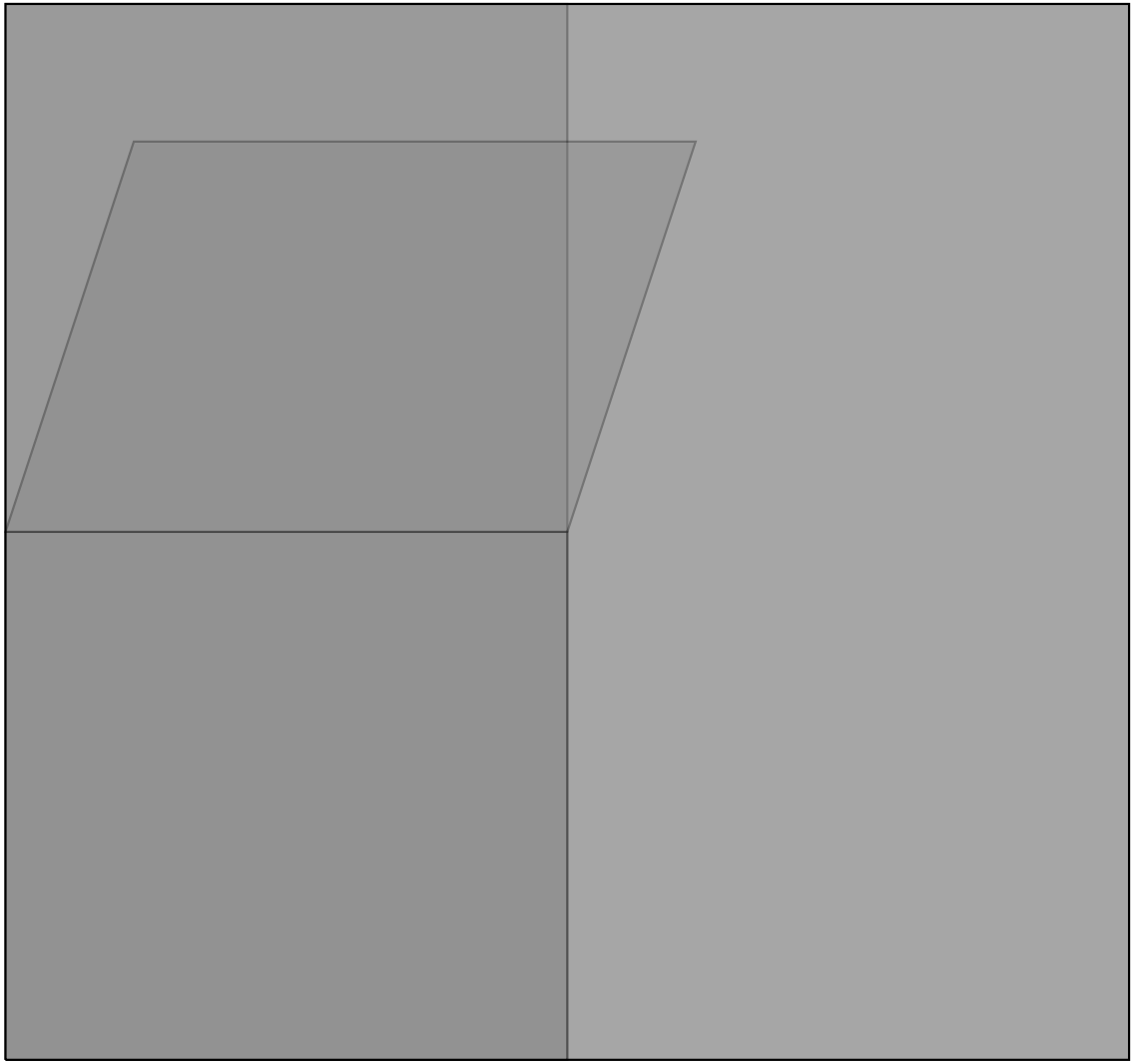

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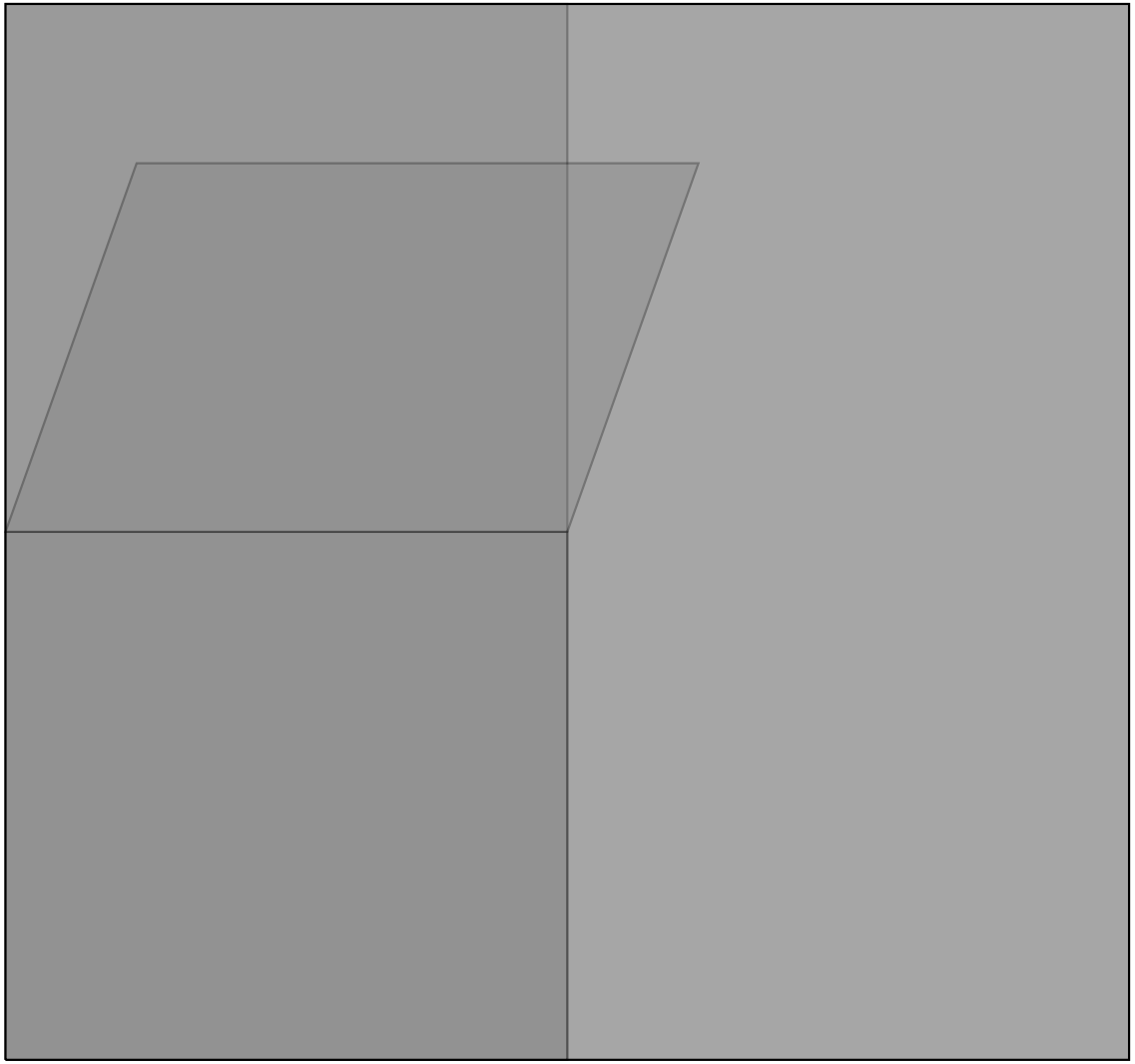

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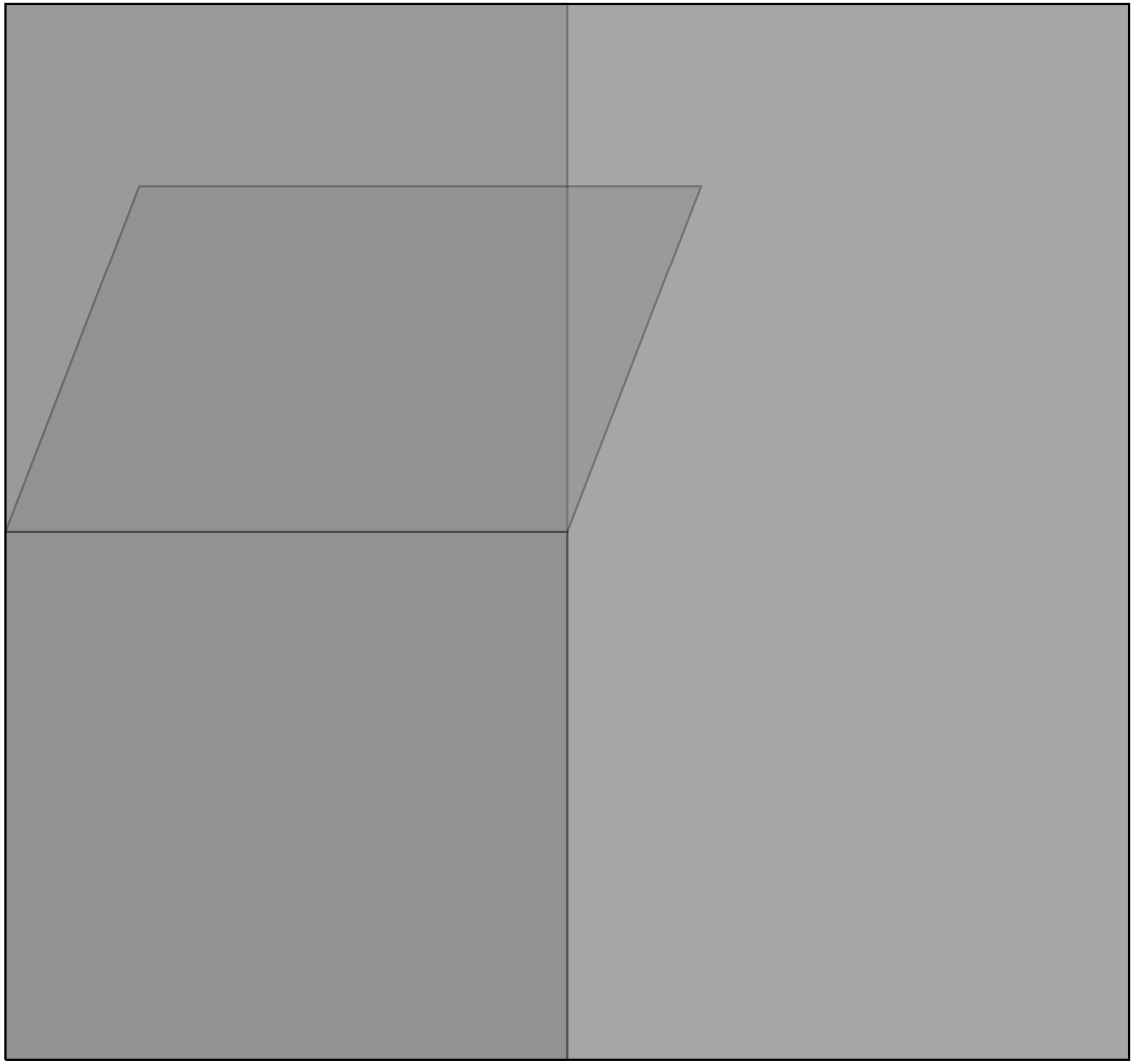

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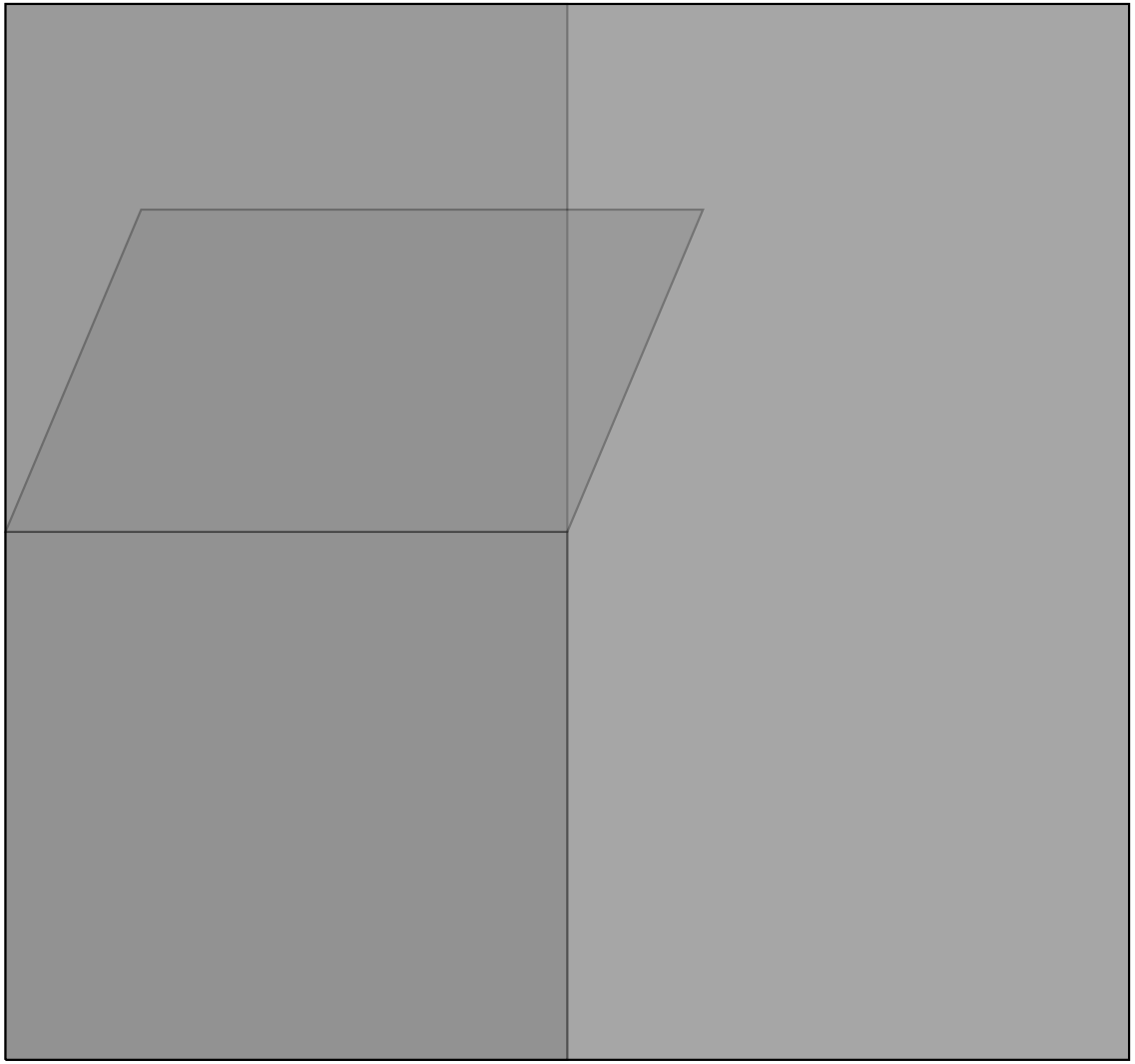

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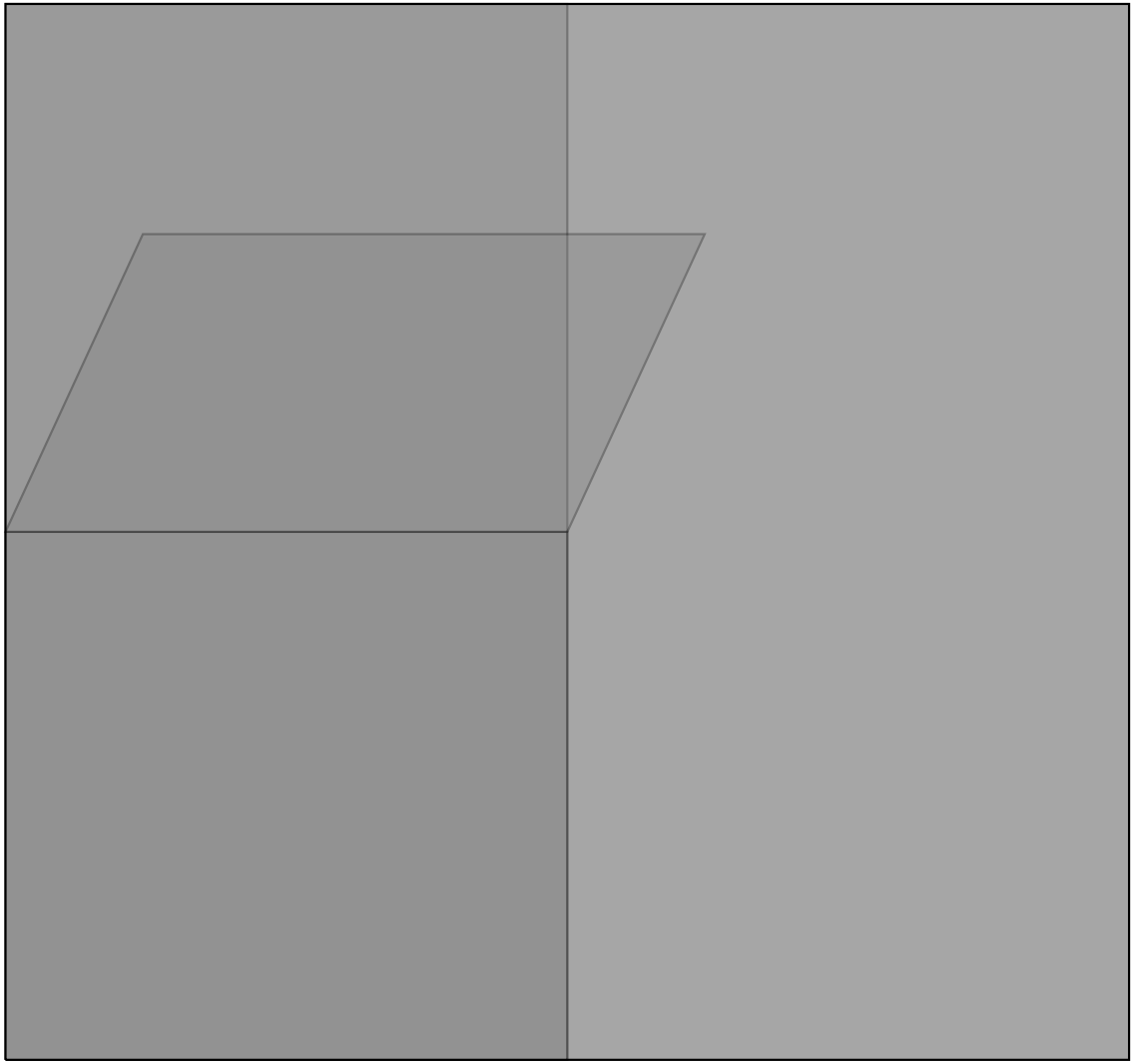

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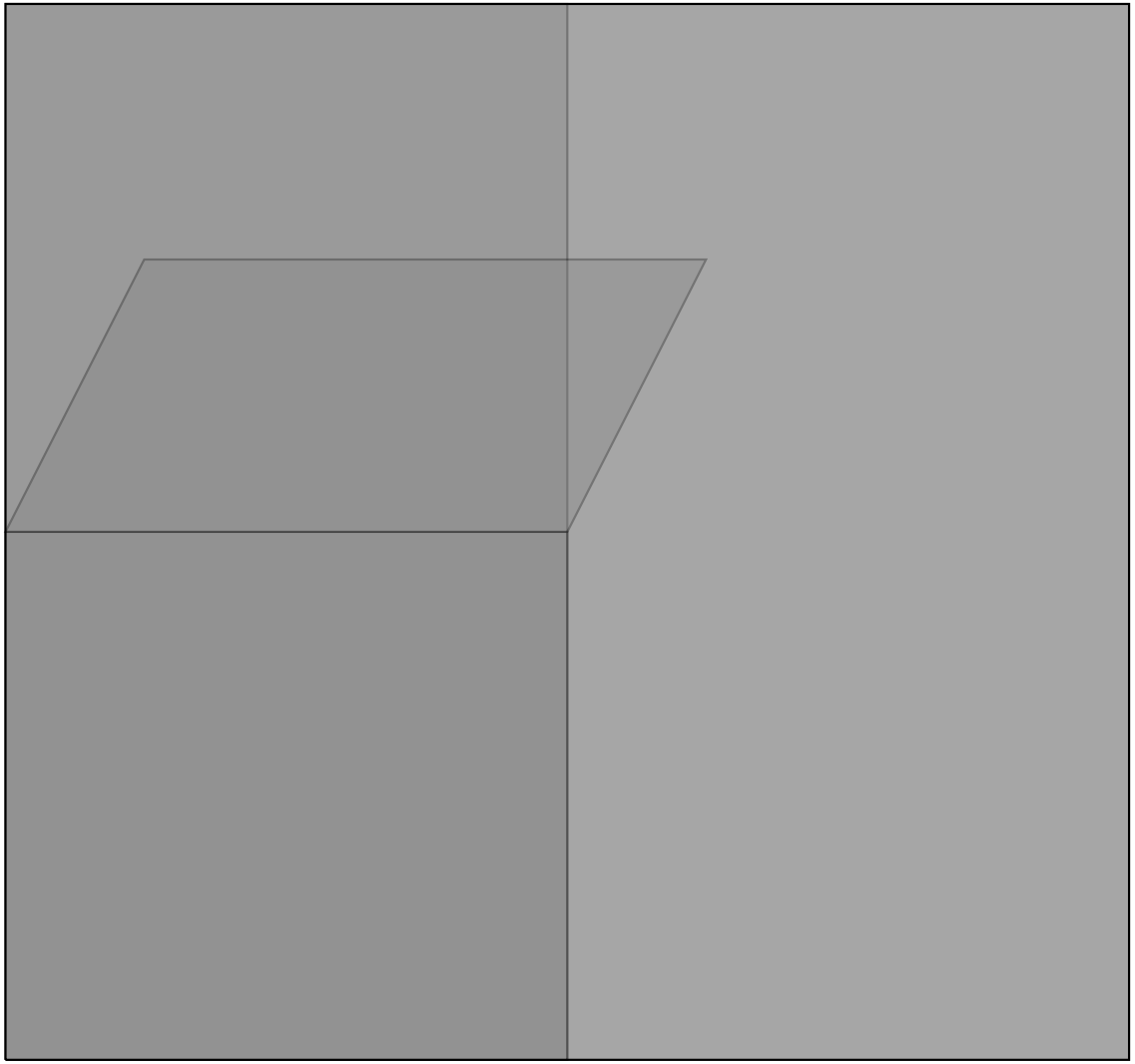

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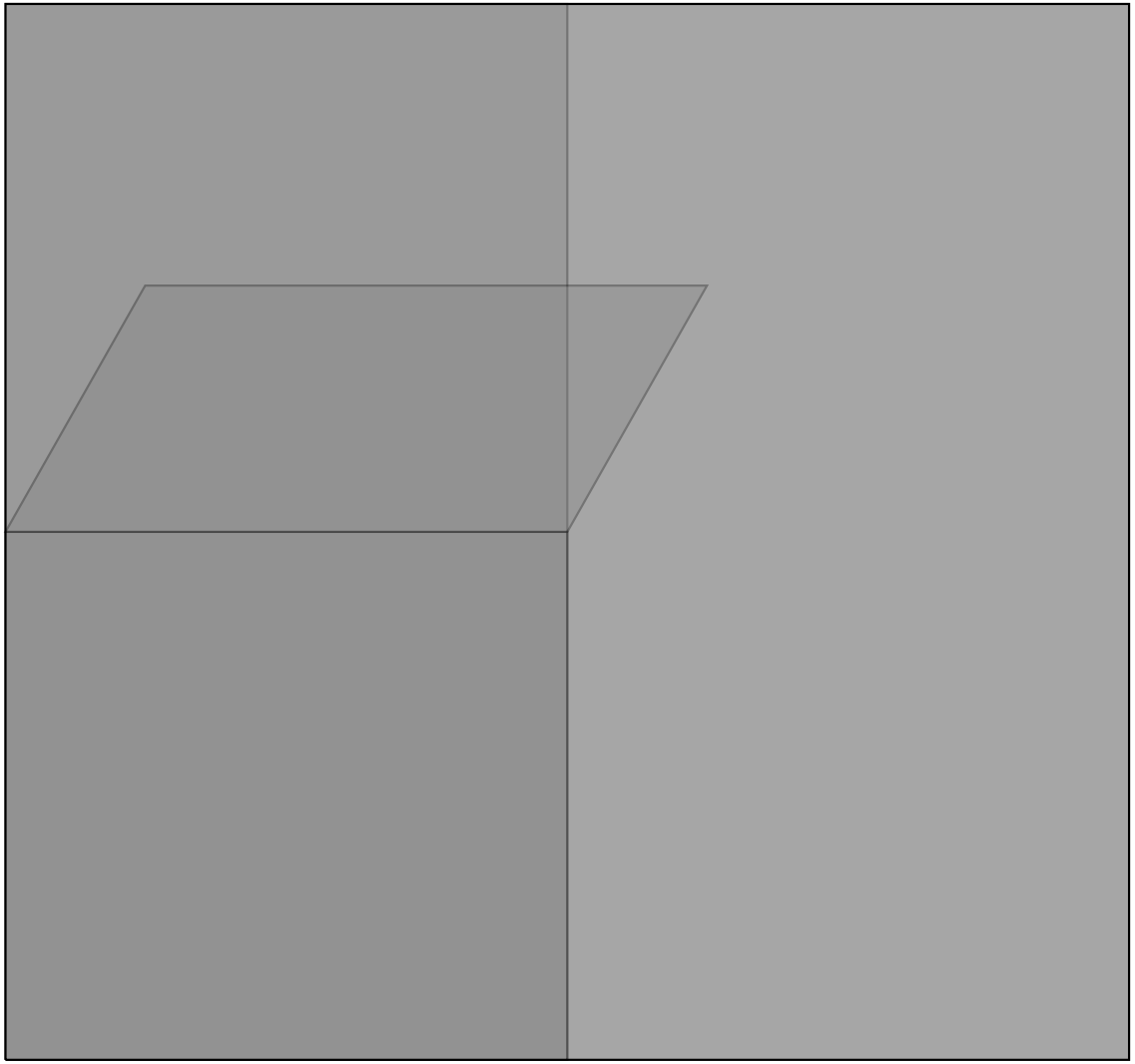

