

Bias and Estimation under Misspecification of the Risk Period in Self-Controlled Case Series Studies

**Supplemental Materials: Simulation Results for Models with One
Age Group**

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Additional simulation results

We provide results from additional simulations for the estimation of the relative incidence, $\hat{R} = \exp(\hat{\beta})$, and optimal risk length, $\hat{\tau}$, under different exposure models with no age effects.

Tables 1-2, 3-4, 5-6, 7-8, 9-10 and 11-12 show the results in estimation for the single Uniformly distributed exposure model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 1-6.

Tables 13-14, 15-16, 17-18, 19-20, 21-22 and 23-24 show the results in estimation for the multiple Uniformly distributed exposures model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 7-12.

Tables 25-26, 27-28, 29-30, 31-32, 33-34, 35-36 show the results in estimation for the single Normally distributed exposure model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 13-18.

Tables 37-38, 39-40, 41-42, 43-44, 45-46 and 47-48 show the results in estimation for the multiple Normally distributed exposures model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 19-24.

Table 1: **Single Uniformly distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.695	0.005	0.15	0.15	0.732	0.032	0.184	0.185	0.524	0.176	0.088	0.119	0.763	0.063	0.207	0.211
15	200	0.712	0.012	0.088	0.088	0.711	0.011	0.099	0.1	0.529	0.171	0.076	0.105	0.623	0.077	0.191	0.197
15	400	0.698	0.002	0.039	0.039	0.706	0.006	0.045	0.045	0.575	0.125	0.042	0.058	0.616	0.084	0.067	0.074
15	800	0.704	0.004	0.024	0.024	0.709	0.009	0.027	0.027	0.617	0.083	0.024	0.031	0.64	0.06	0.03	0.033
30	100	0.707	0.007	0.087	0.087	0.735	0.035	0.112	0.113	0.52	0.18	0.065	0.097	0.615	0.085	0.187	0.194
30	200	0.661	0.039	0.04	0.041	0.658	0.042	0.049	0.051	0.517	0.183	0.041	0.075	0.571	0.129	0.078	0.095
30	400	0.697	0.003	0.027	0.027	0.686	0.014	0.029	0.03	0.595	0.105	0.026	0.037	0.625	0.075	0.035	0.04
30	800	0.703	0.003	0.01	0.01	0.698	0.002	0.013	0.013	0.629	0.071	0.012	0.018	0.646	0.054	0.011	0.014
45	100	0.727	0.027	0.063	0.064	0.75	0.05	0.083	0.086	0.547	0.153	0.05	0.073	0.636	0.064	0.146	0.15
45	200	0.695	0.005	0.031	0.031	0.692	0.008	0.046	0.046	0.573	0.127	0.032	0.048	0.618	0.082	0.057	0.064
45	400	0.698	0.002	0.013	0.013	0.697	0.003	0.019	0.019	0.613	0.087	0.016	0.023	0.638	0.062	0.021	0.025
45	800	0.7	0	0.007	0.007	0.697	0.003	0.009	0.009	0.641	0.059	0.008	0.012	0.663	0.037	0.007	0.009

Table 2: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.7	0.7	9.67	10.15	14.18	0.82	19.82	20.5	15.8	0.8	20.05	20.69
15	200	15.14	0.14	8.87	8.89	12.81	2.19	18.63	23.41	15.06	0.06	22.14	22.15
15	400	14.92	0.08	8.97	8.97	12.58	2.42	17.3	23.14	14.83	0.17	20.08	20.11
15	800	15.07	0.07	8.21	8.21	12.4	2.6	16.08	22.83	14.68	0.32	16.15	16.25
30	100	30.13	0.13	45.51	45.52	26.35	3.65	78.5	91.81	31.21	1.21	95.34	96.81
30	200	30.07	0.07	38.22	38.23	24.48	5.52	69.9	100.4	29.29	0.71	85.13	85.63
30	400	29.64	0.36	41.7	41.82	24.46	5.54	64.06	94.72	30.29	0.29	74.14	74.23
30	800	29.61	0.39	28.36	28.51	23.95	6.05	52.05	88.65	29.03	0.97	59.44	60.39
45	100	43.81	1.19	106.8	108.22	36.36	8.64	206.56	281.27	44.4	0.6	232.17	232.53
45	200	44.36	0.64	99.22	99.64	38.35	6.65	208.24	252.5	46.27	1.27	197.49	199.1
45	400	43.54	1.46	86.27	88.39	35.11	9.89	157.35	255.16	45.18	0.18	162.43	162.46
45	800	44.73	0.27	83.32	83.39	33.78	11.22	143.7	269.5	45.9	0.9	124.63	125.44

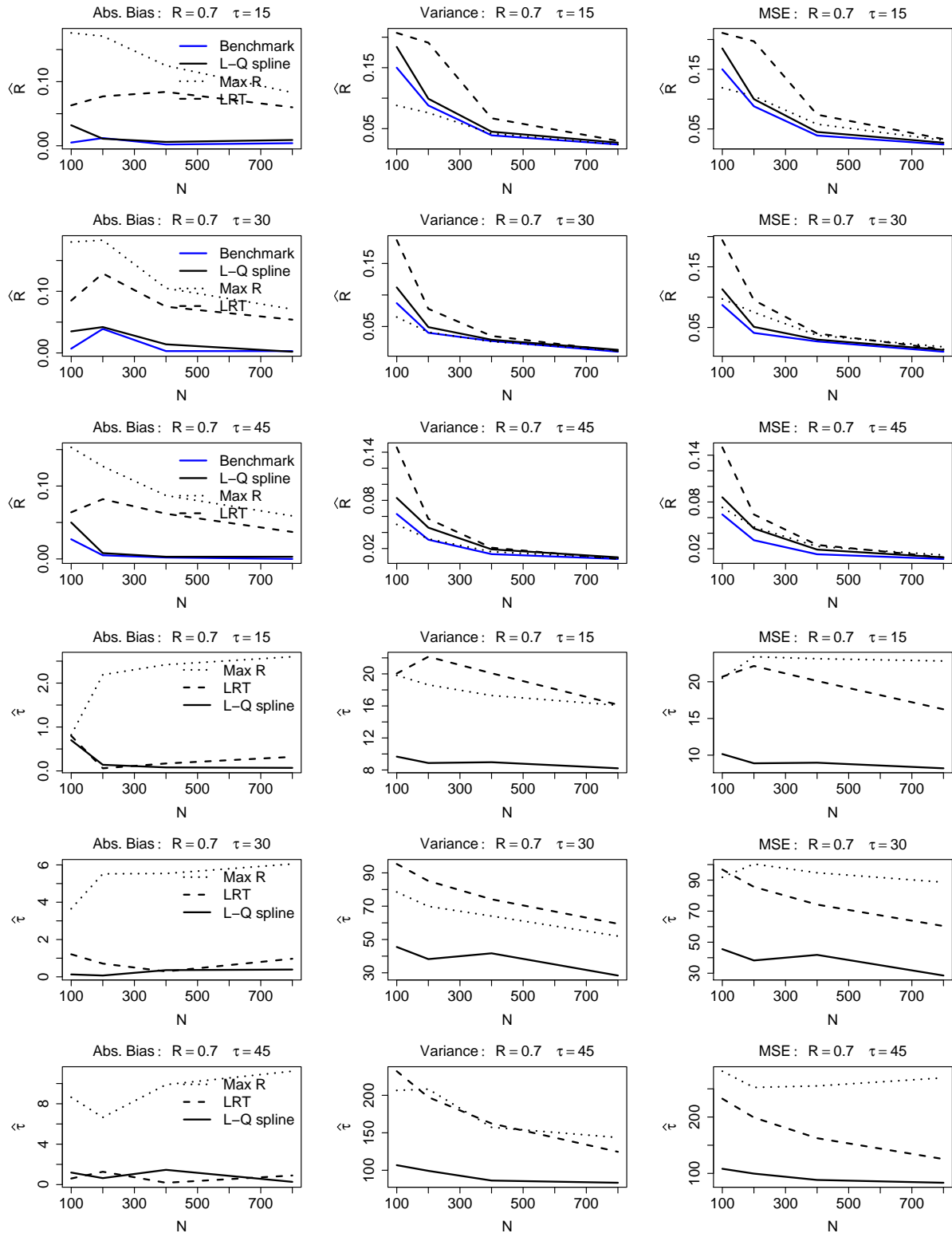


Figure 1: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 3: **Single Uniformly distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.948	0.048	0.227	0.229	1.009	0.109	0.299	0.311	0.7	0.2	0.143	0.183	1.007	0.107	0.585	0.596
15	200	0.939	0.039	0.138	0.14	0.973	0.073	0.169	0.174	0.712	0.188	0.102	0.137	0.921	0.021	0.298	0.299
15	400	0.902	0.002	0.063	0.063	0.924	0.024	0.076	0.076	0.743	0.157	0.056	0.081	0.87	0.03	0.156	0.157
15	800	0.917	0.017	0.024	0.025	0.926	0.026	0.032	0.032	0.804	0.096	0.022	0.032	0.878	0.022	0.059	0.06
30	100	0.943	0.043	0.13	0.132	0.995	0.095	0.162	0.171	0.698	0.202	0.102	0.142	0.942	0.042	0.347	0.349
30	200	0.938	0.038	0.069	0.071	0.962	0.062	0.087	0.091	0.761	0.139	0.061	0.08	0.93	0.03	0.18	0.181
30	400	0.91	0.01	0.03	0.03	0.91	0.01	0.036	0.036	0.775	0.125	0.03	0.045	0.872	0.028	0.078	0.078
30	800	0.889	0.011	0.016	0.016	0.892	0.008	0.019	0.019	0.808	0.092	0.016	0.024	0.858	0.042	0.035	0.037
45	100	0.926	0.026	0.084	0.085	0.939	0.039	0.123	0.125	0.688	0.212	0.08	0.125	0.924	0.024	0.285	0.286
45	200	0.89	0.01	0.037	0.038	0.897	0.003	0.053	0.053	0.72	0.18	0.041	0.073	0.849	0.051	0.126	0.129
45	400	0.899	0.001	0.022	0.022	0.899	0.001	0.031	0.031	0.782	0.118	0.024	0.038	0.867	0.033	0.063	0.064
45	800	0.901	0.001	0.012	0.012	0.893	0.007	0.015	0.015	0.82	0.08	0.013	0.019	0.867	0.033	0.03	0.031

Table 4: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.1	0.1	9.42	9.43	14.6	0.4	22.05	22.21	14.92	0.08	20.85	20.86
15	200	14.35	0.65	9.01	9.44	13.73	1.27	24.52	26.14	15.19	0.19	21.27	21.3
15	400	14.52	0.48	9.25	9.49	13.48	1.52	22.54	24.86	14.88	0.12	25.4	25.42
15	800	14.38	0.62	8.89	9.28	13.35	1.65	23.99	26.72	14.92	0.08	24.52	24.52
30	100	29.22	0.78	40.13	40.74	27.56	2.44	113.04	118.98	29.28	0.72	98.76	99.29
30	200	29.5	0.5	45.88	46.13	26.38	3.62	95.79	108.92	30.14	0.14	101.81	101.83
30	400	28.88	1.12	45.06	46.32	25.29	4.71	102.54	124.75	27.69	2.31	110.55	115.89
30	800	29.93	0.07	39.93	39.93	25.24	4.76	80.8	103.44	29.2	0.8	83.51	84.16
45	100	43.32	1.68	105.04	107.86	39.91	5.09	280.2	306.06	39.7	5.3	263.2	291.3
45	200	42.18	2.82	115.22	123.17	37.18	7.82	241.24	302.38	39.56	5.44	256.14	285.75
45	400	42.94	2.06	120.98	125.23	35.96	9.04	202.49	284.12	41.53	3.47	271.73	283.75
45	800	42.9	2.1	115.1	119.49	37.6	7.4	229.44	284.23	42.61	2.39	251.24	256.94

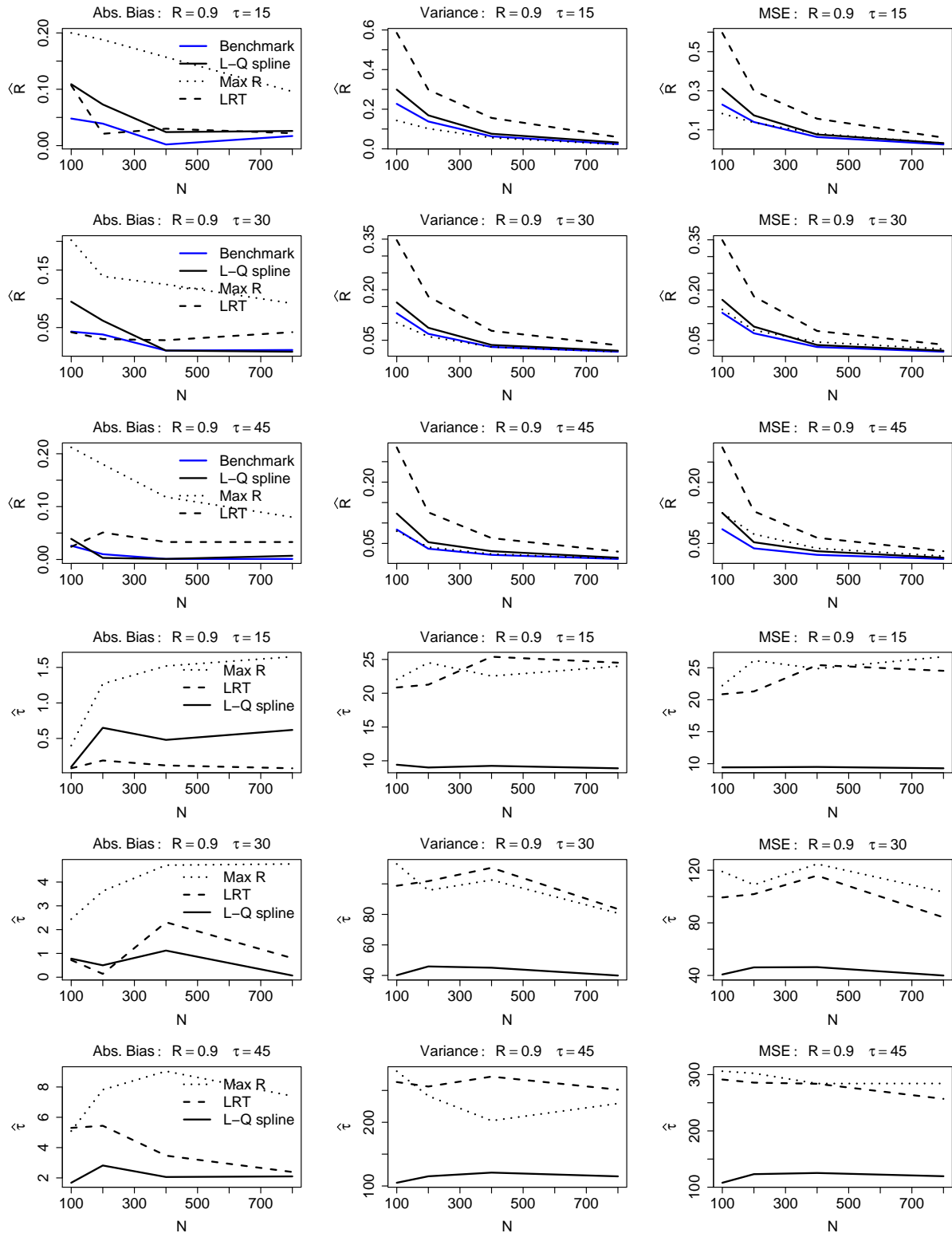


Figure 2: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 5: **Single Uniformly distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.232	0.032	0.292	0.293	1.292	0.092	0.372	0.381	1.604	0.404	0.42	0.584	1.377	0.177	0.692	0.723
15	200	1.253	0.053	0.151	0.154	1.311	0.111	0.204	0.217	1.509	0.309	0.193	0.288	1.368	0.168	0.333	0.362
15	400	1.2	0	0.078	0.078	1.214	0.014	0.1	0.1	1.368	0.168	0.095	0.123	1.264	0.064	0.176	0.18
15	800	1.19	0.01	0.037	0.037	1.21	0.01	0.05	0.05	1.304	0.104	0.044	0.055	1.249	0.049	0.071	0.074
30	100	1.256	0.056	0.154	0.158	1.328	0.128	0.239	0.255	1.582	0.382	0.245	0.392	1.414	0.214	0.437	0.482
30	200	1.194	0.006	0.081	0.081	1.209	0.009	0.108	0.108	1.396	0.196	0.1	0.138	1.282	0.082	0.19	0.196
30	400	1.193	0.007	0.038	0.038	1.219	0.019	0.055	0.055	1.343	0.143	0.054	0.074	1.289	0.089	0.087	0.095
30	800	1.206	0.006	0.021	0.021	1.216	0.016	0.026	0.027	1.3	0.1	0.023	0.033	1.266	0.066	0.032	0.037
45	100	1.19	0.01	0.126	0.126	1.235	0.035	0.18	0.181	1.463	0.263	0.167	0.236	1.28	0.08	0.339	0.346
45	200	1.211	0.011	0.071	0.071	1.253	0.053	0.101	0.104	1.443	0.243	0.087	0.146	1.372	0.172	0.144	0.173
45	400	1.211	0.011	0.035	0.035	1.231	0.031	0.045	0.046	1.344	0.144	0.045	0.066	1.292	0.092	0.067	0.076
45	800	1.221	0.021	0.016	0.016	1.237	0.037	0.023	0.024	1.314	0.114	0.022	0.035	1.287	0.087	0.027	0.035

Table 6: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.04	0.96	8.12	9.04	13.52	1.48	21.53	23.72	14.58	0.42	24.28	24.46
15	200	14.26	0.74	9.1	9.64	12.8	2.2	17.58	22.42	14.22	0.78	20.7	21.32
15	400	14.51	0.49	7.46	7.69	13.13	1.87	20.39	23.9	14.08	0.92	21.52	22.37
15	800	14.07	0.93	8.65	9.51	13.22	1.78	20.22	23.39	14.57	0.43	20.55	20.73
30	100	28.83	1.17	39.3	40.66	25.24	4.76	95.68	118.33	28.42	1.58	104.68	107.17
30	200	28.68	1.32	42.93	44.68	25.4	4.6	86.27	107.41	28.2	1.8	101.4	104.66
30	400	28.08	1.92	41.44	45.13	25.39	4.61	87.82	109.1	27.81	2.19	88.93	93.73
30	800	29.41	0.59	42.64	42.99	24.39	5.61	69.87	101.37	29.39	0.61	76.99	77.36
45	100	40.94	4.06	110.1	126.58	37.26	7.74	250.61	310.58	39.97	5.03	258.98	284.23
45	200	42.25	2.75	123.61	131.2	35.38	9.62	217.95	310.56	40.27	4.73	256.27	278.67
45	400	43.44	1.56	111.45	113.89	35.11	9.89	209.17	307.07	41.96	3.04	210.08	219.32
45	800	42.48	2.52	89.66	95.99	33.63	11.37	158.91	288.23	42.17	2.83	180.59	188.6

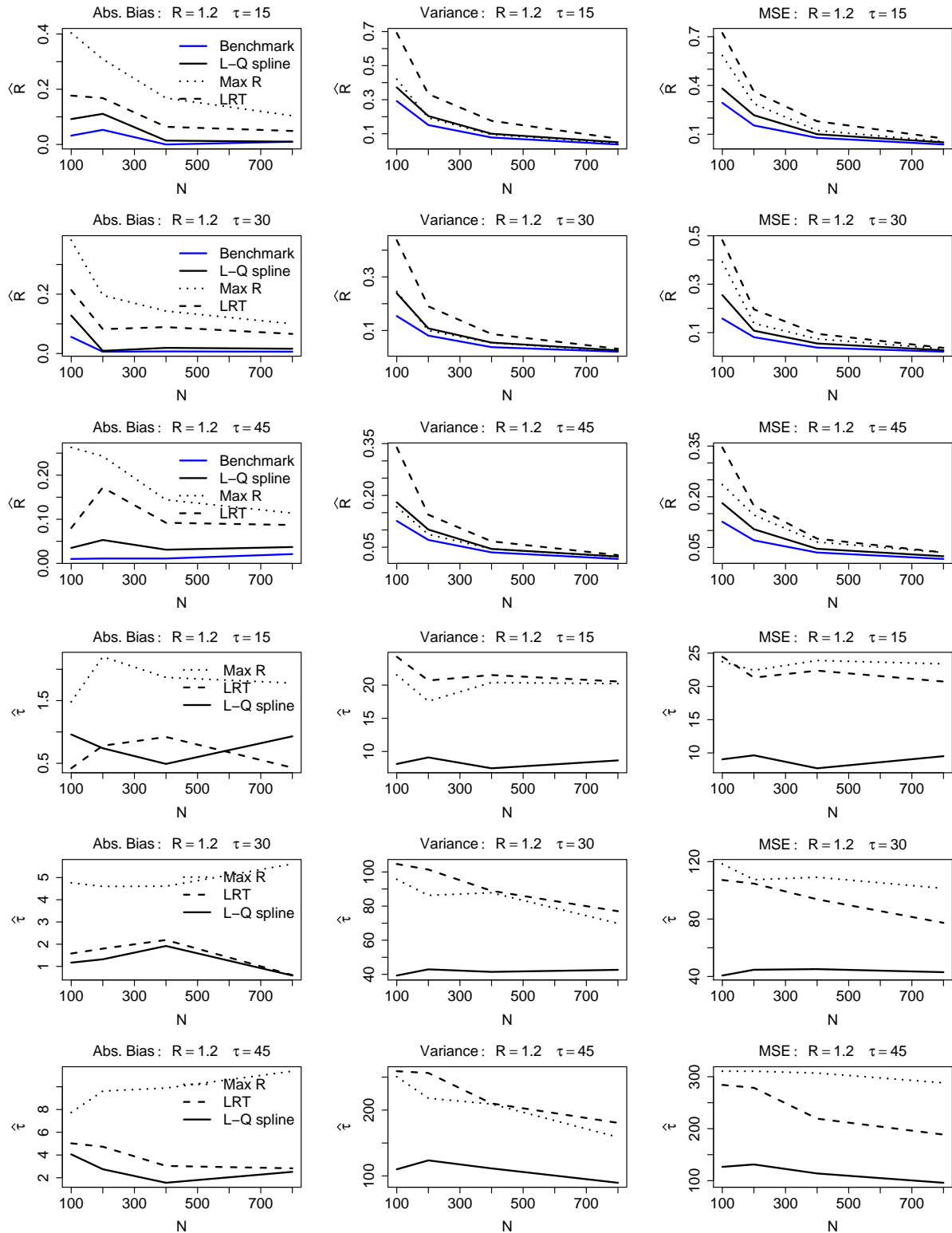


Figure 3: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 7: **Single Uniformly distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.59	0.09	0.375	0.383	1.656	0.156	0.455	0.479	1.993	0.493	0.578	0.821	1.85	0.35	0.777	0.899
15	200	1.544	0.044	0.222	0.224	1.607	0.107	0.266	0.278	1.8	0.3	0.295	0.385	1.695	0.195	0.37	0.408
15	400	1.504	0.004	0.106	0.106	1.548	0.048	0.126	0.128	1.698	0.198	0.142	0.181	1.659	0.159	0.144	0.169
15	800	1.494	0.006	0.055	0.055	1.517	0.017	0.065	0.066	1.613	0.113	0.066	0.079	1.58	0.08	0.064	0.07
30	100	1.5	0	0.222	0.222	1.569	0.069	0.299	0.304	1.811	0.311	0.357	0.454	1.679	0.179	0.49	0.522
30	200	1.463	0.037	0.112	0.113	1.514	0.014	0.157	0.158	1.695	0.195	0.159	0.197	1.639	0.139	0.179	0.198
30	400	1.524	0.024	0.054	0.055	1.554	0.054	0.072	0.074	1.663	0.163	0.072	0.099	1.624	0.124	0.066	0.082
30	800	1.521	0.021	0.025	0.025	1.541	0.041	0.03	0.032	1.619	0.119	0.034	0.048	1.584	0.084	0.031	0.038
45	100	1.547	0.047	0.165	0.167	1.613	0.113	0.231	0.243	1.851	0.351	0.223	0.346	1.744	0.244	0.328	0.387
45	200	1.51	0.01	0.091	0.091	1.549	0.049	0.108	0.11	1.709	0.209	0.117	0.161	1.648	0.148	0.141	0.163
45	400	1.502	0.002	0.04	0.04	1.53	0.03	0.046	0.047	1.63	0.13	0.053	0.07	1.591	0.091	0.055	0.063
45	800	1.504	0.004	0.021	0.021	1.519	0.019	0.019	0.02	1.573	0.073	0.022	0.027	1.542	0.042	0.021	0.023

