

Bias and Estimation under Misspecification of the Risk Period in Self-Controlled Case Series Studies

**Supplemental Materials: Simulation Results for Models with Two
Age Groups**

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Additional simulation results

We provide results from additional simulations for the estimation of the relative incidence, $\hat{R} = \exp(\hat{\beta})$, and optimal risk length, $\hat{\tau}$, under different exposure models with one age effect.

Tables 1-2, 3-4, 5-6, 7-8, 9-10 and 11-12 show the results in estimation for the single Uniformly distributed exposure model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 1-6.

Tables 13-14, 15-16, 17-18, 19-20, 21-22 and 23-24 show the results in estimation for the multiple Uniformly distributed exposures model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 7-12.

Tables 25-26, 27-28, 29-30, 31-32, 33-34, 35-36 show the results in estimation for the single Normally distributed exposure model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 13-18.

Tables 37-38, 39-40, 41-42, 43-44, 45-46 and 47-48 show the results in estimation for the multiple Normally distributed exposures model and relative incidence of $R = 0.7, 0.9, 1.2, 1.5, 2$ and 4 respectively. These also correspond respectively to Figures 19-24.

Table 1: **Single Uniformly distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.731	0.031	0.139	0.14	0.769	0.069	0.184	0.189	0.543	0.157	0.089	0.113	0.789	0.089	0.235	0.243
15	200	0.719	0.019	0.09	0.09	0.742	0.042	0.114	0.116	0.548	0.152	0.07	0.093	0.638	0.062	0.182	0.186
15	400	0.708	0.008	0.04	0.04	0.71	0.01	0.048	0.048	0.579	0.121	0.044	0.058	0.634	0.066	0.076	0.08
15	800	0.702	0.002	0.016	0.016	0.696	0.004	0.02	0.02	0.611	0.089	0.019	0.027	0.632	0.068	0.023	0.028
30	100	0.703	0.003	0.079	0.079	0.738	0.038	0.108	0.109	0.525	0.175	0.066	0.096	0.637	0.063	0.209	0.213
30	200	0.71	0.01	0.041	0.041	0.707	0.007	0.054	0.054	0.574	0.126	0.036	0.052	0.624	0.076	0.07	0.076
30	400	0.699	0.001	0.019	0.019	0.696	0.004	0.025	0.025	0.594	0.106	0.019	0.03	0.614	0.086	0.026	0.033
30	800	0.698	0.002	0.01	0.01	0.692	0.008	0.014	0.014	0.632	0.068	0.011	0.016	0.653	0.047	0.011	0.013
45	100	0.714	0.014	0.065	0.065	0.725	0.025	0.087	0.088	0.52	0.18	0.05	0.083	0.606	0.094	0.132	0.141
45	200	0.682	0.018	0.03	0.031	0.679	0.021	0.039	0.04	0.543	0.157	0.025	0.05	0.58	0.12	0.035	0.05
45	400	0.71	0.01	0.015	0.015	0.703	0.003	0.018	0.018	0.611	0.089	0.016	0.024	0.636	0.064	0.019	0.023
45	800	0.696	0.004	0.008	0.008	0.686	0.014	0.011	0.011	0.635	0.065	0.009	0.013	0.656	0.044	0.008	0.01