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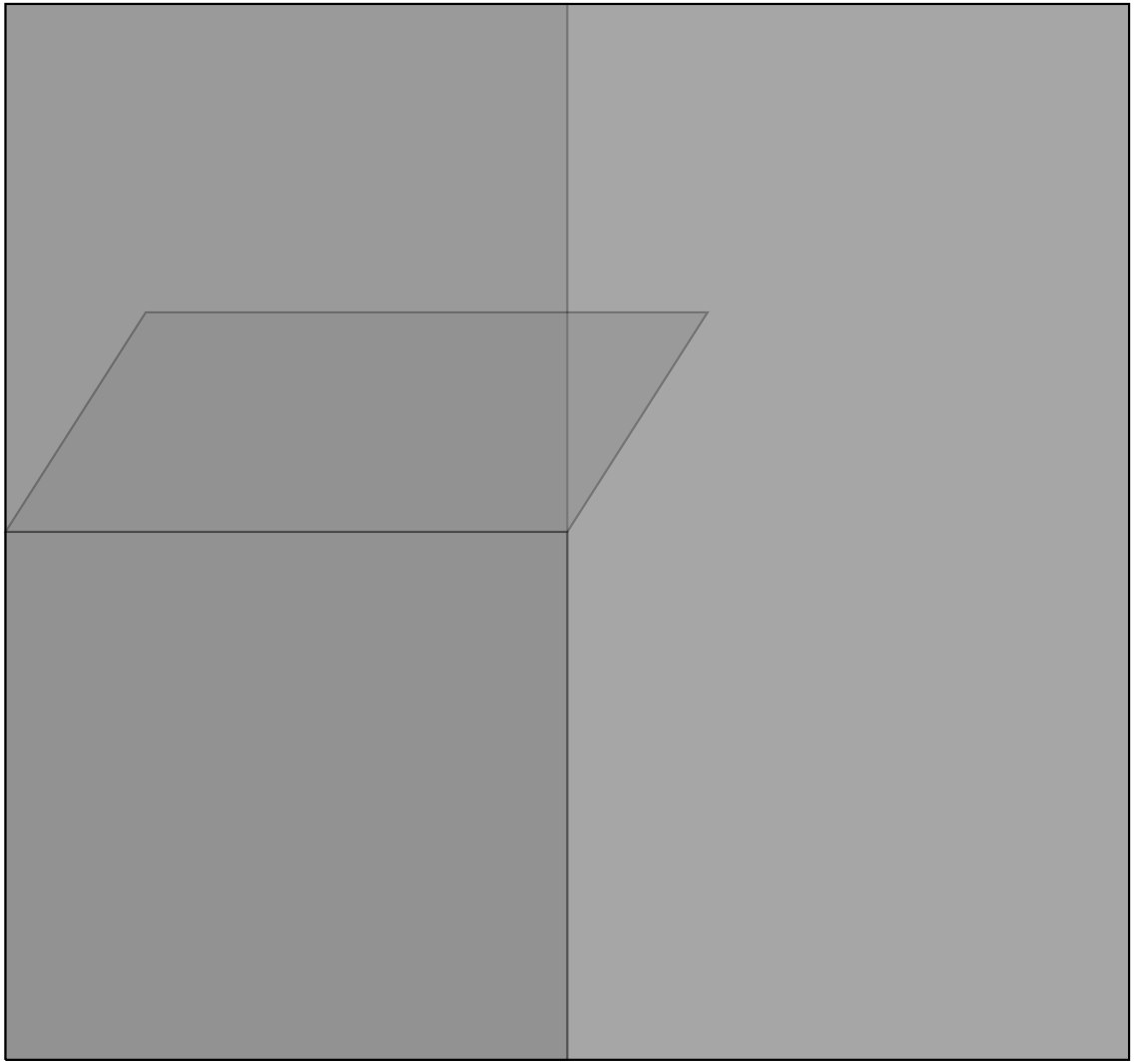

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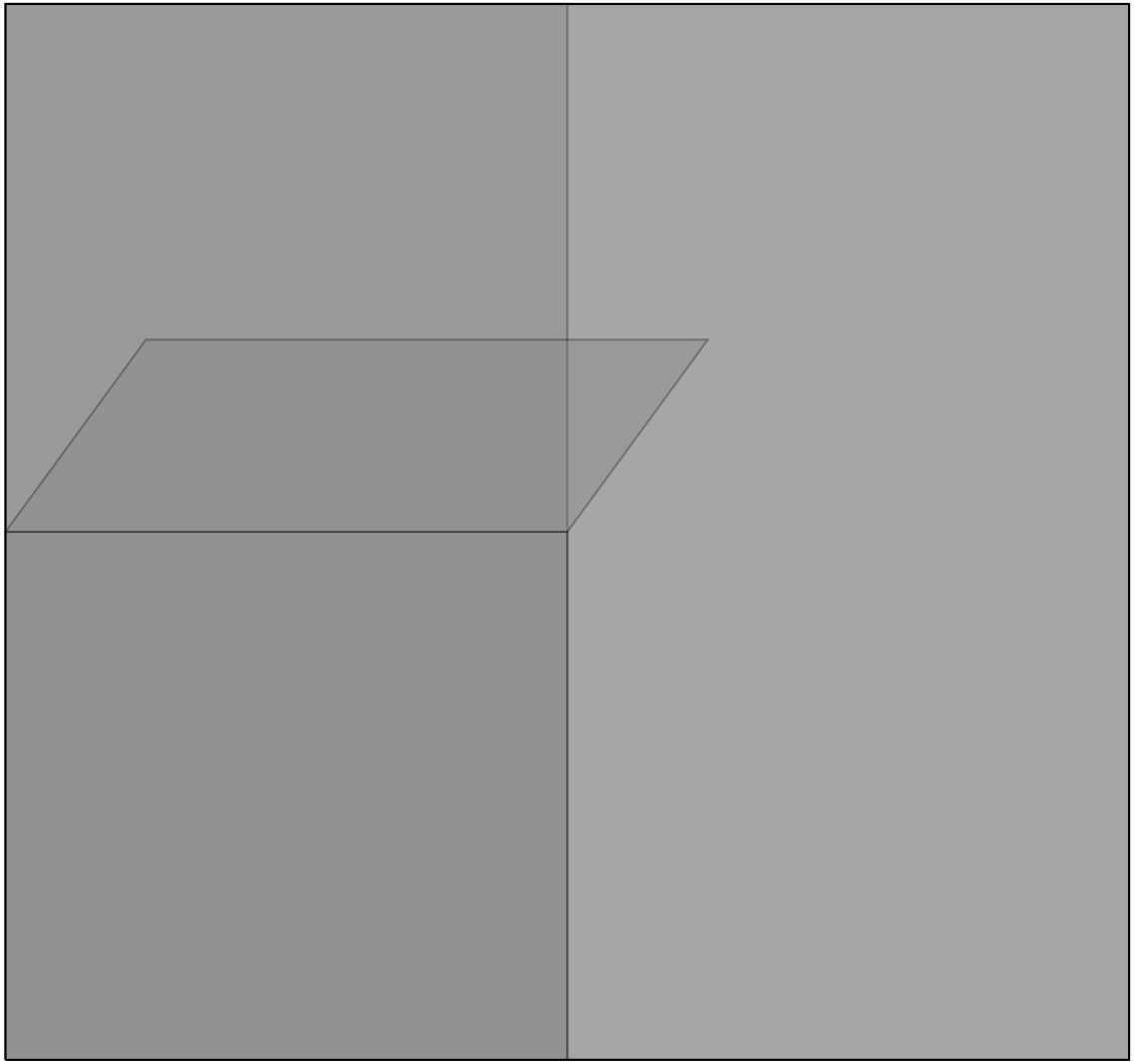

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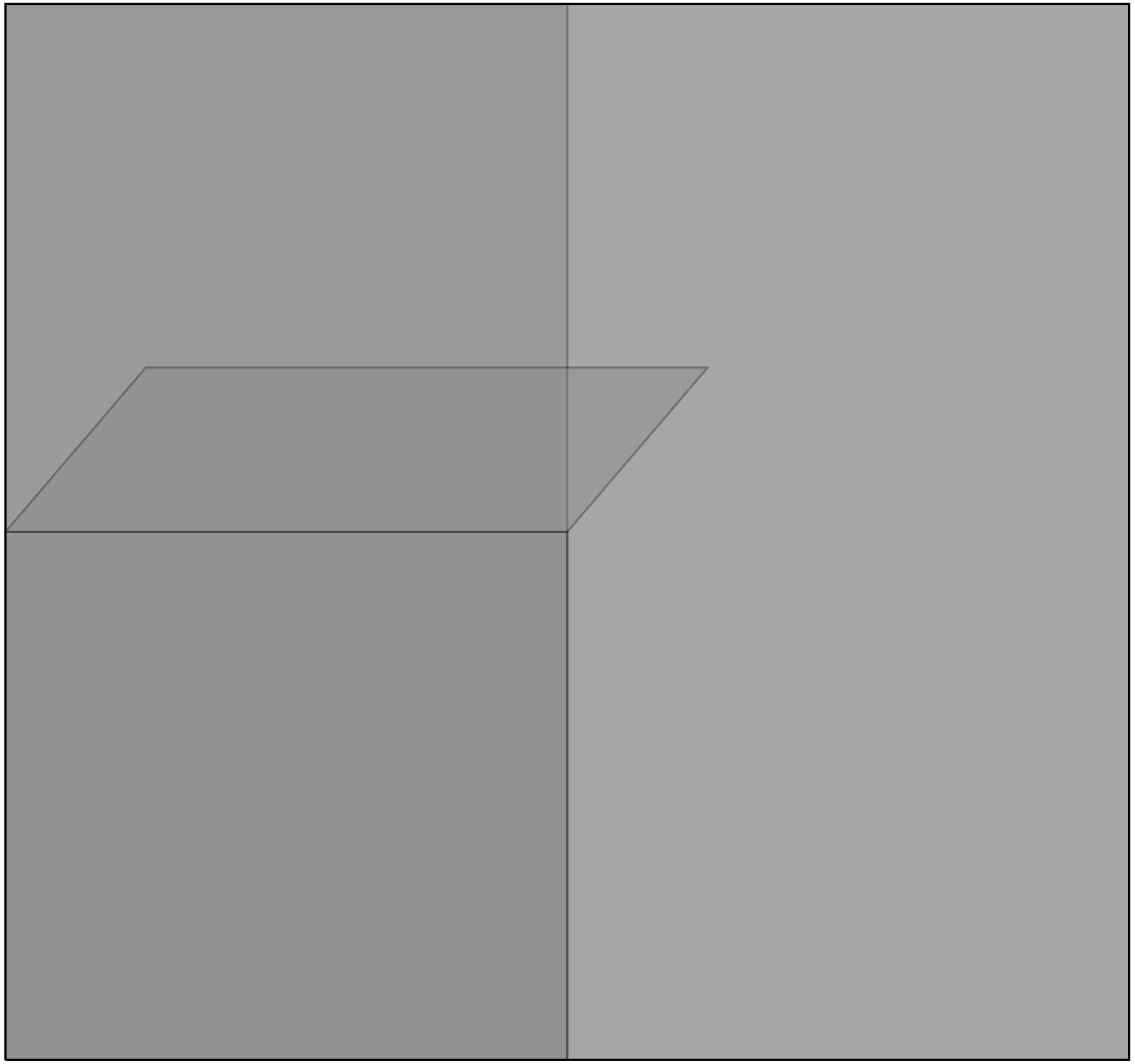

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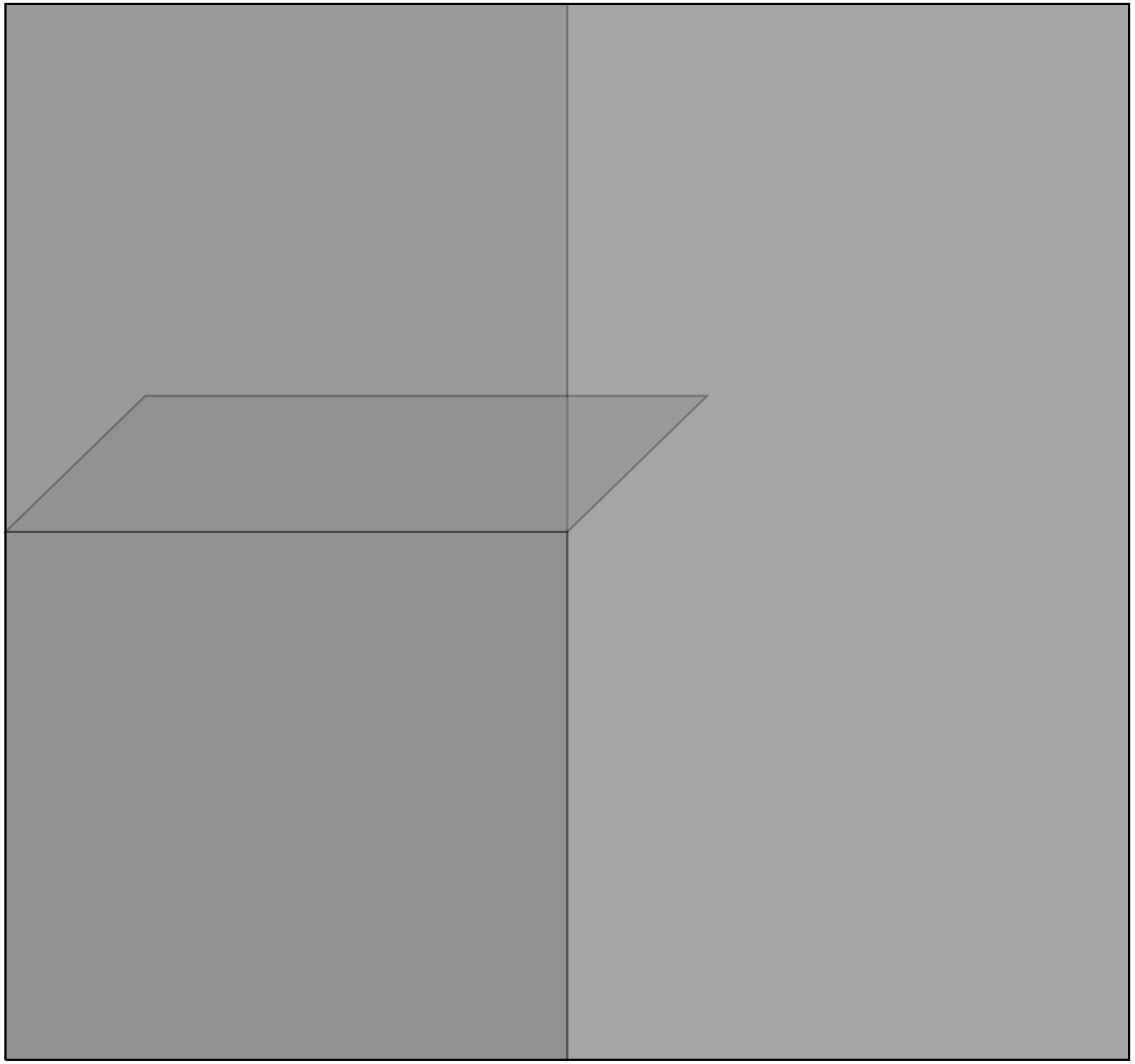

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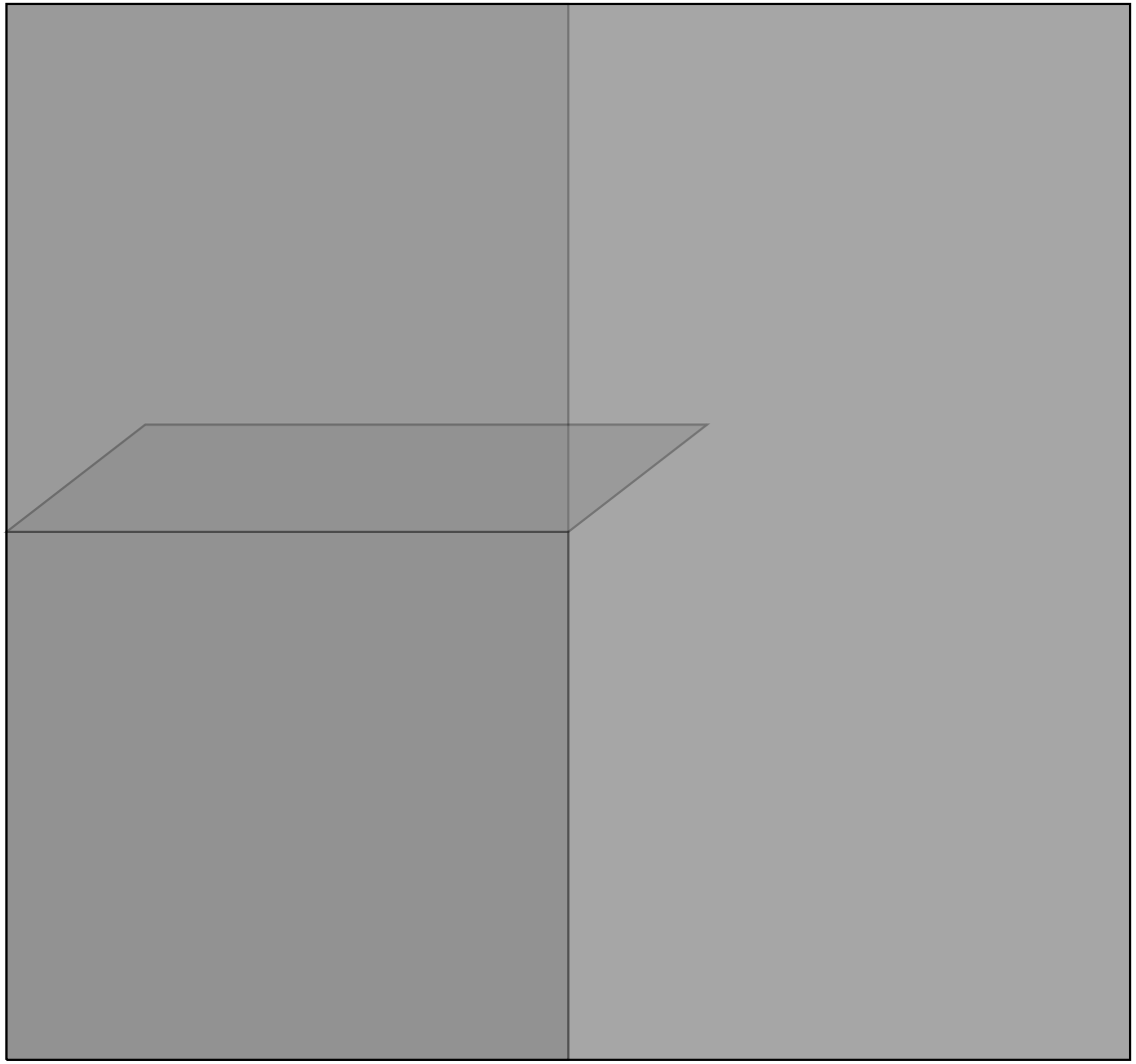

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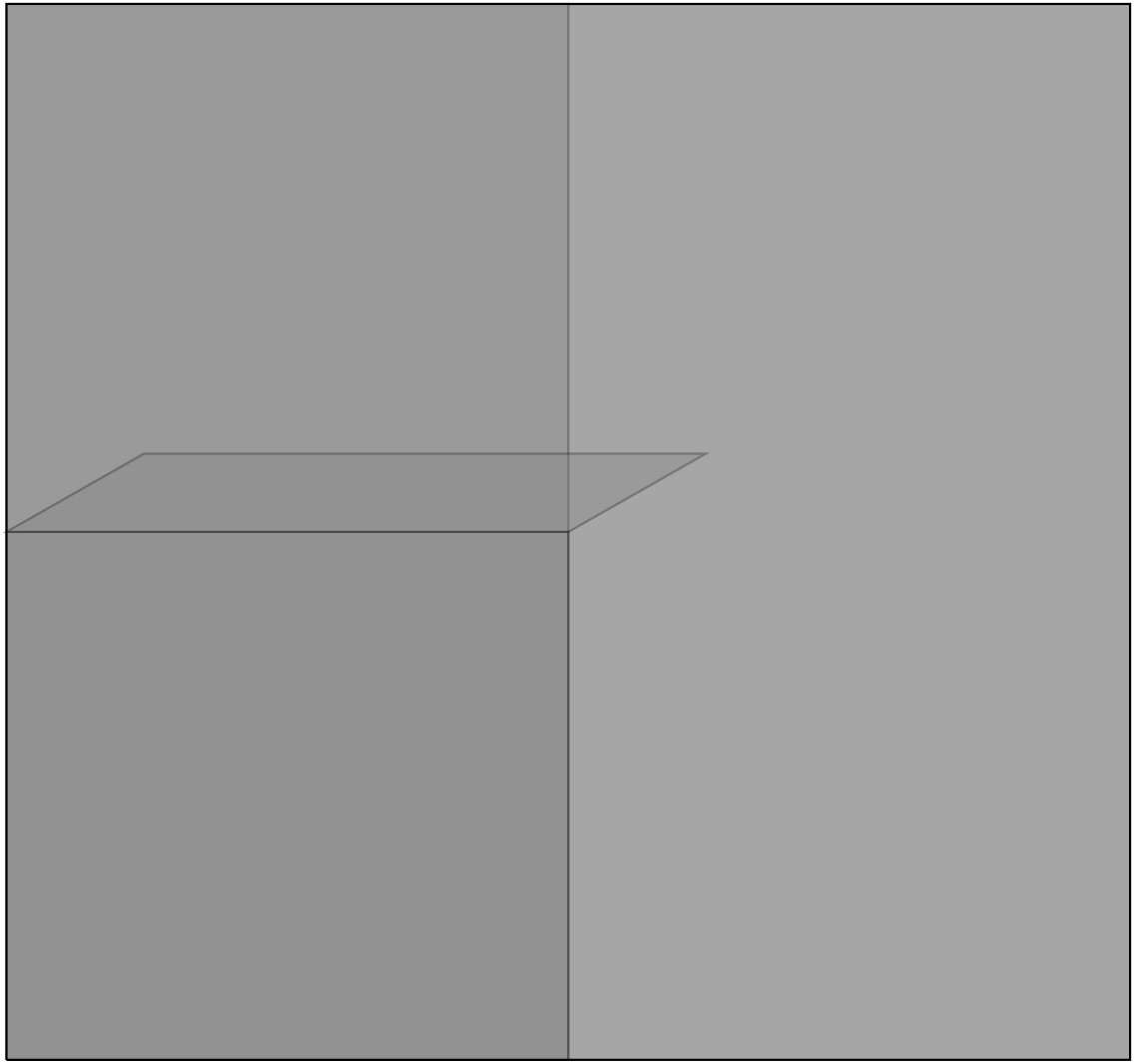

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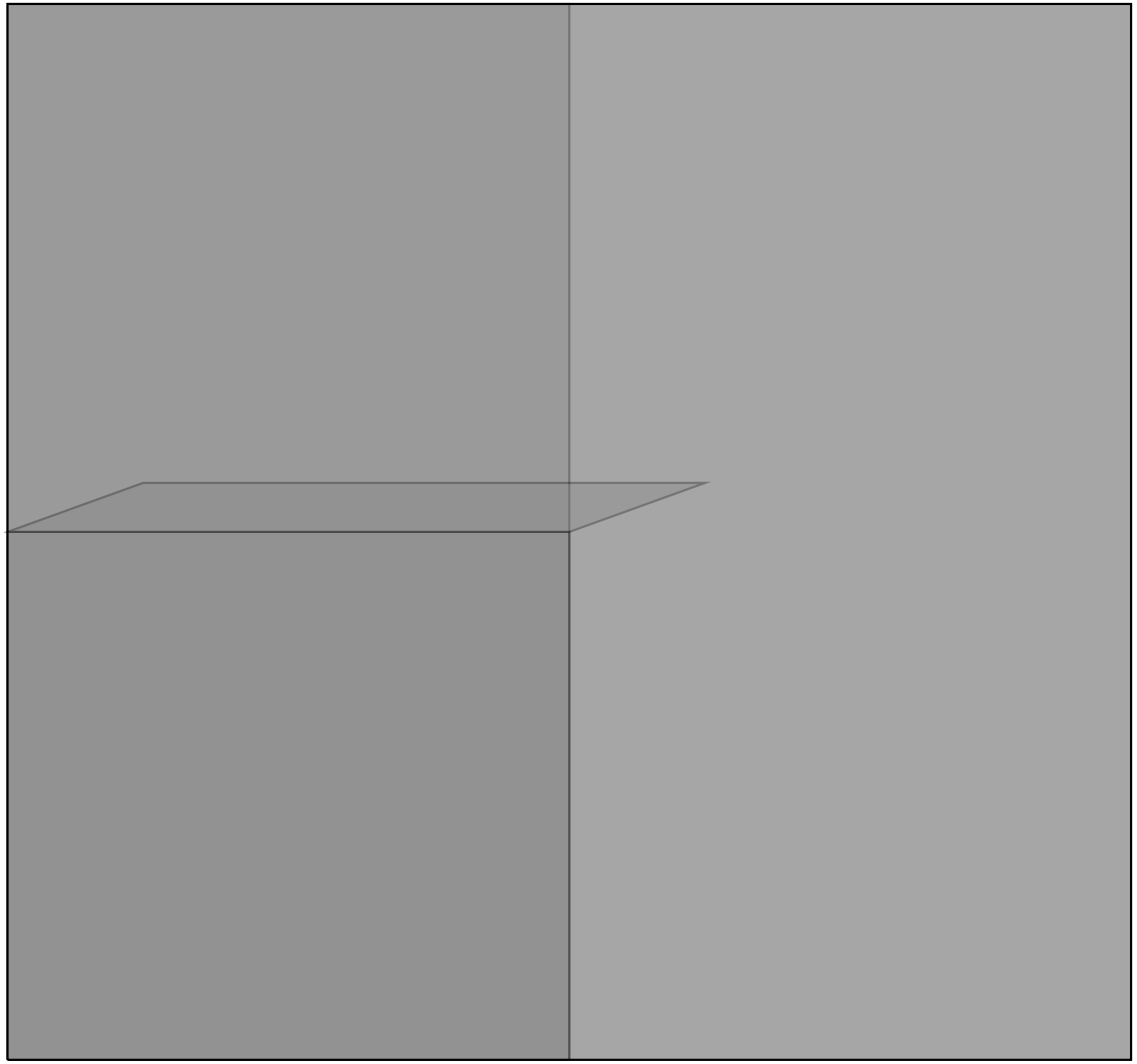

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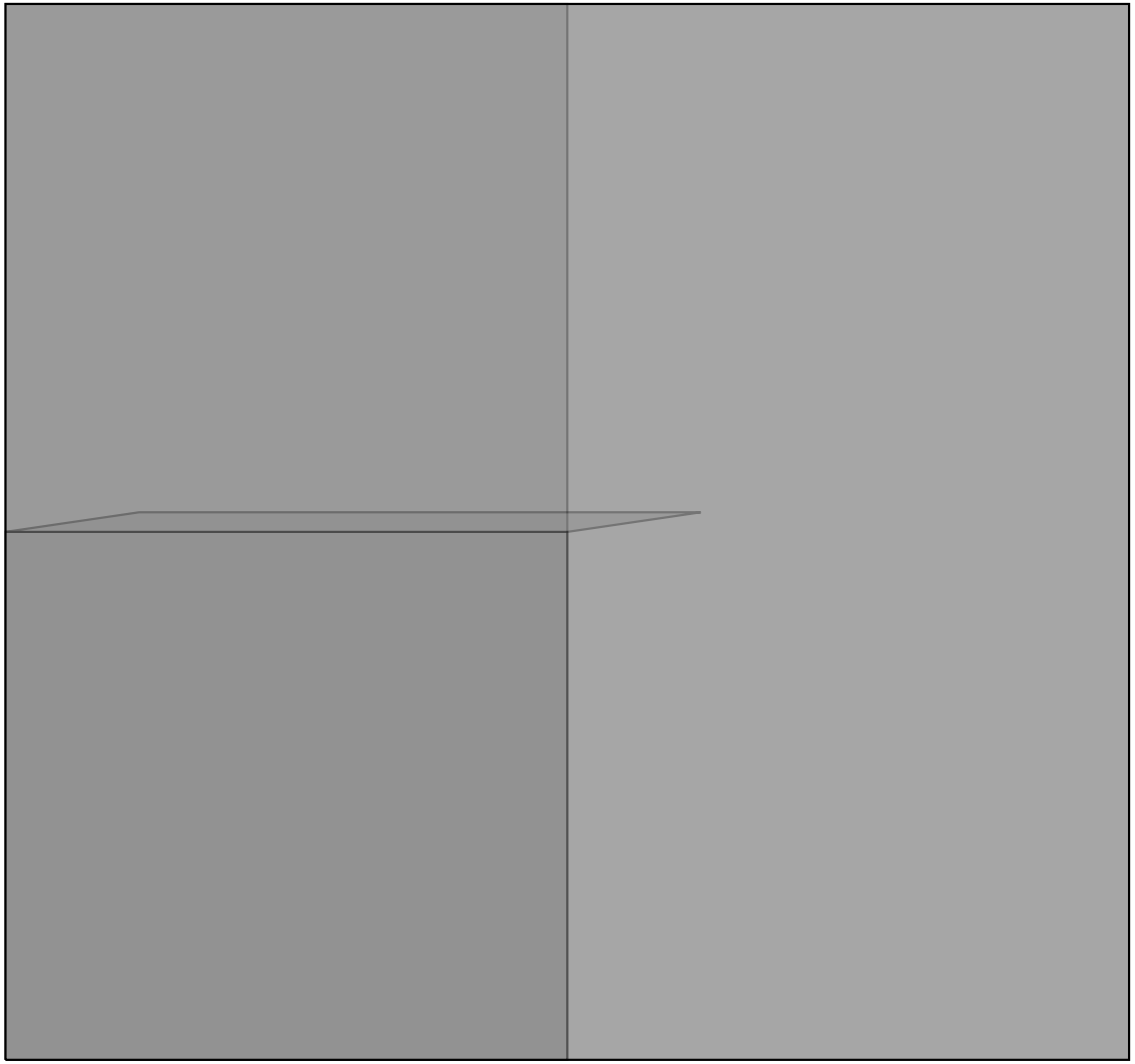