Table 8: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.31	0.69	8.67	9.14	12.46	2.54	18.89	25.33	14.35	0.65	23.2	23.63
15	200	14.15	0.85	9.06	9.79	12.48	2.52	16.18	22.54	14.07	0.93	20.47	21.33
15	400	14.28	0.72	7.4	7.92	11.59	3.41	11.86	23.5	14.17	0.83	17.01	17.71
15	800	14.65	0.35	6.63	6.75	12.08	2.92	11.01	19.57	14.62	0.38	13.58	13.73
30	100	28.4	1.6	39.88	42.43	24.54	5.46	74.71	104.49	28.89	1.11	90.24	91.46
30	200	28.4	1.6	36.28	38.83	24.07	5.93	69.18	104.34	28.36	1.64	72.72	75.41
30	400	28.97	1.03	35.95	37.01	23.5	6.5	55.29	97.57	29.47	0.53	60.32	60.6
30	800	29.28	0.72	26.23	26.75	22.92	7.08	38.92	89.05	29.24	0.76	42.12	42.7
45	100	43.24	1.76	97.28	100.37	34.7	10.3	190.96	297.09	40.89	4.11	197.96	214.82
45	200	41.27	3.73	84.72	98.66	33.96	11.04	156.98	278.75	42.28	2.72	186.44	193.86
45	400	43.03	1.97	81.63	85.51	34.88	10.12	151.11	253.43	44.2	0.8	142.51	143.15
45	800	43.11	1.89	60.54	64.11	34.49	10.51	93.08	203.49	46.04	1.04	83.67	84.74

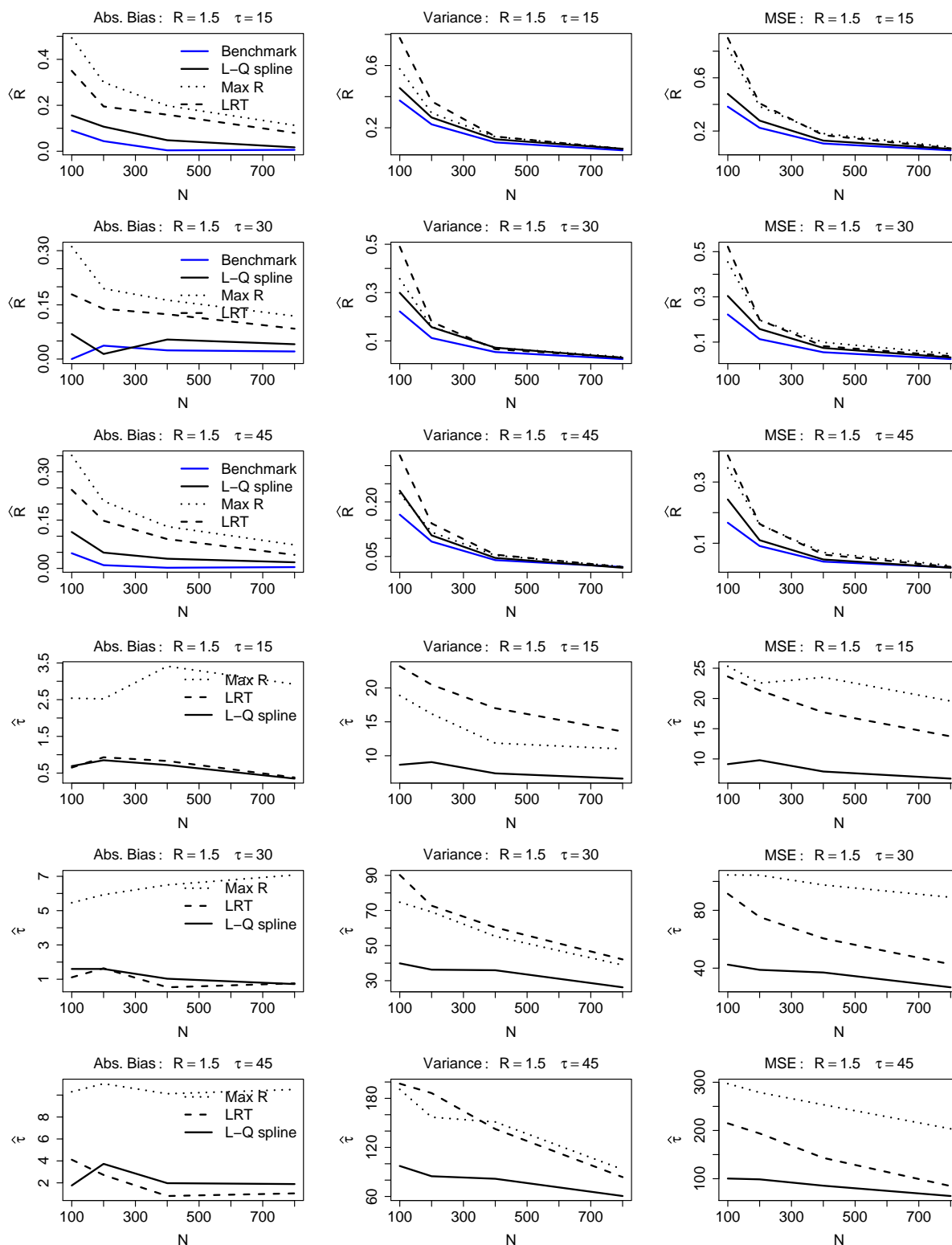


Figure 4: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 9: **Single Uniformly distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.049	0.049	0.619	0.622	2.136	0.136	0.676	0.695	2.499	0.499	0.759	1.009	2.367	0.367	0.847	0.982
15	200	2.051	0.051	0.282	0.284	2.151	0.151	0.375	0.398	2.324	0.324	0.435	0.54	2.235	0.235	0.433	0.488
15	400	1.991	0.009	0.14	0.14	2.037	0.037	0.164	0.165	2.173	0.173	0.163	0.193	2.099	0.099	0.149	0.158
15	800	2.014	0.014	0.072	0.073	2.039	0.039	0.081	0.083	2.137	0.137	0.086	0.104	2.062	0.062	0.078	0.082
30	100	1.972	0.028	0.283	0.283	2.077	0.077	0.382	0.388	2.329	0.329	0.389	0.497	2.247	0.247	0.395	0.456
30	200	2.02	0.02	0.157	0.157	2.085	0.085	0.192	0.199	2.234	0.234	0.191	0.245	2.183	0.183	0.177	0.211
30	400	1.995	0.005	0.077	0.077	2.032	0.032	0.084	0.085	2.134	0.134	0.095	0.113	2.066	0.066	0.086	0.091
30	800	2.011	0.011	0.032	0.032	2.009	0.009	0.034	0.034	2.102	0.102	0.037	0.047	2.038	0.038	0.032	0.033
45	100	1.99	0.01	0.253	0.253	2.069	0.069	0.302	0.306	2.289	0.289	0.338	0.422	2.202	0.202	0.368	0.409
45	200	2.02	0.02	0.117	0.118	2.074	0.074	0.133	0.138	2.216	0.216	0.136	0.183	2.151	0.151	0.125	0.147
45	400	1.991	0.009	0.058	0.058	1.993	0.007	0.064	0.065	2.093	0.093	0.069	0.078	2.041	0.041	0.063	0.065
45	800	2.033	0.033	0.027	0.028	2.03	0.03	0.028	0.029	2.085	0.085	0.028	0.035	2.048	0.048	0.027	0.029

Table 10: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.26	0.74	7.34	7.89	11.66	3.34	13.58	24.71	14.19	0.81	17.38	18.04
15	200	14.08	0.92	6.75	7.59	11.84	3.16	10.42	20.41	14.78	0.22	13.6	13.65
15	400	14.53	0.47	6.4	6.62	11.42	3.58	9.33	22.17	14.89	0.11	11.17	11.18
15	800	14.77	0.23	4.82	4.87	11.58	3.42	8.24	19.92	15.23	0.23	7.09	7.14
30	100	28.35	1.65	37.65	40.36	23.44	6.56	56.2	99.27	28.93	1.07	65.09	66.22
30	200	28.86	1.14	29.5	30.8	24.29	5.71	42.67	75.31	29.01	0.99	46.87	47.85
30	400	29.27	0.73	27.27	27.8	23.25	6.75	34.51	80.12	30.09	0.09	28.24	28.25
30	800	29.88	0.12	13.72	13.73	23.11	6.89	34.25	81.78	30.27	0.27	17.61	17.69
45	100	42.9	2.1	91.58	95.97	35.23	9.77	139.98	235.41	42.37	2.63	128.56	135.46
45	200	43.51	1.49	66.49	68.7	33.33	11.67	104.71	240.98	43.19	1.81	83.96	87.24
45	400	45.3	0.3	37.33	37.42	35.93	9.07	88.2	170.47	44.91	0.09	44.09	44.1
45	800	45.05	0.05	16.69	16.69	36.78	8.22	86.23	153.73	45.53	0.53	13.81	14.09

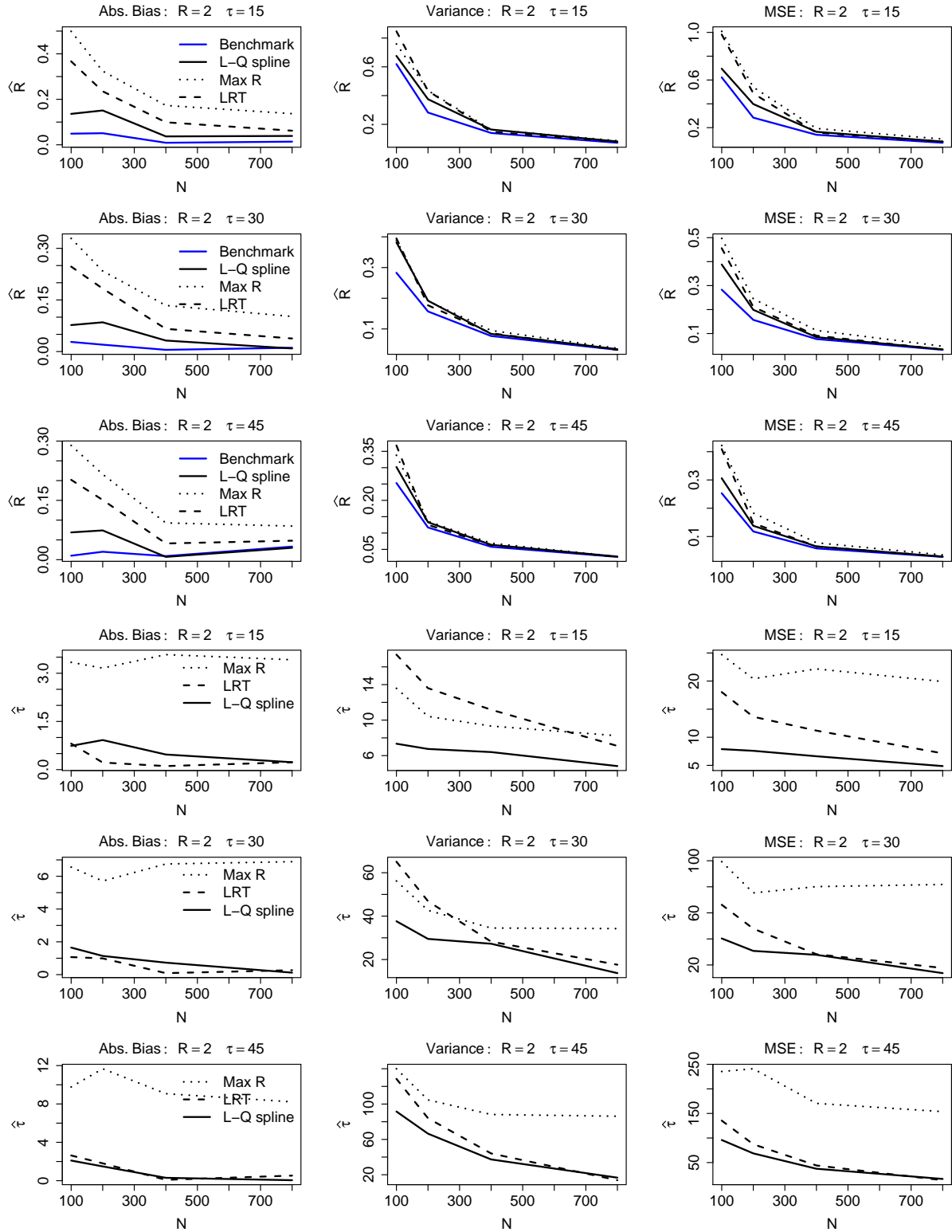


Figure 5: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 11: **Single Uniformly distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	3.956	0.044	0.967	0.969	4.168	0.168	1.213	1.242	4.436	0.436	1.175	1.365	4.267	0.267	1.158	1.229
15	200	4.045	0.045	0.733	0.735	4.087	0.087	0.733	0.741	4.381	0.381	0.835	0.98	4.163	0.163	0.807	0.834
15	400	4.013	0.013	0.361	0.361	4.035	0.035	0.367	0.369	4.215	0.215	0.422	0.468	4.067	0.067	0.37	0.375
15	800	3.971	0.029	0.155	0.156	3.97	0.03	0.161	0.162	4.093	0.093	0.172	0.181	3.985	0.015	0.155	0.155
30	100	4.004	0.004	0.721	0.721	4.091	0.091	0.792	0.801	4.334	0.334	0.801	0.913	4.205	0.205	0.81	0.852
30	200	4.085	0.085	0.454	0.461	4.067	0.067	0.497	0.501	4.258	0.258	0.488	0.554	4.15	0.15	0.465	0.487
30	400	3.997	0.003	0.19	0.19	3.978	0.022	0.198	0.199	4.102	0.102	0.191	0.202	4.018	0.018	0.192	0.192
30	800	4.001	0.001	0.084	0.084	3.986	0.014	0.087	0.088	4.049	0.049	0.084	0.087	4.01	0.01	0.084	0.084
45	100	4.117	0.117	0.71	0.724	4.155	0.155	0.792	0.816	4.389	0.389	0.792	0.943	4.283	0.283	0.81	0.89
45	200	4.061	0.061	0.325	0.329	4.033	0.033	0.316	0.317	4.172	0.172	0.314	0.344	4.103	0.103	0.321	0.331
45	400	4.022	0.022	0.166	0.167	3.989	0.011	0.177	0.177	4.083	0.083	0.166	0.173	4.048	0.048	0.168	0.17
45	800	3.978	0.022	0.095	0.096	3.937	0.063	0.096	0.1	3.992	0.008	0.094	0.094	3.983	0.017	0.096	0.096

Table 12: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.08	0.92	5.66	6.51	11.68	3.32	8.98	19.98	14.9	0.1	9.12	9.13
15	200	14.71	0.29	4.06	4.15	11.7	3.3	7.83	18.69	14.97	0.03	5.31	5.31
15	400	14.86	0.14	1.98	2	12.19	2.81	7.03	14.94	14.87	0.13	1.83	1.85
15	800	15	0	0.75	0.75	12.07	2.93	7.32	15.93	14.98	0.02	0.56	0.56
30	100	29.1	0.9	16.23	17.04	24.31	5.69	34.42	66.84	29.2	0.8	17.71	18.35
30	200	30.01	0.01	8.19	8.19	26.07	3.93	24.52	39.96	29.96	0.04	8.18	8.19
30	400	30.15	0.15	4.63	4.65	25.62	4.38	26.04	45.24	29.96	0.04	1.95	1.95
30	800	29.99	0.01	0.99	0.99	27.09	2.91	17.32	25.78	29.94	0.06	0.35	0.35
45	100	44.46	0.54	33.67	33.95	38.59	6.41	79.37	120.42	44.77	0.23	37.76	37.81
45	200	45.27	0.27	10.57	10.64	39.81	5.19	57	83.94	45.18	0.18	8.01	8.05
45	400	45.39	0.39	5.16	5.31	41.52	3.48	39.93	52.02	44.75	0.25	2.14	2.2
45	800	45.5	0.5	1.52	1.77	44.05	0.95	6.61	7.51	44.97	0.03	0.3	0.3

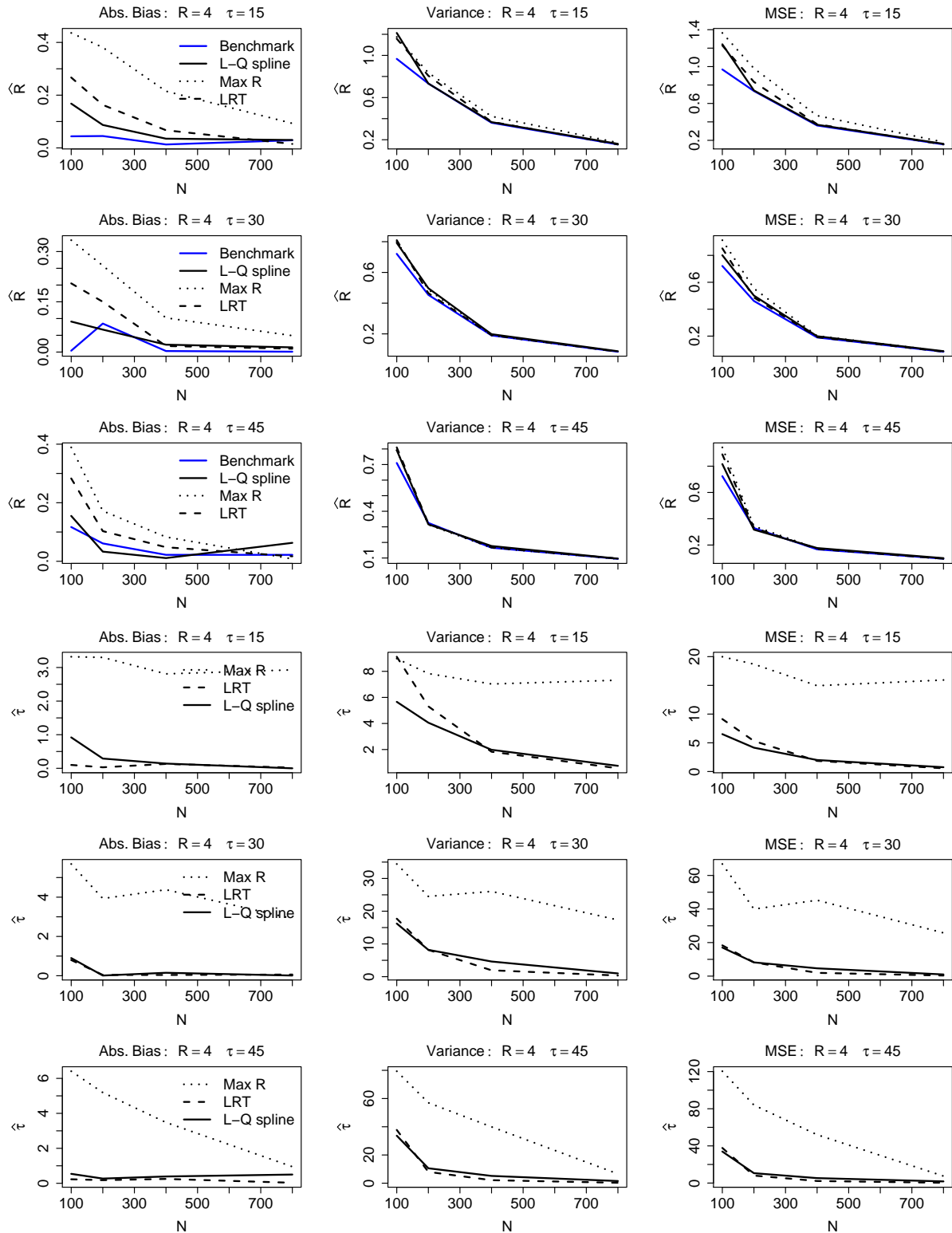


Figure 6: **Single Uniformly distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 13: **Multiple Uniformly distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.715	0.015	0.102	0.102	0.744	0.044	0.141	0.143	0.527	0.173	0.064	0.094	0.643	0.057	0.247	0.251
15	200	0.707	0.007	0.058	0.058	0.716	0.016	0.069	0.069	0.557	0.143	0.053	0.073	0.62	0.08	0.115	0.122
15	400	0.698	0.002	0.027	0.027	0.694	0.006	0.032	0.032	0.601	0.099	0.032	0.042	0.634	0.066	0.049	0.053
15	800	0.696	0.004	0.015	0.015	0.693	0.007	0.021	0.021	0.625	0.075	0.017	0.023	0.64	0.06	0.016	0.02
30	100	0.703	0.003	0.065	0.065	0.73	0.03	0.082	0.082	0.533	0.167	0.047	0.075	0.617	0.083	0.131	0.138
30	200	0.701	0.001	0.037	0.037	0.696	0.004	0.048	0.048	0.58	0.12	0.034	0.049	0.627	0.073	0.067	0.073
30	400	0.715	0.015	0.019	0.019	0.716	0.016	0.026	0.026	0.635	0.065	0.021	0.025	0.66	0.04	0.026	0.027
30	800	0.695	0.005	0.009	0.009	0.693	0.007	0.011	0.011	0.642	0.058	0.009	0.013	0.662	0.038	0.009	0.01
45	100	0.688	0.012	0.049	0.049	0.707	0.007	0.068	0.068	0.529	0.171	0.038	0.067	0.585	0.115	0.081	0.094
45	200	0.71	0.01	0.022	0.022	0.706	0.006	0.028	0.028	0.589	0.111	0.024	0.037	0.625	0.075	0.041	0.046
45	400	0.707	0.007	0.014	0.014	0.698	0.002	0.016	0.016	0.635	0.065	0.014	0.018	0.656	0.044	0.016	0.018
45	800	0.7	0	0.004	0.004	0.697	0.003	0.005	0.005	0.653	0.047	0.005	0.008	0.675	0.025	0.004	0.005