Table 2: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.18	0.18	10.49	10.53	14.15	0.85	21.31	22.04	15.79	0.79	21.3	21.92
15	200	14.95	0.05	7.94	7.94	13.25	1.75	19.66	22.74	14.59	0.41	19.21	19.37
15	400	14.78	0.22	8.6	8.65	12.67	2.33	17.32	22.73	14.38	0.62	21.46	21.84
15	800	14.97	0.03	8.52	8.52	12.28	2.72	15.03	22.45	14.89	0.11	17.92	17.93
30	100	30.14	0.14	52.07	52.09	26.35	3.65	85.86	99.17	31.28	1.28	104.15	105.79
30	200	29.27	0.73	34.29	34.82	26.95	3.05	83.99	93.29	30.66	0.66	88.72	89.16
30	400	29.43	0.57	36.02	36.35	24.5	5.5	65.99	96.27	28.36	1.64	73.18	75.88
30	800	29.4	0.6	30.86	31.23	23.53	6.47	58.36	100.19	30.59	0.59	60.1	60.45
45	100	43.91	1.09	111.22	112.4	37.53	7.47	221.25	277.02	44.86	0.14	234.64	234.66
45	200	43.92	1.08	105.53	106.68	35.05	9.95	191.57	290.56	45.7	0.7	224.68	225.17
45	400	43.85	1.15	82.37	83.69	33.4	11.6	153.79	288.42	43.95	1.05	181.24	182.35
45	800	43.82	1.18	71.77	73.16	33.93	11.07	119.92	242.48	45.68	0.68	132.61	133.08