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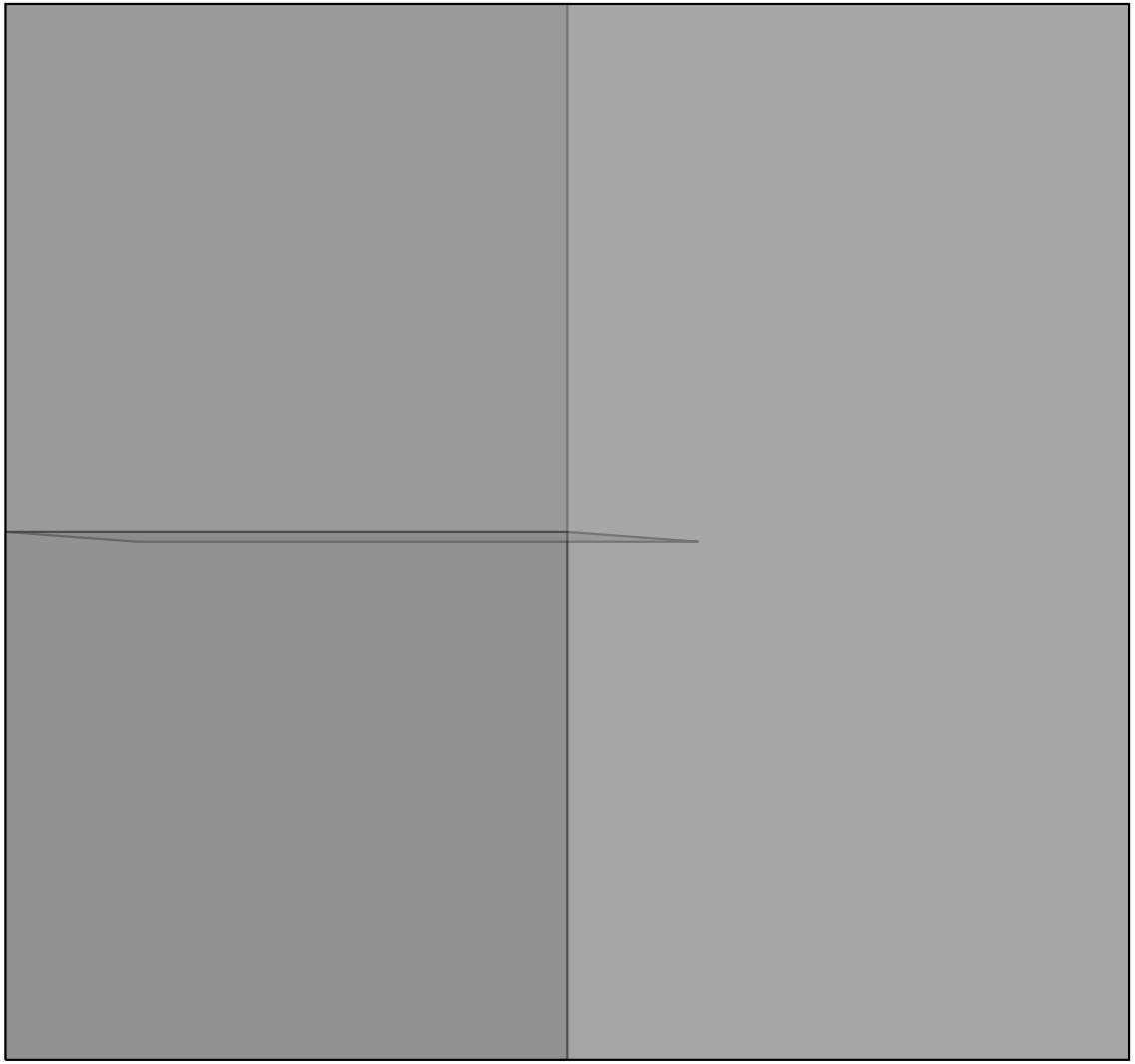

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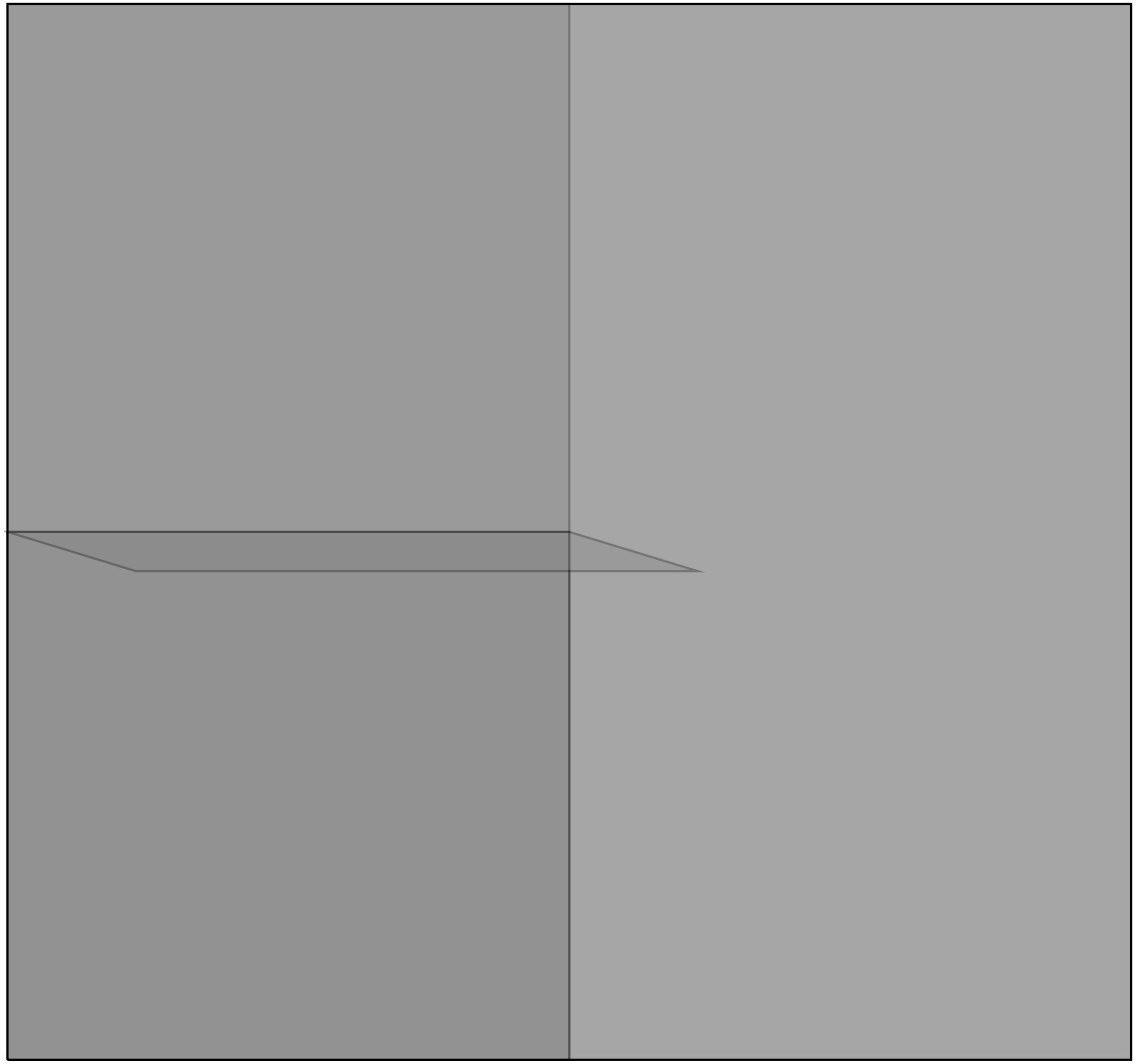

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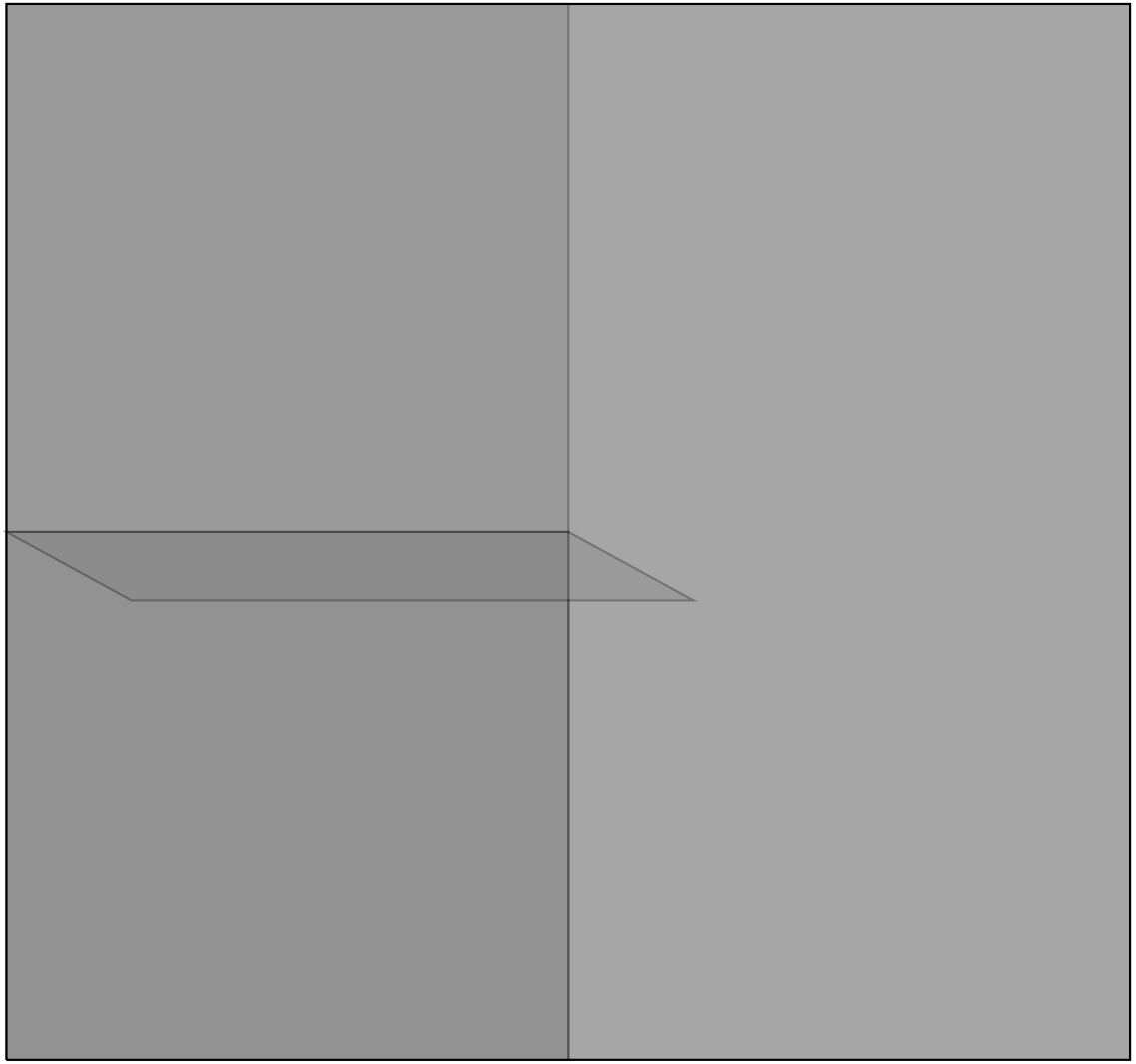

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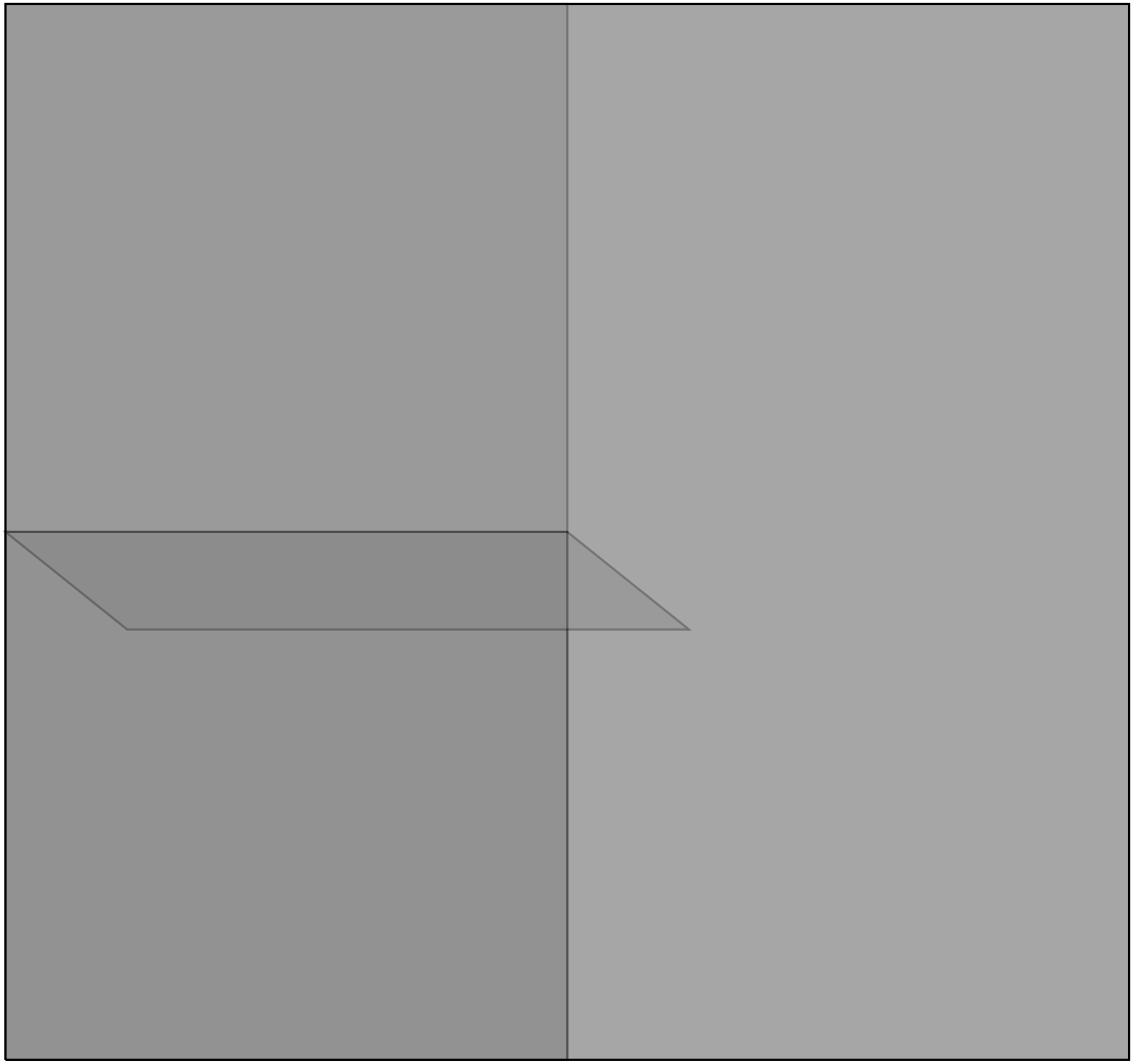

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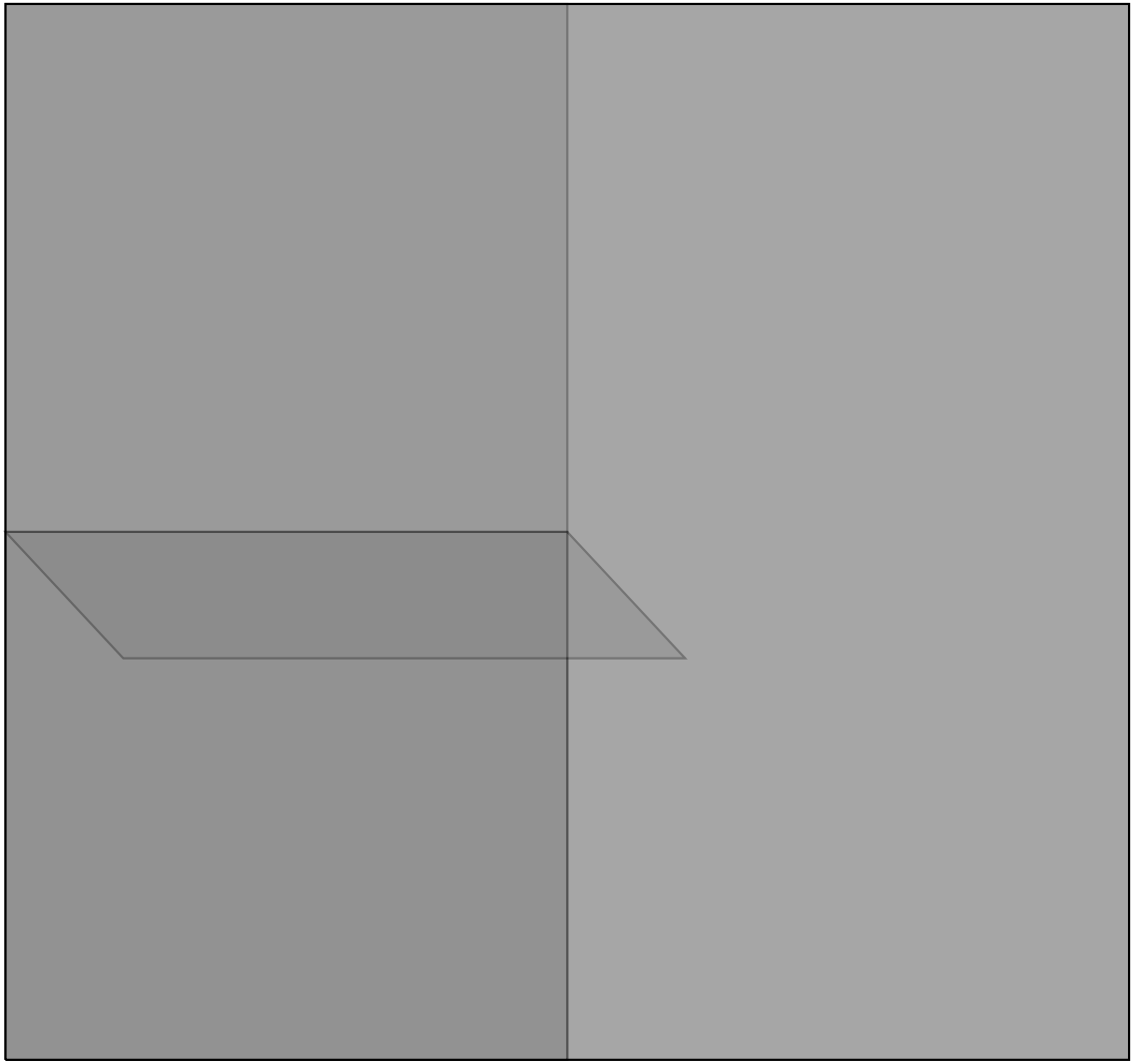

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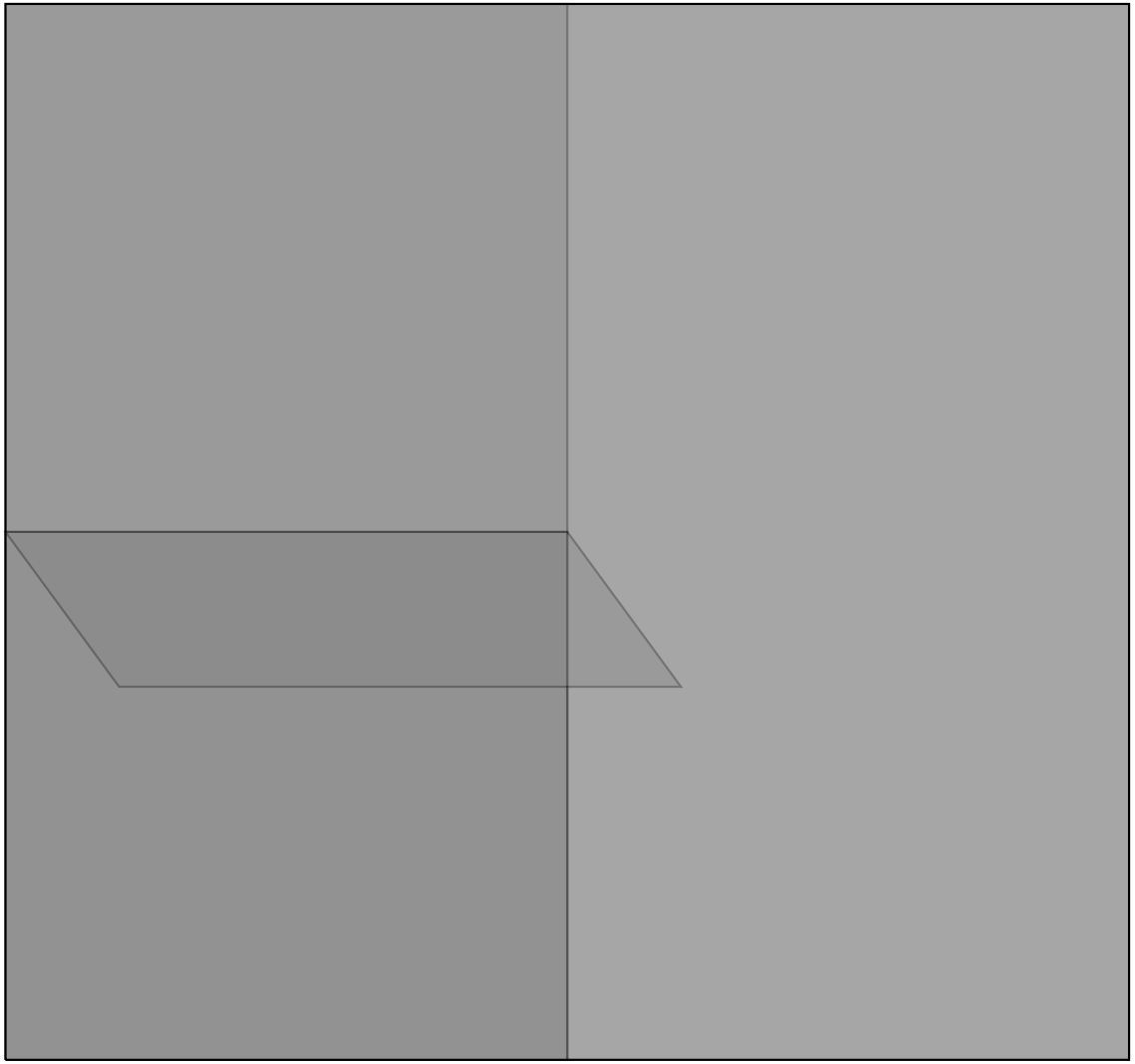

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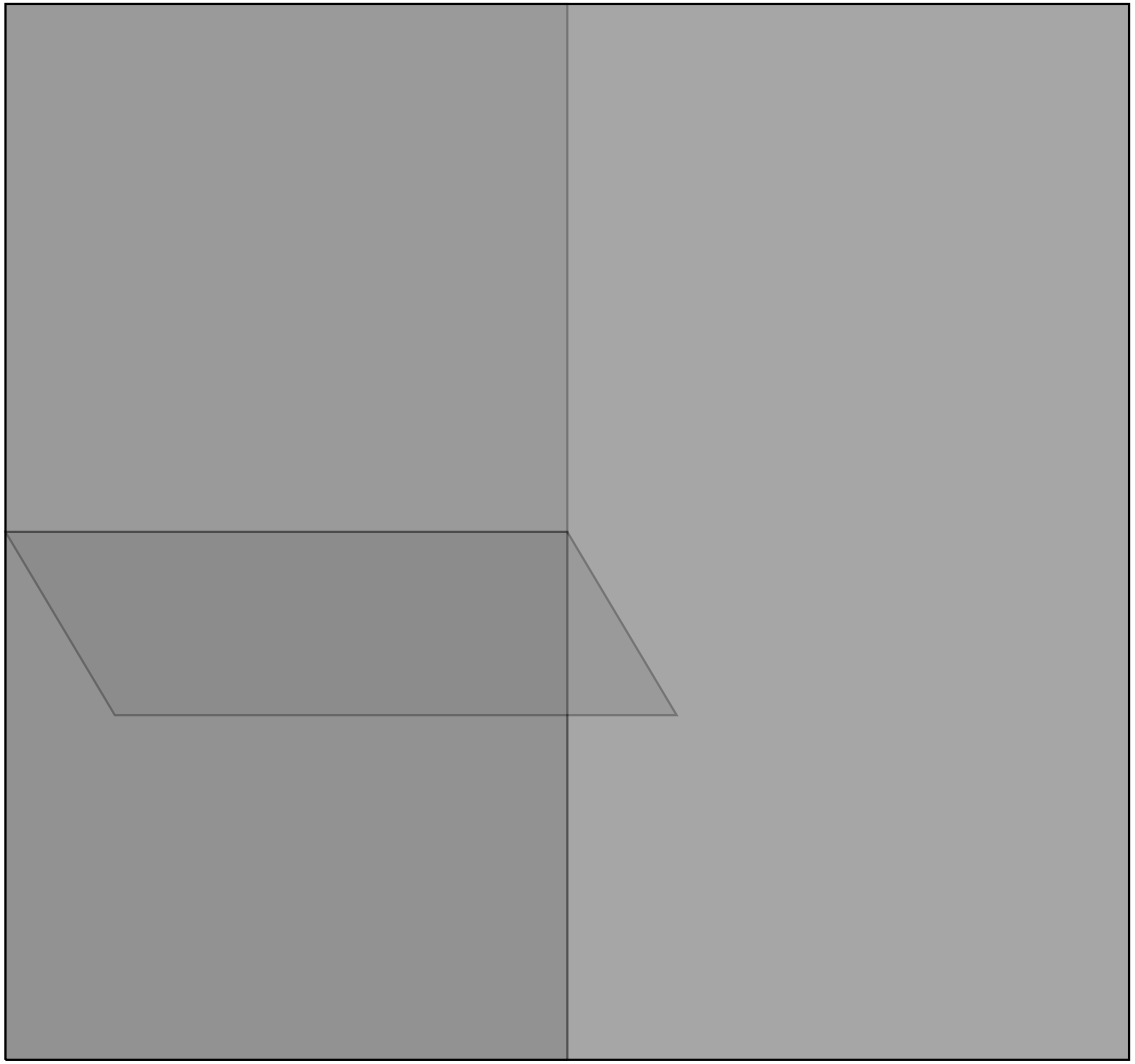