Table 14: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	15.18	0.18	9.32	9.36	13.52	1.48	21.12	23.32	15.42	0.42	23.94	24.12	15.42	0.42	23.94	24.12
15	200	14.89	0.11	8.7	8.71	13.04	1.96	18.44	22.28	14.79	0.21	20.09	20.14	14.79	0.21	20.09	20.14
15	400	14.97	0.03	7.7	7.7	12.84	2.16	17.02	21.69	14.62	0.38	17.08	17.23	14.62	0.38	17.08	17.23
15	800	15.17	0.17	9.06	9.09	12.06	2.94	11.37	20.02	14.46	0.54	13.82	14.11	14.46	0.54	13.82	14.11
30	100	30.09	0.09	41.33	41.34	26.73	3.27	91.88	102.58	30.89	0.89	97.85	98.65	30.89	0.89	97.85	98.65
30	200	30.74	0.74	34.73	35.29	25.14	4.86	74.96	98.57	29.51	0.49	78.51	78.75	29.51	0.49	78.51	78.75
30	400	29.94	0.06	33.59	33.6	25.29	4.71	70.27	92.44	31.1	1.1	67.07	68.28	31.1	1.1	67.07	68.28
30	800	30.38	0.38	26.55	26.69	23.63	6.37	49.74	90.28	30.49	0.49	48.75	48.99	30.49	0.49	48.75	48.99
45	100	43.92	1.08	106.15	107.31	37.5	7.5	213.43	269.71	45.71	0.71	226.9	227.41	45.71	0.71	226.9	227.41
45	200	44.43	0.57	103.18	103.51	36.48	8.52	185.49	258.04	42.77	2.23	200.45	205.4	42.77	2.23	200.45	205.4
45	400	43.43	1.57	60.3	62.75	37.67	7.33	154.03	207.71	44.84	0.16	135.34	135.36	44.84	0.16	135.34	135.36
45	800	45.16	0.16	60.63	60.66	34.56	10.44	128.84	237.88	46.84	1.84	87.21	90.59	46.84	1.84	87.21	90.59

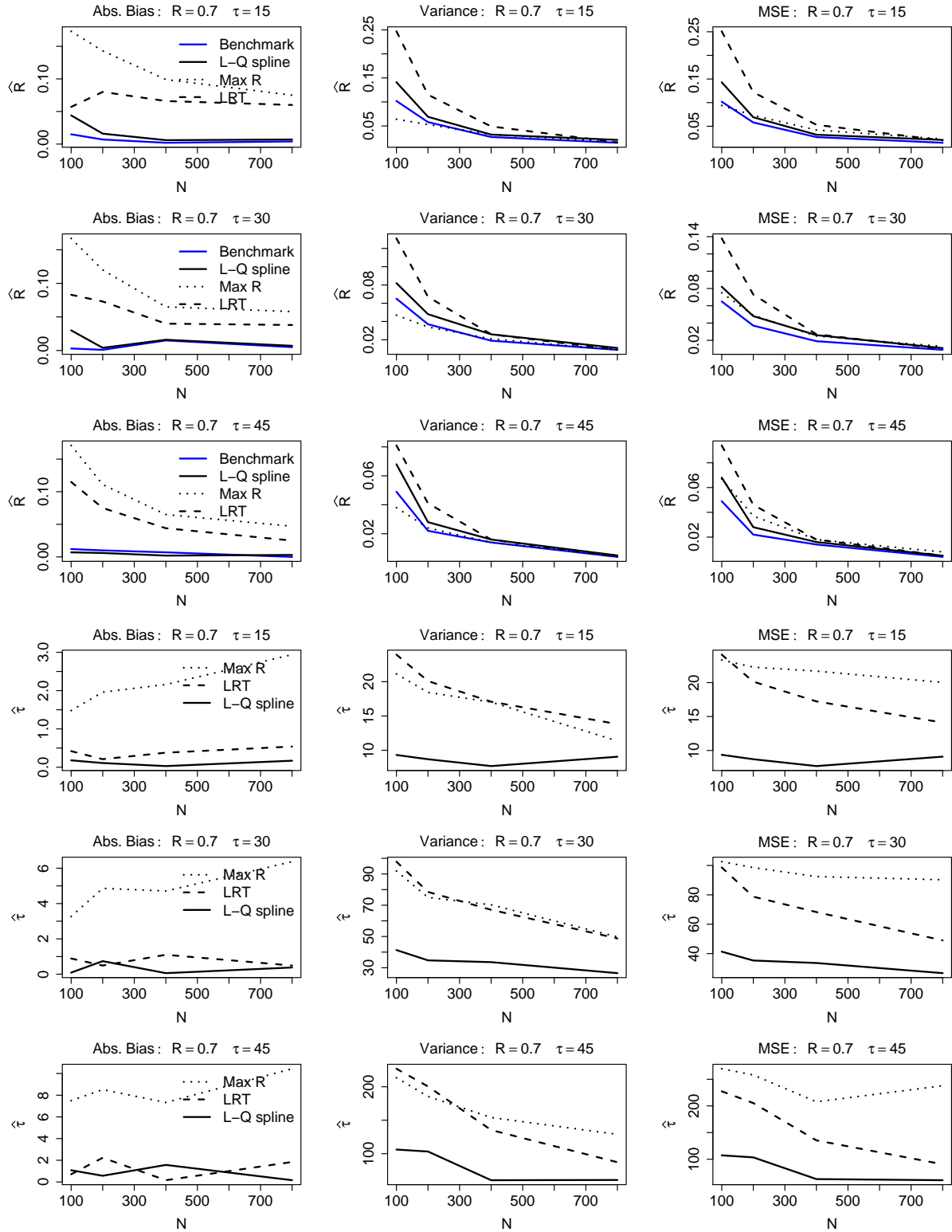


Figure 7: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 15: **Multiple Uniformly distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.94	0.04	0.177	0.178	0.996	0.096	0.206	0.216	0.674	0.226	0.128	0.179	0.909	0.009	0.45	0.45
15	200	0.905	0.005	0.089	0.089	0.917	0.017	0.11	0.111	0.702	0.198	0.072	0.111	0.851	0.049	0.206	0.208
15	400	0.908	0.008	0.041	0.041	0.913	0.013	0.053	0.053	0.772	0.128	0.038	0.055	0.874	0.026	0.099	0.099
15	800	0.915	0.015	0.018	0.018	0.923	0.023	0.023	0.024	0.815	0.085	0.02	0.027	0.881	0.019	0.049	0.05
30	100	0.889	0.011	0.088	0.088	0.91	0.01	0.119	0.119	0.679	0.221	0.071	0.12	0.856	0.044	0.236	0.238
30	200	0.916	0.016	0.045	0.046	0.929	0.029	0.065	0.065	0.748	0.152	0.042	0.065	0.877	0.023	0.127	0.128
30	400	0.902	0.002	0.023	0.023	0.898	0.002	0.031	0.031	0.788	0.112	0.022	0.034	0.869	0.031	0.061	0.062
30	800	0.901	0.001	0.012	0.012	0.899	0.001	0.015	0.015	0.825	0.075	0.012	0.017	0.867	0.033	0.028	0.029
45	100	0.897	0.003	0.067	0.067	0.919	0.019	0.099	0.1	0.695	0.205	0.058	0.1	0.871	0.029	0.206	0.207
45	200	0.892	0.008	0.034	0.034	0.887	0.013	0.045	0.045	0.736	0.164	0.028	0.055	0.833	0.067	0.088	0.093
45	400	0.899	0.001	0.012	0.012	0.899	0.001	0.018	0.018	0.796	0.104	0.013	0.024	0.855	0.045	0.039	0.041
45	800	0.898	0.002	0.009	0.009	0.896	0.004	0.013	0.013	0.827	0.073	0.01	0.016	0.865	0.035	0.023	0.025

Table 16: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.66	0.34	9.35	9.47	13.99	1.01	24.79	25.8	14.72	0.28	23.53	23.61
15	200	14.65	0.35	8.63	8.75	13.25	1.75	23.98	27.06	14.8	0.2	22.27	22.31
15	400	14.65	0.35	9.92	10.04	13.41	1.59	23.53	26.07	14.34	0.66	22.97	23.41
15	800	15.24	0.24	8.11	8.17	12.96	2.04	20.92	25.09	14.44	0.56	23	23.31
30	100	29.52	0.48	38.7	38.93	27.11	2.89	89.21	97.56	28.47	1.53	84.03	86.36
30	200	29.62	0.38	43.39	43.54	26.19	3.81	100.38	114.88	28.18	1.82	100.52	103.83
30	400	28.38	1.62	37.49	40.13	25.48	4.52	85.27	105.73	27.53	2.47	94.53	100.64
30	800	29.2	0.8	36.62	37.26	25.13	4.87	83.78	107.49	27.8	2.2	93.45	98.28
45	100	43.05	1.95	113.08	116.9	37.2	7.8	243.81	304.71	41.11	3.89	253.5	268.67
45	200	43.14	1.86	115.03	118.48	37.02	7.98	226.16	289.91	41.22	3.78	244.75	259.03
45	400	42.98	2.02	119.6	123.68	37.31	7.69	238.2	297.39	42.58	2.42	269.84	275.71
45	800	42.1	2.9	104.66	113.1	38.72	6.28	251.35	290.8	42.87	2.13	230.05	234.57

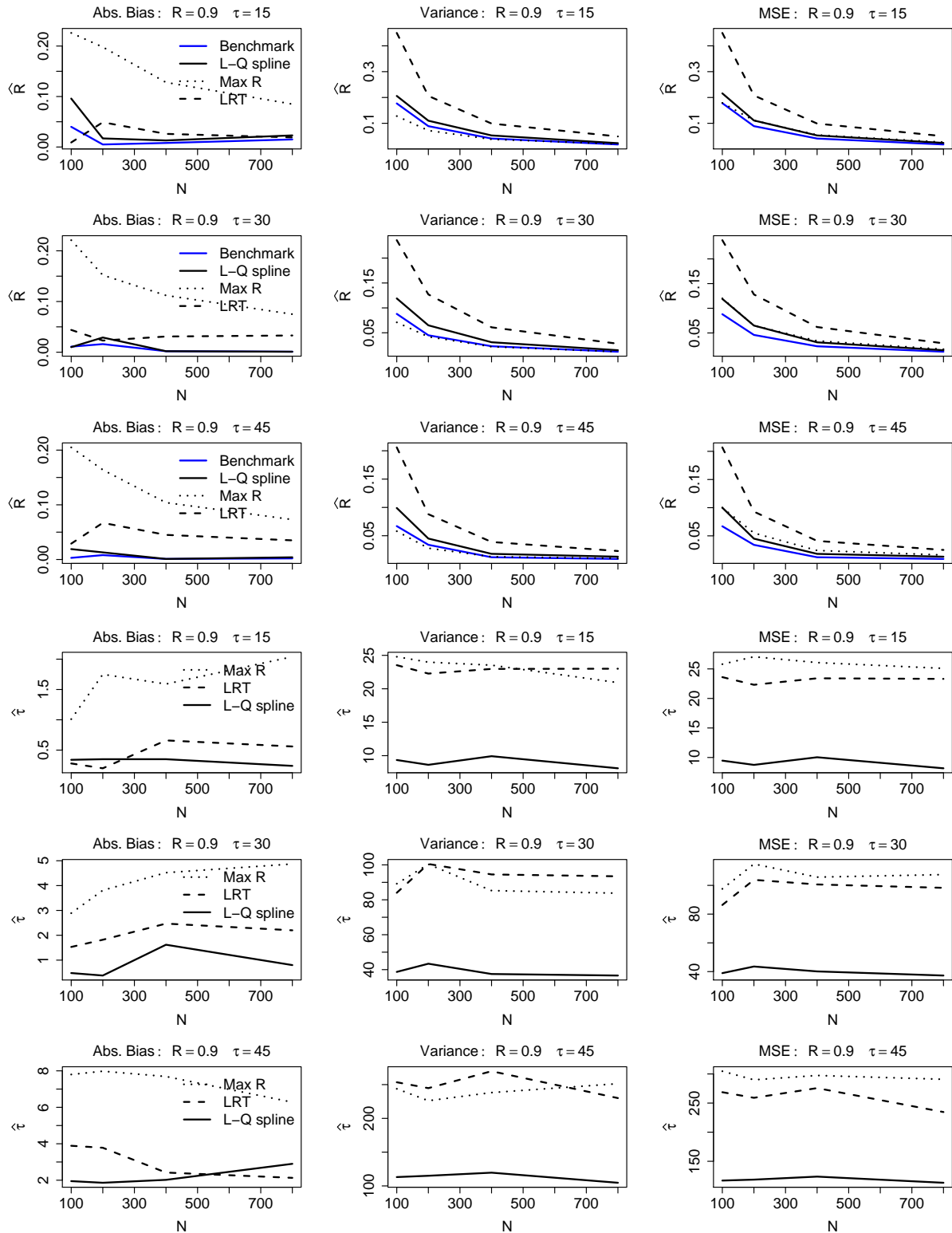


Figure 8: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 17: **Multiple Uniformly distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.164	0.036	0.226	0.228	1.209	0.009	0.304	0.304	1.466	0.266	0.318	0.388	1.237	0.037	0.559	0.56
15	200	1.185	0.015	0.116	0.117	1.218	0.018	0.154	0.154	1.393	0.193	0.144	0.181	1.261	0.061	0.257	0.261
15	400	1.19	0.01	0.052	0.052	1.208	0.008	0.064	0.064	1.334	0.134	0.057	0.075	1.246	0.046	0.113	0.115
15	800	1.195	0.005	0.031	0.031	1.211	0.011	0.041	0.041	1.297	0.097	0.039	0.048	1.252	0.052	0.06	0.062
30	100	1.207	0.007	0.137	0.138	1.249	0.049	0.187	0.189	1.488	0.288	0.198	0.281	1.348	0.148	0.336	0.358
30	200	1.194	0.006	0.06	0.061	1.219	0.019	0.082	0.083	1.368	0.168	0.074	0.102	1.275	0.075	0.135	0.141
30	400	1.217	0.017	0.034	0.034	1.222	0.022	0.04	0.041	1.328	0.128	0.043	0.059	1.275	0.075	0.068	0.073
30	800	1.187	0.013	0.013	0.013	1.196	0.004	0.017	0.017	1.274	0.074	0.015	0.02	1.256	0.056	0.02	0.023
45	100	1.261	0.061	0.103	0.107	1.29	0.09	0.146	0.154	1.522	0.322	0.145	0.248	1.388	0.188	0.275	0.311
45	200	1.2	0	0.057	0.057	1.21	0.01	0.078	0.078	1.351	0.151	0.064	0.087	1.258	0.058	0.133	0.136
45	400	1.191	0.009	0.025	0.025	1.213	0.013	0.033	0.034	1.315	0.115	0.032	0.045	1.277	0.077	0.053	0.059
45	800	1.195	0.005	0.013	0.013	1.21	0.01	0.016	0.016	1.271	0.071	0.015	0.02	1.256	0.056	0.017	0.021

Table 18: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	14.74	0.26	10.06	10.13	13.81	1.19	23.11	24.53	14.38	0.62	22.75	23.13	14.38	0.62	22.75	23.13
15	200	14.57	0.43	9.44	9.63	12.98	2.02	20.26	24.34	13.96	1.04	21.3	22.37	13.96	1.04	21.3	22.37
15	400	14.6	0.4	9.68	9.83	13.68	1.32	22.29	24.04	14.93	0.07	24.45	24.45	14.93	0.07	24.45	24.45
15	800	14.17	0.83	7.8	8.49	12.58	2.42	15.77	21.64	14.27	0.73	21.37	21.91	14.27	0.73	21.37	21.91
30	100	28.65	1.35	41.98	43.79	24.54	5.46	87.55	117.38	27.98	2.02	97.62	101.7	27.98	2.02	97.62	101.7
30	200	28.58	1.42	39.3	41.32	24.74	5.26	83.4	111.03	28.55	1.45	91.19	93.29	28.55	1.45	91.19	93.29
30	400	29.58	0.42	33.1	33.28	25.22	4.78	78.22	101.06	29.91	0.09	84.58	84.59	29.91	0.09	84.58	84.59
30	800	29.21	0.79	43.33	43.96	24.59	5.41	71.69	100.98	28.38	1.62	80.96	83.59	28.38	1.62	80.96	83.59
45	100	42.21	2.79	99.18	106.98	38.36	6.64	247.2	291.26	40.62	4.38	235.49	254.65	40.62	4.38	235.49	254.65
45	200	43.28	1.72	125.1	128.08	40.53	4.47	231.75	251.75	42.9	2.1	248.4	252.79	42.9	2.1	248.4	252.79
45	400	42.33	2.67	116.19	123.31	35.63	9.37	189.6	277.43	40.43	4.57	222.77	243.63	40.43	4.57	222.77	243.63
45	800	43.21	1.79	94.77	97.97	36.06	8.94	168.49	248.41	42.8	2.2	173.23	178.06	42.8	2.2	173.23	178.06

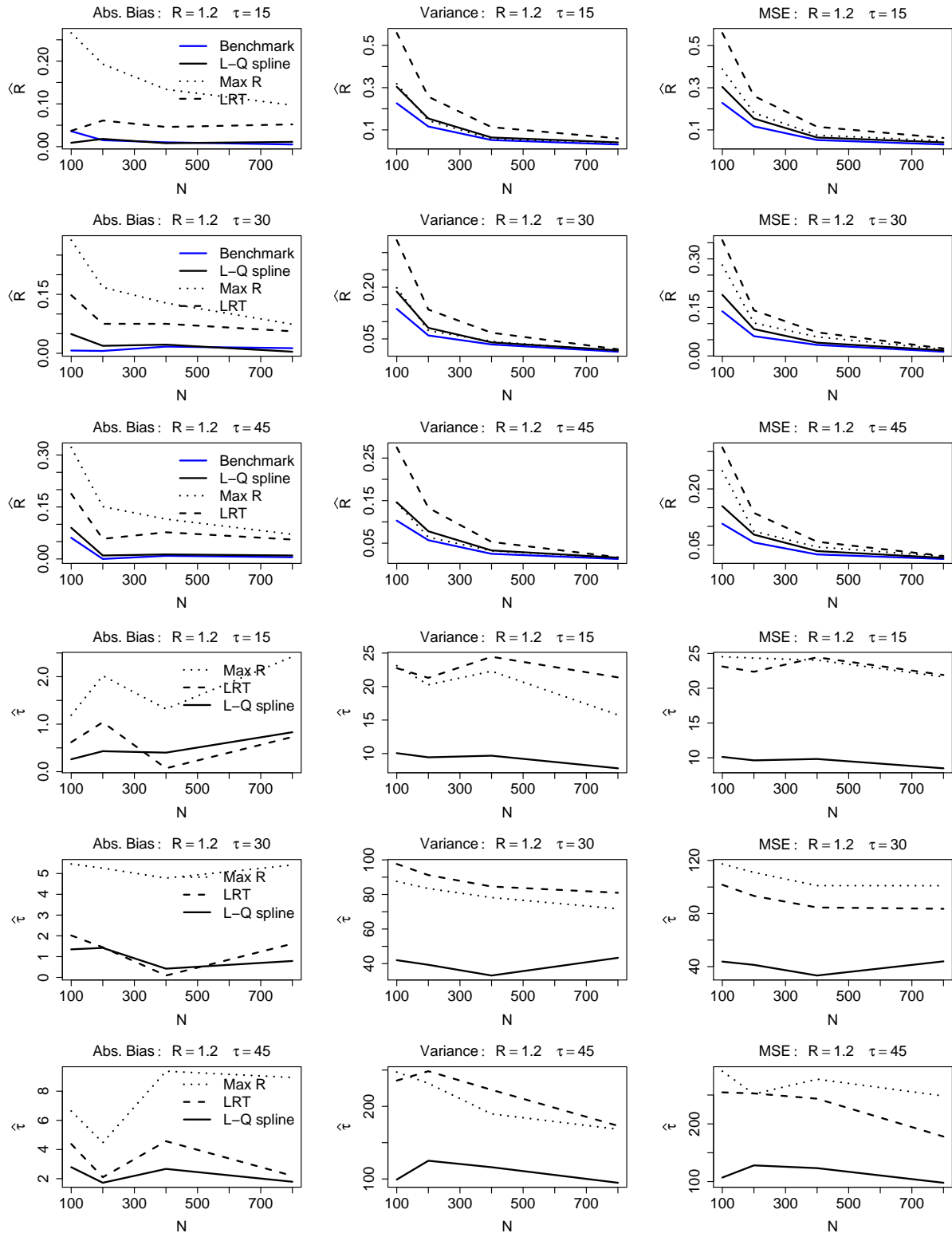


Figure 9: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 19: **Multiple Uniformly distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.481	0.019	0.249	0.249	1.568	0.068	0.337	0.341	1.83	0.33	0.369	0.478	1.667	0.167	0.58	0.607
15	200	1.503	0.003	0.154	0.154	1.55	0.05	0.184	0.186	1.741	0.241	0.197	0.255	1.671	0.171	0.252	0.281
15	400	1.508	0.008	0.078	0.078	1.541	0.041	0.093	0.095	1.668	0.168	0.104	0.132	1.628	0.128	0.116	0.133
15	800	1.508	0.008	0.036	0.036	1.53	0.03	0.041	0.042	1.607	0.107	0.041	0.053	1.567	0.067	0.038	0.042
30	100	1.506	0.006	0.172	0.172	1.563	0.063	0.214	0.218	1.768	0.268	0.198	0.27	1.676	0.176	0.293	0.324
30	200	1.547	0.047	0.086	0.089	1.605	0.105	0.113	0.124	1.743	0.243	0.116	0.175	1.693	0.193	0.128	0.166
30	400	1.513	0.013	0.046	0.046	1.547	0.047	0.061	0.063	1.635	0.135	0.056	0.075	1.607	0.107	0.052	0.063
30	800	1.502	0.002	0.017	0.017	1.513	0.013	0.019	0.019	1.571	0.071	0.02	0.025	1.547	0.047	0.02	0.022
45	100	1.547	0.047	0.157	0.159	1.568	0.068	0.19	0.194	1.803	0.303	0.18	0.272	1.744	0.244	0.234	0.294
45	200	1.499	0.001	0.057	0.057	1.537	0.037	0.073	0.074	1.655	0.155	0.077	0.101	1.615	0.115	0.096	0.11
45	400	1.502	0.002	0.032	0.032	1.519	0.019	0.042	0.042	1.601	0.101	0.037	0.047	1.581	0.081	0.034	0.041
45	800	1.505	0.005	0.015	0.015	1.506	0.006	0.016	0.016	1.557	0.057	0.016	0.019	1.532	0.032	0.015	0.016