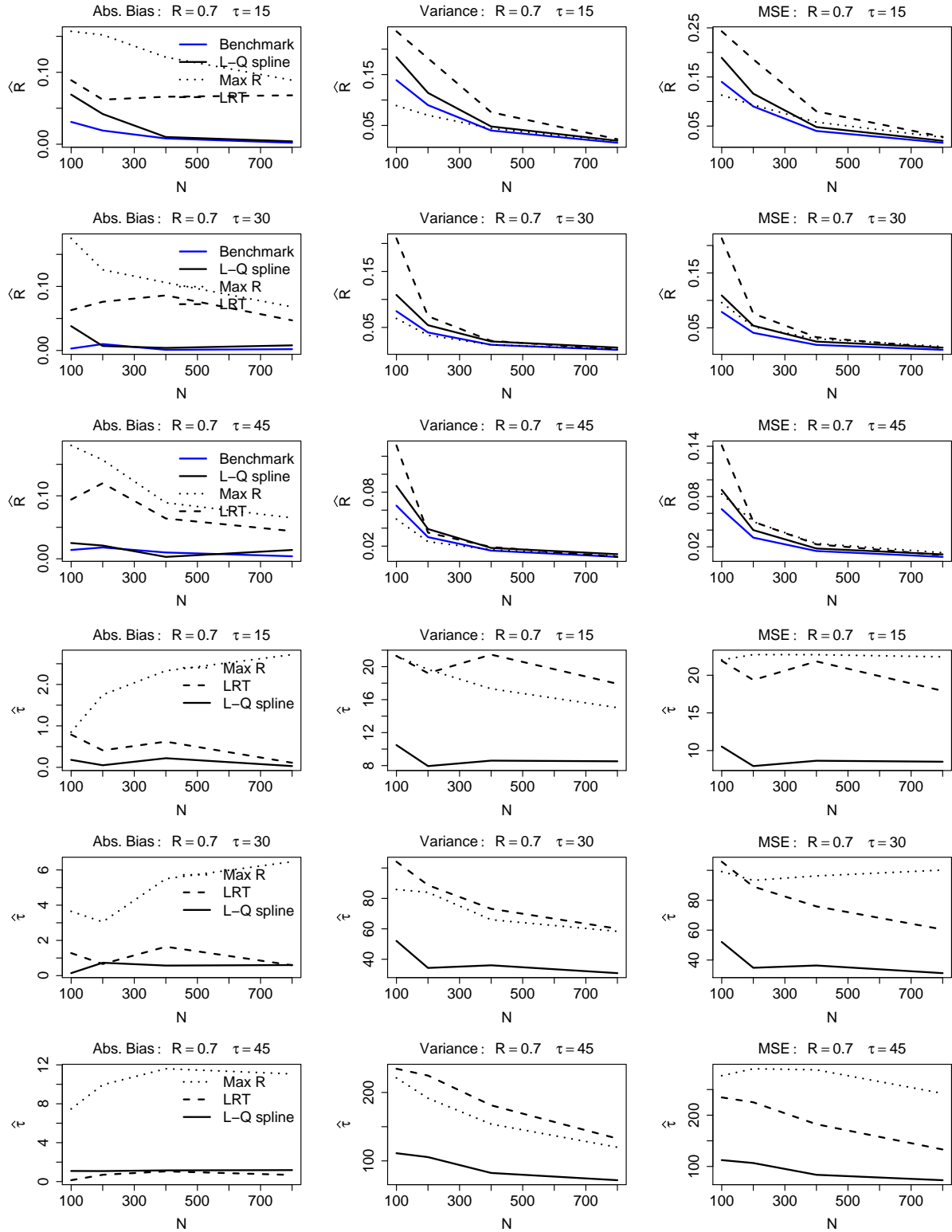


Figure 1: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 3: **Single Uniformly distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.929	0.029	0.249	0.249	0.977	0.077	0.307	0.313	0.686	0.214	0.16	0.206	0.99	0.09	0.594	0.602
15	200	0.873	0.027	0.109	0.109	0.89	0.01	0.136	0.136	0.668	0.232	0.088	0.141	0.845	0.055	0.255	0.258
15	400	0.892	0.008	0.043	0.043	0.906	0.006	0.057	0.057	0.743	0.157	0.051	0.076	0.868	0.032	0.138	0.139
15	800	0.917	0.017	0.024	0.024	0.934	0.034	0.033	0.034	0.801	0.099	0.025	0.034	0.884	0.016	0.063	0.063
30	100	0.888	0.012	0.115	0.115	0.921	0.021	0.149	0.149	0.666	0.234	0.089	0.144	0.91	0.01	0.366	0.366
30	200	0.917	0.017	0.071	0.072	0.923	0.023	0.087	0.087	0.741	0.159	0.065	0.09	0.902	0.002	0.181	0.181
30	400	0.918	0.018	0.03	0.03	0.927	0.027	0.043	0.044	0.791	0.109	0.031	0.043	0.896	0.004	0.086	0.086
30	800	0.903	0.003	0.018	0.018	0.898	0.002	0.021	0.021	0.82	0.08	0.017	0.023	0.874	0.026	0.038	0.039
45	100	0.911	0.011	0.093	0.094	0.951	0.051	0.139	0.142	0.653	0.247	0.071	0.132	0.888	0.012	0.286	0.286
45	200	0.925	0.025	0.042	0.043	0.949	0.049	0.067	0.07	0.761	0.139	0.042	0.062	0.922	0.022	0.141	0.142
45	400	0.908	0.008	0.018	0.018	0.911	0.011	0.026	0.026	0.786	0.114	0.019	0.032	0.869	0.031	0.056	0.057
45	800	0.913	0.013	0.012	0.012	0.91	0.01	0.016	0.016	0.832	0.068	0.013	0.017	0.88	0.02	0.029	0.029

Table 4: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.97	0.03	9.57	9.57	14.8	0.2	21.98	22.02	15.28	0.28	23.75	23.83
15	200	14.95	0.05	9.63	9.63	13.78	1.22	25.5	26.99	14.73	0.27	24.23	24.3
15	400	14.72	0.28	9.19	9.27	13.65	1.35	24.26	26.07	14.35	0.65	27.11	27.54
15	800	14.5	0.5	9.55	9.8	13.25	1.75	24.24	27.3	14.66	0.34	23.88	24
30	100	28.54	1.46	37.27	39.41	27.82	2.18	109.65	114.4	28.46	1.54	109.3	111.68
30	200	29.17	0.83	41.55	42.24	26.82	3.18	99.31	109.4	28.84	1.16	102.38	103.72
30	400	29.31	0.69	47.42	47.89	25.66	4.34	97.2	116.05	28.03	1.97	96.9	100.8
30	800	28.5	1.5	39.32	41.56	25.67	4.33	84.16	102.92	29.67	0.33	102.71	102.82
45	100	41.74	3.26	110.72	121.35	37.72	7.28	273.88	326.82	40.64	4.36	260	279.03
45	200	41.61	3.39	114.08	125.58	39.74	5.26	267.48	295.16	40.53	4.47	256.68	276.68
45	400	42.19	2.81	109.77	117.69	36.57	8.43	251.61	322.63	42.04	2.96	262.95	271.71
45	800	42.67	2.33	101.18	106.6	35.17	9.83	208.9	305.61	41.31	3.69	234.19	247.83