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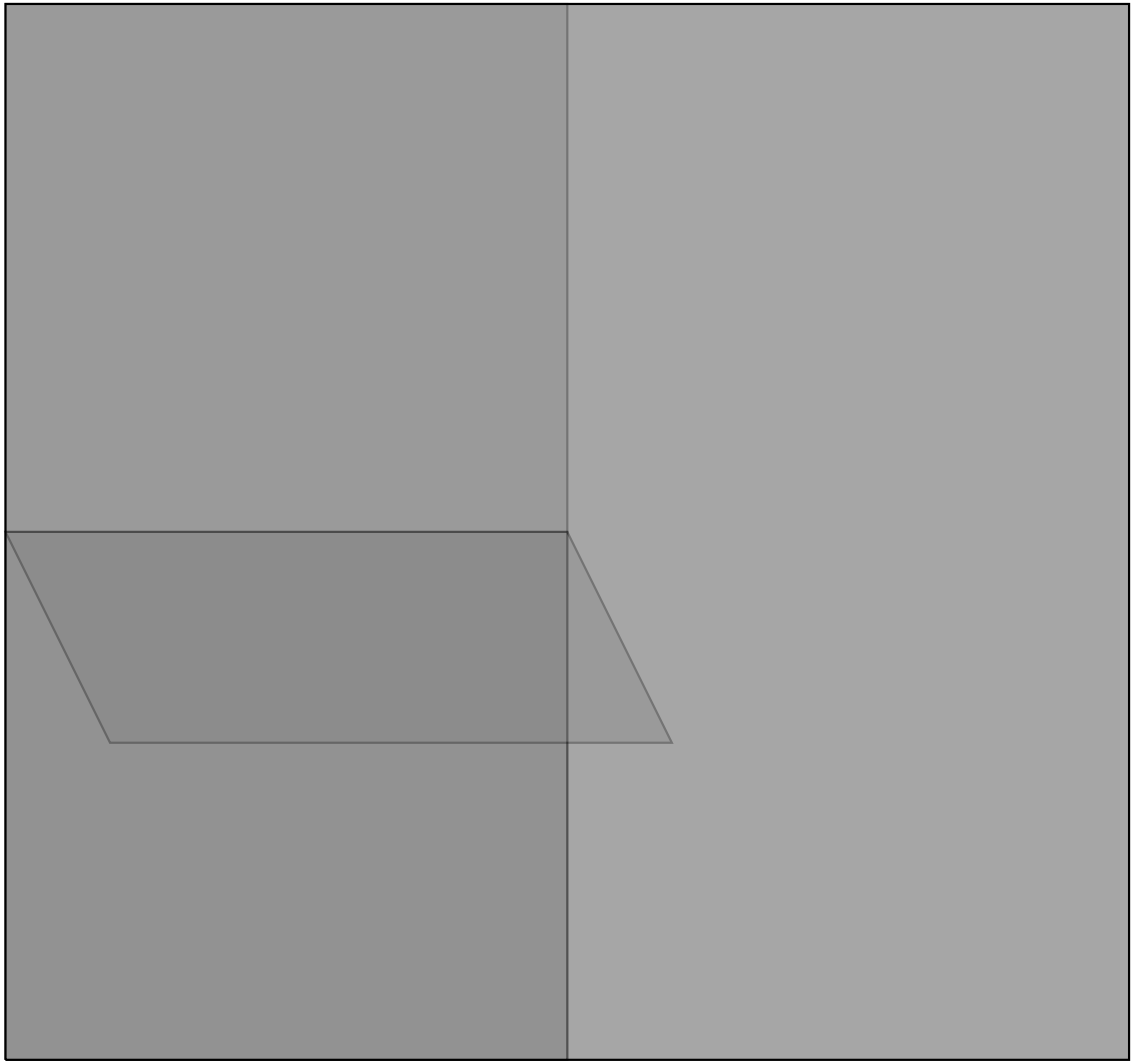

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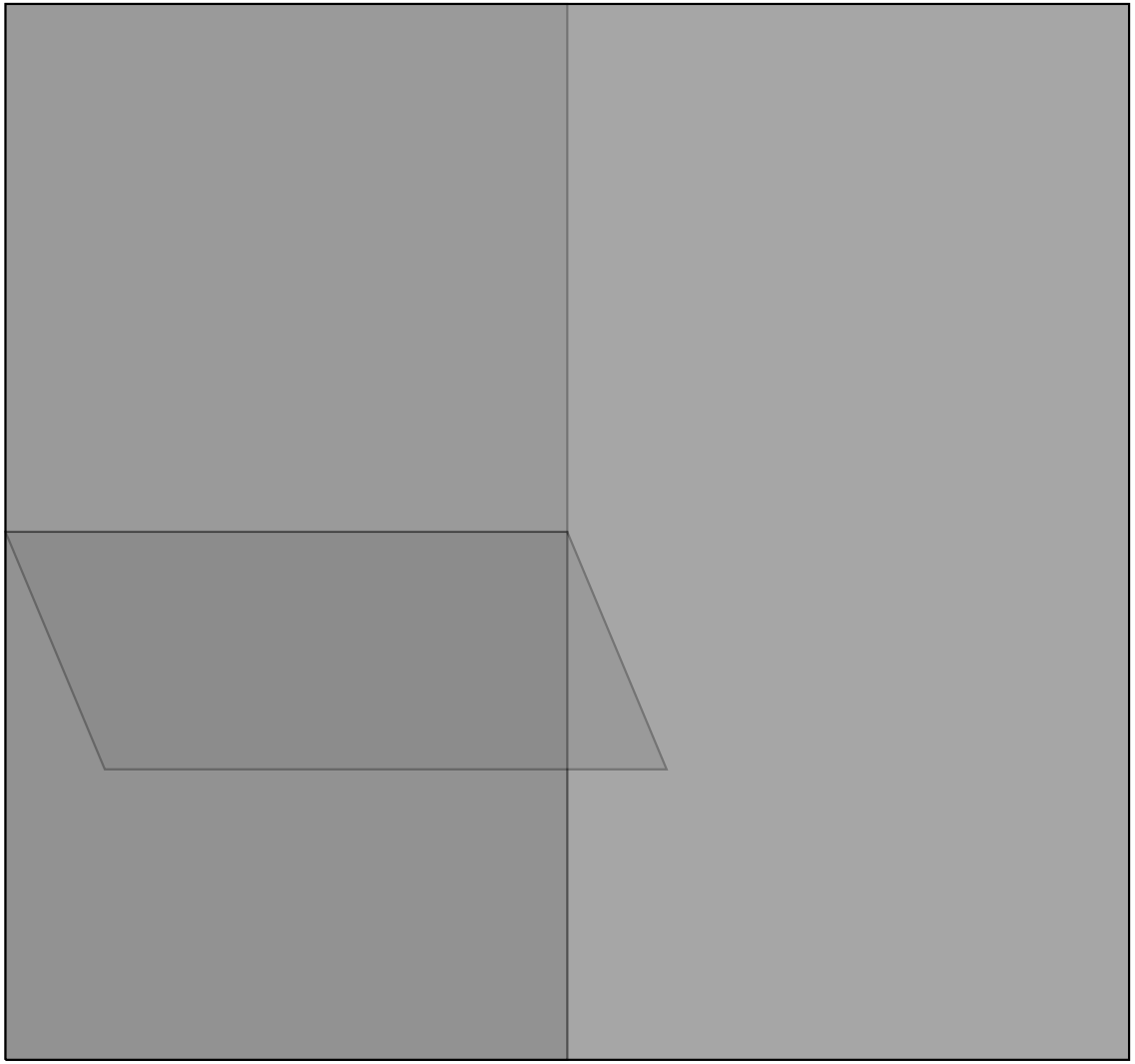

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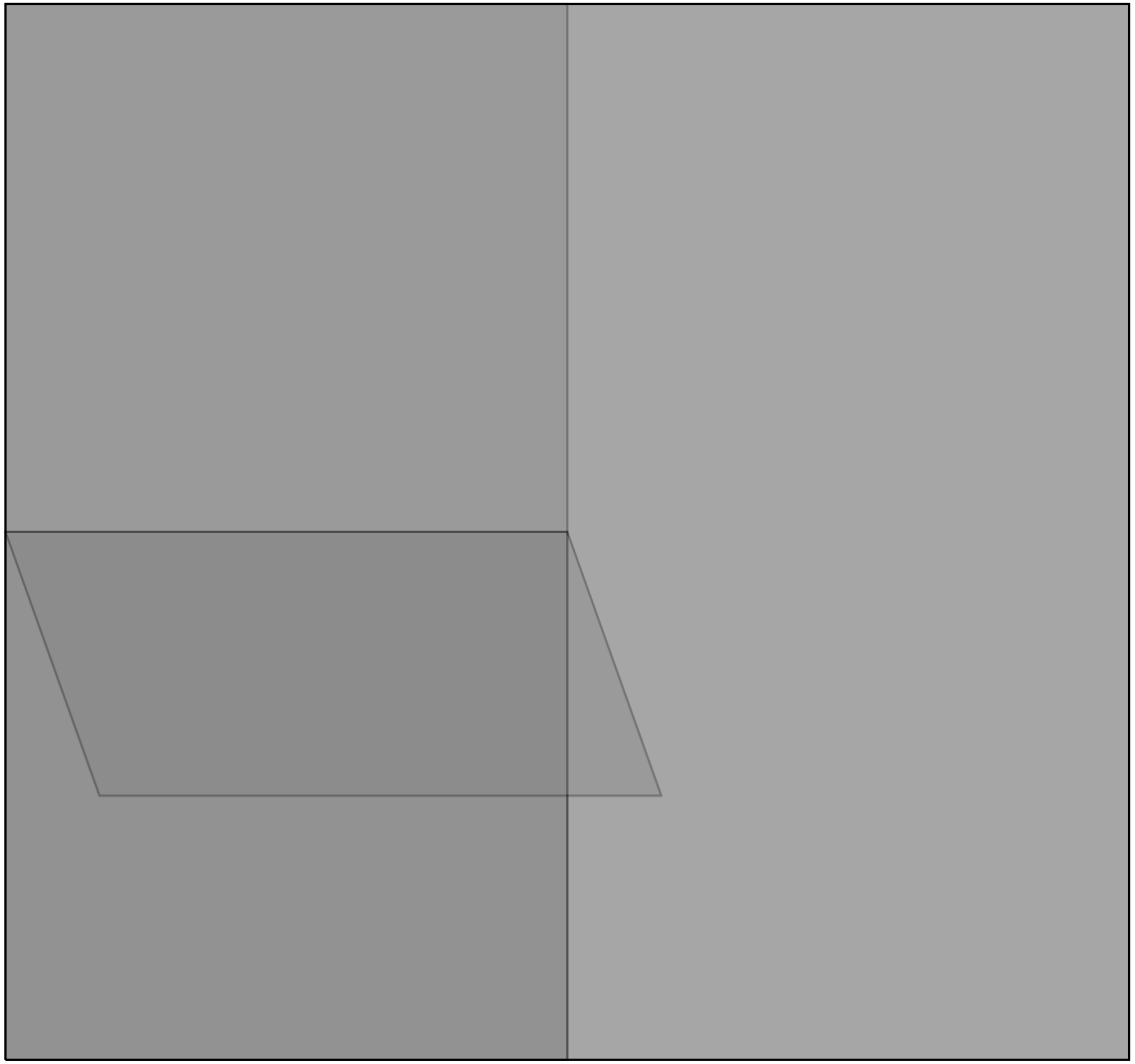

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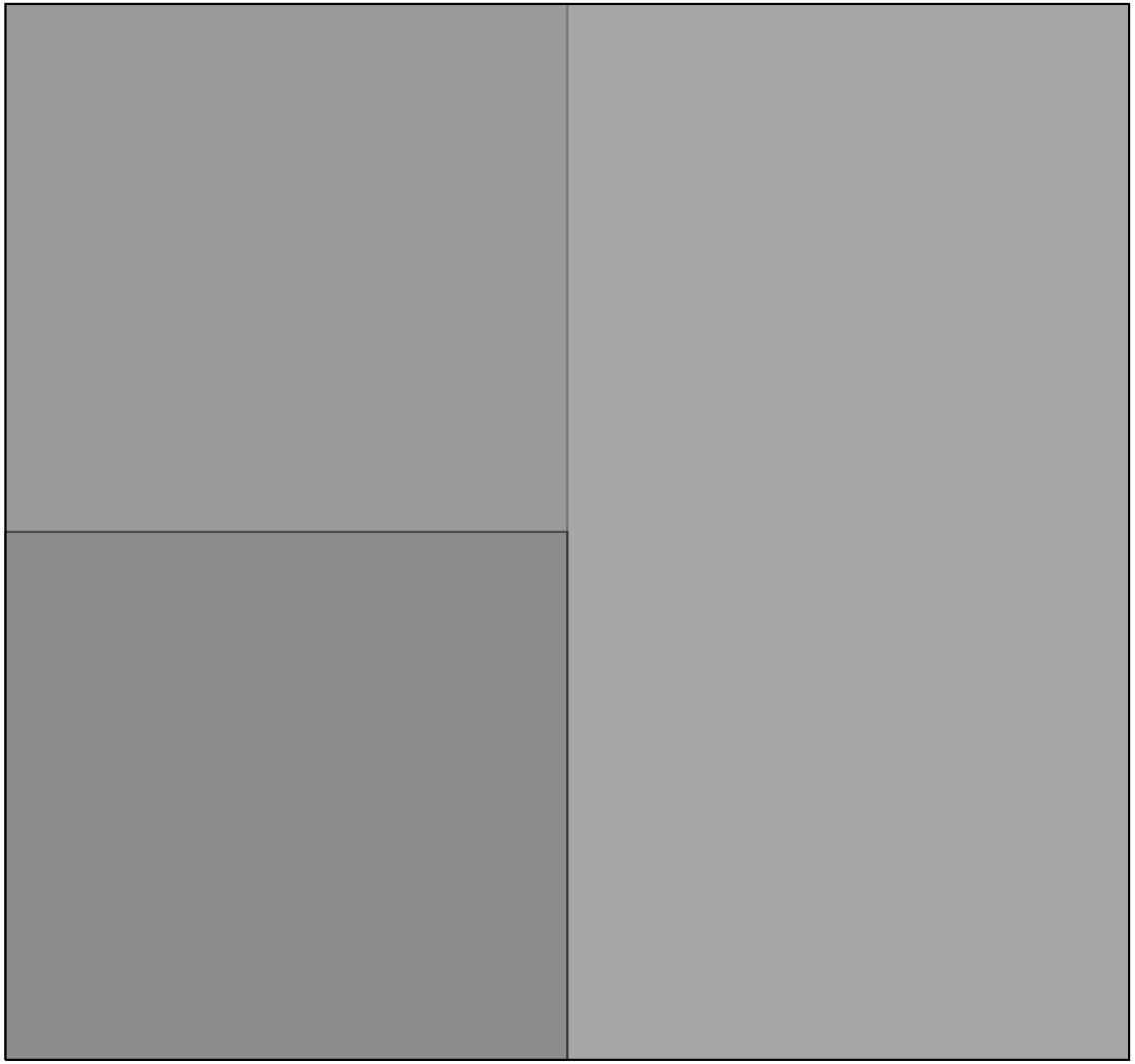

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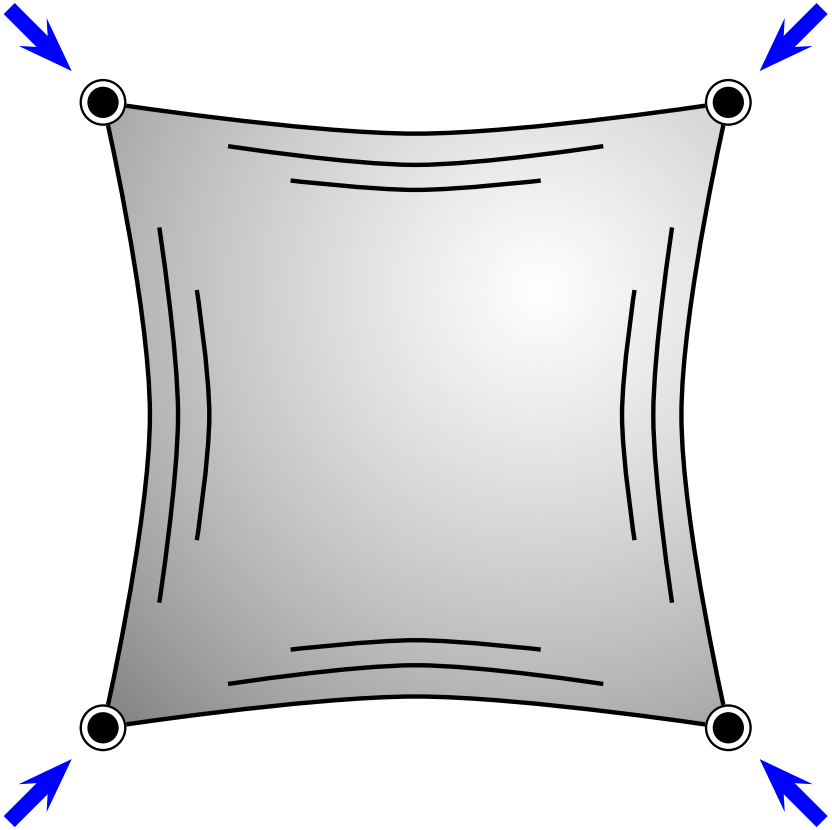
$\mathbb{Z}_2$ orbifold pillow



☞ An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points

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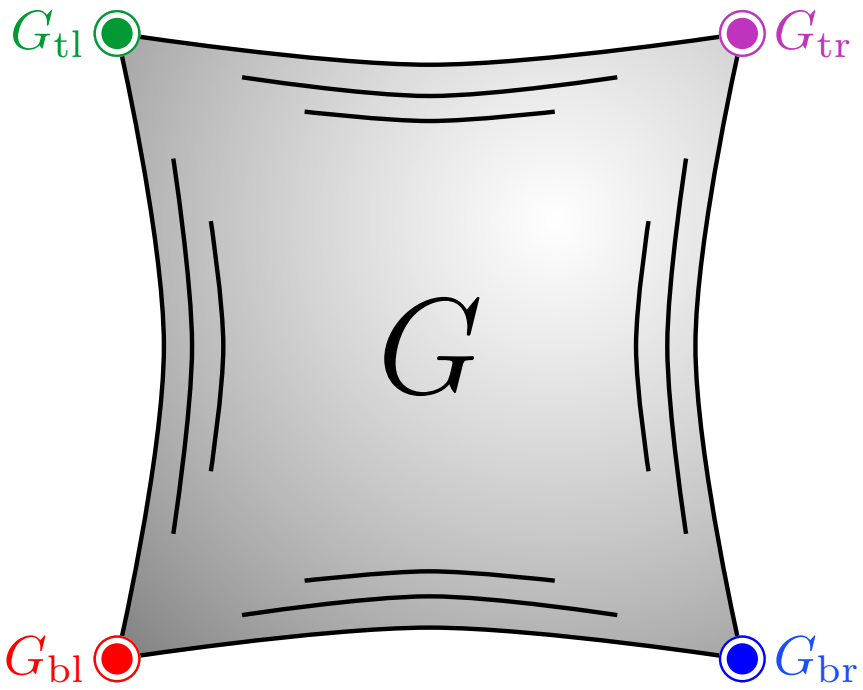

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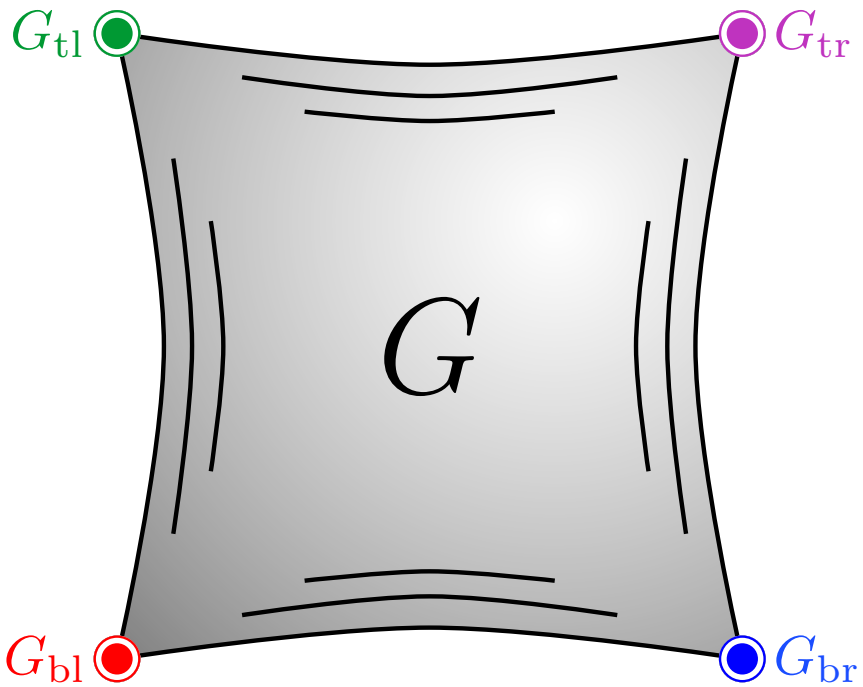

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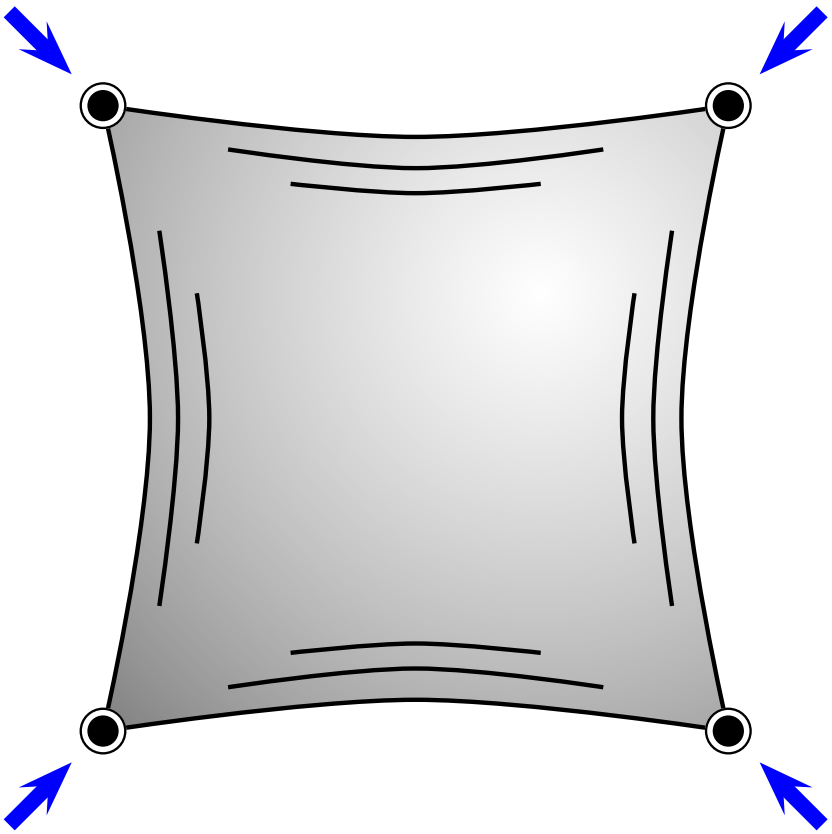
What is an orbifold?



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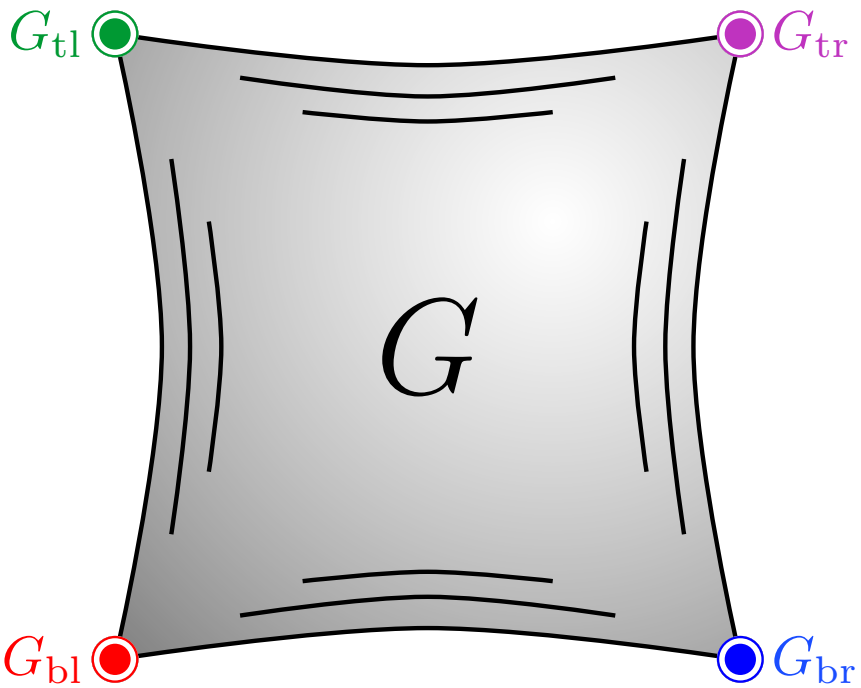

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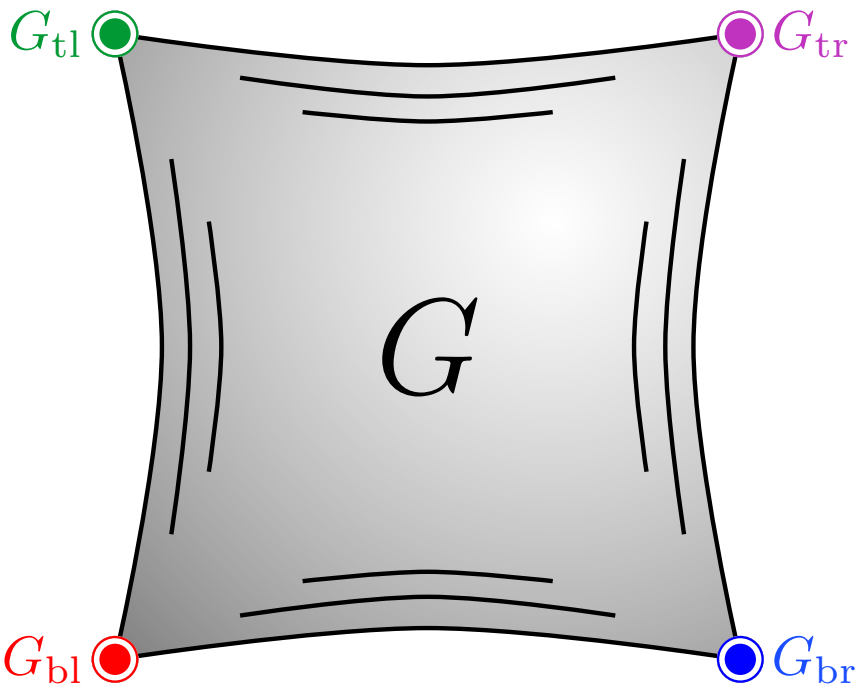

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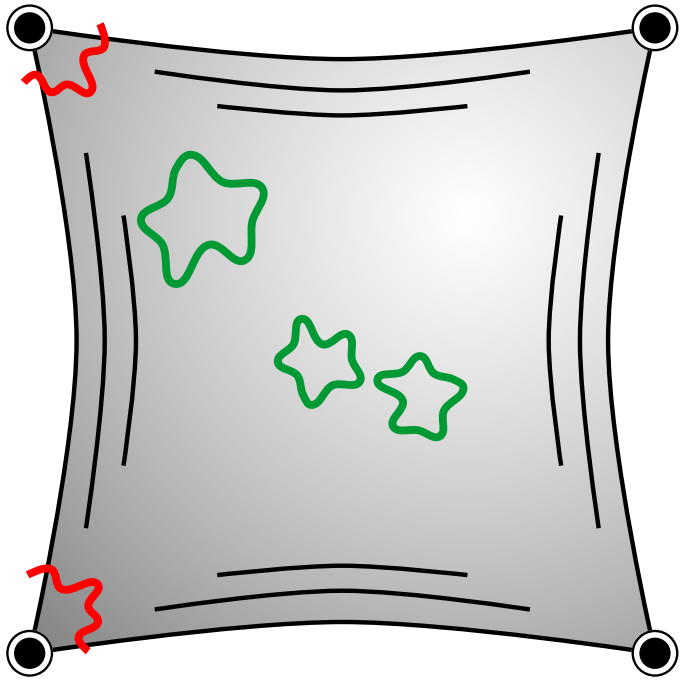
Strings on orbifolds

heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
'twisted' sectors = strings which are only closed on the orbifold	'brane fields' (hard to understand in field–theoretical framework)