Table 20: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	14.05	0.95	8.32	9.22	12.72	2.28	17.46	22.66	14.25	0.75	21.12	21.68	14.25	0.75	21.12	21.68
15	200	14.28	0.72	7.35	7.86	12.23	2.77	14.86	22.53	13.9	1.1	17.56	18.77	13.9	1.1	17.56	18.77
15	400	14.33	0.67	7.84	8.29	12.28	2.72	13.77	21.19	14.42	0.58	18.83	19.16	14.42	0.58	18.83	19.16
15	800	14.72	0.28	6.82	6.9	11.92	3.08	11.49	20.98	15.27	0.27	11.67	11.74	15.27	0.27	11.67	11.74
30	100	28.51	1.49	39.56	41.77	25.44	4.56	76.13	96.9	28.7	1.3	85.53	87.21	28.7	1.3	85.53	87.21
30	200	27.61	2.39	32.5	38.22	23.67	6.33	54.93	94.96	27.83	2.17	63.11	67.82	27.83	2.17	63.11	67.82
30	400	29.07	0.93	35.03	35.89	24.35	5.65	56.11	88.07	29.14	0.86	53.19	53.94	29.14	0.86	53.19	53.94
30	800	29.24	0.76	21.2	21.77	24.26	5.74	38.59	71.58	29.85	0.15	34.54	34.56	29.85	0.15	34.54	34.56
45	100	43.86	1.14	95.9	97.2	36.8	8.2	213.63	280.81	43.1	1.9	225.6	229.23	43.1	1.9	225.6	229.23
45	200	42.95	2.05	82.55	86.75	37.54	7.46	160.51	216.12	44.51	0.49	154.26	154.5	44.51	0.49	154.26	154.5
45	400	43.88	1.12	74.95	76.21	35.71	9.29	121	207.33	43.72	1.28	119.1	120.74	43.72	1.28	119.1	120.74
45	800	45.36	0.36	44.25	44.38	36.06	8.94	116.74	196.66	45.78	0.78	50.94	51.54	45.78	0.78	50.94	51.54

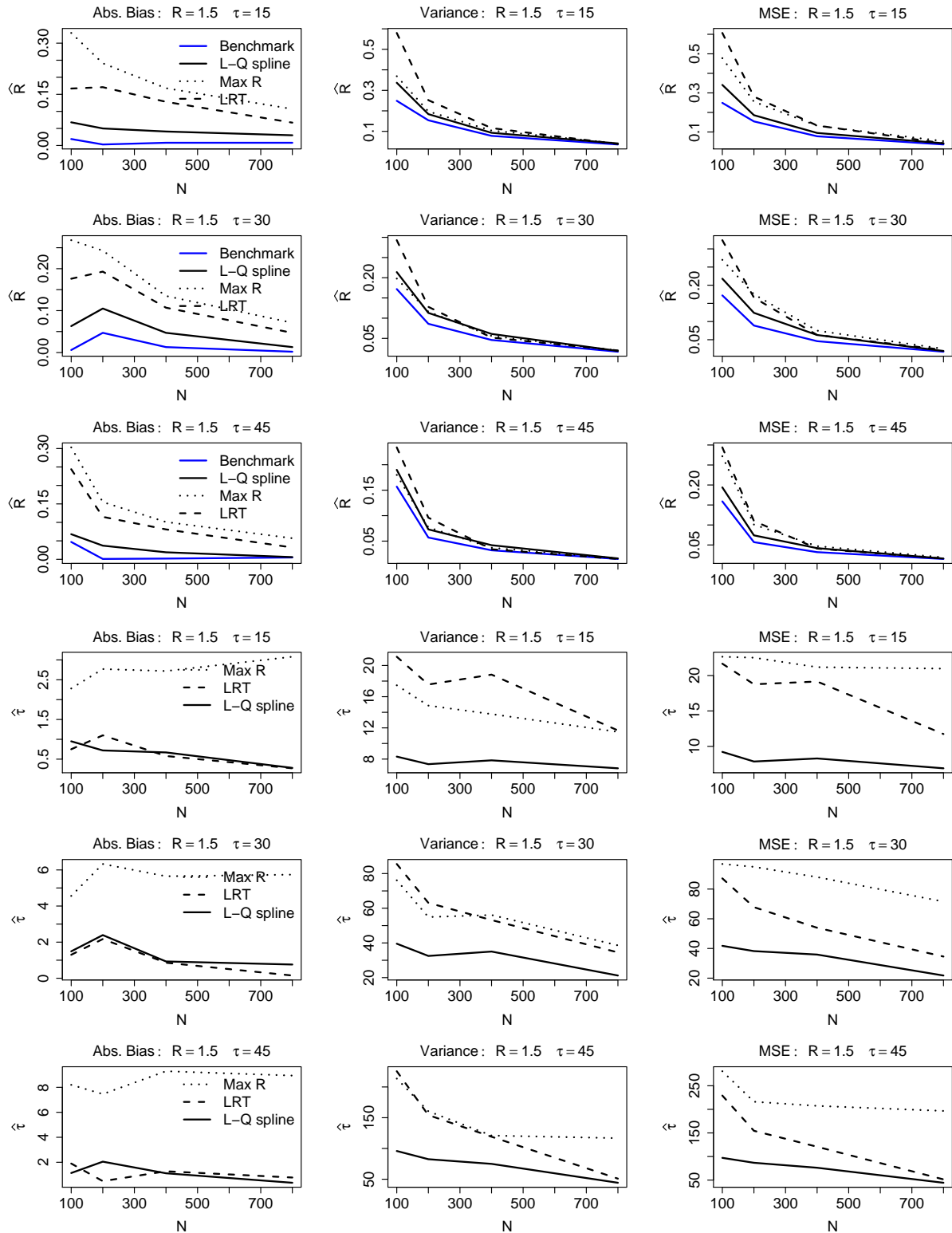


Figure 10: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 21: **Multiple Uniformly distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT			
		Est.	Bias	MSE	Est.	Bias	MSE	Est.	Bias	MSE	Est.	Bias	MSE	
15	100	2.106	0.106	0.48	2.194	0.194	0.538	2.458	0.458	0.573	2.368	0.368	0.607	0.743
15	200	2.016	0.016	0.222	2.092	0.092	0.264	2.259	0.259	0.279	2.191	0.191	0.277	0.313
15	400	2.032	0.032	0.123	2.065	0.065	0.138	2.203	0.203	0.149	2.134	0.134	0.133	0.151
15	800	1.996	0.004	0.053	2.012	0.012	0.058	2.083	0.083	0.066	2.023	0.023	0.058	0.058
30	100	2.102	0.102	0.259	2.193	0.193	0.317	2.402	0.402	0.307	2.339	0.339	0.315	0.43
30	200	2.019	0.019	0.143	2.063	0.063	0.173	2.198	0.198	0.167	2.15	0.15	0.162	0.184
30	400	2.01	0.01	0.07	2.037	0.037	0.079	2.118	0.118	0.072	2.065	0.065	0.072	0.076
30	800	2.019	0.019	0.027	2.02	0.02	0.027	2.073	0.073	0.031	2.048	0.048	0.028	0.03
45	100	2.058	0.058	0.234	2.12	0.12	0.275	2.293	0.293	0.266	2.247	0.247	0.249	0.31
45	200	2.018	0.018	0.094	2.051	0.051	0.101	2.173	0.173	0.101	2.137	0.137	0.103	0.122
45	400	2.005	0.005	0.055	2.016	0.016	0.059	2.087	0.087	0.056	2.061	0.061	0.057	0.06
45	800	1.973	0.027	0.028	1.971	0.029	0.028	2.014	0.014	0.03	1.991	0.009	0.028	0.028

Table 22: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT				
		Est.	Bias	MSE	Est.	Bias	MSE	Est.	Bias	MSE		
15	100	14.31	0.69	7.39	12.07	2.93	20.91	14.36	0.64	15.85	16.26	16.26
15	200	14.26	0.74	7.69	12.07	2.93	20.37	14.48	0.52	13.58	13.85	13.85
15	400	14.7	0.3	6.51	11.59	3.41	21.41	14.59	0.41	8.97	9.14	9.14
15	800	14.87	0.13	3.39	11.95	3.05	17.24	15.21	0.21	4.82	4.86	4.86
30	100	28.74	1.26	33.46	23.46	6.54	88.08	28.68	1.32	55.01	56.76	56.76
30	200	28.98	1.02	27.28	24.87	5.13	68.76	29.88	0.12	44.99	45	45
30	400	29.06	0.94	22.21	24.1	5.9	69.22	30.09	0.09	25.68	25.69	25.69
30	800	29.97	0.03	8.36	25.66	4.34	44.29	29.49	0.51	8.87	9.13	9.13
45	100	43.63	1.37	87.22	39.24	5.76	167.33	44.72	0.28	94.57	94.65	94.65
45	200	44.63	0.37	62.19	37.12	7.88	173.77	43.87	1.13	92.71	93.98	93.98
45	400	44.09	0.91	28.32	38.07	6.93	136.38	44.13	0.87	43.56	44.32	44.32
45	800	45.29	0.29	14.14	39.36	5.64	97.17	44.91	0.09	14.85	14.85	14.85

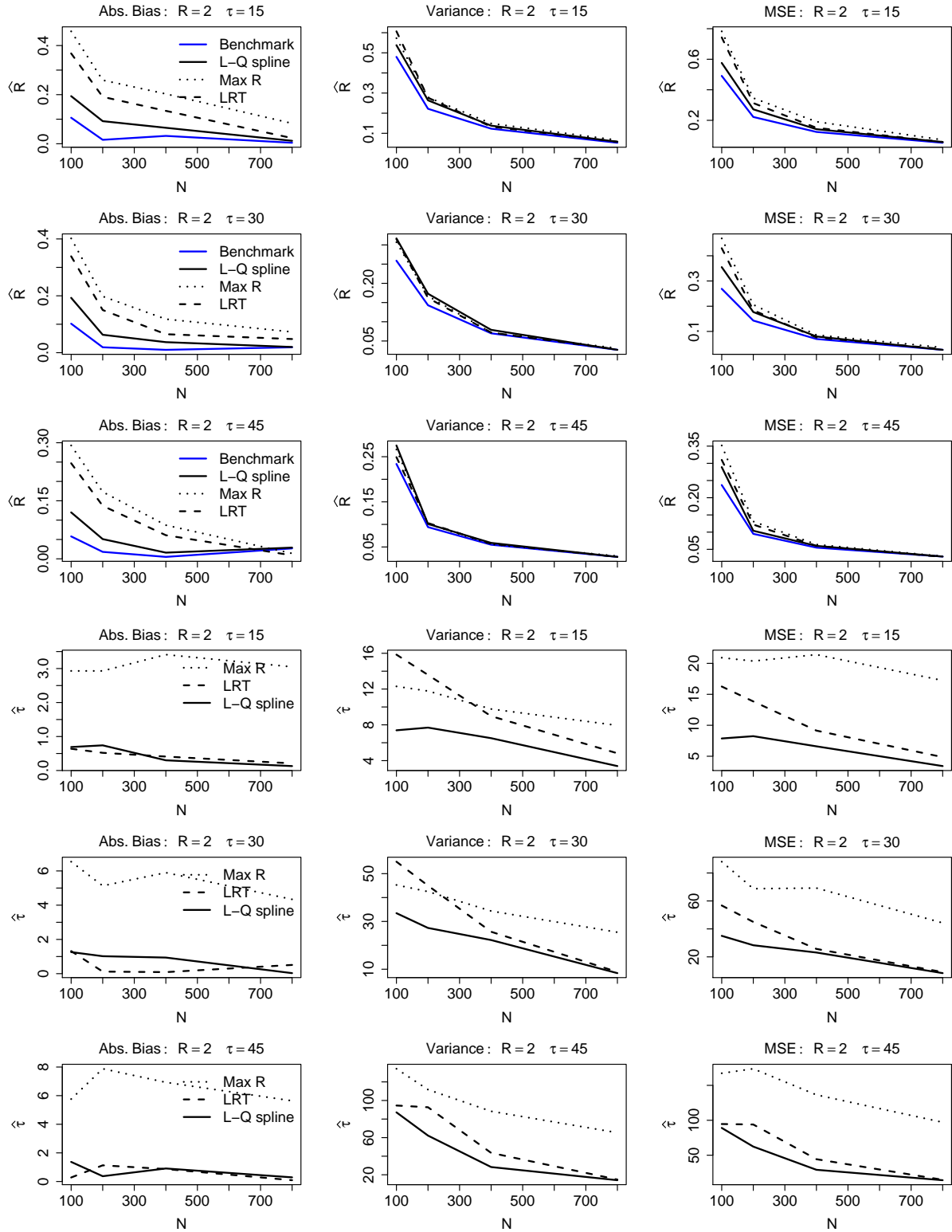


Figure 11: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 23: **Multiple Uniformly distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	4.081	0.081	0.971	0.978	4.233	0.233	0.97	1.024	4.473	0.473	1.142	1.366	4.326	0.326	1.077	1.183
15	200	3.991	0.009	0.516	0.516	4.026	0.026	0.501	0.502	4.194	0.194	0.526	0.564	4.056	0.056	0.48	0.483
15	400	4.046	0.046	0.262	0.264	4.052	0.052	0.271	0.274	4.187	0.187	0.282	0.317	4.066	0.066	0.255	0.26
15	800	3.968	0.032	0.13	0.131	3.956	0.044	0.133	0.134	4.034	0.034	0.148	0.149	3.974	0.026	0.135	0.135
30	100	3.965	0.035	0.745	0.747	4.002	0.002	0.786	0.786	4.257	0.257	0.898	0.965	4.166	0.166	0.869	0.896
30	200	4.071	0.071	0.368	0.373	4.069	0.069	0.353	0.358	4.172	0.172	0.365	0.395	4.131	0.131	0.365	0.383
30	400	3.972	0.028	0.221	0.222	3.959	0.041	0.217	0.219	4.026	0.026	0.214	0.215	3.992	0.008	0.22	0.22
30	800	3.99	0.01	0.095	0.095	3.971	0.029	0.095	0.095	4.004	0.004	0.096	0.096	3.994	0.006	0.094	0.094
45	100	4.116	0.116	1.036	1.049	4.121	0.121	1.067	1.082	4.296	0.296	1.041	1.129	4.251	0.251	1.027	1.09
45	200	3.995	0.005	0.407	0.407	3.98	0.02	0.428	0.429	4.094	0.094	0.403	0.411	4.062	0.062	0.426	0.43
45	400	4.01	0.01	0.17	0.17	3.973	0.027	0.174	0.175	4.035	0.035	0.174	0.175	4.025	0.025	0.174	0.175
45	800	4.01	0.01	0.077	0.077	3.977	0.023	0.073	0.074	4.021	0.021	0.076	0.076	4.016	0.016	0.076	0.076

Table 24: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.1	0.9	5.04	5.86	11.96	3.04	7.31	16.56	14.62	0.38	6.57	6.71
15	200	14.88	0.12	2.75	2.76	12.54	2.46	6.33	12.37	15.02	0.02	1.85	1.85
15	400	14.94	0.06	1.29	1.3	12.61	2.39	6.36	12.08	15.02	0.02	0.88	0.88
15	800	15.07	0.07	0.47	0.48	13.35	1.65	5.01	7.73	15	0	0.2	0.2
30	100	29.51	0.49	18.21	18.45	25.61	4.39	31.56	50.81	29.27	0.73	21.82	22.35
30	200	29.97	0.03	5.92	5.92	27.5	2.5	15.89	22.13	29.77	0.23	3.98	4.04
30	400	30.02	0.02	1.81	1.81	27.46	2.54	17.86	24.3	29.96	0.04	1.61	1.61
30	800	30.05	0.05	0.91	0.91	29.29	0.71	2.98	3.49	30.01	0.01	0.26	0.26
45	100	44.79	0.21	34.6	34.64	41.25	3.75	54.79	68.88	45.14	0.14	32.23	32.25
45	200	45.16	0.16	13.6	13.63	41.4	3.6	42.58	55.53	44.68	0.32	10.82	10.92
45	400	45.65	0.65	4.01	4.43	44.01	0.99	7.42	8.41	45.16	0.16	2.4	2.42
45	800	45.42	0.42	1.44	1.61	44.39	0.61	3.53	3.9	44.96	0.04	0.44	0.44

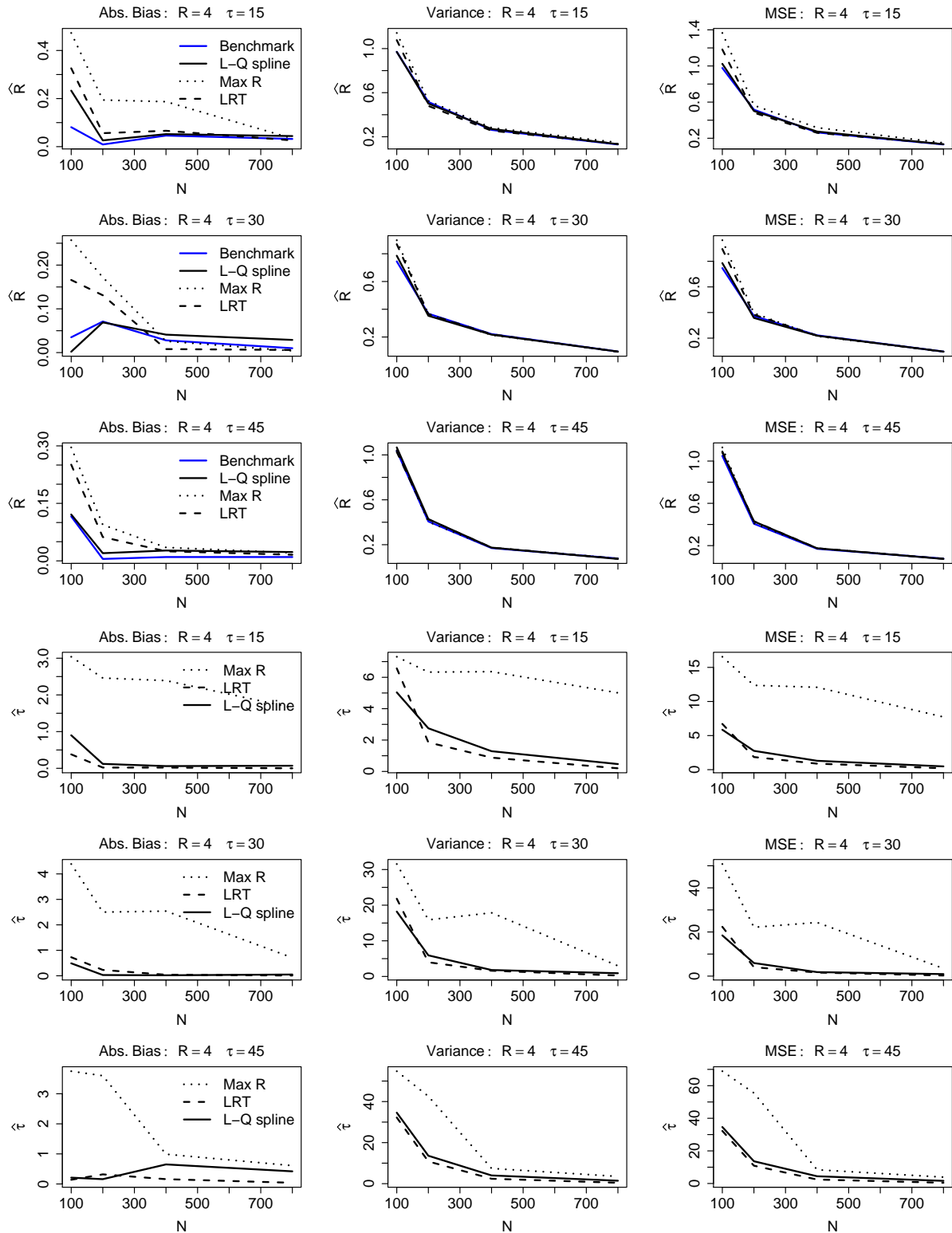


Figure 12: **Multiple Uniformly distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 25: **Single Normally distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.758	0.058	0.155	0.158	0.775	0.075	0.209	0.215	0.541	0.159	0.085	0.11	0.807	0.107	0.147	0.159
15	200	0.71	0.01	0.098	0.098	0.737	0.037	0.128	0.13	0.532	0.168	0.081	0.109	0.634	0.066	0.209	0.214
15	400	0.685	0.015	0.044	0.044	0.69	0.01	0.055	0.055	0.564	0.136	0.045	0.063	0.613	0.087	0.073	0.081
15	800	0.712	0.012	0.023	0.023	0.702	0.002	0.027	0.027	0.622	0.078	0.024	0.03	0.65	0.05	0.034	0.037
30	100	0.712	0.012	0.075	0.075	0.75	0.05	0.107	0.11	0.49	0.21	0.06	0.105	0.595	0.105	0.201	0.212
30	200	0.715	0.015	0.043	0.043	0.722	0.022	0.059	0.059	0.564	0.136	0.041	0.059	0.617	0.083	0.08	0.087
30	400	0.694	0.006	0.022	0.022	0.689	0.011	0.026	0.026	0.59	0.11	0.022	0.034	0.62	0.08	0.029	0.035
30	800	0.709	0.009	0.013	0.013	0.709	0.009	0.015	0.015	0.629	0.071	0.014	0.019	0.652	0.048	0.016	0.018
45	100	0.701	0.001	0.055	0.055	0.714	0.014	0.081	0.081	0.521	0.179	0.052	0.084	0.631	0.069	0.158	0.163
45	200	0.714	0.014	0.031	0.031	0.714	0.014	0.041	0.041	0.59	0.11	0.031	0.043	0.653	0.047	0.06	0.063
45	400	0.693	0.007	0.013	0.013	0.689	0.011	0.017	0.018	0.605	0.095	0.013	0.022	0.627	0.073	0.015	0.02
45	800	0.703	0.003	0.008	0.008	0.698	0.002	0.009	0.009	0.644	0.056	0.008	0.011	0.668	0.032	0.007	0.008