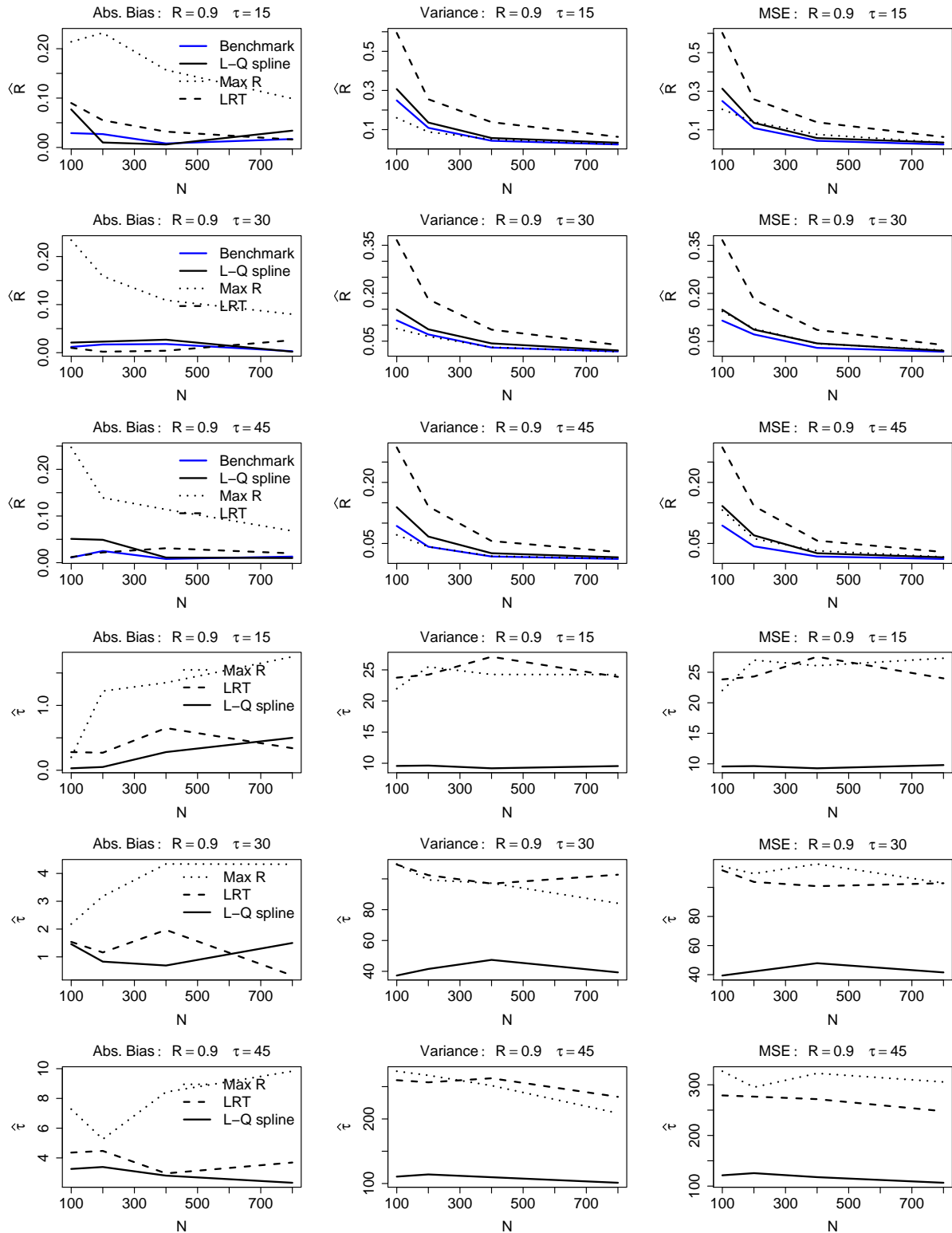


Figure 2: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 5: **Single Uniformly distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.226	0.026	0.289	0.289	1.332	0.132	0.41	0.427	1.619	0.419	0.451	0.627	1.305	0.105	0.542	0.553
15	200	1.253	0.053	0.155	0.158	1.304	0.104	0.204	0.215	1.504	0.304	0.216	0.308	1.35	0.15	0.371	0.394
15	400	1.195	0.005	0.071	0.071	1.201	0.001	0.084	0.084	1.362	0.162	0.092	0.118	1.261	0.061	0.16	0.164
15	800	1.215	0.015	0.044	0.044	1.227	0.027	0.05	0.051	1.337	0.137	0.049	0.068	1.282	0.082	0.073	0.08
30	100	1.22	0.02	0.172	0.172	1.282	0.082	0.235	0.242	1.546	0.346	0.24	0.359	1.356	0.156	0.44	0.464
30	200	1.218	0.018	0.079	0.08	1.255	0.055	0.104	0.107	1.43	0.23	0.093	0.146	1.314	0.114	0.184	0.197
30	400	1.204	0.004	0.048	0.048	1.223	0.023	0.057	0.058	1.339	0.139	0.061	0.08	1.267	0.067	0.099	0.104
30	800	1.213	0.013	0.017	0.017	1.224	0.024	0.023	0.023	1.308	0.108	0.02	0.032	1.286	0.086	0.027	0.035
45	100	1.242	0.042	0.124	0.125	1.271	0.071	0.18	0.185	1.509	0.309	0.154	0.249	1.311	0.111	0.338	0.351
45	200	1.228	0.028	0.059	0.06	1.253	0.053	0.087	0.089	1.445	0.245	0.091	0.151	1.356	0.156	0.161	0.185