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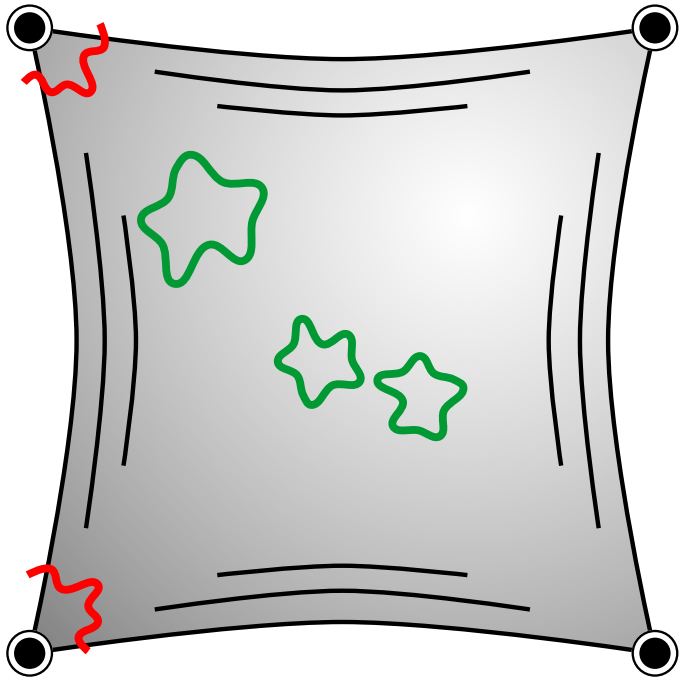
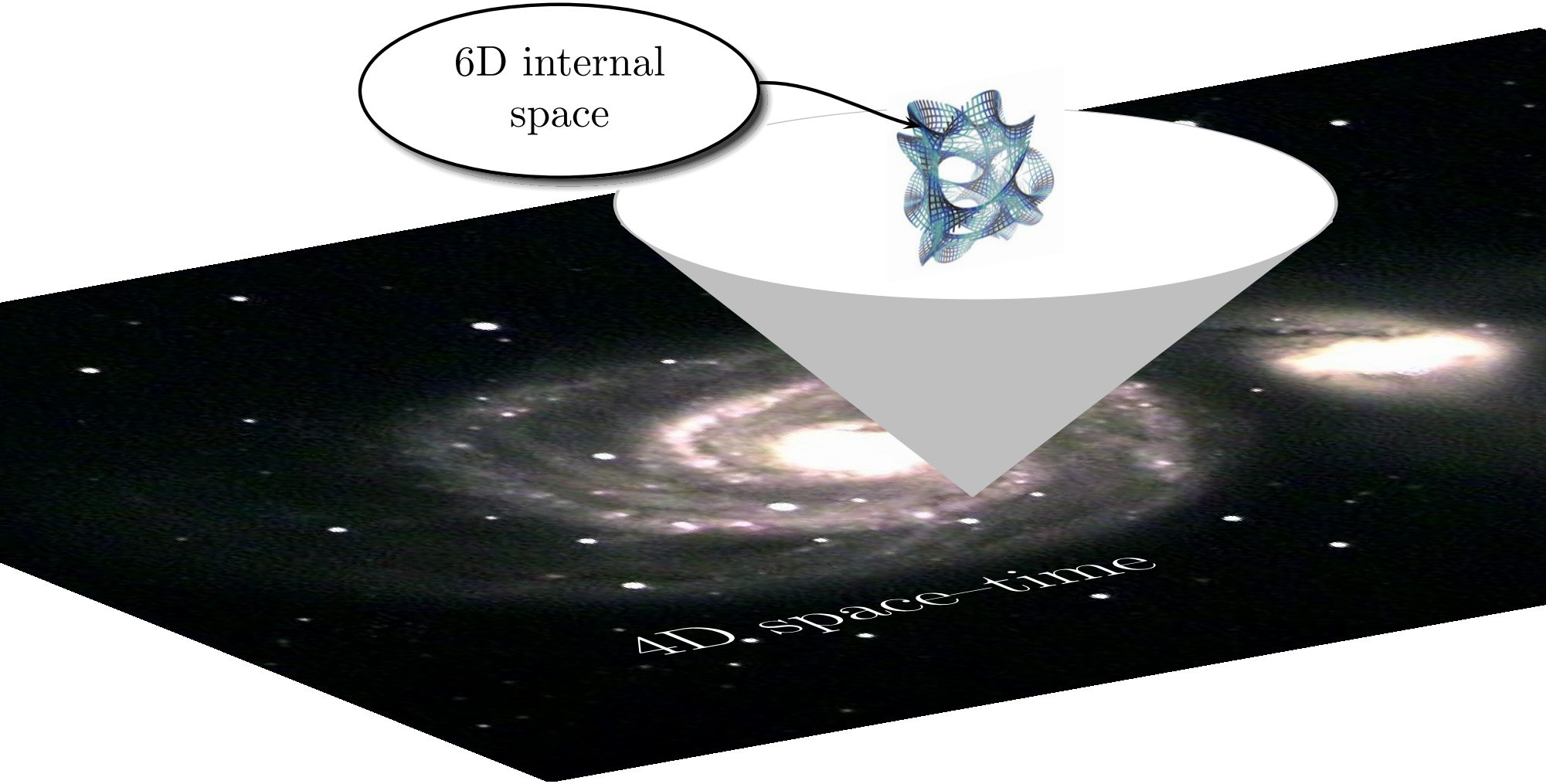


- ☞ ('Brane') Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry
- ➥ E.g. if the electron lives at a point with SO(10) symmetry also u and d quarks live there

String compactifications with local $\mathrm{SO}(10)$ GUTs

String compactifications with local SO(10) GUTs



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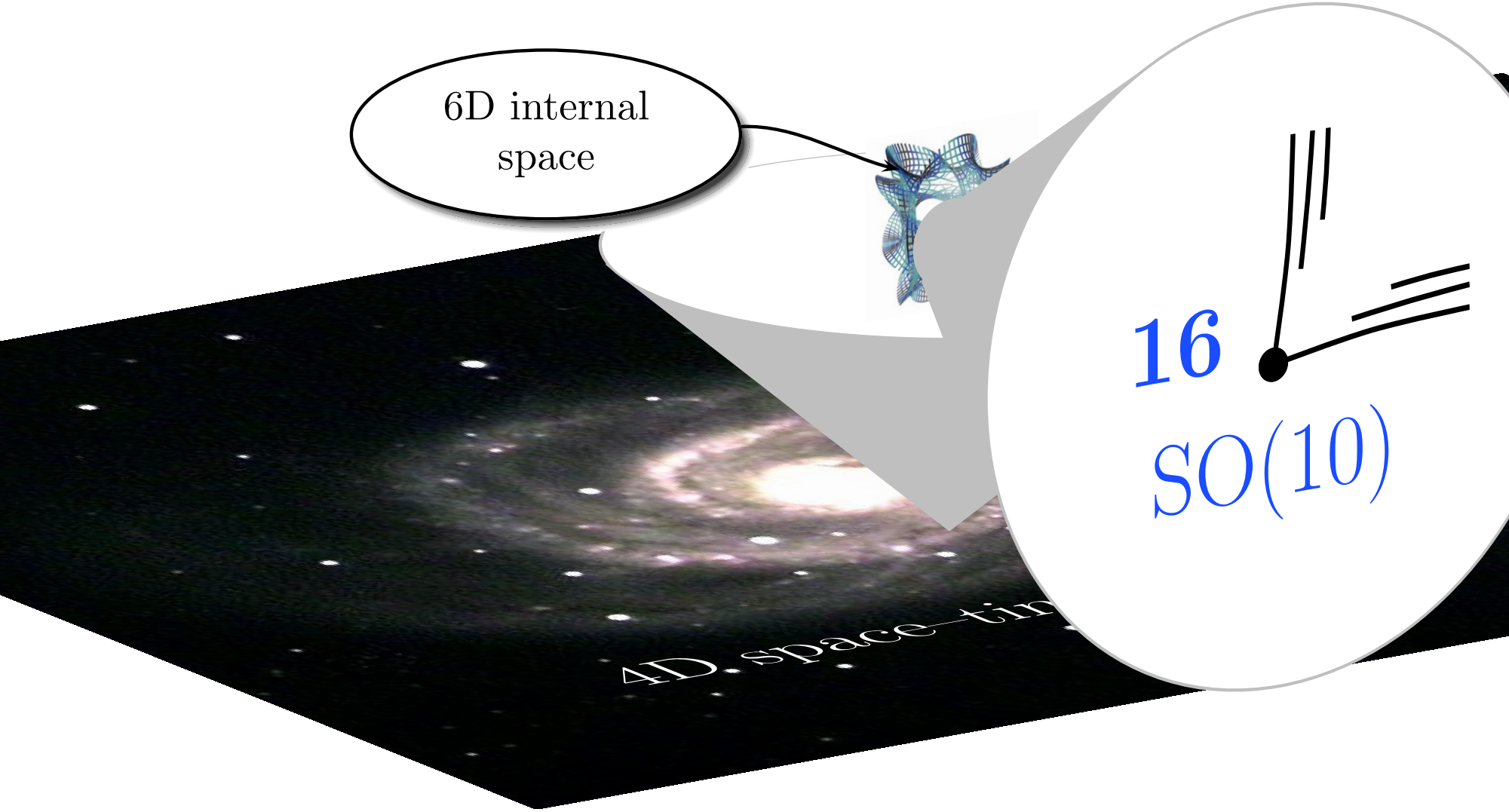

Residual R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions

Residual R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
  and are arguably on the same footing as the fundamental symmetries C, P and T
- $\mathbb{Z}_4^R$ originates from $\mathbb{Z}_2$ orbifold plane

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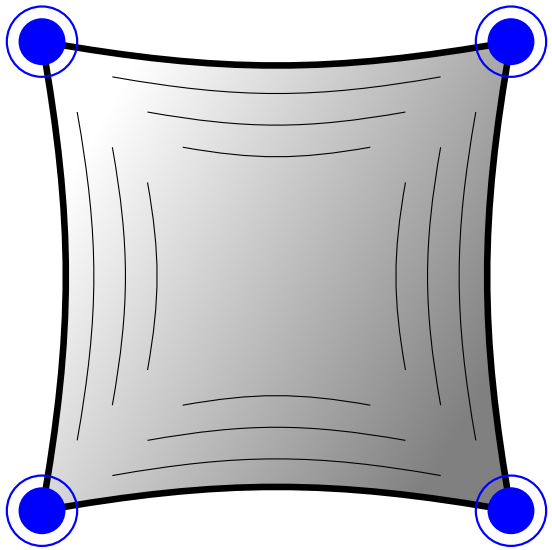

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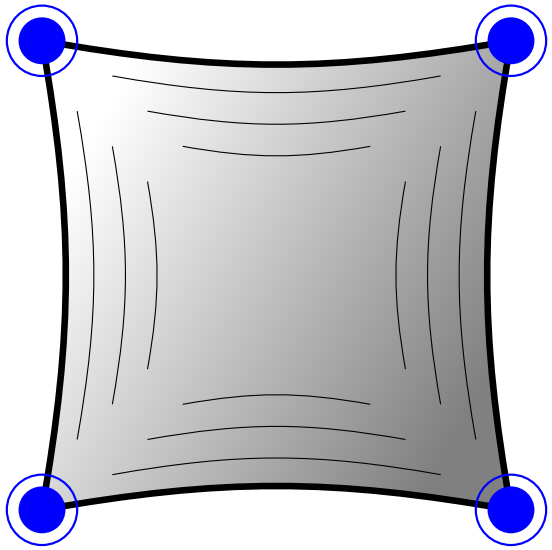

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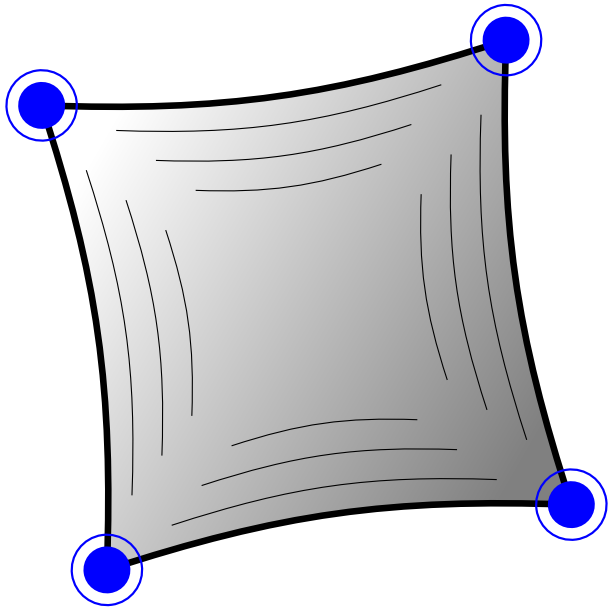

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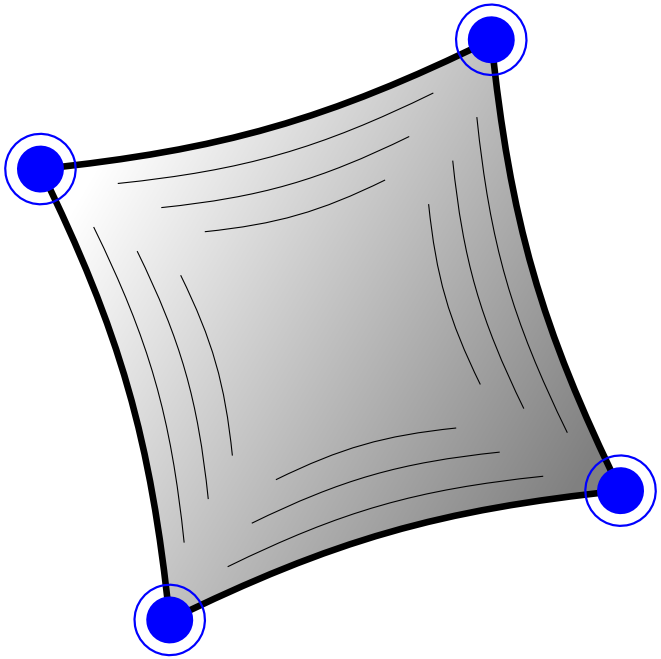

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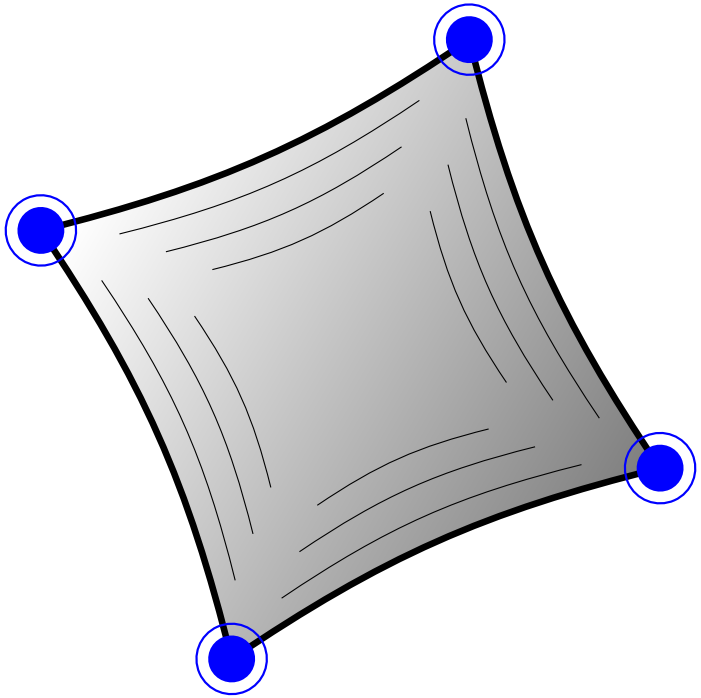

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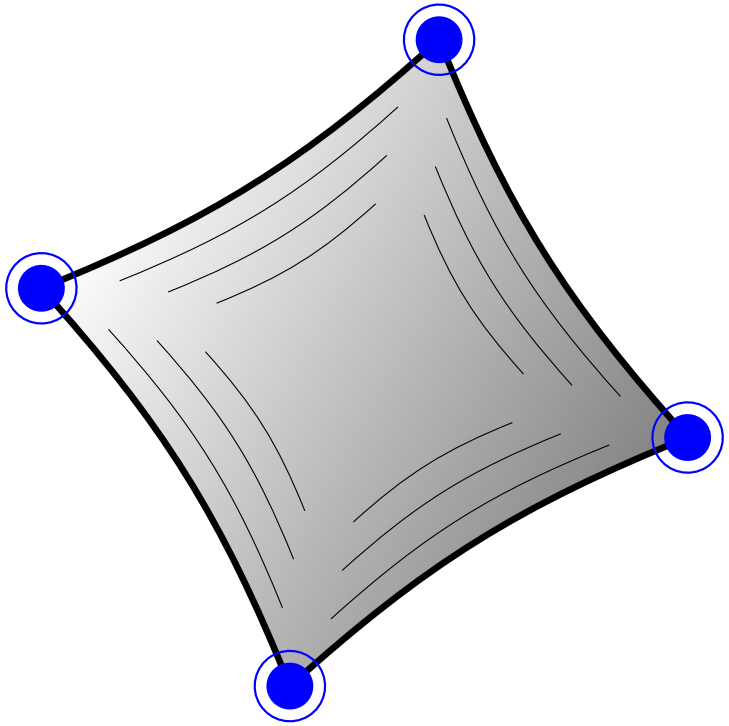

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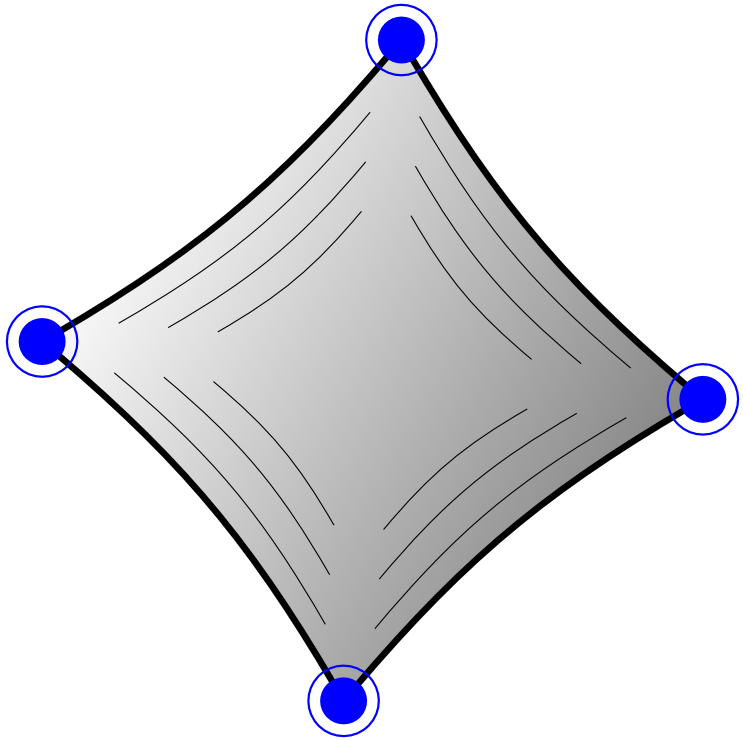

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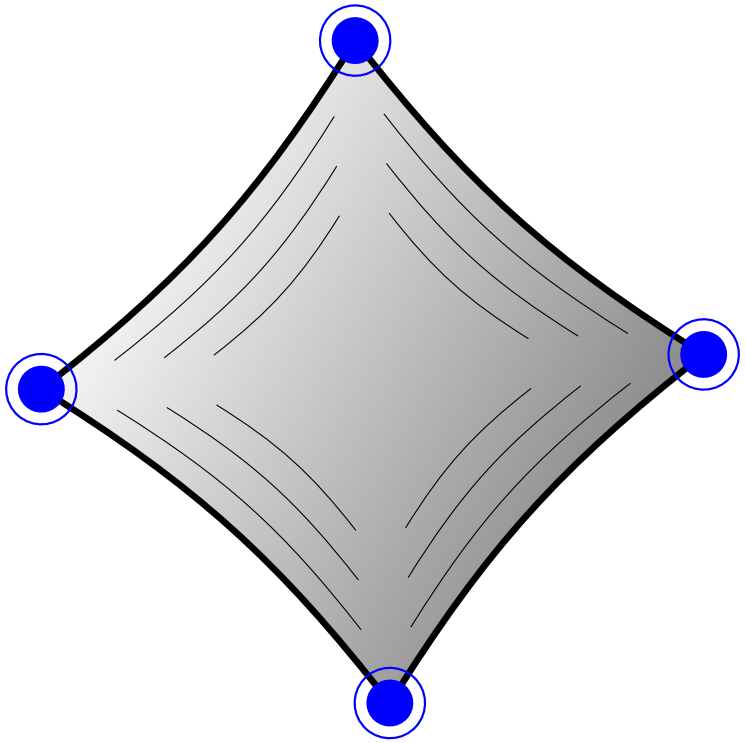

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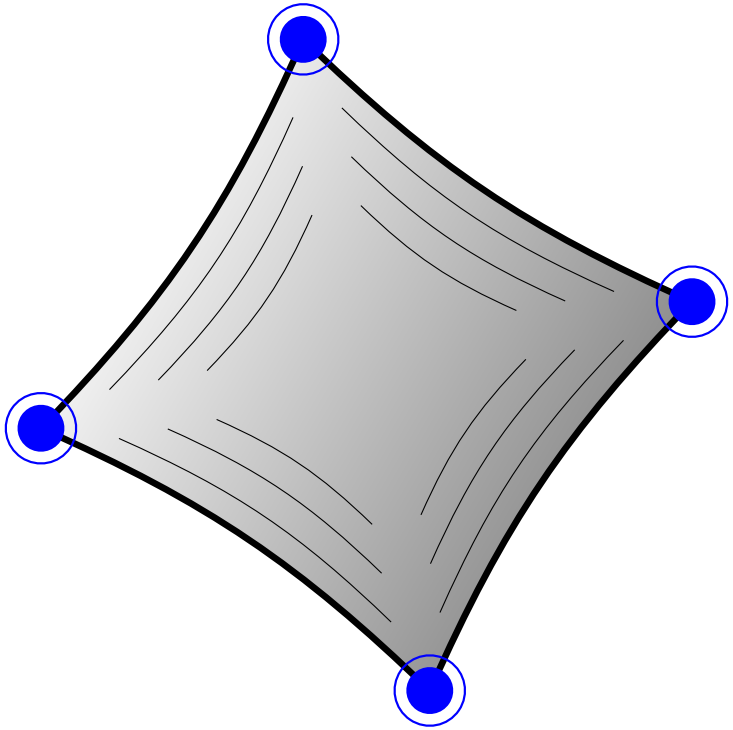

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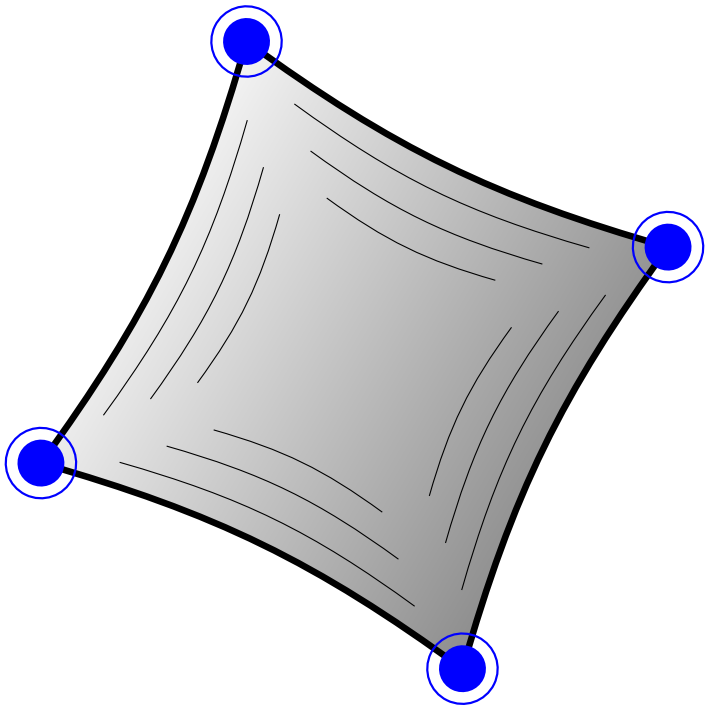

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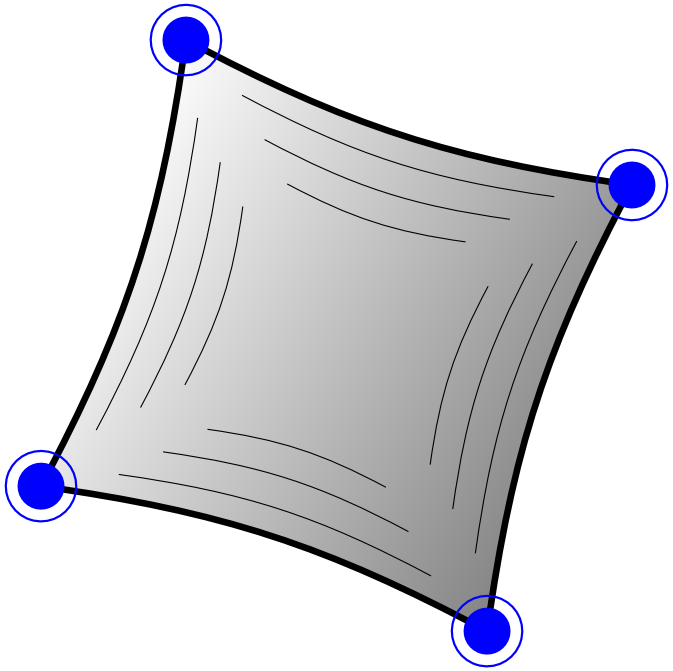

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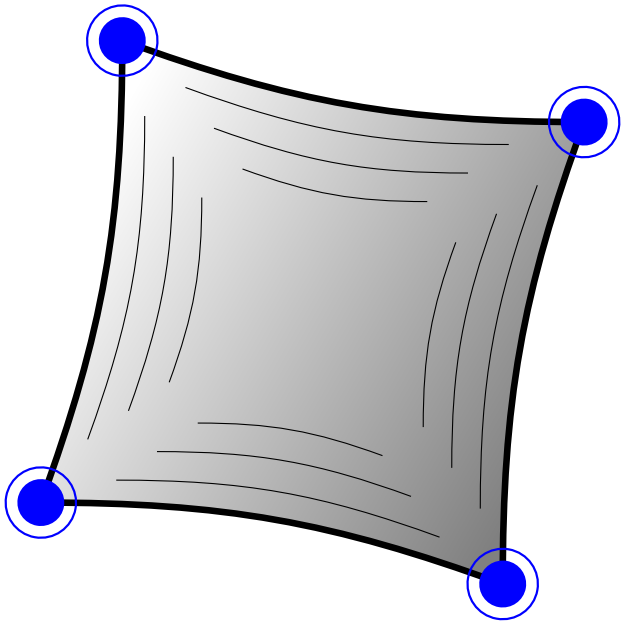