Table 26: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.32	0.32	11.16	11.26	14.52	0.48	23.56	23.79	15.87	0.87	22.88	23.64
15	200	14.68	0.32	9.95	10.05	13.78	1.22	20.95	22.44	14.93	0.07	22.66	22.67
15	400	14.66	0.34	7.63	7.75	13.4	1.6	20.65	23.22	15.35	0.35	20.66	20.78
15	800	14.83	0.17	8.12	8.15	12.57	2.43	14.3	20.21	14.89	0.11	18.17	18.19
30	100	29.57	0.43	41.42	41.6	25.51	4.49	83.34	103.52	29.18	0.82	102.26	102.93
30	200	28.82	1.18	39.54	40.92	25.69	4.31	79.3	97.89	29.21	0.79	91.88	92.51
30	400	30.13	0.13	39.95	39.96	24.38	5.62	58.71	90.27	29.95	0.05	77.54	77.54
30	800	30.09	0.09	29.7	29.71	22.66	7.34	50.79	104.69	28.34	1.66	65.56	68.33
45	100	43.03	1.97	116.26	120.14	37.29	7.71	206.66	266.08	44.56	0.44	249.32	249.51
45	200	42.91	2.09	88.5	92.85	36.37	8.63	190.64	265.08	44.73	0.27	193.48	193.56
45	400	44.99	0.01	96.56	96.56	34.67	10.33	166.2	272.94	44.73	0.27	174.13	174.2
45	800	45.4	0.4	71.56	71.71	34.69	10.31	127.41	233.74	45.78	0.78	119.27	119.89

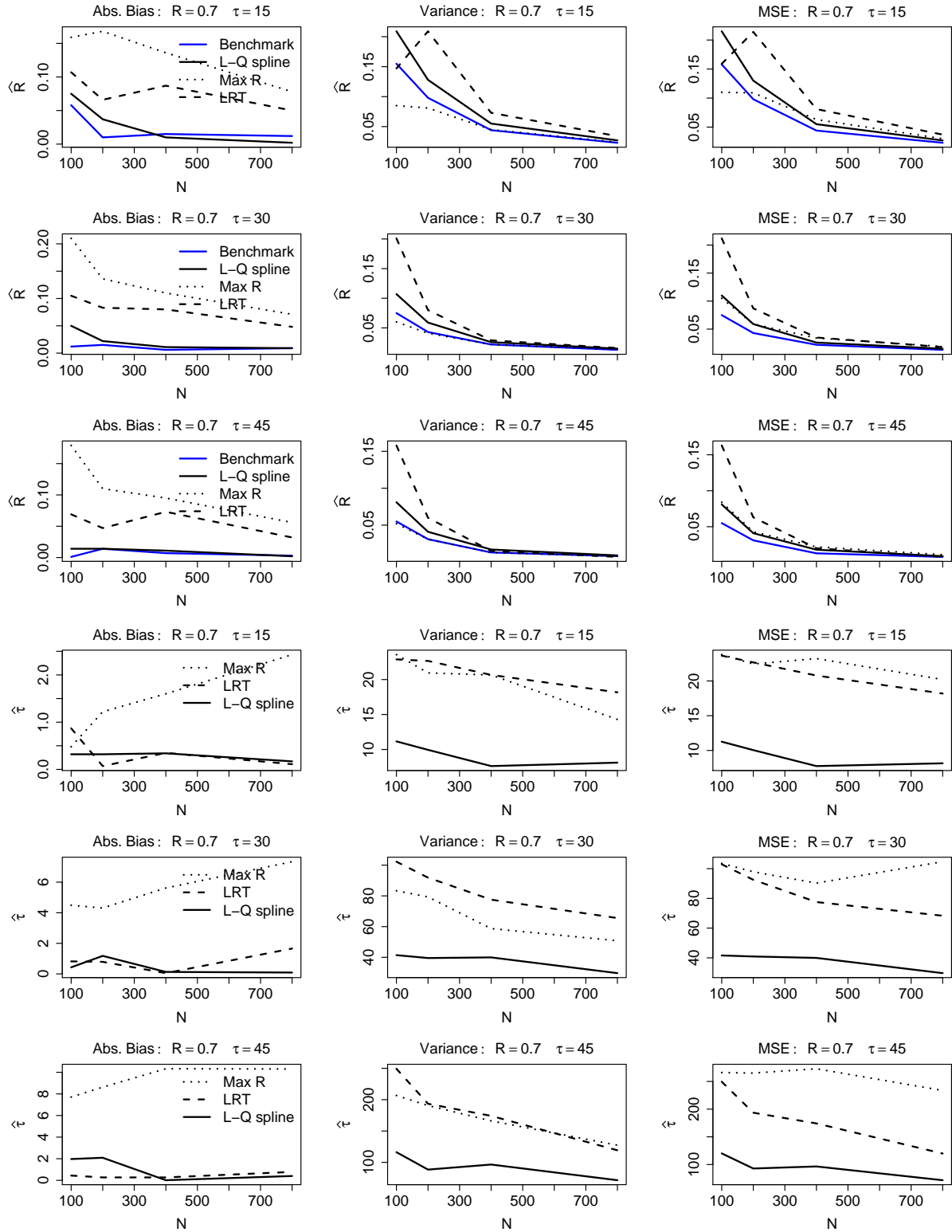


Figure 13: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 27: **Single Normally distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	0.928	0.028	0.221	0.222	1.009	0.109	0.31	0.322	0.684	0.216	0.154	0.201	0.976	0.076	0.568	0.574
15	200	0.868	0.032	0.102	0.103	0.898	0.002	0.126	0.126	0.671	0.229	0.083	0.135	0.855	0.045	0.272	0.274
15	400	0.904	0.004	0.057	0.057	0.921	0.021	0.071	0.071	0.757	0.143	0.045	0.065	0.874	0.026	0.125	0.126
15	800	0.901	0.001	0.03	0.03	0.904	0.004	0.041	0.041	0.79	0.11	0.028	0.041	0.878	0.022	0.075	0.075
30	100	0.894	0.006	0.113	0.113	0.918	0.018	0.168	0.169	0.641	0.259	0.095	0.162	0.849	0.051	0.323	0.326
30	200	0.9	0	0.045	0.045	0.91	0.01	0.065	0.065	0.721	0.179	0.042	0.075	0.868	0.032	0.143	0.144
30	400	0.901	0.001	0.03	0.03	0.899	0.001	0.042	0.042	0.763	0.137	0.029	0.048	0.855	0.045	0.075	0.077
30	800	0.9	0	0.014	0.014	0.902	0.002	0.019	0.019	0.812	0.088	0.016	0.024	0.872	0.028	0.037	0.038
45	100	0.904	0.004	0.089	0.089	0.935	0.035	0.135	0.136	0.68	0.22	0.066	0.114	0.9	0	0.256	0.256
45	200	0.913	0.013	0.04	0.041	0.925	0.025	0.058	0.059	0.749	0.151	0.041	0.063	0.892	0.008	0.135	0.135
45	400	0.899	0.001	0.022	0.022	0.897	0.003	0.029	0.029	0.783	0.117	0.022	0.036	0.87	0.03	0.061	0.062
45	800	0.908	0.008	0.01	0.01	0.908	0.008	0.014	0.014	0.829	0.071	0.012	0.017	0.879	0.021	0.03	0.03

Table 28: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.89	0.11	10.83	10.84	14.38	0.62	23.04	23.43	15.3	0.3	22.45	22.54
15	200	14.43	0.57	10.28	10.61	14.74	0.26	25.47	25.54	14.97	0.03	25.89	25.89
15	400	14.53	0.47	9.22	9.44	13.42	1.58	21.17	23.66	14.71	0.29	22.55	22.63
15	800	14.3	0.7	8.02	8.51	13.99	1.01	23.38	24.39	14.71	0.29	23.77	23.85
30	100	28.65	1.35	41.77	43.59	26.71	3.29	95.44	106.28	29.32	0.68	98.54	99.01
30	200	28.83	1.17	38.96	40.33	26.97	3.03	100.47	109.65	27.79	2.21	95.35	100.22
30	400	29.34	0.66	44.35	44.78	24.83	5.17	92.71	119.4	29.3	0.7	97.35	97.85
30	800	29.13	0.87	38.84	39.61	24.54	5.46	88.79	118.57	30.42	0.42	100.12	100.3
45	100	43.44	1.56	116.69	119.13	37.88	7.12	241.88	292.59	41.71	3.29	255.43	266.27
45	200	43.07	1.93	122.57	126.32	37.44	7.56	276.72	333.91	40.8	4.2	260.12	277.72
45	400	43.65	1.35	113.14	114.95	37.11	7.89	242.76	305.08	42.2	2.8	237.86	245.72
45	800	43.33	1.67	115.52	118.32	37.43	7.57	238.01	295.36	41.77	3.23	239.19	249.63

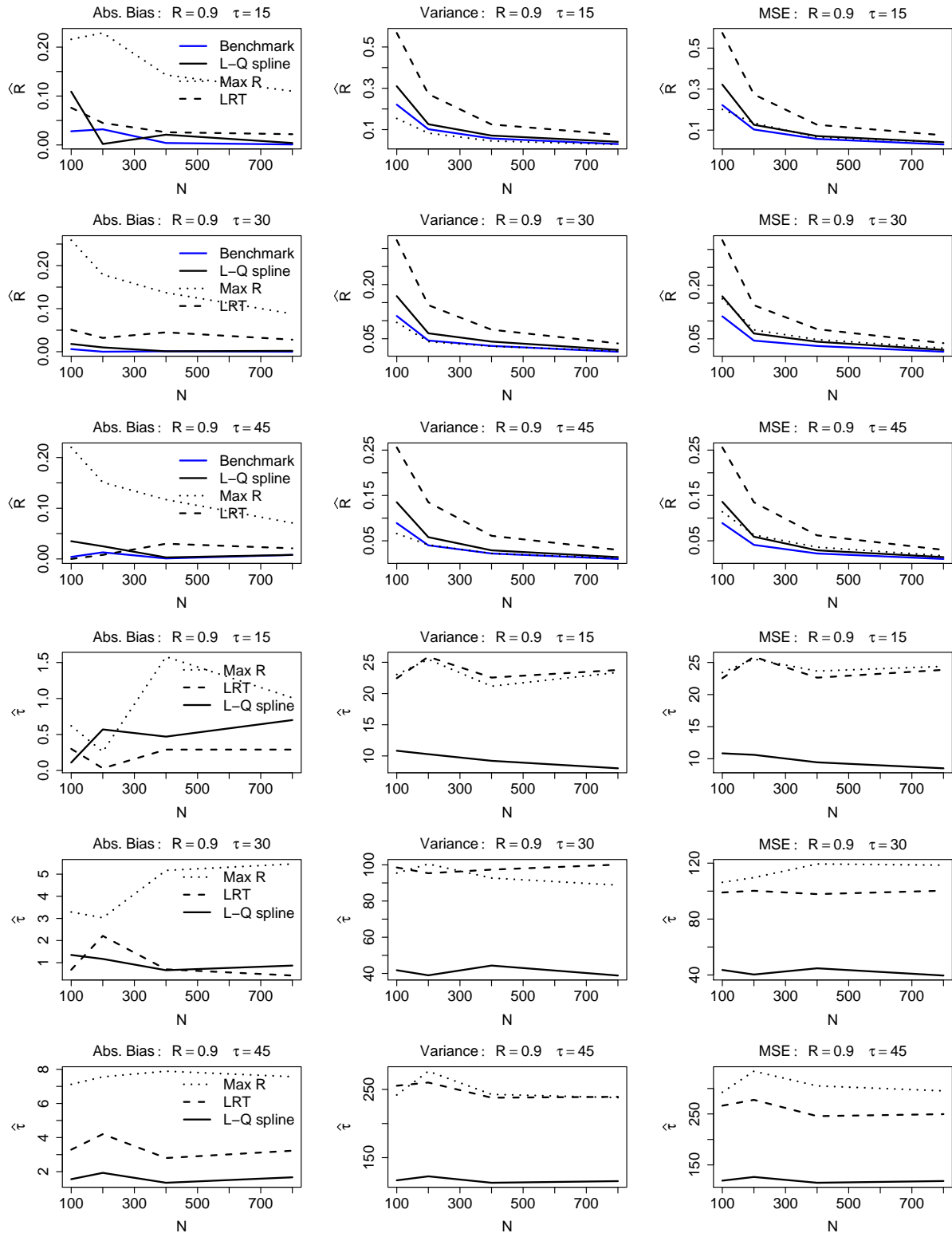


Figure 14: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 29: **Single Normally distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.218	0.018	0.312	0.313	1.295	0.095	0.4	0.409	1.626	0.426	0.5	0.682	1.373	0.173	0.764	0.794
15	200	1.204	0.004	0.172	0.172	1.252	0.052	0.217	0.22	1.471	0.271	0.215	0.288	1.32	0.12	0.364	0.378
15	400	1.194	0.006	0.072	0.072	1.227	0.027	0.092	0.093	1.367	0.167	0.09	0.118	1.269	0.069	0.158	0.163
15	800	1.195	0.005	0.04	0.04	1.211	0.011	0.049	0.05	1.317	0.117	0.048	0.062	1.255	0.055	0.077	0.08
30	100	1.238	0.038	0.159	0.161	1.288	0.088	0.205	0.213	1.543	0.343	0.227	0.344	1.366	0.166	0.418	0.446
30	200	1.221	0.021	0.089	0.089	1.247	0.047	0.122	0.124	1.437	0.237	0.108	0.164	1.323	0.123	0.196	0.211
30	400	1.202	0.002	0.037	0.037	1.225	0.025	0.046	0.047	1.351	0.151	0.05	0.072	1.283	0.083	0.087	0.094
30	800	1.208	0.008	0.023	0.023	1.221	0.021	0.027	0.028	1.305	0.105	0.028	0.039	1.274	0.074	0.041	0.046
45	100	1.197	0.003	0.11	0.11	1.256	0.056	0.146	0.149	1.511	0.311	0.164	0.261	1.323	0.123	0.334	0.349
45	200	1.207	0.007	0.06	0.06	1.241	0.041	0.101	0.102	1.421	0.221	0.087	0.136	1.332	0.132	0.161	0.179
45	400	1.218	0.018	0.03	0.03	1.229	0.029	0.041	0.042	1.356	0.156	0.037	0.061	1.313	0.113	0.058	0.071
45	800	1.201	0.001	0.015	0.015	1.213	0.013	0.017	0.018	1.285	0.085	0.019	0.027	1.257	0.057	0.022	0.025

Table 30: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.71	0.29	8.87	8.95	12.13	2.87	17.57	25.8	14.66	0.34	21.59	21.71
15	200	14.44	0.56	9.02	9.34	12.97	2.03	21.97	26.08	14.21	0.79	21.85	22.48
15	400	14.59	0.41	9.48	9.65	12.36	2.64	17.39	24.37	13.67	1.33	20.53	22.31
15	800	14.27	0.73	9.48	10.02	12.54	2.46	17.16	23.23	14.34	0.66	20.19	20.63
30	100	28.93	1.07	41.66	42.79	23.84	6.16	71.52	109.41	26.88	3.12	86.39	96.13
30	200	28.52	1.48	41.15	43.34	25.06	4.94	88.02	112.47	28.57	1.43	93.31	95.35
30	400	29.39	0.61	44.52	44.89	24.35	5.65	76.15	108.05	28.52	1.48	91.05	93.24
30	800	28.71	1.29	35.78	37.45	23.79	6.21	58.87	97.45	27.89	2.11	75.26	79.72
45	100	42.26	2.74	98.84	106.37	36.43	8.57	255.29	328.78	42.2	2.8	298.16	305.99
45	200	42.37	2.63	121.78	128.69	36.65	8.35	221.16	290.83	41.02	3.98	239.1	254.93
45	400	42.49	2.51	111.83	118.14	36.67	8.33	222.14	291.47	42.44	2.56	217.6	224.17
45	800	42.54	2.46	93.92	99.98	34.26	10.74	146.74	262.17	43.7	1.3	175.81	177.5

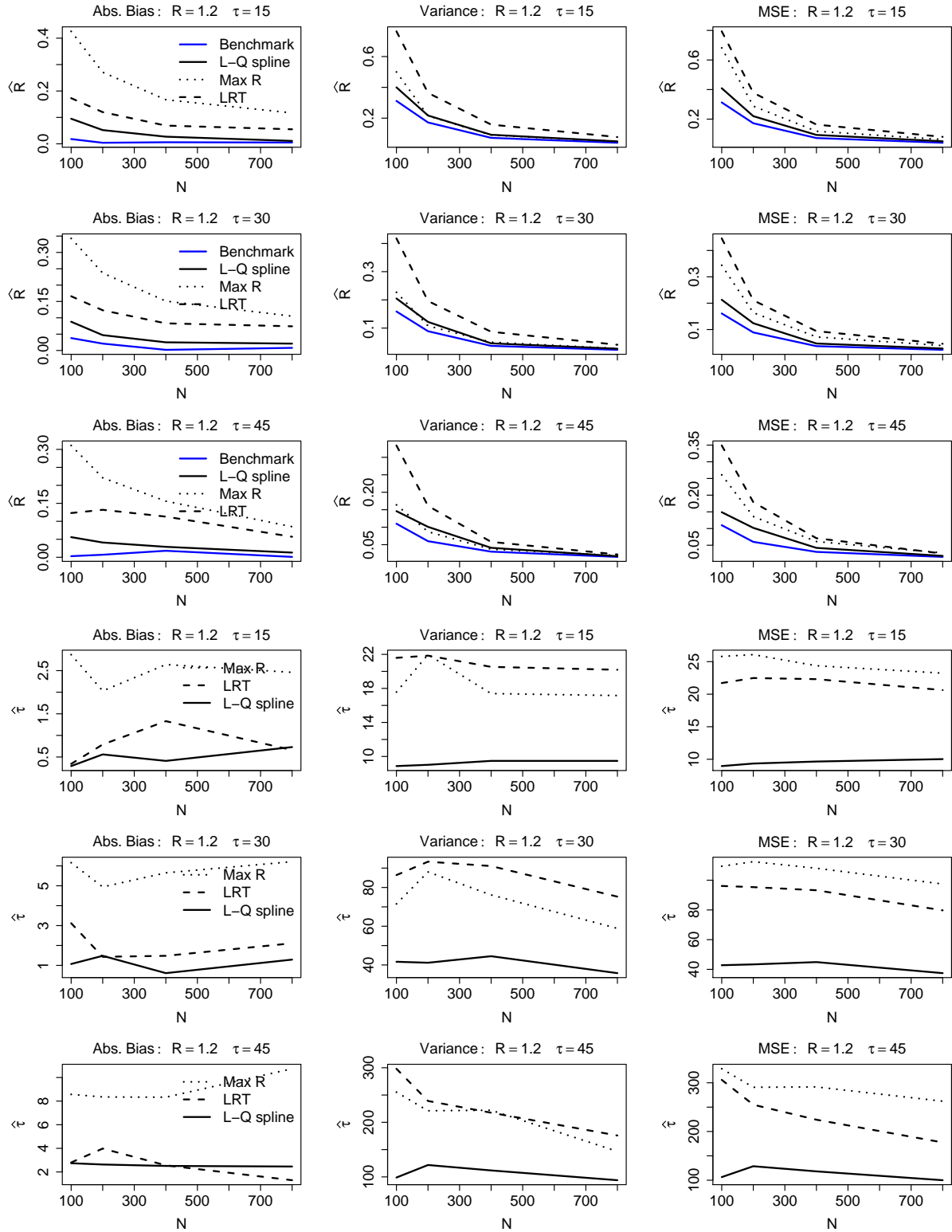


Figure 15: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 31: **Single Normally distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.567	0.067	0.424	0.428	1.678	0.178	0.515	0.546	1.95	0.45	0.556	0.758	1.776	0.276	0.721	0.797
15	200	1.511	0.011	0.188	0.189	1.572	0.072	0.242	0.247	1.787	0.287	0.255	0.337	1.704	0.204	0.315	0.357
15	400	1.485	0.015	0.103	0.103	1.511	0.011	0.123	0.123	1.666	0.166	0.137	0.165	1.601	0.101	0.183	0.193
15	800	1.5	0	0.045	0.045	1.532	0.032	0.056	0.057	1.641	0.141	0.06	0.08	1.6	0.1	0.055	0.065
30	100	1.526	0.026	0.252	0.253	1.62	0.12	0.31	0.324	1.836	0.336	0.324	0.437	1.714	0.214	0.447	0.492
30	200	1.517	0.017	0.099	0.1	1.559	0.059	0.14	0.143	1.741	0.241	0.136	0.194	1.691	0.191	0.159	0.195
30	400	1.481	0.019	0.045	0.045	1.506	0.006	0.052	0.053	1.607	0.107	0.053	0.064	1.564	0.064	0.051	0.055
30	800	1.495	0.005	0.022	0.022	1.515	0.015	0.026	0.027	1.577	0.077	0.026	0.032	1.54	0.04	0.023	0.025
45	100	1.524	0.024	0.144	0.145	1.578	0.078	0.197	0.203	1.808	0.308	0.195	0.29	1.711	0.211	0.307	0.351
45	200	1.492	0.008	0.074	0.074	1.539	0.039	0.086	0.087	1.673	0.173	0.088	0.118	1.633	0.133	0.096	0.114
45	400	1.499	0.001	0.035	0.035	1.53	0.03	0.047	0.048	1.631	0.131	0.049	0.066	1.59	0.09	0.044	0.052
45	800	1.5	0	0.018	0.018	1.51	0.01	0.022	0.022	1.58	0.08	0.024	0.03	1.545	0.045	0.022	0.024

Table 32: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	14.35	0.65	8.35	8.77	12.32	2.68	15.4	22.58	14.56	0.44	20.52	20.71	14.56	0.44	20.52	20.71
15	200	14.29	0.71	8.05	8.55	12.27	2.73	15.59	23.04	14.19	0.81	19.93	20.59	14.19	0.81	19.93	20.59
15	400	14.1	0.9	8.66	9.48	12.4	2.6	14.75	21.53	14.1	0.9	16.32	17.14	14.1	0.9	16.32	17.14
15	800	14.39	0.61	7.04	7.42	11.36	3.64	9.6	22.83	14.2	0.8	14.62	15.27	14.2	0.8	14.62	15.27
30	100	28.34	1.66	48.37	51.12	24.75	5.25	75.76	103.33	28.58	1.42	85.19	87.21	28.58	1.42	85.19	87.21
30	200	29.02	0.98	42.37	43.34	23.85	6.15	61.69	99.46	28.39	1.61	76.86	79.46	28.39	1.61	76.86	79.46
30	400	28.41	1.59	30.81	33.35	24.34	5.66	57.46	89.53	29.81	0.19	58.86	58.9	29.81	0.19	58.86	58.9
30	800	29.05	0.95	27.7	28.61	23.65	6.35	40.49	80.77	30.8	0.8	39.66	40.3	30.8	0.8	39.66	40.3
45	100	42.74	2.26	106.43	111.52	36.42	8.58	184.82	258.4	41.21	3.79	211.5	225.86	41.21	3.79	211.5	225.86
45	200	41.59	3.41	94.17	105.81	33.84	11.16	138.75	263.31	42.63	2.37	160.24	165.87	42.63	2.37	160.24	165.87
45	400	42.66	2.34	80.69	86.17	34.01	10.99	124.66	245.55	44.47	0.53	132.18	132.46	44.47	0.53	132.18	132.46
45	800	44.89	0.11	58.24	58.25	33.62	11.38	94.71	224.26	44.77	0.23	86.55	86.6	44.77	0.23	86.55	86.6

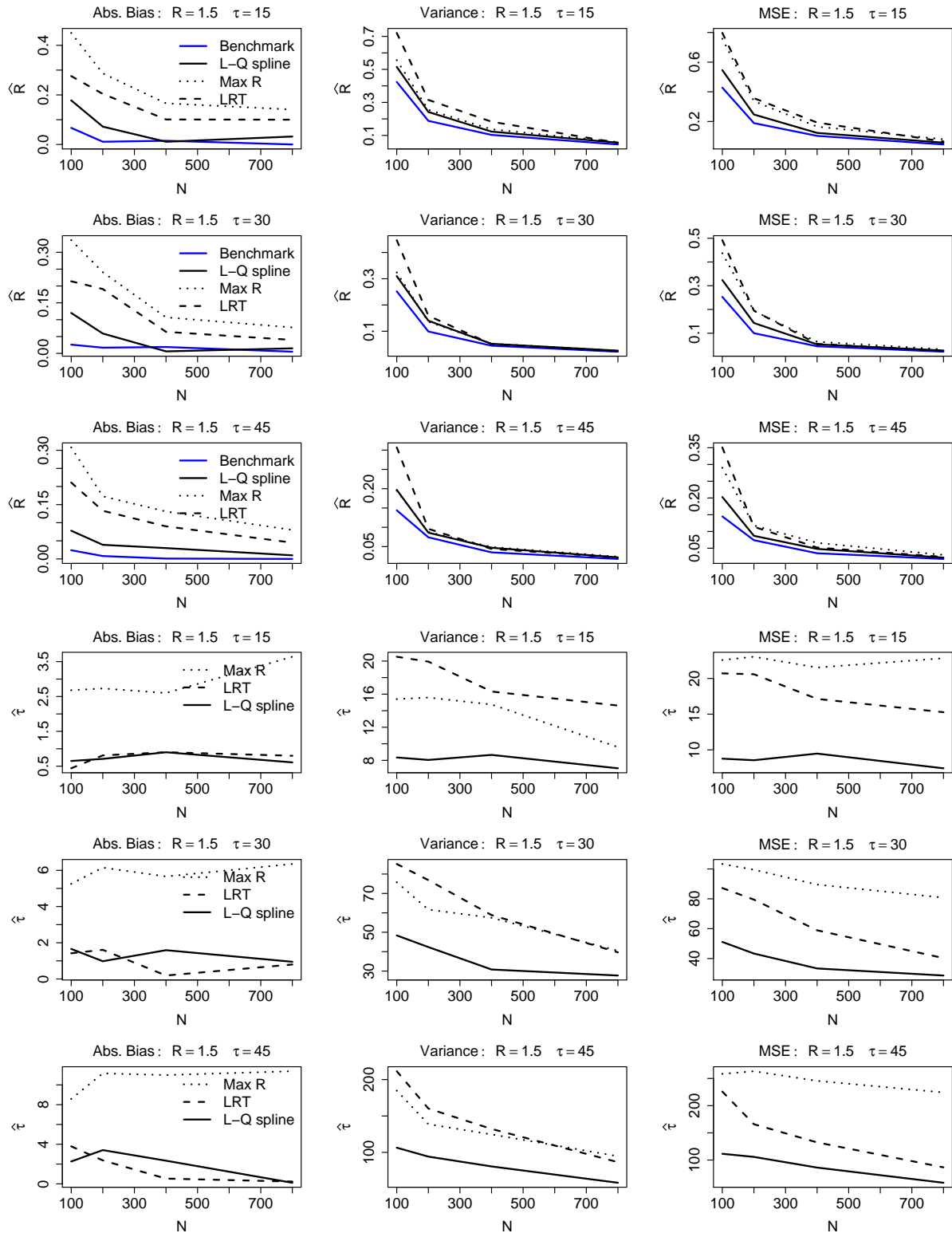


Figure 16: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 33: **Single Normally distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.008	0.008	0.513	0.513	2.115	0.115	0.649	0.662	2.437	0.437	0.637	0.828	2.321	0.321	0.691	0.794
15	200	2	0	0.306	0.306	2.098	0.098	0.357	0.366	2.302	0.302	0.38	0.471	2.193	0.193	0.371	0.408
15	400	2.071	0.071	0.15	0.155	2.11	0.11	0.174	0.186	2.264	0.264	0.185	0.255	2.2	0.2	0.159	0.199
15	800	1.988	0.012	0.062	0.063	2.011	0.011	0.073	0.073	2.107	0.107	0.079	0.091	2.036	0.036	0.068	0.07
30	100	2.017	0.017	0.284	0.285	2.132	0.132	0.379	0.396	2.338	0.338	0.399	0.513	2.258	0.258	0.39	0.456
30	200	2.007	0.007	0.161	0.161	2.049	0.049	0.2	0.203	2.227	0.227	0.198	0.25	2.151	0.151	0.191	0.214
30	400	1.992	0.008	0.084	0.084	2.007	0.007	0.098	0.098	2.134	0.134	0.097	0.114	2.063	0.063	0.088	0.092
30	800	1.986	0.014	0.037	0.037	1.994	0.006	0.042	0.042	2.07	0.07	0.048	0.053	2.013	0.013	0.042	0.042
45	100	2.019	0.019	0.239	0.239	2.095	0.095	0.302	0.311	2.298	0.298	0.298	0.387	2.22	0.22	0.306	0.355
45	200	2.038	0.038	0.111	0.112	2.077	0.077	0.125	0.131	2.22	0.22	0.13	0.178	2.146	0.146	0.124	0.145
45	400	1.981	0.019	0.055	0.055	1.998	0.002	0.06	0.06	2.082	0.082	0.061	0.068	2.034	0.034	0.057	0.058
45	800	1.998	0.002	0.027	0.027	1.998	0.002	0.029	0.029	2.061	0.061	0.031	0.035	2.015	0.015	0.028	0.028

Table 34: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.24	0.76	7.73	8.31	12	15.15	24.15	24.15	14.01	0.99	17.86	18.84
15	200	14.02	0.98	7.37	8.34	12.12	2.88	11.86	20.18	14.51	0.49	14.59	14.83
15	400	14.63	0.37	6.97	7.11	11.83	3.17	9.12	19.15	14.24	0.76	11.3	11.87
15	800	14.59	0.41	5.21	5.38	11.67	3.33	8.65	19.72	14.99	0.01	6.17	6.17
30	100	27.99	2.01	34.66	38.68	23.2	6.8	43.73	89.95	28.62	1.38	57.54	59.44
30	200	28.83	1.17	34.13	35.49	23.26	6.74	42.36	87.84	29.54	0.46	49.21	49.42
30	400	29.63	0.37	26.43	26.57	22.53	7.47	39.57	95.33	29.79	0.21	27.88	27.92
30	800	29.63	0.37	12.15	12.29	23.42	6.58	33.88	77.15	29.96	0.04	9.5	9.5
45	100	43.54	1.46	75.62	77.75	34.95	10.05	125.55	226.46	43.42	1.58	124.4	126.89
45	200	42.93	2.07	64.54	68.82	34.41	10.59	110.96	223.06	45.19	0.19	101.98	102.02
45	400	44.79	0.21	45.24	45.28	35.86	9.14	81.99	165.45	44.53	0.47	37.51	37.72
45	800	44.81	0.19	15.2	15.23	35.99	9.01	76.83	158.02	45.06	0.06	9.82	9.82

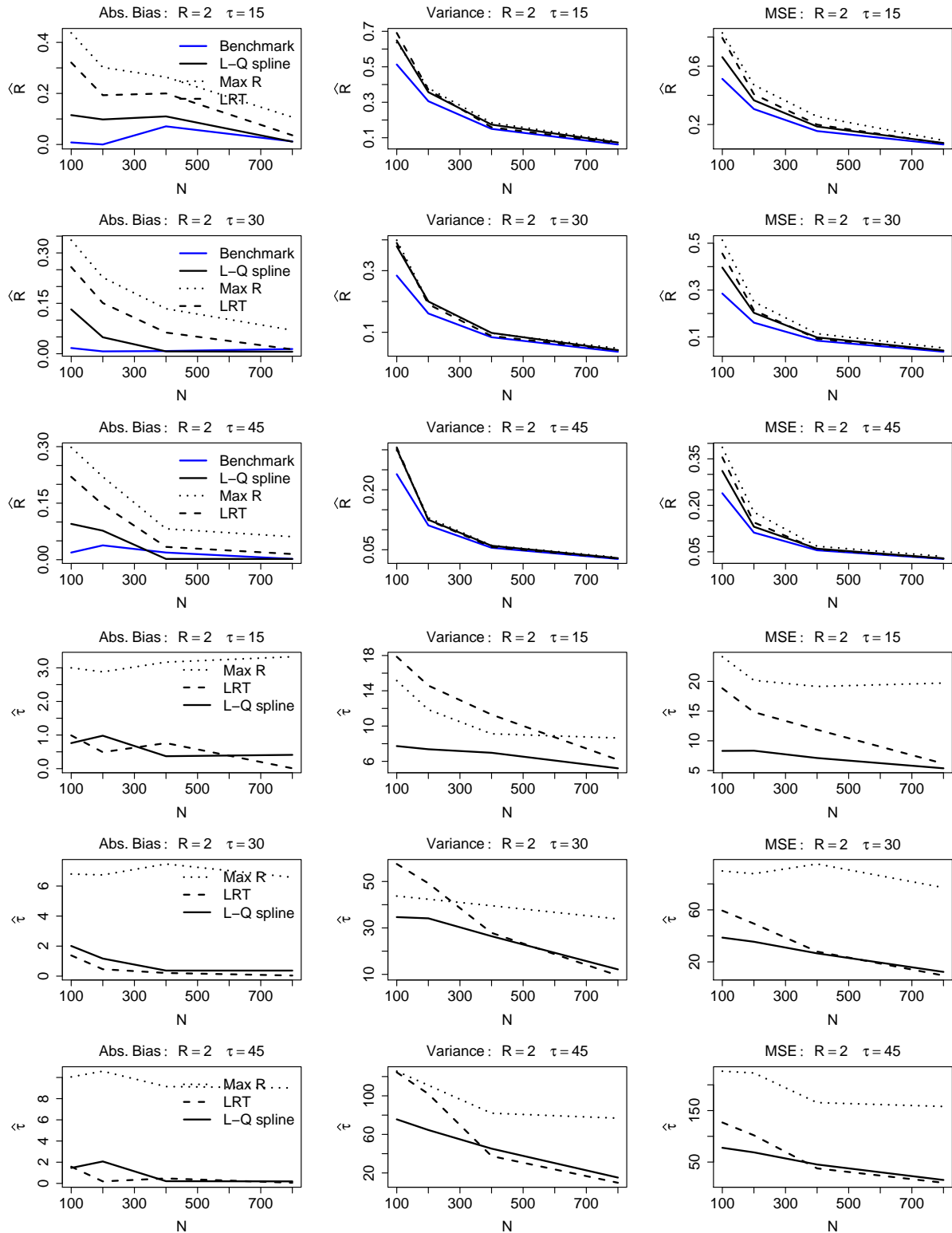


Figure 17: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 35: **Single Normally distributed exposure with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark				(b) Linear-quadratic spline				(c) \widehat{R}_{\max} approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	3.974	0.026	1.221	1.222	4.189	0.189	1.303	1.338	4.492	0.492	1.327	1.569	4.354	0.354	1.29	1.415
15	200	4.097	0.097	0.651	0.66	4.187	0.187	0.717	0.752	4.395	0.395	0.734	0.89	4.238	0.238	0.764	0.821
15	400	4.04	0.04	0.347	0.349	4.052	0.052	0.379	0.382	4.238	0.238	0.389	0.445	4.084	0.084	0.359	0.366
15	800	3.976	0.024	0.132	0.132	3.98	0.02	0.14	0.14	4.079	0.079	0.152	0.158	3.98	0.02	0.138	0.139
30	100	4.144	0.144	0.994	1.015	4.205	0.205	1.005	1.047	4.475	0.475	1.013	1.239	4.338	0.338	1.06	1.175
30	200	4.117	0.117	0.475	0.489	4.137	0.137	0.507	0.526	4.313	0.313	0.487	0.585	4.193	0.193	0.518	0.555
30	400	4.087	0.087	0.197	0.204	4.066	0.066	0.201	0.205	4.179	0.179	0.191	0.223	4.108	0.108	0.196	0.208
30	800	4.031	0.031	0.102	0.103	4.016	0.016	0.105	0.105	4.068	0.068	0.102	0.106	4.037	0.037	0.104	0.105
45	100	4.062	0.062	0.664	0.668	4.114	0.114	0.687	0.7	4.345	0.345	0.686	0.805	4.245	0.245	0.69	0.75
45	200	4.014	0.014	0.327	0.327	4.013	0.013	0.321	0.321	4.152	0.152	0.351	0.375	4.068	0.068	0.33	0.334
45	400	4.016	0.016	0.135	0.135	3.982	0.018	0.133	0.133	4.064	0.064	0.129	0.133	4.038	0.038	0.131	0.133
45	800	3.989	0.011	0.096	0.097	3.953	0.047	0.097	0.099	4.004	0.004	0.093	0.093	3.993	0.007	0.096	0.096

Table 36: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline				(c) \widehat{R}_{\max} approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.08	0.92	5.83	6.68	11.65	3.35	9.35	20.59	14.21	0.79	9.35	9.97
15	200	14.36	0.64	3.8	4.21	11.85	3.15	6.96	16.89	14.67	0.33	4.38	4.49
15	400	14.82	0.18	2.24	2.27	12.28	2.72	6.71	14.13	14.92	0.08	1.71	1.72
15	800	14.92	0.08	0.46	0.47	12.57	2.43	6.55	12.46	15.07	0.07	0.59	0.59
30	100	29.37	0.63	19.46	19.85	24.3	5.7	34.09	66.62	29.69	0.31	22.06	22.16
30	200	29.83	0.17	8.15	8.18	25.39	4.61	26.29	47.52	29.8	0.2	7.88	7.92
30	400	30.09	0.09	3.15	3.16	26.45	3.55	21.13	33.72	30	0	2.15	2.15
30	800	30.07	0.07	0.81	0.82	28.06	1.94	10.91	14.67	30.02	0.02	0.32	0.32
45	100	44.37	0.63	32.73	33.12	37.15	7.85	78.24	139.92	44.13	0.87	35.09	35.86
45	200	44.7	0.3	12.68	12.77	39.44	5.56	51.59	82.48	45.08	0.08	8.82	8.82
45	400	45.4	0.4	3.89	4.05	42.41	2.59	27.04	33.73	44.97	0.03	3.39	3.39
45	800	45.33	0.33	1.74	1.85	43.81	1.19	6.91	8.31	44.99	0.01	0.42	0.42

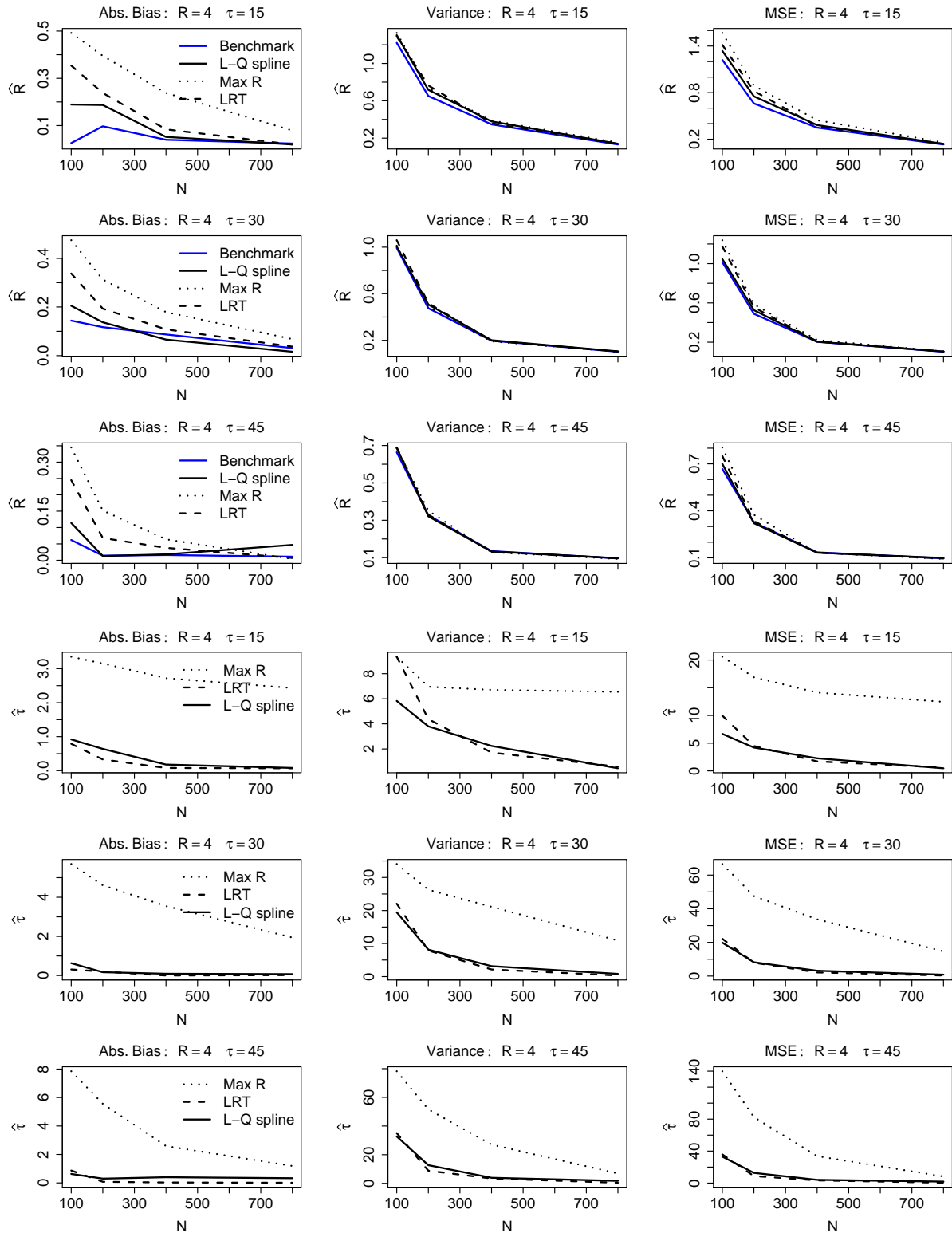


Figure 18: **Single Normally distributed exposure with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 37: **Multiple Normally distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.739	0.039	0.128	0.13	0.759	0.059	0.151	0.154	0.548	0.152	0.094	0.117	0.711	0.011	0.228	0.228
15	200	0.742	0.042	0.058	0.059	0.765	0.065	0.075	0.079	0.59	0.11	0.049	0.061	0.662	0.038	0.124	0.126
15	400	0.69	0.01	0.031	0.031	0.686	0.014	0.036	0.037	0.595	0.105	0.034	0.045	0.631	0.069	0.051	0.055
15	800	0.702	0.002	0.015	0.015	0.705	0.005	0.018	0.018	0.627	0.073	0.016	0.021	0.647	0.053	0.016	0.019
30	100	0.716	0.016	0.076	0.076	0.75	0.05	0.102	0.104	0.543	0.157	0.068	0.093	0.675	0.025	0.196	0.196
30	200	0.737	0.037	0.037	0.039	0.731	0.031	0.045	0.046	0.612	0.088	0.036	0.044	0.657	0.043	0.068	0.069
30	400	0.703	0.003	0.017	0.017	0.703	0.003	0.021	0.021	0.625	0.075	0.018	0.024	0.657	0.043	0.026	0.028
30	800	0.705	0.005	0.009	0.009	0.703	0.003	0.011	0.011	0.649	0.051	0.011	0.014	0.668	0.032	0.01	0.011
45	100	0.689	0.011	0.042	0.042	0.692	0.008	0.05	0.05	0.523	0.177	0.04	0.071	0.585	0.115	0.085	0.098
45	200	0.711	0.011	0.023	0.023	0.714	0.014	0.033	0.034	0.592	0.108	0.024	0.036	0.63	0.07	0.039	0.044
45	400	0.708	0.008	0.011	0.011	0.703	0.003	0.014	0.014	0.642	0.058	0.012	0.015	0.662	0.038	0.011	0.013
45	800	0.701	0.001	0.005	0.005	0.695	0.005	0.006	0.006	0.658	0.042	0.005	0.007	0.675	0.025	0.005	0.006