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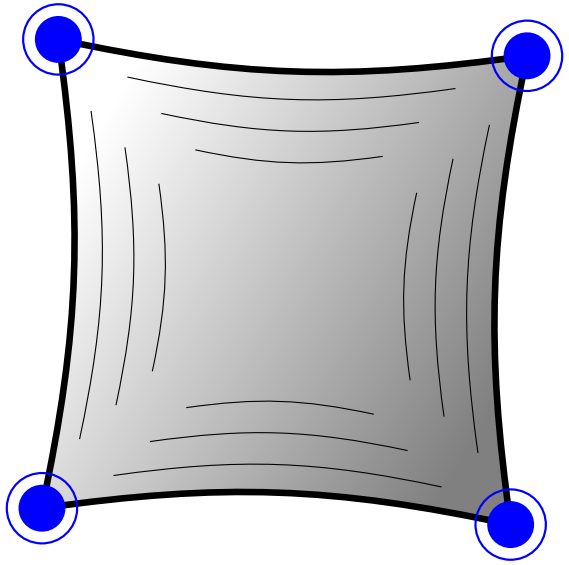

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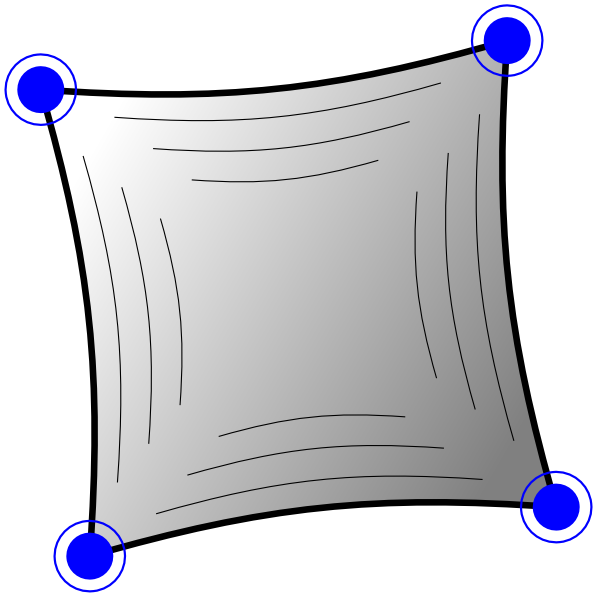

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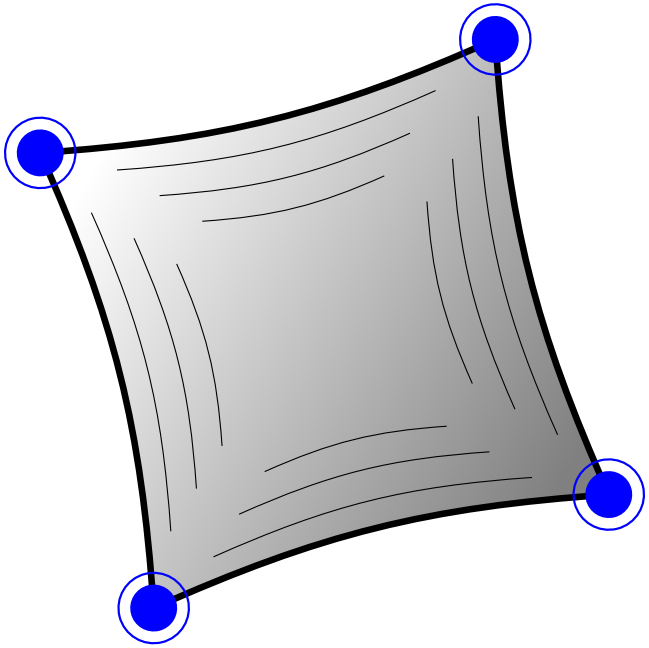

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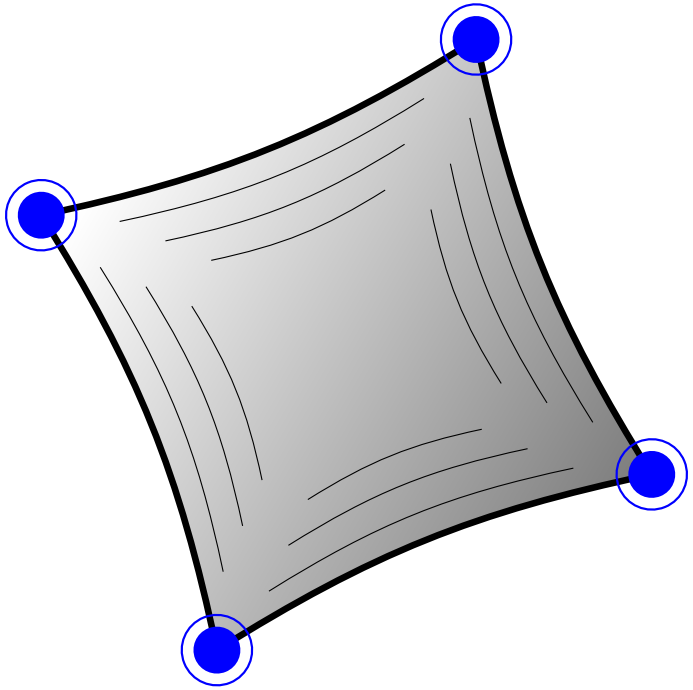

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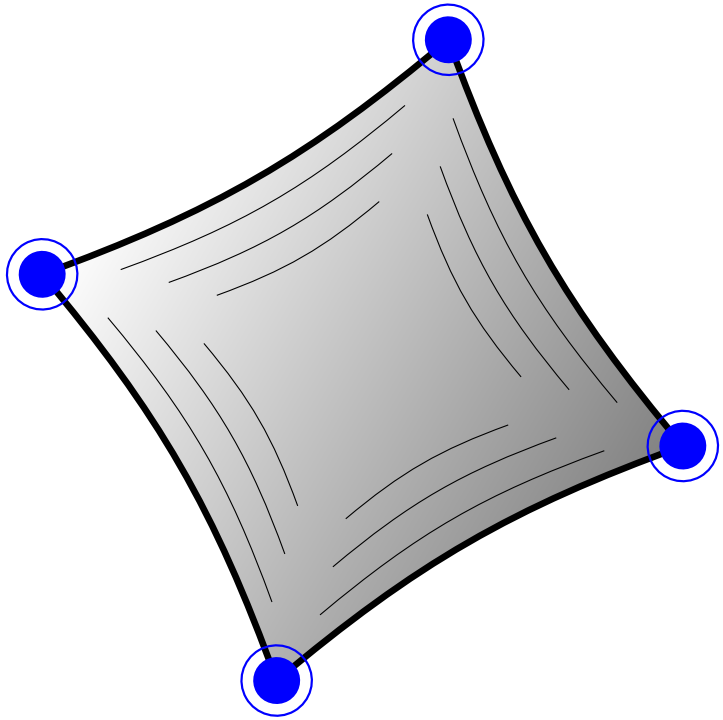

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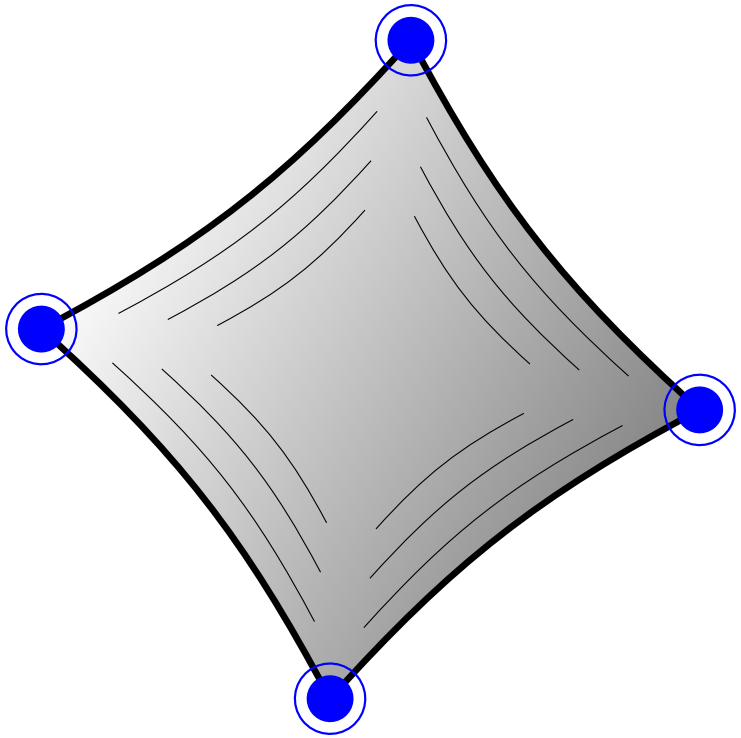

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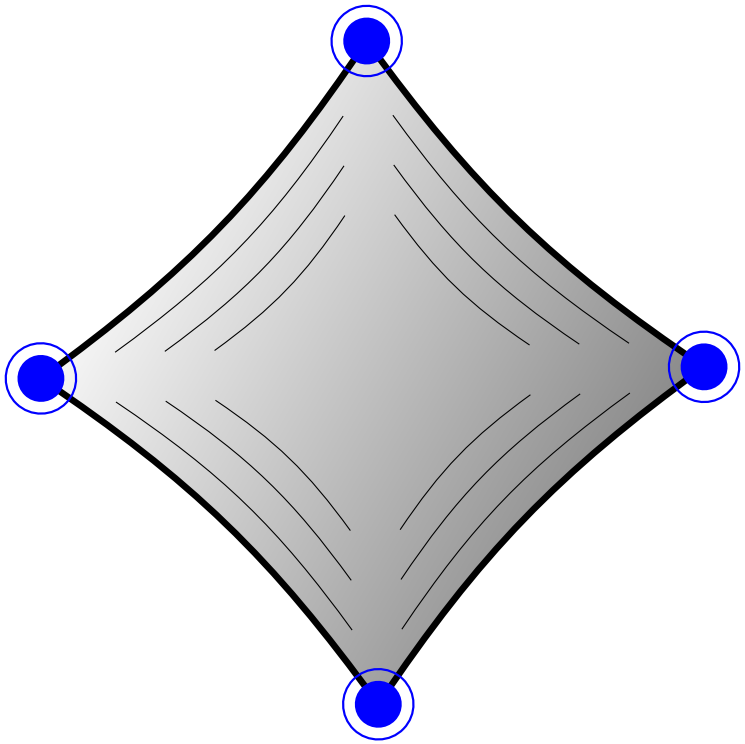

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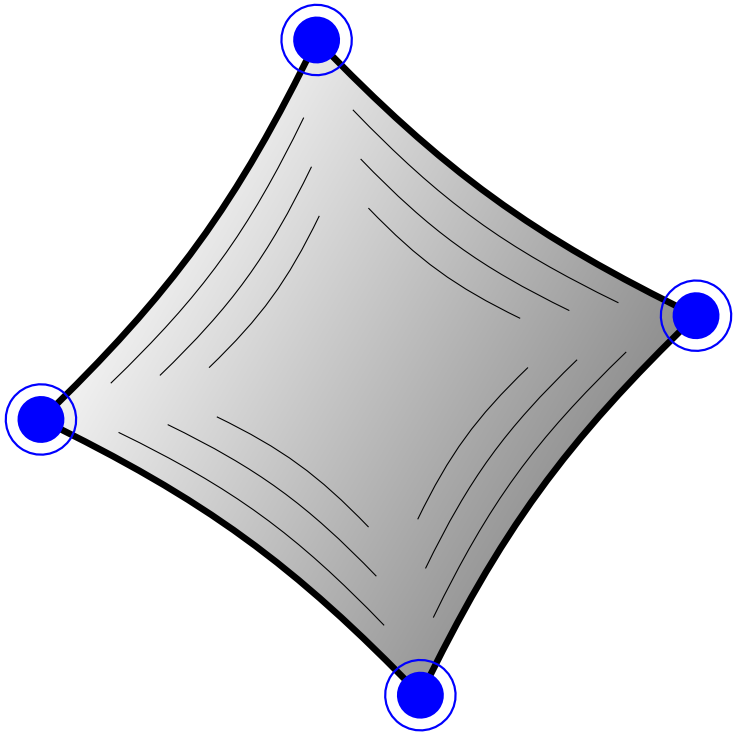

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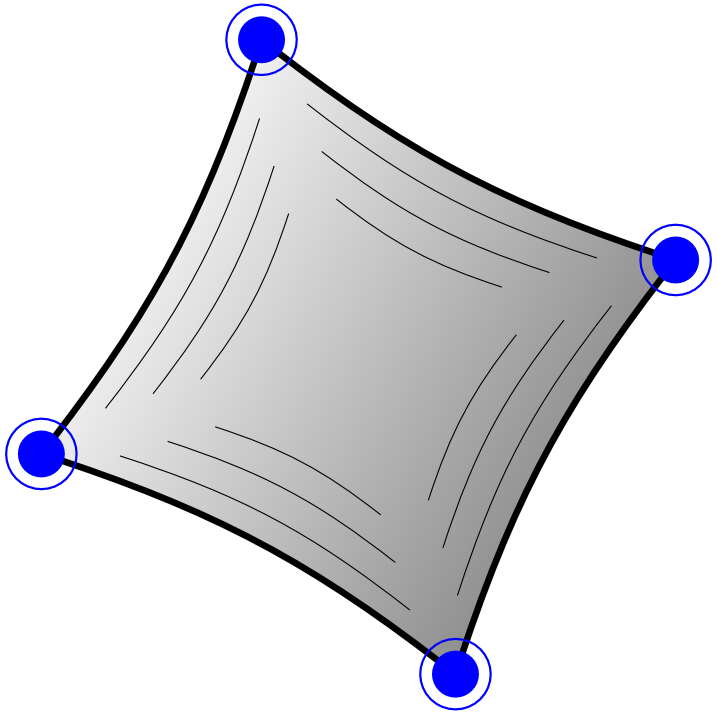

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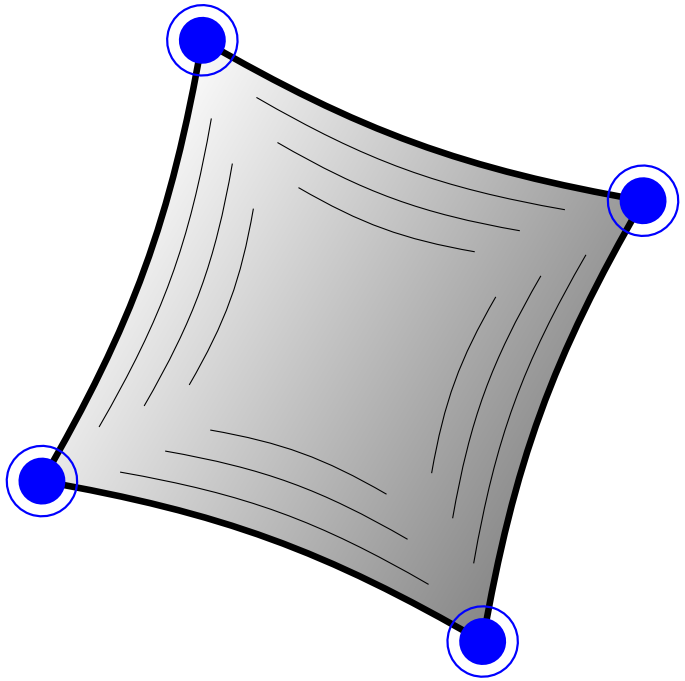

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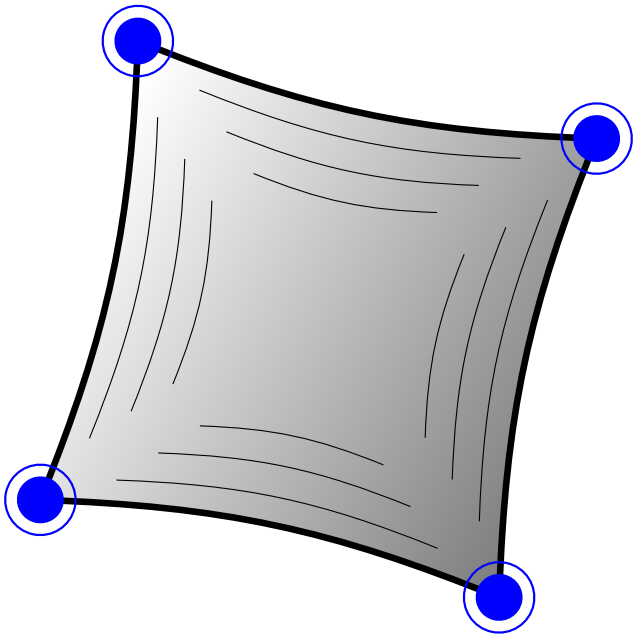

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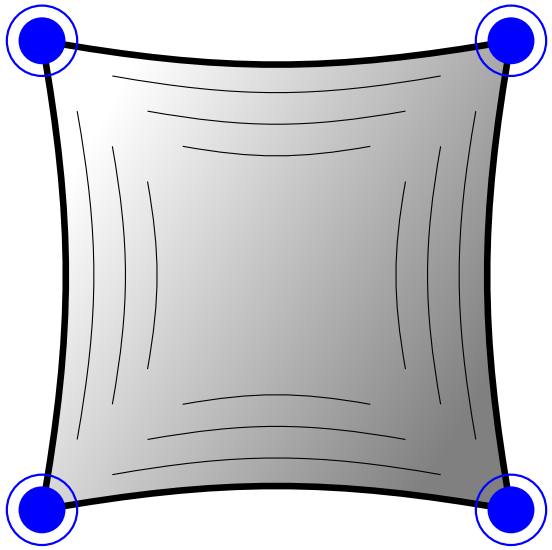

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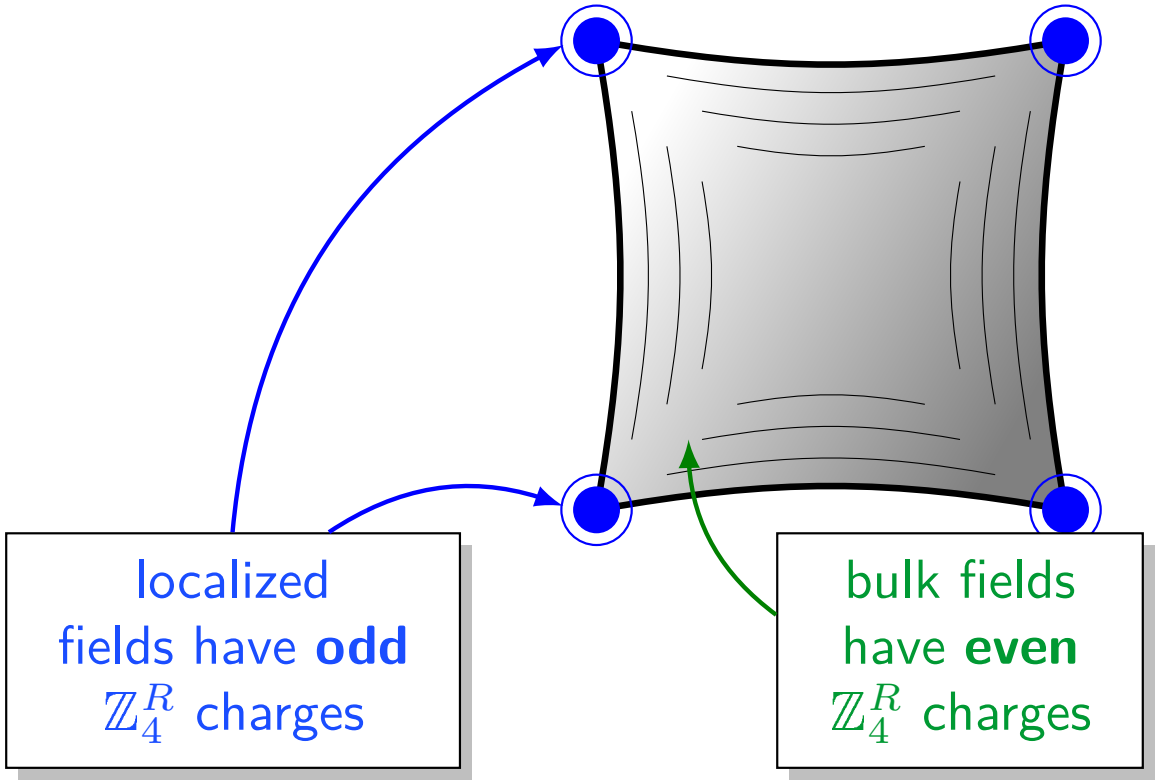


Features

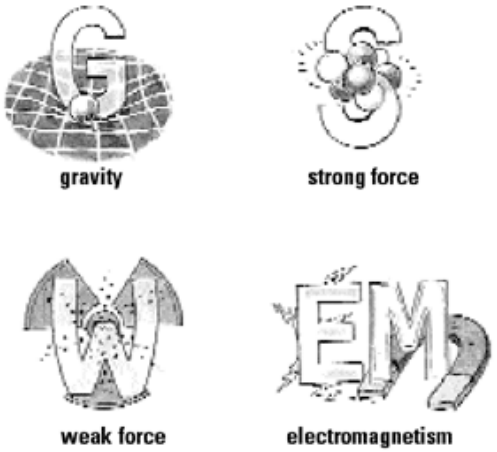
1. $3 \times \mathbf{16}$ + Higgs + nothing

No exotics

Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007a]

Features

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$



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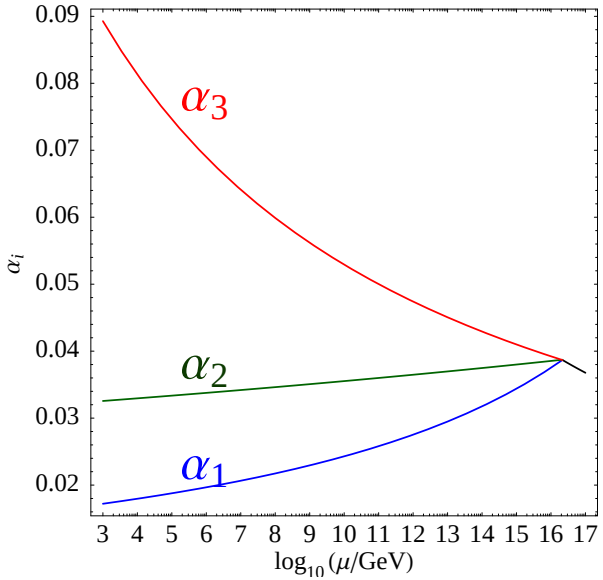
1. $3 \times \mathbf{16} +$ Higgs $+$ nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
   precision gauge unification (PGU)
   from non–local GUT breaking



Raby, M.R. & Schmidt-Hoberg [2010], Krippendorf, Nilles, M.R. & Winkler [2013]

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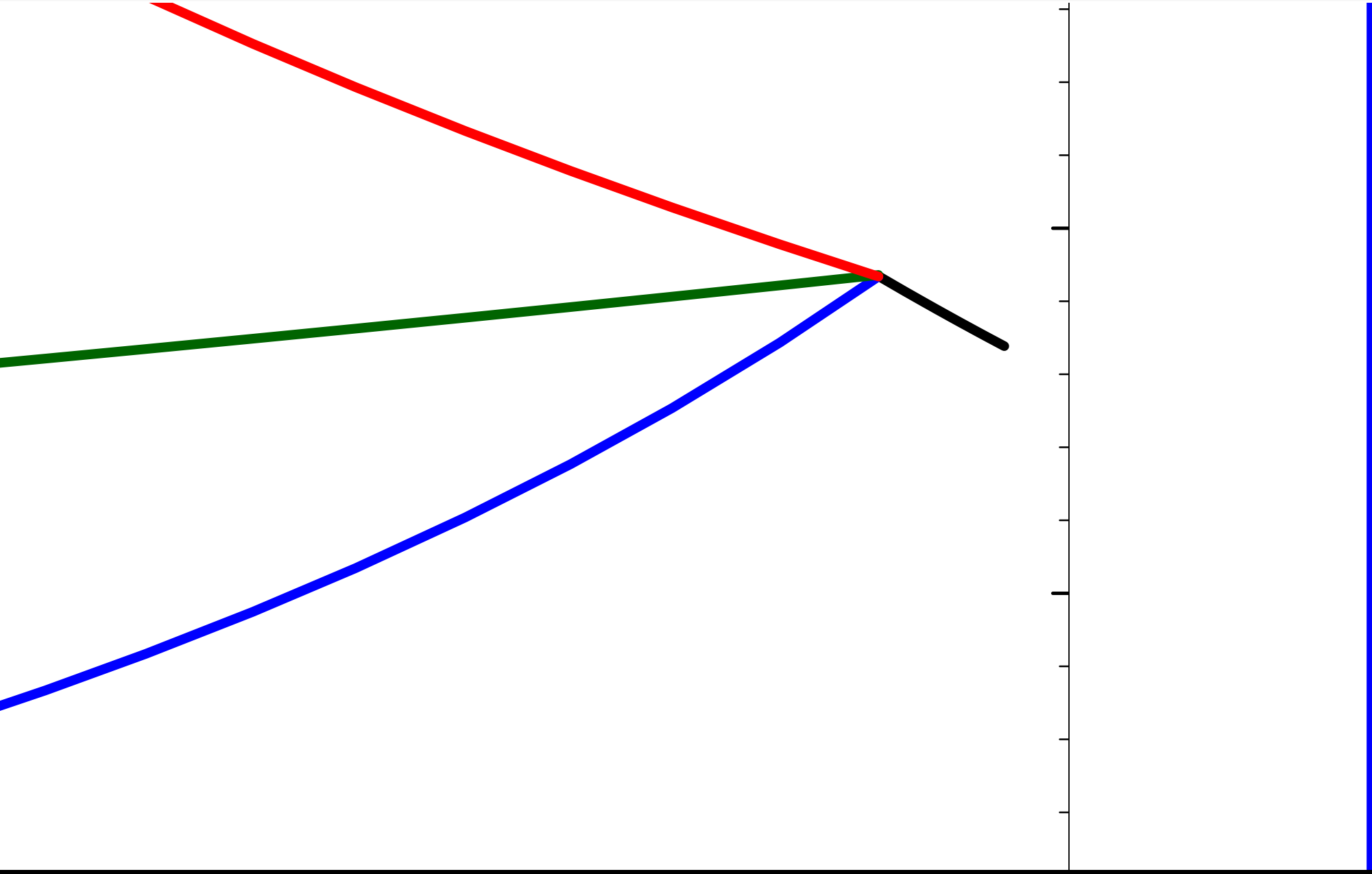

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3. Unification
4. R parity & $\mathbb{Z}_4^R$

$\cancel{\overline{u}\,\overline{d}\,\overline{d}}$ $\cancel{q\,\overline{d}\,\ell}$

$\cancel{\ell\,\ell\,\overline{e}}$ $\cancel{\ell\,H_u}$

$\curvearrowright$ proton long–lived

$\curvearrowright$ DM stable

Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007b], Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]

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5. See–saw

$\curvearrowright$ suppressed ν masses

Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez & M.R. [2007]

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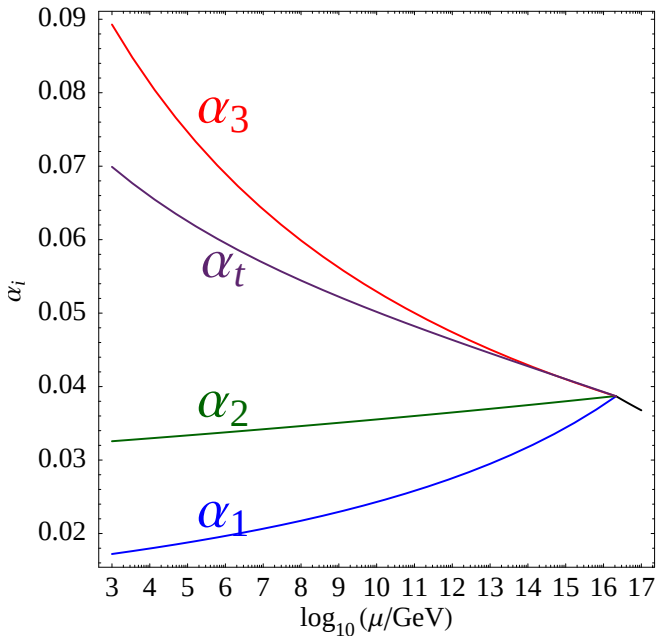
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen



$\curvearrowright$ realistic top mass

Hosteins, Kappl, M.R. & Schmidt-Hoberg [2009]

Features

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2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
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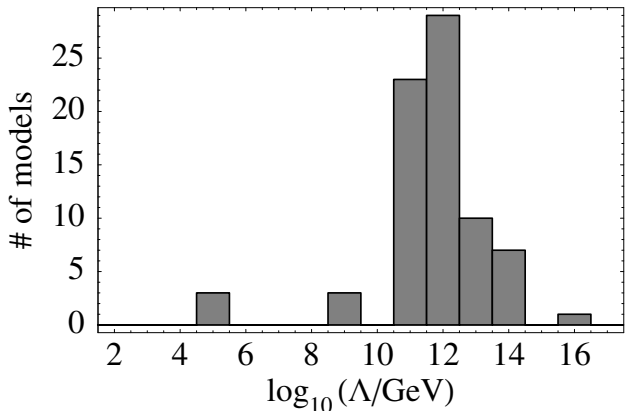
7. 'Realistic' hidden sector scale of hidden sector strong dynamics is consistent with TeV–scale soft masses



Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007a]

Features & "stringy surprises"

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4. R parity & $\mathbb{Z}_4^R$
5. See–saw
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen
7. 'Realistic' hidden sector
8. Solution to the μ problem

$\mu \sim \langle \mathscr{W} \rangle$

$\langle \mathscr{W} \rangle \ll M_{\mathrm{P}}^3$ from approximate $\mathrm{U}(1)_R$ symmetries

$\curvearrowright$ light Higgs

Kappl, Nilles, Ramos-Sánchez, M.R., Schmidt-Hoberg & Vaudrevange [2009], Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

Features & "stringy surprises"

1. $3 \times \mathbf{16}$ + Higgs + nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$

that's what we searched for...