Table 38: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.23	0.23	9.32	9.37	13.43	1.57	18.27	20.73	15.97	0.97	19.38	20.32
15	200	15.12	0.12	8.32	8.33	13.71	1.29	22.64	24.31	15.41	0.41	21.85	22.02
15	400	15.05	0.05	8.19	8.2	13.39	1.61	18.71	21.3	15.47	0.47	19.16	19.38
15	800	15.37	0.37	7.65	7.79	11.97	3.03	11.86	21.04	14.98	0.02	14.64	14.64
30	100	29.78	0.22	39.5	39.54	27.07	2.93	96.43	105.01	30.14	0.14	95.2	95.22
30	200	30.16	0.16	40.98	41	26.08	3.92	86.27	101.64	29.43	0.57	81.57	81.89
30	400	29.66	0.34	40.19	40.31	23.77	6.23	68.28	107.11	30.77	0.77	73.33	73.92
30	800	30.04	0.04	32.25	32.25	24.44	5.56	52.92	83.81	30.49	0.49	48.12	48.36
45	100	45.27	0.27	114.69	114.76	34.63	10.37	186.94	294.41	42.96	2.04	229.81	233.97
45	200	44.55	0.45	104.86	105.07	35.36	9.64	181.49	274.38	44.01	0.99	188.58	189.57
45	400	43.91	1.09	85.04	86.22	35.52	9.48	162.56	252.48	47.26	2.26	129.49	134.6
45	800	44.86	0.14	60.93	60.95	34.73	10.27	99	204.4	45.52	0.52	82.45	82.72

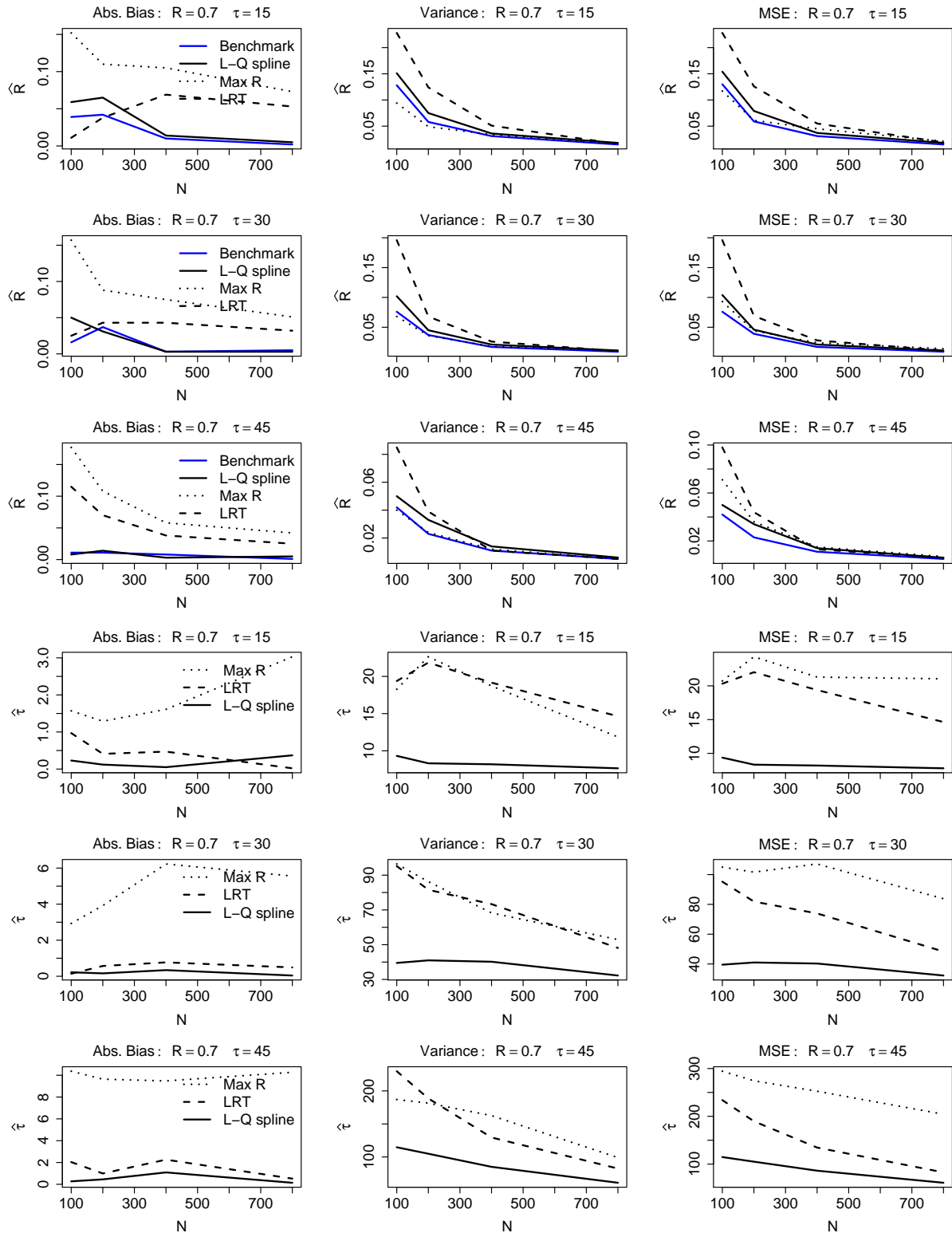


Figure 19: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 39: **Multiple Normally distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.937	0.037	0.156	0.157	0.978	0.078	0.203	0.209	0.71	0.19	0.114	0.15	0.963	0.063	0.437	0.441
15	200	0.937	0.037	0.089	0.09	0.952	0.052	0.101	0.104	0.731	0.169	0.076	0.105	0.929	0.029	0.227	0.227
15	400	0.911	0.011	0.044	0.044	0.92	0.02	0.049	0.049	0.781	0.119	0.041	0.055	0.898	0.002	0.108	0.108
15	800	0.903	0.003	0.022	0.022	0.906	0.006	0.026	0.026	0.807	0.093	0.023	0.031	0.865	0.035	0.048	0.05
30	100	0.887	0.013	0.091	0.091	0.903	0.003	0.129	0.129	0.661	0.239	0.068	0.125	0.879	0.021	0.249	0.25
30	200	0.921	0.021	0.042	0.042	0.931	0.031	0.055	0.056	0.753	0.147	0.038	0.059	0.881	0.019	0.113	0.114
30	400	0.897	0.003	0.021	0.021	0.889	0.011	0.027	0.027	0.786	0.114	0.019	0.032	0.837	0.063	0.041	0.045
30	800	0.909	0.009	0.013	0.013	0.903	0.003	0.016	0.016	0.838	0.062	0.013	0.017	0.881	0.019	0.028	0.028
45	100	0.926	0.026	0.067	0.068	0.955	0.055	0.105	0.108	0.725	0.175	0.06	0.09	0.943	0.043	0.228	0.23
45	200	0.895	0.005	0.023	0.023	0.9	0	0.036	0.036	0.744	0.156	0.023	0.047	0.834	0.066	0.076	0.08
45	400	0.909	0.009	0.016	0.016	0.912	0.012	0.023	0.023	0.813	0.087	0.014	0.021	0.891	0.009	0.045	0.045
45	800	0.895	0.005	0.007	0.007	0.887	0.013	0.011	0.012	0.825	0.075	0.007	0.013	0.857	0.043	0.017	0.019

Table 40: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.67	0.33	9	9.11	14.11	0.89	23.54	24.33	14.44	0.56	21.62	21.93
15	200	15.25	0.25	8.49	8.55	13.12	1.88	21.78	25.32	14.36	0.64	22.41	22.82
15	400	14.7	0.3	9.16	9.25	13.29	1.71	22.55	25.47	14.79	0.21	23.05	23.1
15	800	14.57	0.43	8.51	8.69	13.44	1.56	21.82	24.25	14.35	0.65	22.17	22.59
30	100	29.1	0.9	36.82	37.63	26.2	3.8	97.83	112.3	28.81	1.19	92.65	94.07
30	200	29.74	0.26	46.06	46.13	26.25	3.75	99.63	113.68	29.21	0.79	102.32	102.94
30	400	28.96	1.04	43.34	44.42	25.74	4.26	93.69	111.8	30.84	0.84	94.68	95.38
30	800	28.79	1.21	42.92	44.37	25.86	4.14	82.54	99.69	29.42	0.58	83.02	83.35
45	100	43.39	1.61	122.19	124.79	39.12	5.88	267.56	302.19	40.85	4.15	272.03	289.26
45	200	43.35	1.65	103.5	106.24	36.69	8.31	252.33	321.41	40.79	4.21	255.25	272.94
45	400	43.18	1.82	110.4	113.72	36.77	8.23	219.5	287.26	42.93	2.07	247.45	251.74
45	800	43.6	1.4	102.64	104.61	37.6	7.4	185.46	240.25	43.72	1.28	213.05	214.68

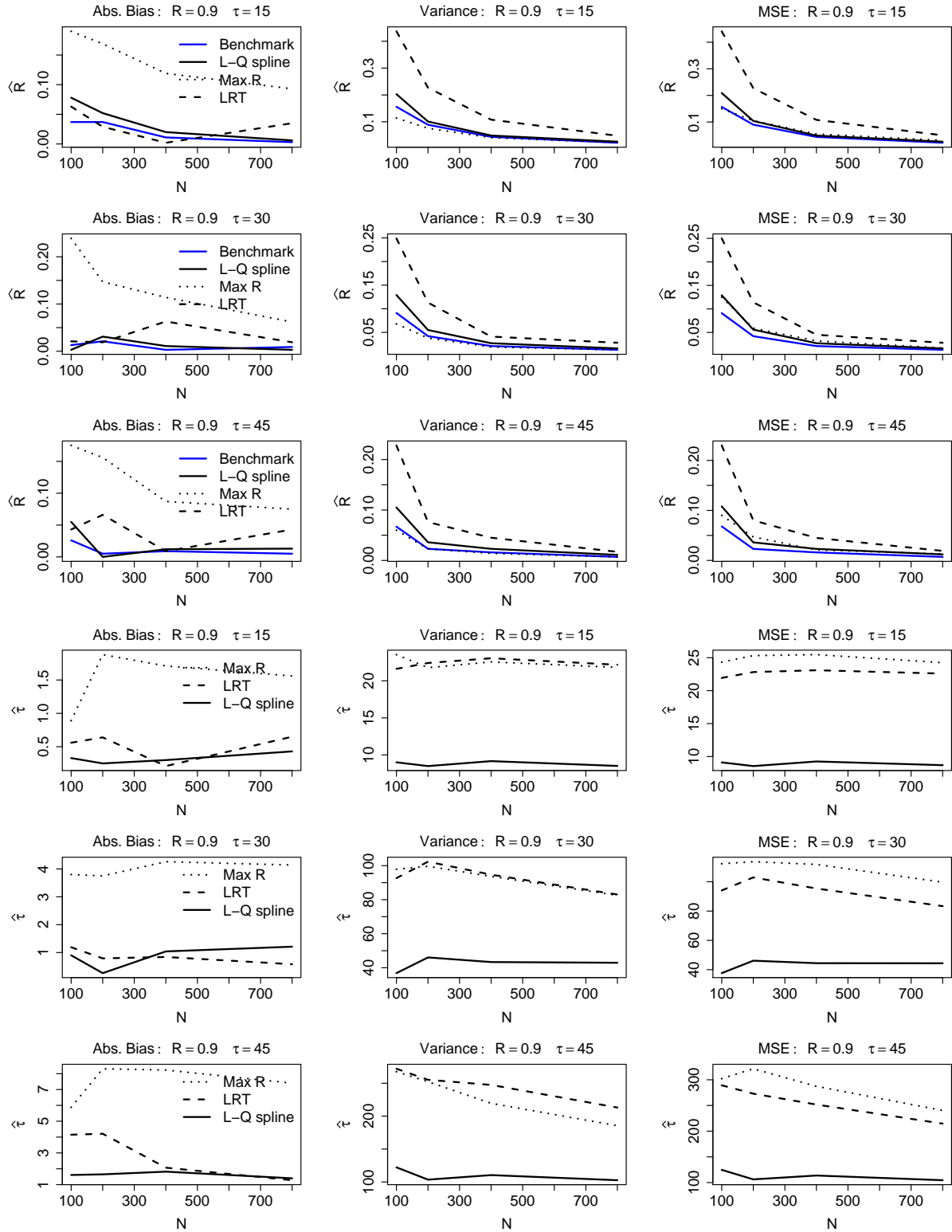


Figure 20: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 41: **Multiple Normally distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.18	0.02	0.229	0.23	1.26	0.06	0.276	0.28	1.543	0.343	0.331	0.448	1.314	0.114	0.581	0.594
15	200	1.218	0.018	0.129	0.129	1.266	0.066	0.17	0.175	1.442	0.242	0.16	0.219	1.327	0.127	0.264	0.28
15	400	1.189	0.011	0.066	0.066	1.196	0.004	0.079	0.079	1.334	0.134	0.074	0.092	1.246	0.046	0.131	0.133
15	800	1.215	0.015	0.028	0.028	1.227	0.027	0.036	0.037	1.314	0.114	0.033	0.046	1.269	0.069	0.046	0.051
30	100	1.236	0.036	0.135	0.136	1.269	0.069	0.158	0.162	1.526	0.326	0.162	0.268	1.39	0.19	0.306	0.342
30	200	1.222	0.022	0.068	0.069	1.25	0.05	0.088	0.09	1.408	0.208	0.091	0.134	1.309	0.109	0.159	0.171
30	400	1.198	0.002	0.032	0.032	1.225	0.025	0.039	0.039	1.323	0.123	0.04	0.055	1.274	0.074	0.064	0.069
30	800	1.2	0	0.014	0.014	1.208	0.008	0.018	0.018	1.288	0.088	0.019	0.026	1.267	0.067	0.021	0.026
45	100	1.219	0.019	0.11	0.111	1.255	0.055	0.139	0.142	1.469	0.269	0.128	0.2	1.329	0.129	0.261	0.278
45	200	1.211	0.011	0.045	0.045	1.228	0.028	0.063	0.064	1.388	0.188	0.066	0.101	1.299	0.099	0.122	0.132
45	400	1.21	0.01	0.026	0.026	1.23	0.03	0.03	0.031	1.326	0.126	0.032	0.047	1.286	0.086	0.049	0.056
45	800	1.197	0.003	0.011	0.011	1.205	0.005	0.015	0.015	1.271	0.071	0.013	0.018	1.251	0.051	0.018	0.02

Table 42: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	14.48	0.52	9.45	9.72	12.84	2.16	21.42	26.07	14.14	0.86	24.66	25.4	14.14	0.86	24.66	25.4
15	200	14.2	0.8	7.73	8.37	13.06	1.94	19.45	23.22	13.79	1.21	22.19	23.65	13.79	1.21	22.19	23.65
15	400	14.73	0.27	9.97	10.05	13.55	1.45	21.75	23.85	14.84	0.16	23	23.02	14.84	0.16	23	23.02
15	800	14.54	0.46	8.29	8.5	12.26	2.74	16.05	23.55	14.88	0.12	18.79	18.8	14.88	0.12	18.79	18.8
30	100	28.86	1.14	42.73	44.04	26.51	3.49	105.29	117.45	28.54	1.46	103.4	105.53	28.54	1.46	103.4	105.53
30	200	29.15	0.85	40.3	41.02	24.87	5.13	82.71	109.03	28.99	1.01	89.62	90.63	28.99	1.01	89.62	90.63
30	400	28.57	1.43	40.47	42.52	24.52	5.48	71.47	101.47	28.51	1.49	88.56	90.77	28.51	1.49	88.56	90.77
30	800	29.81	0.19	39.53	39.57	23.86	6.14	63.69	101.33	28.9	1.1	79.08	80.28	28.9	1.1	79.08	80.28
45	100	43.1	1.9	114.11	117.73	36.58	8.42	221.08	292.01	39.74	5.26	232.42	260.11	39.74	5.26	232.42	260.11
45	200	42.64	2.36	81.24	86.8	37.67	7.33	242.81	296.57	41.99	3.01	237.24	246.3	41.99	3.01	237.24	246.3
45	400	42.55	2.45	106.22	112.21	36.91	8.09	220.14	285.6	42.34	2.66	232.52	239.58	42.34	2.66	232.52	239.58
45	800	43.91	1.09	92.56	93.74	36.31	8.69	188.12	263.7	44.59	0.41	191.07	191.24	44.59	0.41	191.07	191.24

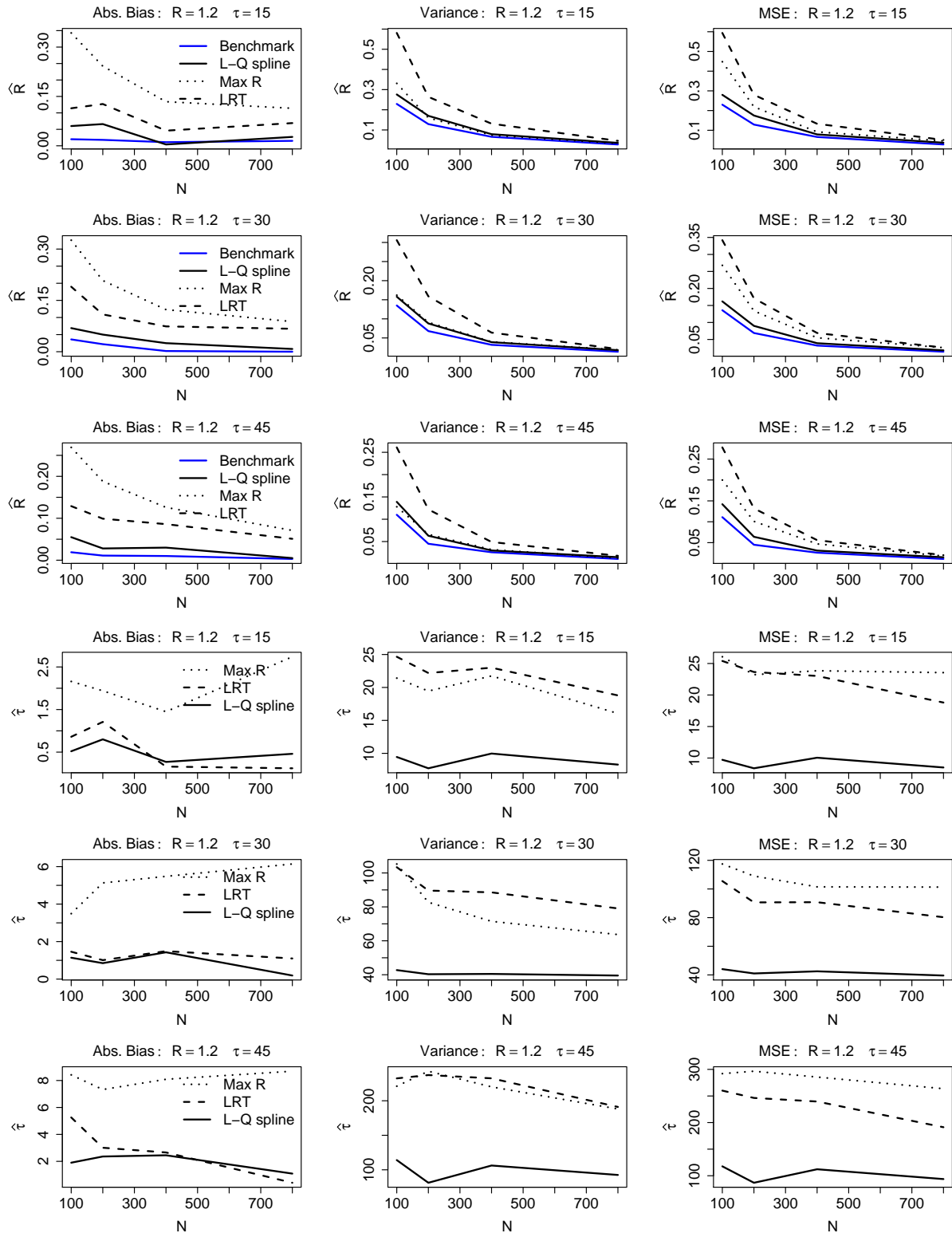


Figure 21: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 43: **Multiple Normally distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.62	0.12	0.329	0.343	1.71	0.21	0.406	0.45	1.935	0.435	0.403	0.593	1.778	0.278	0.555	0.632
15	200	1.518	0.018	0.161	0.161	1.555	0.055	0.2	0.203	1.753	0.253	0.197	0.261	1.674	0.174	0.243	0.273
15	400	1.532	0.032	0.076	0.077	1.57	0.07	0.09	0.095	1.674	0.174	0.087	0.118	1.628	0.128	0.086	0.102
15	800	1.477	0.023	0.035	0.036	1.488	0.012	0.043	0.043	1.581	0.081	0.044	0.05	1.543	0.043	0.04	0.042
30	100	1.517	0.017	0.176	0.177	1.603	0.103	0.23	0.241	1.787	0.287	0.228	0.311	1.701	0.201	0.316	0.357
30	200	1.528	0.028	0.09	0.09	1.563	0.063	0.115	0.119	1.702	0.202	0.109	0.15	1.654	0.154	0.117	0.141
30	400	1.488	0.012	0.037	0.037	1.523	0.023	0.051	0.052	1.613	0.113	0.052	0.064	1.583	0.083	0.055	0.062
30	800	1.492	0.008	0.021	0.021	1.512	0.012	0.025	0.026	1.565	0.065	0.025	0.03	1.536	0.036	0.023	0.024
45	100	1.544	0.044	0.186	0.188	1.614	0.114	0.244	0.257	1.821	0.321	0.228	0.332	1.751	0.251	0.286	0.349
45	200	1.534	0.034	0.071	0.073	1.571	0.071	0.082	0.088	1.685	0.185	0.078	0.112	1.649	0.149	0.08	0.102
45	400	1.513	0.013	0.035	0.035	1.528	0.028	0.037	0.038	1.608	0.108	0.036	0.048	1.581	0.081	0.033	0.039
45	800	1.511	0.011	0.015	0.015	1.515	0.015	0.017	0.017	1.567	0.067	0.017	0.021	1.547	0.047	0.016	0.018