5. See–saw
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen
7. 'Realistic' hidden sector
8. Solution to the μ problem

...that's what we got 'for free'

"stringy surprises"

$\mathcal{CP}$ violation

from finite groups

$\mathcal{CP}$ violation in Nature

- ☞ $\cancel{\mathcal{CP}}$ so far only observed in flavor sector

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huge literature

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huge literature

here:

non–Abelian discrete (flavor) symmetry $G \leftrightarrow \cancel{\mathcal{CP}}$

Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner [2014]

group G with automorphisms u

Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner [2014]

group G with automorphisms u → there is a u for which no $\mathrm{FS}_u^{(n)}$ is 0

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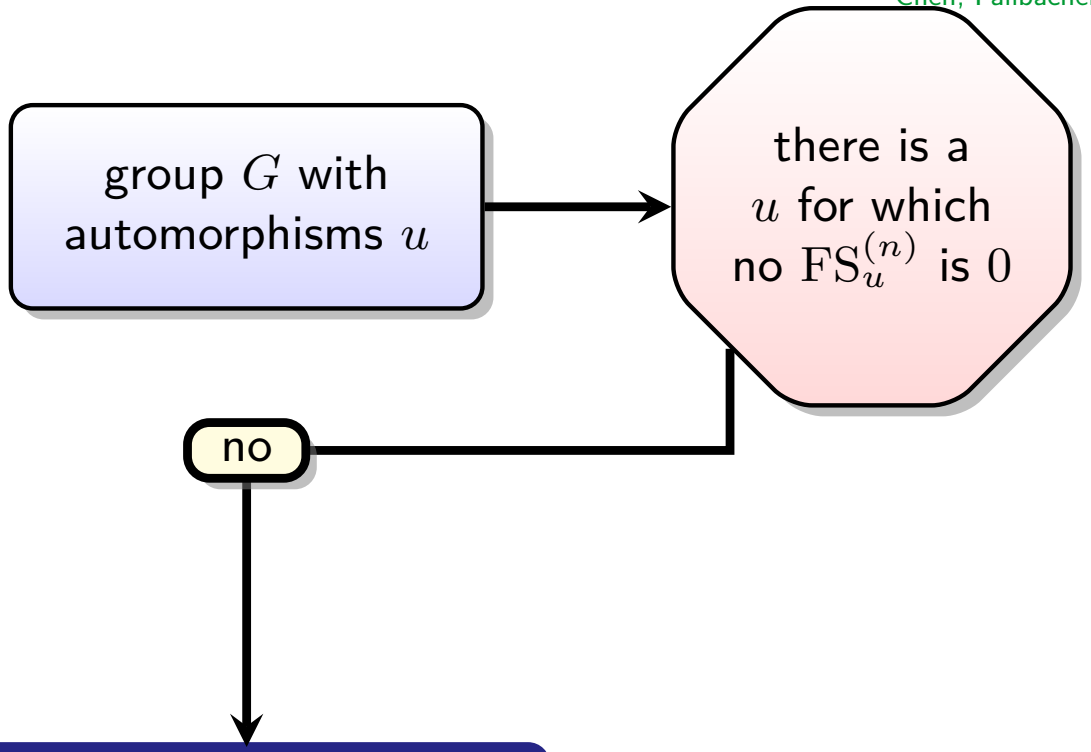

Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical $\mathcal{CP}$ transformation

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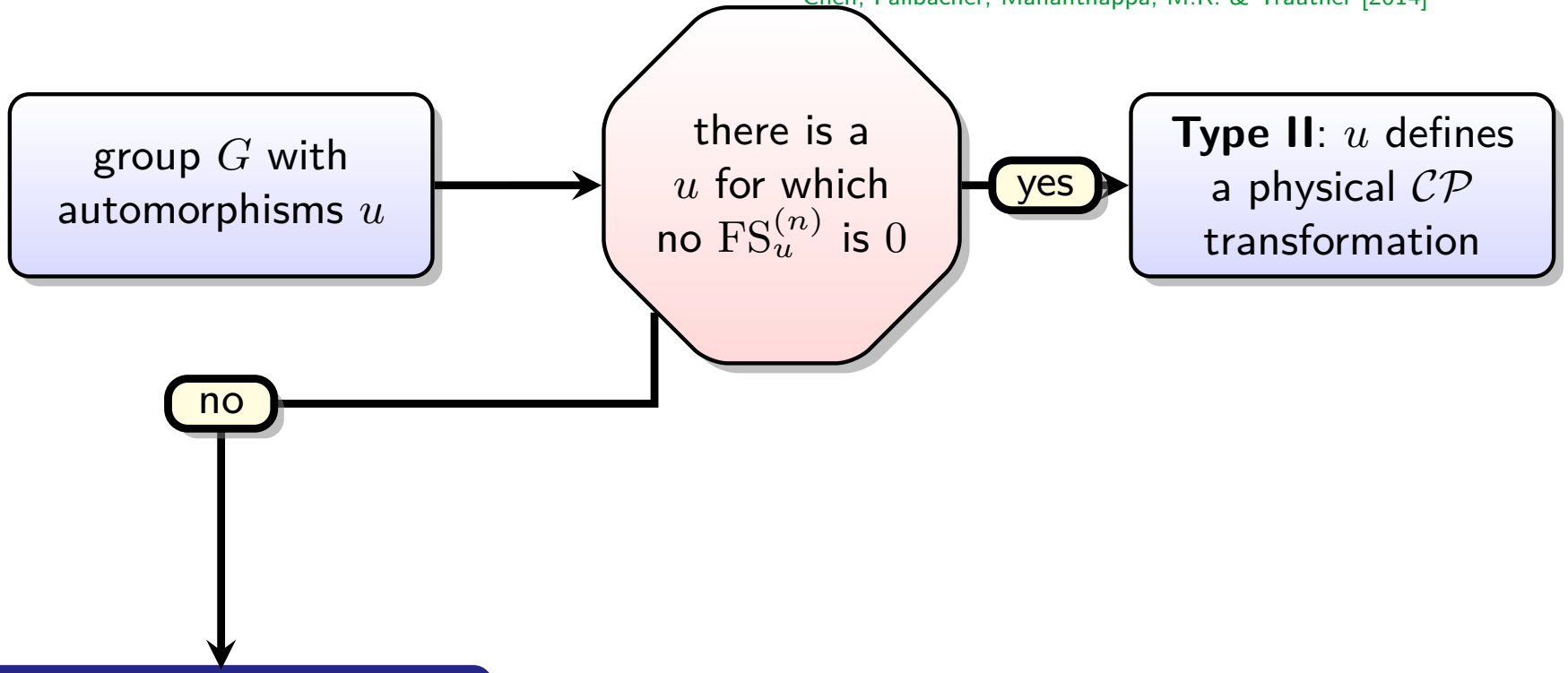

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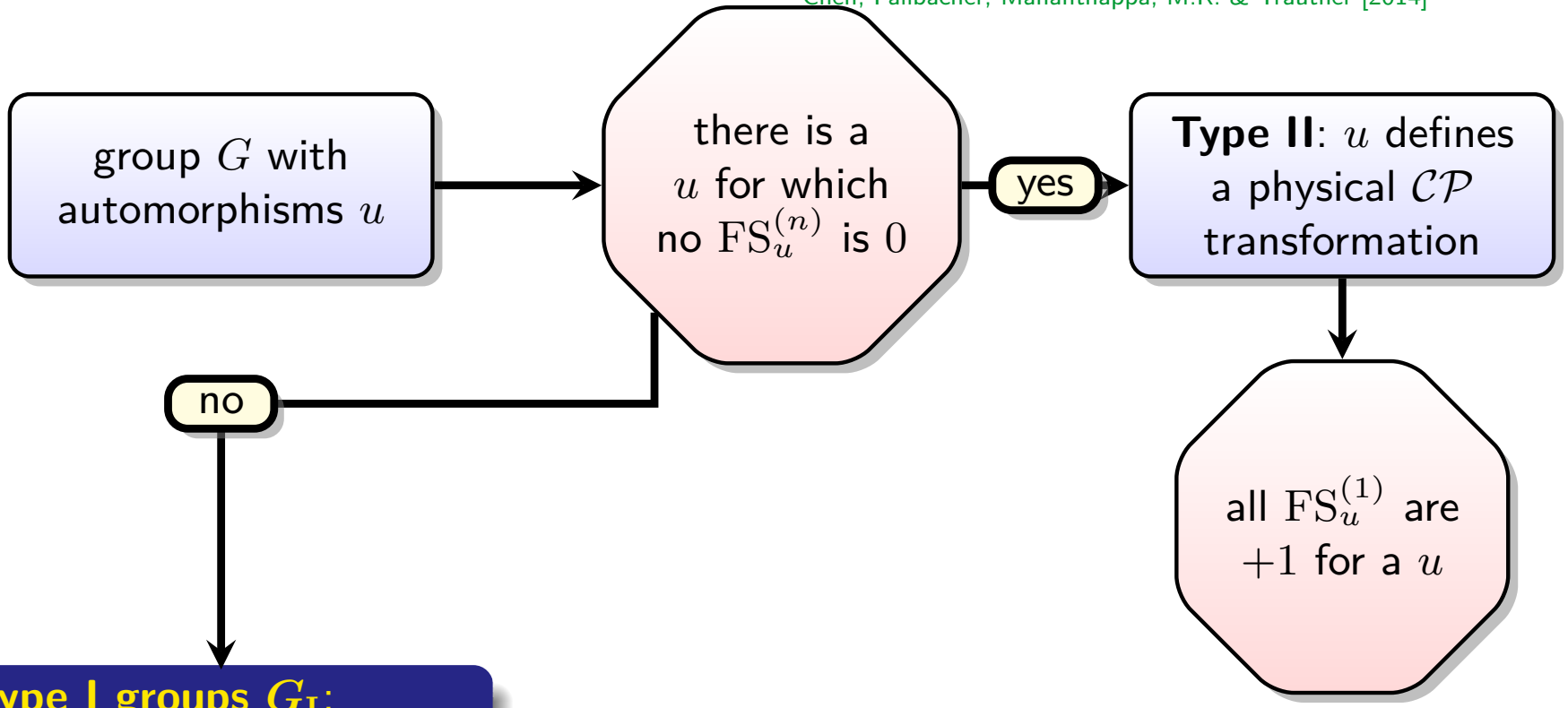

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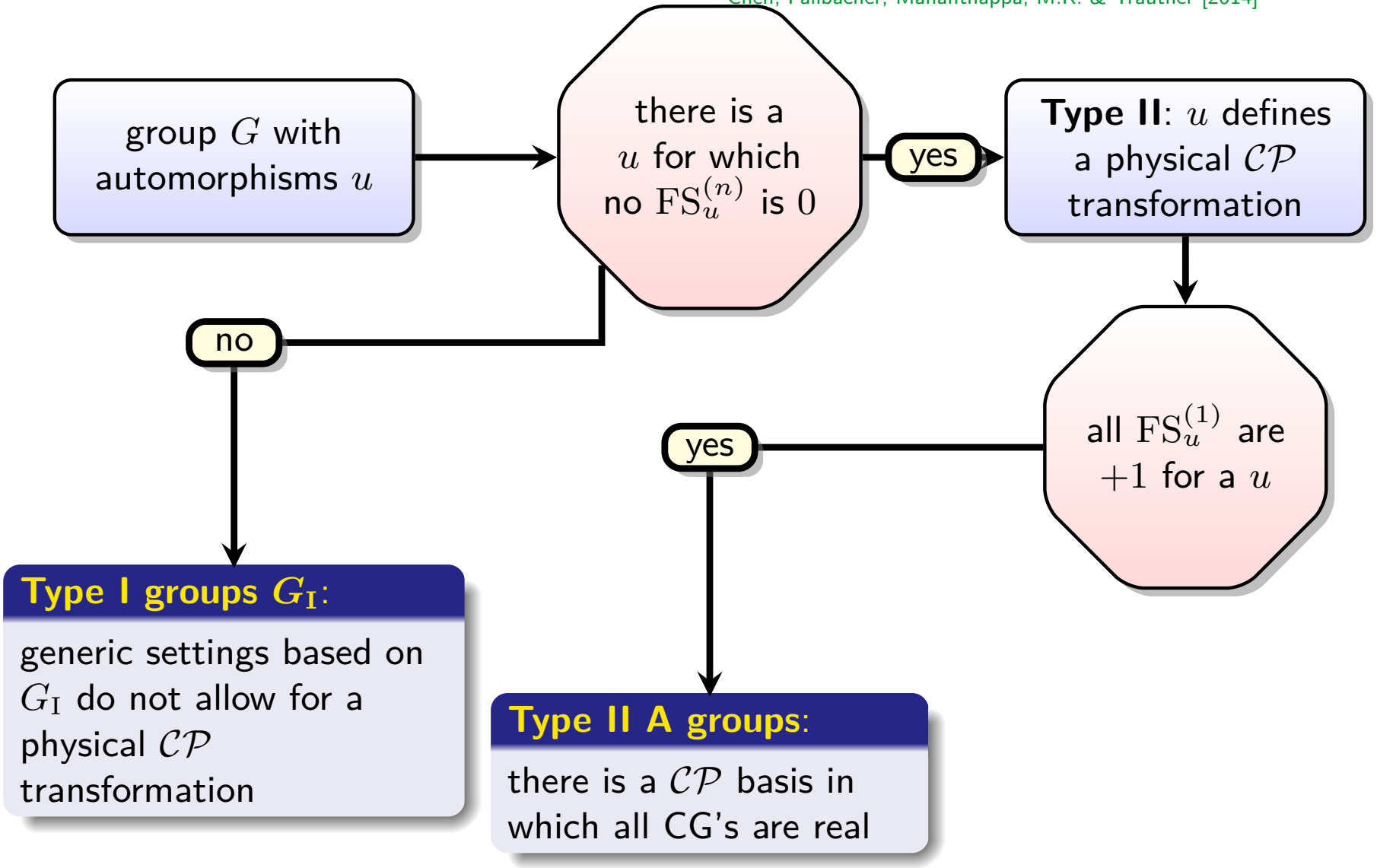

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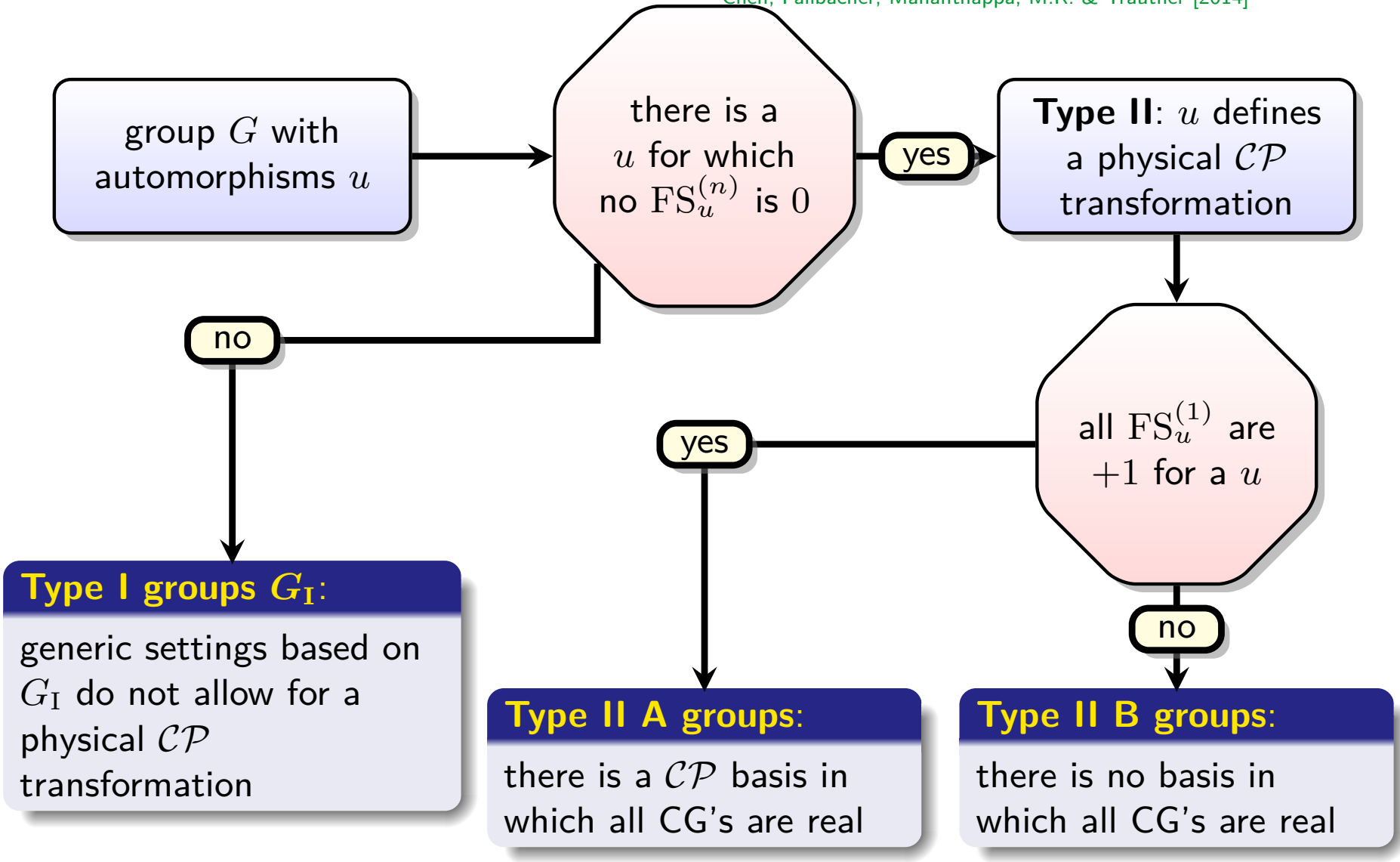

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group G with automorphisms u

Upshot:

Type I groups violate $\mathcal{CP}$!

e II: u defines physical $\mathcal{CP}$ nsformation

no

all $\mathrm{FS}_u^{(1)}$ are $+1$ for a u

yes

no

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Type II A groups:

there is a $\mathcal{CP}$ basis in which all CG's are real

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there is no basis in which all CG's are real

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Remainder of this talk:
Even the very first orbifold models had type–I flavor symmetries!

T
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w

groups:
basis in
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$\mathcal{CP}$ violation

from

strings

First 3 family models from stringy orbifolds

Ibáñez, Kim, Nilles & Quevedo [1987]

☞ Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold

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- Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold
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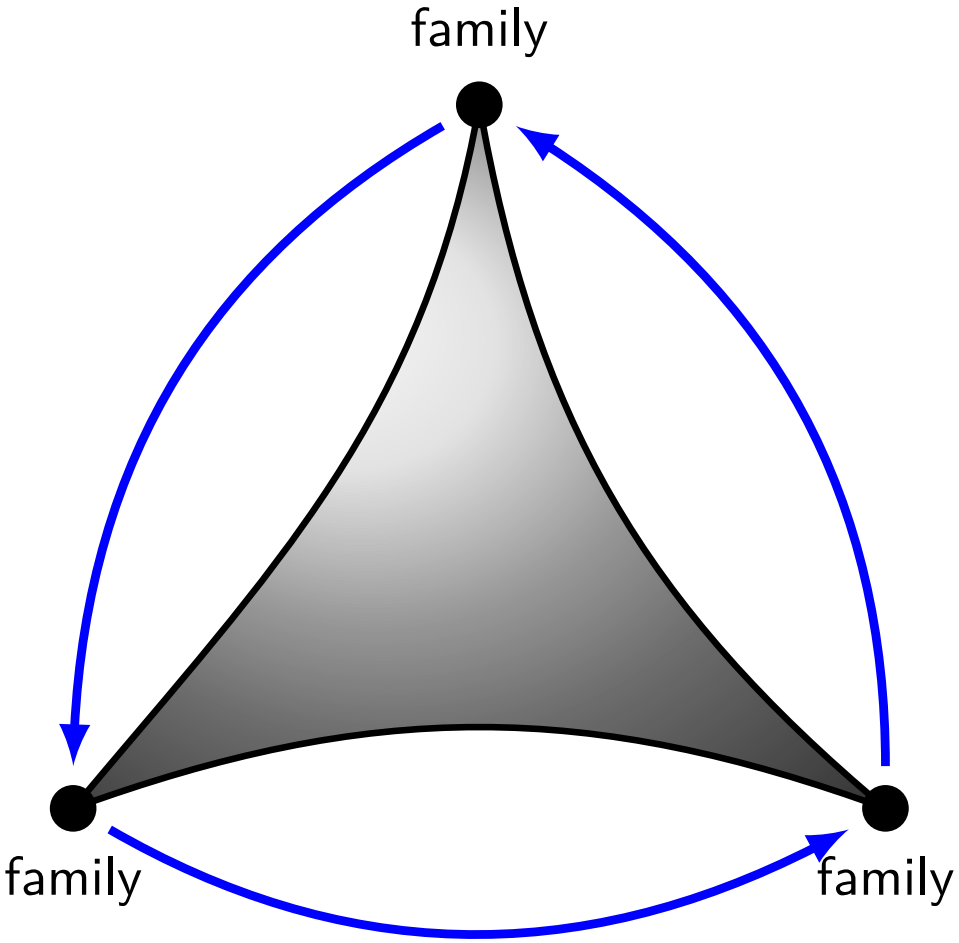

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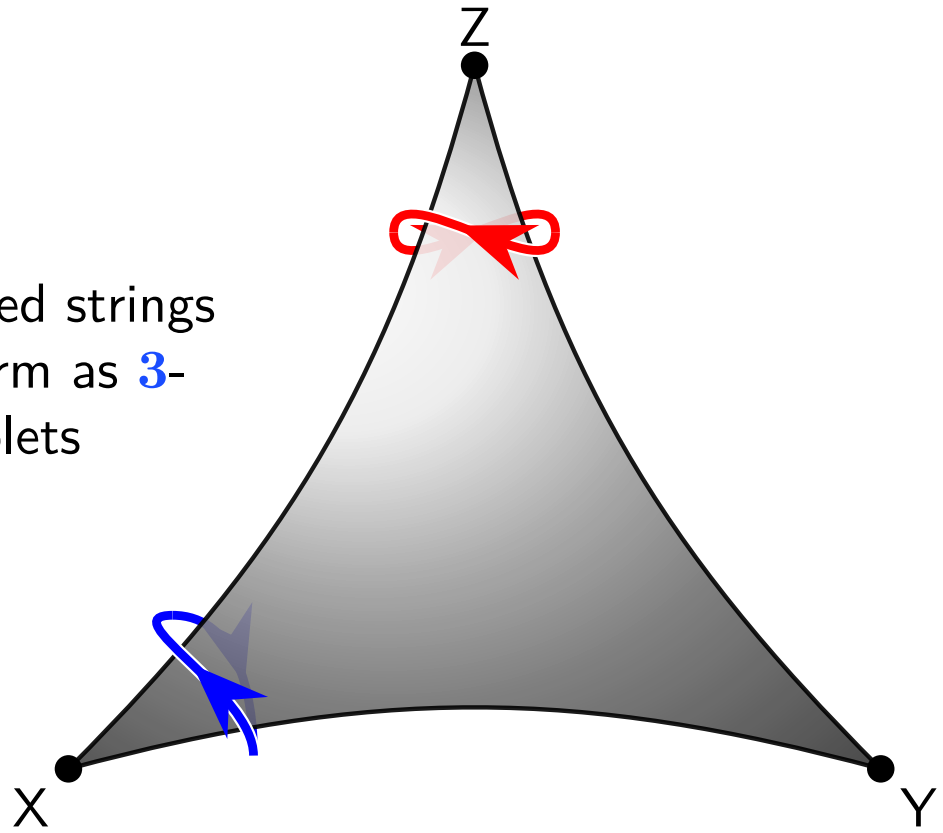
  - ➥ flavor/family symmetry

localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets



$\Delta(54)$

from a

$\mathbb{Z}_3$ orbifold plane

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

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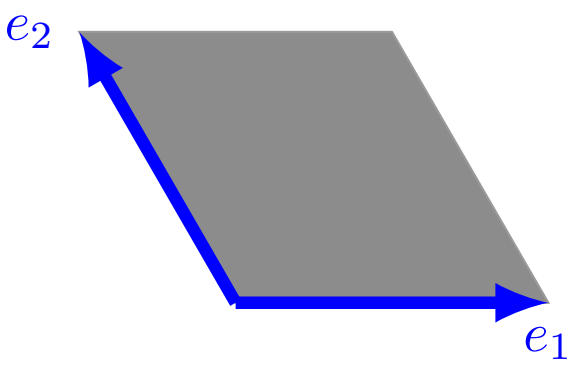

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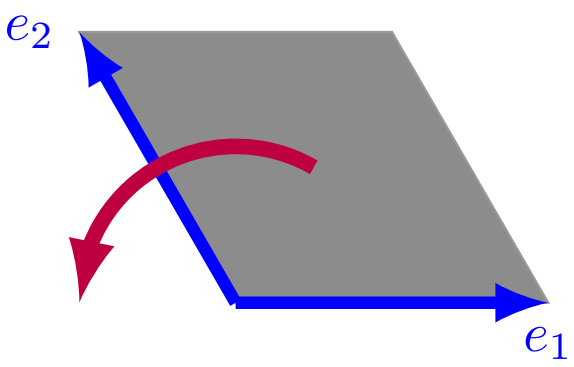

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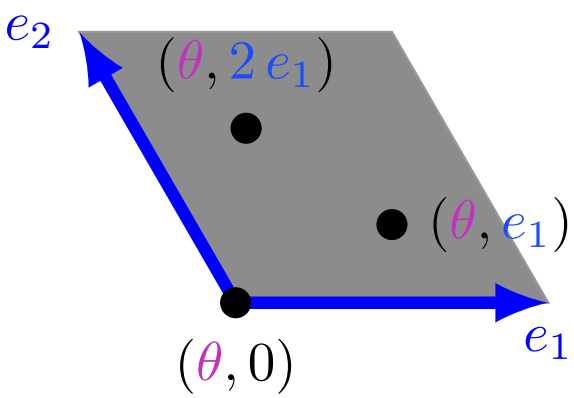

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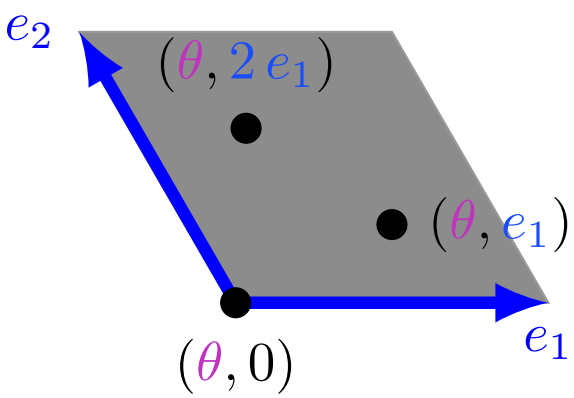

Hamidi & Vafa [1987]
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☞ coupling between n localized states $|(\theta, m^{(j)}\, e_1)\rangle$ only allowed if

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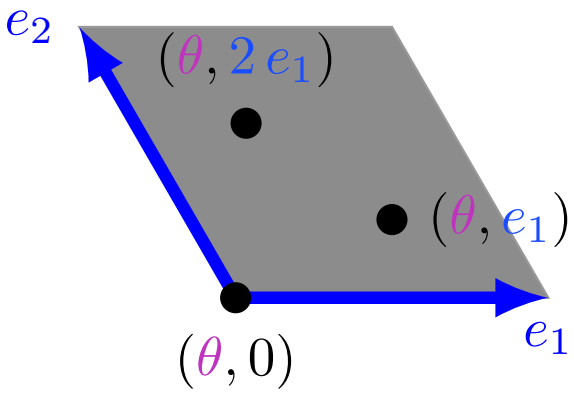

$$\begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \rightarrow \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} c|(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

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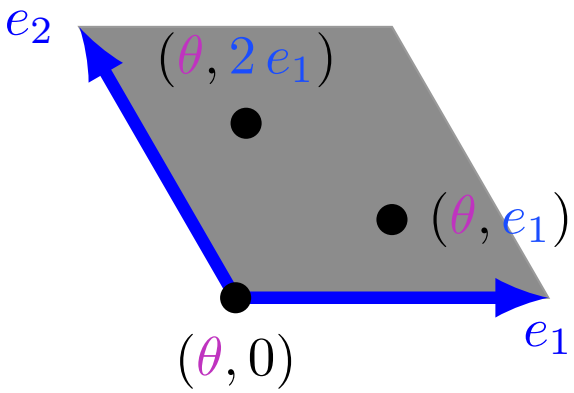

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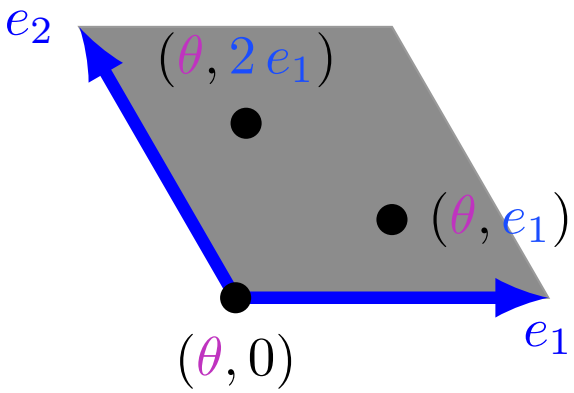

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➥ flavor symmetry

$$S_3 \cup (\mathbb{Z}_3 \times \mathbb{Z}_3) \;=\; S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) \;=\; \Delta(54)$$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]

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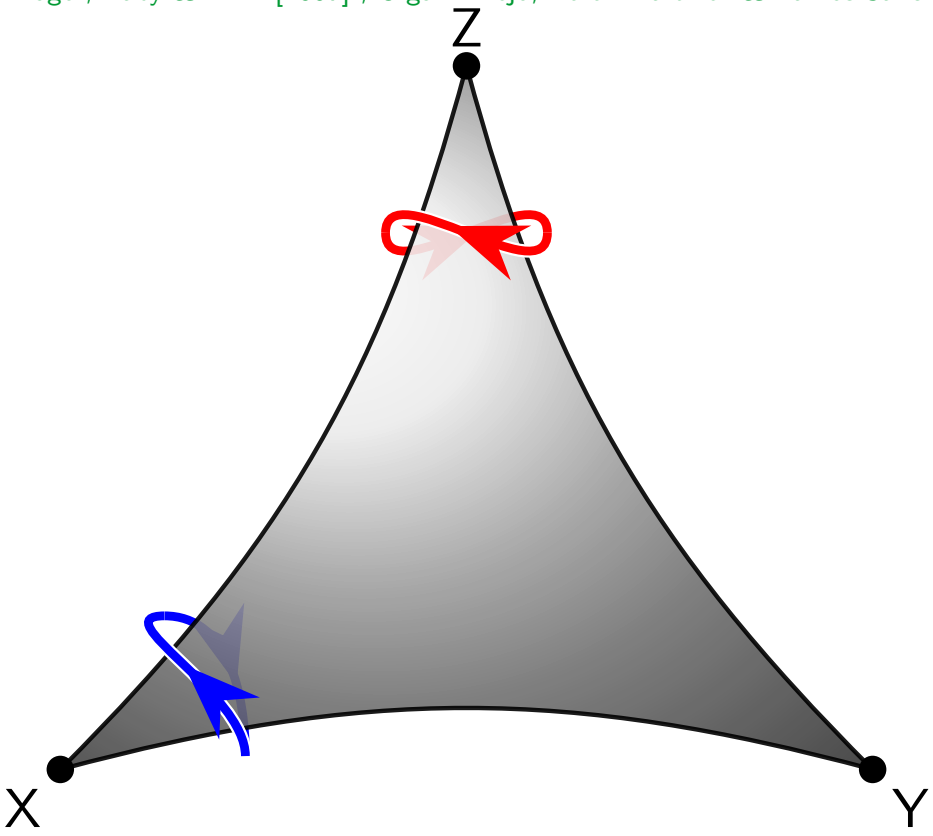

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- explicit model Carballo-Perez, Peinado & Ramos-Sánchez [2016]

#	irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\overline{u}_i$
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\overline{d}_i$
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\overline{e}_i$
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\overline{\nu}_i$

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$$\exists \text{ out } : \ \mathbf{3}_i \overset{\text{out}}{\longleftrightarrow} \overline{\mathbf{3}}_i \quad \text{and} \quad \mathbf{1}_i \overset{\text{out}}{\longleftrightarrow} \overline{\mathbf{1}}_i$$

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- ☞ at the massless level, only 3- and 1–dimensional representations occur $\curvearrowright$ a class–inverting outer automorphism exists $\curvearrowright$ a $\mathcal{CP}$ candidate exists

$\mathcal{CP}$ violation

in the

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$\mathcal{CP}$ violation from strings

- however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

Nilles, M.R., Trautner & Vaudrevange [2018]

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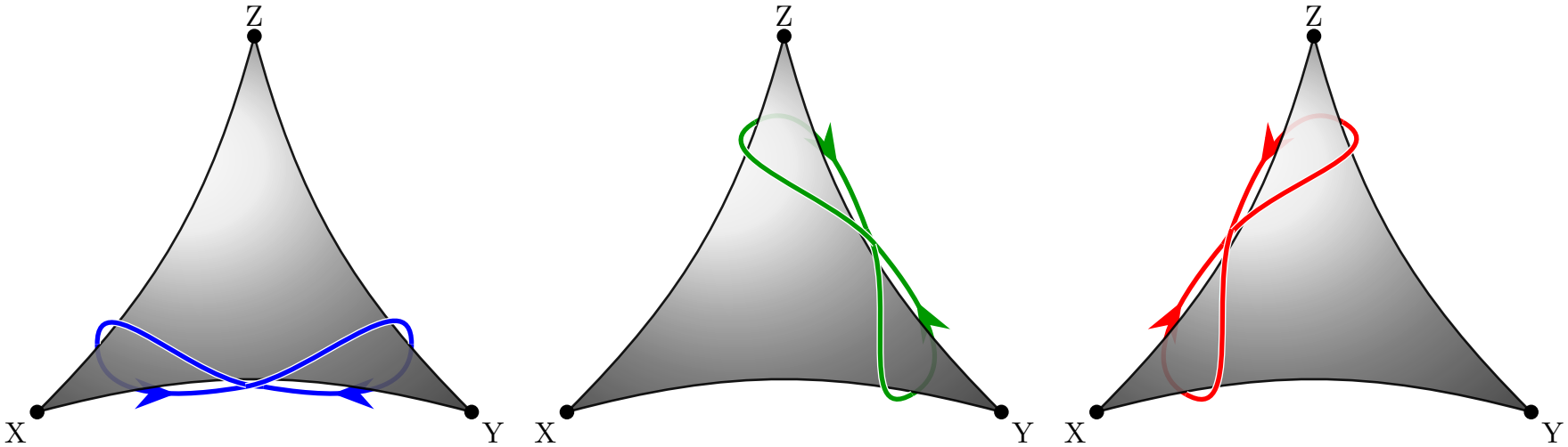

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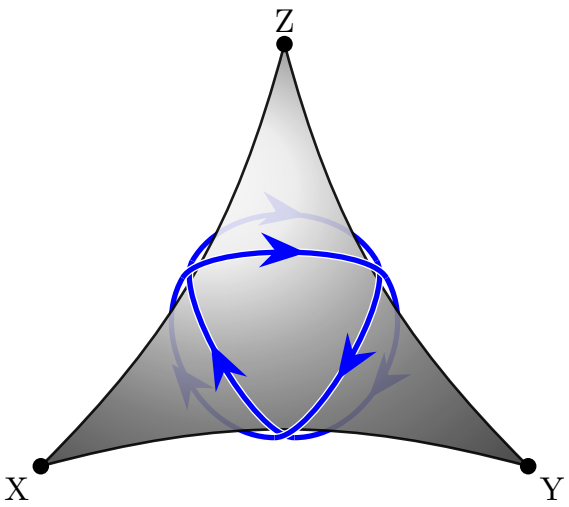

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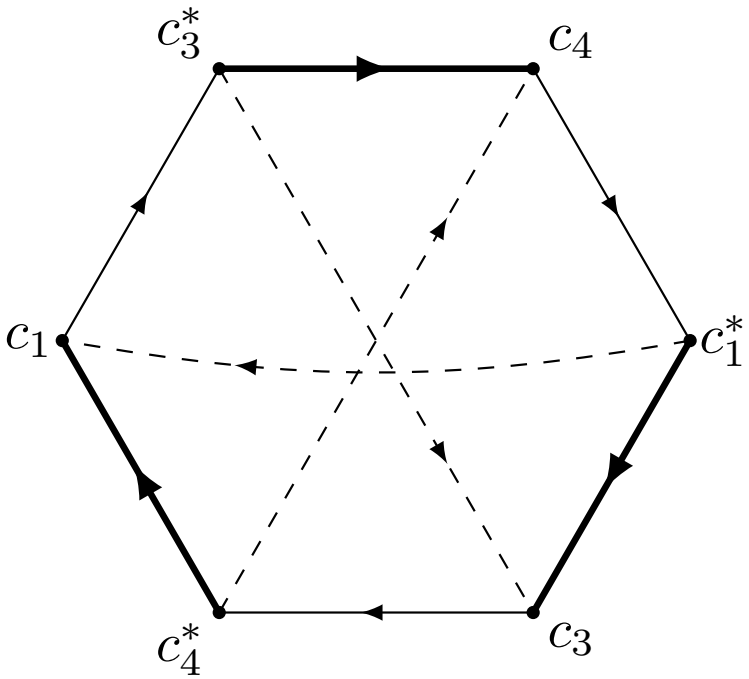

- we follow invariant approach — Bernabéu, Branco & Gronau [1986]
- super powerful tool: `Susyno` — Fonseca [2012]

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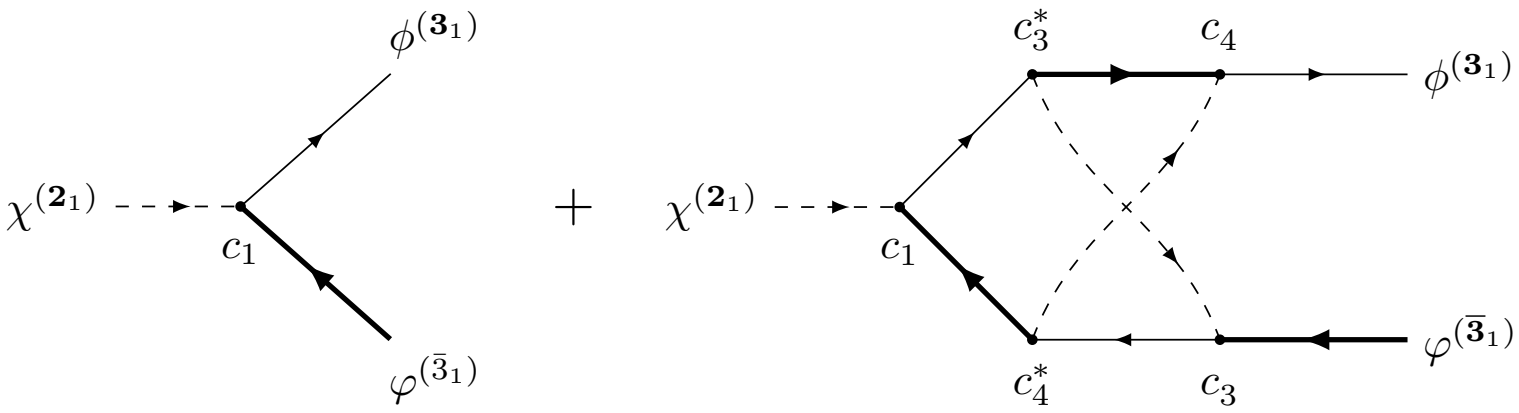

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bottom–line:

$\mathcal{CP}$ violation can come from group theory in UV complete settings in which the origin of the flavor group is fully understood

Summary

and

outlook

Summary

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1. solution of the μ problem

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     c. flavor symmetries: 'crystallography' of compact space

$\mathcal{CP}$ violation from strings

☞ Certain finite groups clash with $\mathcal{CP}$

$\mathcal{CP}$ violation from strings

- Certain finite groups clash with $\mathcal{CP}$
- Even the very first string models of particle physics have $\mathcal{CP}$ violating flavor symmetries

$\mathcal{CP}$ violation from strings

- Certain finite groups clash with $\mathcal{CP}$
- Even the very first string models of particle physics have $\mathcal{CP}$ violating flavor symmetries
- This mechanism is vastly unexplored so far

Outlook

☞ More insights by analyzing known heterotic constructions using by other means (heterotic M–theory, F–theory, . . .)

Outlook

- ☞ More insights by analyzing known heterotic constructions using by other means (heterotic M–theory, F–theory, . . .)
- ☞ More realistic models with $\mathcal{CP}$ violation from finite groups?

Thank you very much!

Anomaly cancellation

Anomaly freedom
w/ or w/o
Green–Schwarz
+
Gauge unification

Anomaly cancellation

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ \text{w/ or w/o}\\ \text{Green–Schwarz}\\ +\\ \text{Gauge unification}\end{array}\right\} \rightarrow \text{“Anomaly universality”}$$

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Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$\begin{aligned} A_{G^2-\mathbb{Z}_N} &= \sum_f \ell^{(f)} \cdot q^{(f)} \\ A_{\mathrm{grav}^2-\mathbb{Z}_N} &= \sum_m q^{(m)} \end{aligned}$$

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Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

sum over all representations of G

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

sum over all fermions

Anomaly cancellation

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Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

Dynkin index

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

discrete charges

Anomaly cancellation

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ \text{w/ or w/o}\\ \text{Green–Schwarz}\\ +\\ \text{Gauge unification}\end{array}\right\} \rightarrow \text{“Anomaly universality”}$$

traditional constraint:

all A coefficients vanish

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

??

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} 0 \mod \eta$$

$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} 0 \mod \eta$$

Anomaly cancellation

Anomaly freedom
w/ or w/o
Green–Schwarz
+
Gauge unification

$\rightarrow$ "Anomaly universality"

universal
shift due to
GS saxion

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} \rho \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

traditional constraint:

all A coefficients vanish

anomaly "universality":

$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = A_{\mathrm{SU}(2)^2-\mathbb{Z}_N}$
if $\mathrm{SU}(3) \times \mathrm{SU}(2) \subset \mathrm{SU}(5) \ldots \mathrm{E}_8$

It has to be an R symmetry

Hall, Nomura & Pierce [2002] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

☞ Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

charge of gth $\mathbf{5}$–plet

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right)$$

Higgs charges

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

charge of gth $\mathbf{10}$–plet

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$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

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bottom–line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

Only discrete R symmetries may do the job

- ☞ Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries
- ☞ There are no anomaly–free continuous R symmetries in the MSSM ?
- ➥ Only remaining option: discrete R symmetries

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

☞ Powerful tool: anomaly matching

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't Hooft [1976] ; Csáki & Murayama [1998]

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

extra

matter

gauginos

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- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

universal

SM gauginos

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \frac{1}{2}\cdot 2\cdot 2\cdot q_\theta$$

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extra gauginos from X, Y bosons

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- Assume now that some mechanism eliminates the extra gauginos
- ➥ Extra stuff must be non–universal (split multiplets)

't Hooft anomaly matching for R symmetries

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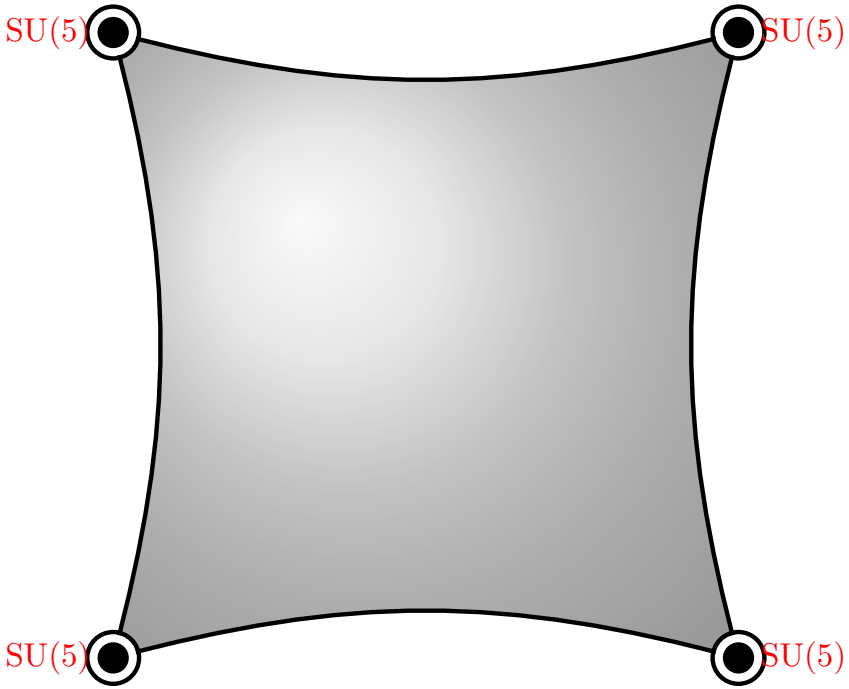
bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

An example

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

? ; Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

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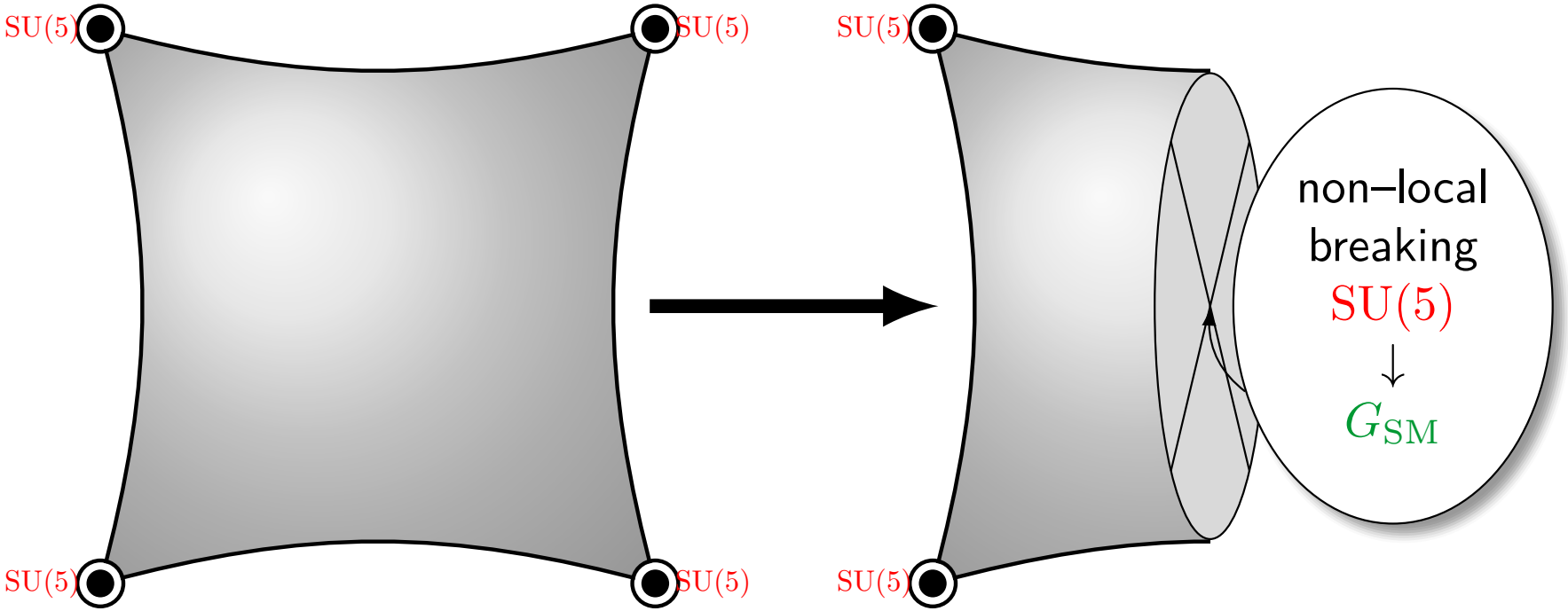

1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry
2. step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:
   - breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\rm L} \times \mathrm{U}(1)_Y$
   - reduces the number of generations to 3

analogous mechanism in CY MSSMs ?
?

Main features

1. GUT symmetry breaking non–local
   $\curvearrowright$ (almost) no 'logarithmic running above the GUT scale'

? ; ?

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
   $\curvearrowright$ complete blow–up without breaking SM gauge symmetry in principle possible

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y \times [\mathrm{SU}(3) \times \mathrm{SU}(2)^2 \times \mathrm{U}(1)^8]$

Main features

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4. massless spectrum

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
3	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

Main features

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4. massless spectrum

   spectrum $= 3 \times$ generation $+$ vector-like

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
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6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

$\mathbb{Z}_4^R$: discriminate between

matter with $\mathbb{Z}_4^R$ charge 1

and

Higgs/exotics with $\mathbb{Z}_4^R$ charge 0 or 2

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
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6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

☞ Many other good features:

- no fractionally charged exotics (i.e. all SM fields come from $\mathrm{SU}(5)$ representations)
- non–trivial full–rank Yukawa couplings
- gauge–top unification
- SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

$\mathbb{Z}_4^R$ summarized

Yukawa couplings ✓

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, \boldsymbol{h}_d \boldsymbol{h}_u + \kappa_i\, \boldsymbol{\ell}_i \boldsymbol{h}_u \\ &+ Y_e^{gf}\, \boldsymbol{\ell}_g \boldsymbol{h}_d \boldsymbol{e}^{\mathcal{C}}{}_f + Y_d^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_d \boldsymbol{d}^{\mathcal{C}}{}_f + Y_u^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_u \boldsymbol{u}^{\mathcal{C}}{}_f \\ &+ \lambda_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{\ell}_f \boldsymbol{e}^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{q}_f \boldsymbol{d}^{\mathcal{C}}{}_k + \lambda''_{gfk}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{d}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \\ &+ \kappa_{gf}\, \boldsymbol{h}_u \boldsymbol{\ell}_g\, \boldsymbol{h}_u \boldsymbol{\ell}_f + \kappa^{(1)}_{gfk\ell}\, \boldsymbol{q}_g \boldsymbol{q}_f \boldsymbol{q}_k \boldsymbol{\ell}_\ell + \kappa^{(2)}_{gfk\ell}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{u}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \boldsymbol{e}^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\; & \mu\, \boldsymbol{h}_d \boldsymbol{h}_u + \kappa_i\, \boldsymbol{\ell}_i \boldsymbol{h}_u \\
& + Y_e^{gf}\, \boldsymbol{\ell}_g \boldsymbol{h}_d \boldsymbol{e}^{\mathcal{C}}{}_f + Y_d^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_d \boldsymbol{d}^{\mathcal{C}}{}_f + Y_u^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_u \boldsymbol{u}^{\mathcal{C}}{}_f \\
& + \lambda_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{\ell}_f \boldsymbol{e}^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{q}_f \boldsymbol{d}^{\mathcal{C}}{}_k + \lambda''_{gfk}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{d}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \\
& + \kappa_{gf}\, \boldsymbol{h}_u \boldsymbol{\ell}_g \boldsymbol{h}_u \boldsymbol{\ell}_f + \kappa^{(1)}_{gfk\ell}\, \boldsymbol{q}_g \boldsymbol{q}_f \boldsymbol{q}_k \boldsymbol{\ell}_\ell + \kappa^{(2)}_{gfk\ell}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{u}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \boldsymbol{e}^{\mathcal{C}}{}_\ell
\end{aligned}
$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

- ☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

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$\mathcal{O}\left(\frac{m_{3/2}}{M_{\rm P}^2}\right)$

- R parity violating couplings forbidden
- μ term of the right size

  order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

  Planck units

- proton decay under control

$\mathbb{Z}_4^R$ summarized

Yukawa couplings ✓

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, \boldsymbol{h}_d \boldsymbol{h}_u + \kappa_i\, \boldsymbol{\ell}_i \boldsymbol{h}_u \\ &+ Y_e^{gf}\, \boldsymbol{\ell}_g \boldsymbol{h}_d \boldsymbol{e}^{\mathcal{C}}{}_f + Y_d^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_d \boldsymbol{d}^{\mathcal{C}}{}_f + Y_u^{gf}\, \boldsymbol{q}_g \boldsymbol{h}_u \boldsymbol{u}^{\mathcal{C}}{}_f \\ &+ \lambda_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{\ell}_f \boldsymbol{e}^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \boldsymbol{\ell}_g \boldsymbol{q}_f \boldsymbol{d}^{\mathcal{C}}{}_k + \lambda''_{gfk}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{d}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \\ &+ \kappa_{gf}\, \boldsymbol{h}_u \boldsymbol{\ell}_g\, \boldsymbol{h}_u \boldsymbol{\ell}_f + \kappa^{(1)}_{gfk\ell}\, \boldsymbol{q}_g \boldsymbol{q}_f \boldsymbol{q}_k \boldsymbol{\ell}_\ell + \kappa^{(2)}_{gfk\ell}\, \boldsymbol{u}^{\mathcal{C}}{}_g \boldsymbol{u}^{\mathcal{C}}{}_f \boldsymbol{d}^{\mathcal{C}}{}_k \boldsymbol{e}^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\; & \mu\, \boldsymbol{h}_d \boldsymbol{h}_u + \kappa_i\, \boldsymbol{\ell}_i \boldsymbol{h}_u \\
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\end{aligned}
$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

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$\mathcal{O}\left(\frac{m_{3/2}}{M_{\mathrm{P}}^2}\right)$

- R parity violating couplings forbidden
- μ term of the right size

  order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

  Planck units

- proton decay under control

$\mathbb{Z}_4^R$ summarized

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