Table 44: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.29	0.71	7.9	8.41	12.41	2.59	15.27	21.96	14.53	0.47	18.25	18.47
15	200	14.47	0.53	8.1	8.38	12.26	2.74	15.32	22.84	14.79	0.21	20.03	20.07
15	400	14.49	0.51	8.6	8.86	12.21	2.79	11.82	19.63	15.04	0.04	15.66	15.66
15	800	14.62	0.38	7.88	8.03	11.84	3.16	11.7	21.69	14.67	0.33	13.37	13.48
30	100	28.19	1.81	39.26	42.55	24.11	5.89	60.84	95.53	27.96	2.04	76.88	81.05
30	200	29.04	0.96	38.55	39.47	23.81	6.19	56.11	94.44	29.54	0.46	67.33	67.55
30	400	28.43	1.57	33.12	35.59	23.87	6.13	51.8	89.32	28.95	1.05	53.72	54.82
30	800	29.74	0.26	25.15	25.22	24.05	5.95	37.52	72.98	29.89	0.11	31.02	31.04
45	100	40.98	4.02	91.78	107.94	35.42	9.58	182.83	274.66	40.6	4.4	187.58	206.95
45	200	43.77	1.23	92.45	93.95	37.63	7.37	154.28	208.55	44.69	0.31	142	142.1
45	400	44.34	0.66	70.35	70.78	36.54	8.46	115.72	187.33	44.66	0.34	97.46	97.58
45	800	45.2	0.2	49.16	49.2	36.02	8.98	99.17	179.8	44.68	0.32	46.14	46.24

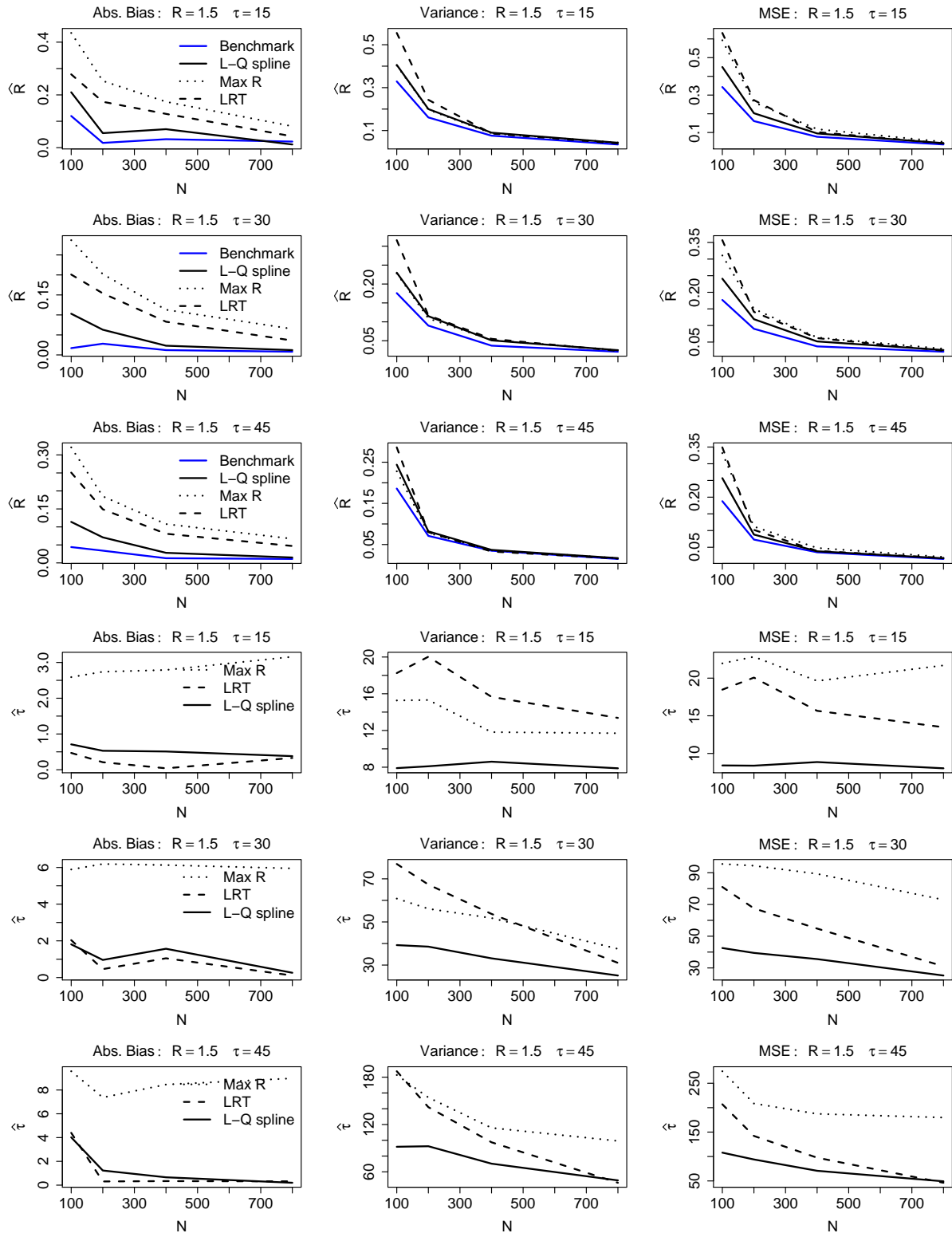


Figure 22: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 45: **Multiple Normally distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	2.034	0.034	0.43	0.431	2.118	0.118	0.51	0.524	2.389	0.389	0.528	0.679	2.311	0.311	0.568	0.665
15	200	2.004	0.004	0.246	0.246	2.077	0.077	0.284	0.289	2.246	0.246	0.284	0.345	2.175	0.175	0.268	0.299
15	400	2.022	0.022	0.11	0.111	2.057	0.057	0.123	0.126	2.177	0.177	0.134	0.166	2.117	0.117	0.122	0.136
15	800	1.986	0.014	0.06	0.06	2.005	0.005	0.064	0.064	2.087	0.087	0.07	0.077	2.027	0.027	0.06	0.061
30	100	2.05	0.05	0.215	0.217	2.139	0.139	0.283	0.303	2.348	0.348	0.278	0.399	2.288	0.288	0.275	0.358
30	200	2.028	0.028	0.117	0.117	2.066	0.066	0.119	0.123	2.224	0.224	0.133	0.183	2.166	0.166	0.124	0.151
30	400	2.016	0.016	0.062	0.062	2.043	0.043	0.067	0.069	2.109	0.109	0.065	0.077	2.065	0.065	0.063	0.067
30	800	1.997	0.003	0.025	0.025	1.998	0.002	0.028	0.028	2.043	0.043	0.025	0.027	2.016	0.016	0.027	0.028
45	100	2.041	0.041	0.2	0.202	2.1	0.1	0.272	0.282	2.305	0.305	0.256	0.349	2.26	0.26	0.247	0.315
45	200	1.989	0.011	0.097	0.097	2.017	0.017	0.102	0.102	2.122	0.122	0.099	0.114	2.082	0.082	0.093	0.1
45	400	2.013	0.013	0.058	0.058	2.019	0.019	0.06	0.06	2.083	0.083	0.064	0.07	2.056	0.056	0.062	0.065
45	800	2.023	0.023	0.027	0.028	2.019	0.019	0.03	0.03	2.054	0.054	0.031	0.034	2.038	0.038	0.029	0.031

Table 46: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.17	0.83	8.11	8.8	12.12	2.88	13.98	22.27	14.23	0.77	16.59	17.18
15	200	14.35	0.65	8.06	8.48	11.88	3.12	11.66	21.37	14.46	0.54	12.1	12.39
15	400	14.37	0.63	5.63	6.03	11.76	3.24	8.53	19.04	14.43	0.57	7.66	7.99
15	800	14.71	0.29	3.82	3.91	11.78	3.22	6.88	17.23	14.93	0.07	4.92	4.92
30	100	28.36	1.64	29.98	32.68	23.91	6.09	57.46	94.49	28.43	1.57	52.61	55.08
30	200	29.68	0.32	26.59	26.69	23.54	6.46	42.34	84.04	29.21	0.79	42.92	43.54
30	400	29.29	0.71	16.6	17.11	24.65	5.35	31	59.64	30.29	0.29	23.82	23.9
30	800	29.83	0.17	7.34	7.37	25.74	4.26	26.4	44.56	29.86	0.14	8.46	8.48
45	100	43.48	1.52	82.45	84.77	36.32	8.68	143.8	219.21	43.62	1.38	130.99	132.9
45	200	43.64	1.36	56.71	58.55	37.23	7.77	100.24	160.59	45.08	0.08	84.01	84.02
45	400	45.04	0.04	26.7	26.7	38.59	6.41	69.72	110.83	44.88	0.12	25.93	25.94
45	800	45.3	0.3	10.86	10.95	40.96	4.04	40.92	57.2	45.09	0.09	9.12	9.13

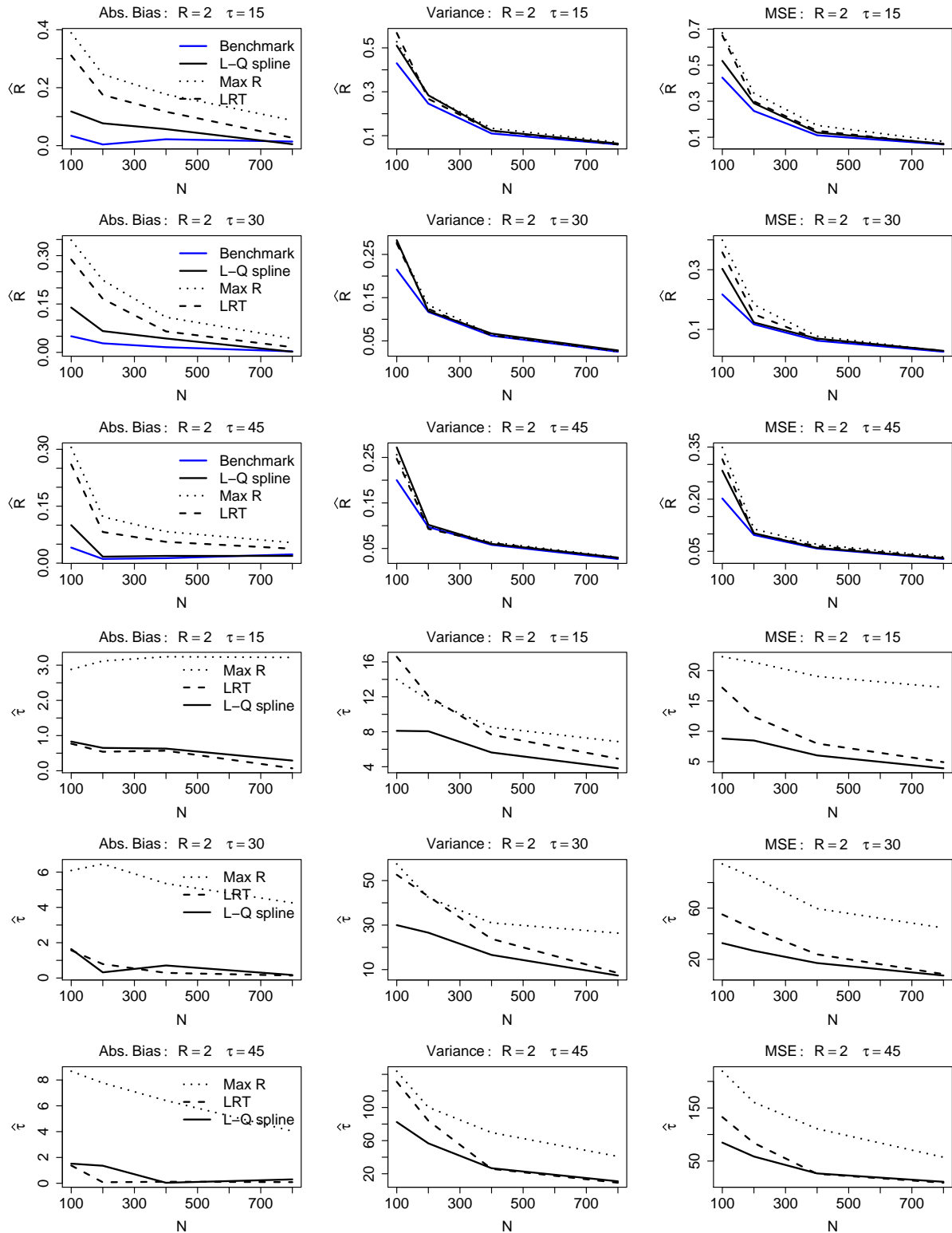


Figure 23: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 47: **Multiple Normally distributed exposures with no age effects.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark				(b) Linear-quadratic spline				(c) \widehat{R}_{\max} approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	4.073	0.073	1.034	1.039	4.201	0.201	1.176	1.217	4.498	0.498	1.24	1.488	4.329	0.329	1.084	1.193
15	200	4.033	0.033	0.545	0.546	4.052	0.052	0.57	0.573	4.245	0.245	0.611	0.671	4.119	0.119	0.565	0.58
15	400	4.027	0.027	0.223	0.223	4.02	0.02	0.233	0.233	4.126	0.126	0.218	0.234	4.036	0.036	0.221	0.222
15	800	4.011	0.011	0.161	0.161	4.007	0.007	0.166	0.166	4.078	0.078	0.163	0.169	4.017	0.017	0.166	0.166
30	100	4.041	0.041	0.639	0.641	4.109	0.109	0.654	0.666	4.288	0.288	0.669	0.753	4.198	0.198	0.662	0.701
30	200	4.032	0.032	0.374	0.375	4.028	0.028	0.393	0.394	4.159	0.159	0.391	0.416	4.091	0.091	0.393	0.401
30	400	4.009	0.009	0.201	0.201	3.986	0.014	0.198	0.198	4.045	0.045	0.195	0.197	4.021	0.021	0.203	0.203
30	800	4.015	0.015	0.101	0.101	3.99	0.01	0.1	0.1	4.029	0.029	0.101	0.102	4.017	0.017	0.102	0.102
45	100	4.038	0.038	0.771	0.773	4.049	0.049	0.725	0.727	4.239	0.239	0.742	0.799	4.205	0.205	0.764	0.807
45	200	3.994	0.006	0.347	0.347	3.964	0.036	0.354	0.355	4.069	0.069	0.341	0.346	4.049	0.049	0.354	0.356
45	400	4.018	0.018	0.178	0.178	3.977	0.023	0.185	0.185	4.044	0.044	0.18	0.182	4.04	0.04	0.181	0.182
45	800	4.049	0.049	0.08	0.082	4.009	0.009	0.079	0.079	4.056	0.056	0.08	0.083	4.054	0.054	0.081	0.084

Table 48: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline				(c) \widehat{R}_{\max} approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.49	0.51	5.55	5.8	12.06	2.94	7.77	16.41	14.45	0.55	6.1	6.4
15	200	14.91	0.09	2.87	2.88	12.54	2.46	6.7	12.74	14.77	0.23	2.1	2.15
15	400	14.97	0.03	1.19	1.19	13.09	1.91	5.42	9.07	15.09	0.09	0.57	0.58
15	800	14.98	0.02	0.33	0.33	13.29	1.71	5.64	8.56	15.01	0.01	0.27	0.27
30	100	29.5	0.5	11.65	11.89	25.77	4.23	31.89	49.75	29.7	0.3	11.79	11.87
30	200	29.9	0.1	7.54	7.55	26.66	3.34	22.72	33.88	29.98	0.02	6.92	6.92
30	400	30.24	0.24	1.87	1.92	28.4	1.6	11.04	13.6	30.18	0.18	1.28	1.31
30	800	30.15	0.15	0.89	0.91	29.35	0.65	2.92	3.35	30.04	0.04	0.2	0.2
45	100	45.13	0.13	34.47	34.48	41.44	3.56	58.57	71.26	44.24	0.76	29.6	30.18
45	200	45.52	0.52	9.36	9.62	42.36	2.64	29.31	36.3	44.69	0.31	9.22	9.31
45	400	45.59	0.59	2.9	3.26	44.11	0.89	4.65	5.45	44.7	0.3	2.39	2.48
45	800	45.56	0.56	1.34	1.65	44.57	0.43	2.88	3.07	44.93	0.07	0.23	0.24

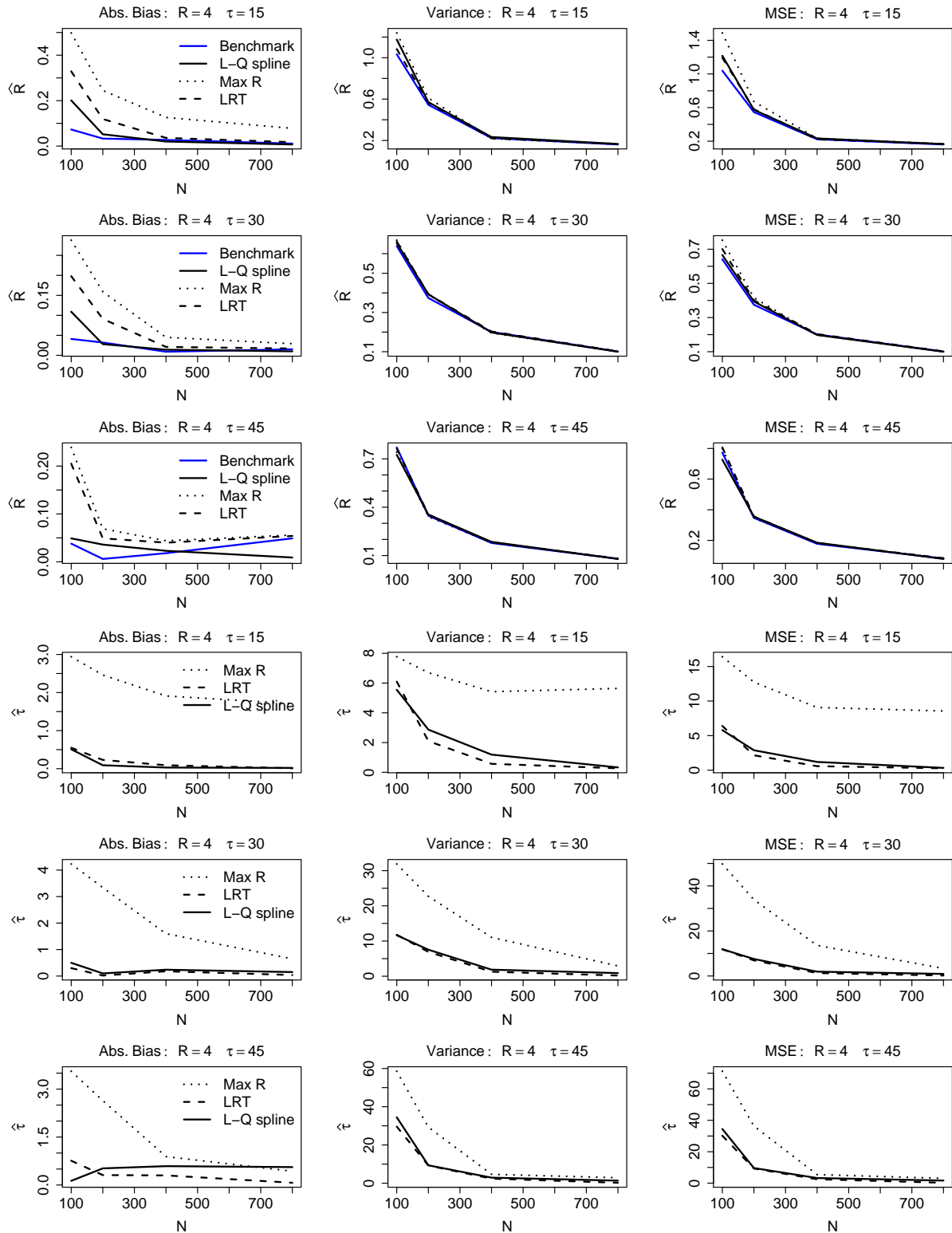


Figure 24: **Multiple Normally distributed exposures with no age effects.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.