45	400	1.207	0.007	0.032	0.032	1.213	0.013	0.04	0.04	1.336	0.136	0.037	0.055	1.289	0.089	0.061	0.069
45	800	1.212	0.012	0.015	0.015	1.225	0.025	0.019	0.019	1.299	0.099	0.018	0.028	1.278	0.078	0.022	0.028

Table 6: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.32	0.68	9.89	10.34	12.72	2.28	20.01	25.22	14.06	0.94	23.49	24.37
15	200	14.17	0.83	9.26	9.96	12.76	2.24	18.86	23.86	14.73	0.27	23.57	23.65
15	400	14.29	0.71	9.56	10.07	13.03	1.97	22.9	26.78	14.51	0.49	20.91	21.16
15	800	14.29	0.71	7.28	7.78	12.82	2.18	18.56	23.29	14.86	0.14	20.6	20.62
30	100	28.4	1.6	41.74	44.31	25.46	4.54	95.03	115.62	28.81	1.19	103.03	104.45
30	200	29.06	0.94	42.77	43.66	24.62	5.38	86.61	115.57	27.25	2.75	86.63	94.19
30	400	28.84	1.16	41.93	43.28	23.93	6.07	69.37	106.16	28.23	1.77	88.2	91.33
30	800	29.58	0.42	40.94	41.11	24.69	5.31	73.86	102.02	28.08	1.92	72.72	76.4
45	100	41.14	3.86	117.15	132.09	39.18	5.82	254.43	288.29	41.82	3.18	272.66	282.74
45	200	42.37	2.63	108.88	115.78	36.6	8.4	229.48	300.07	40.61	4.39	216.15	235.44
45	400	44.38	0.62	110.9	111.29	36.33	8.67	233.43	308.65	42.55	2.45	263.63	269.65
45	800	43.56	1.44	89.1	91.17	35.51	9.49	150.54	240.65	42.11	2.89	170.7	179.05

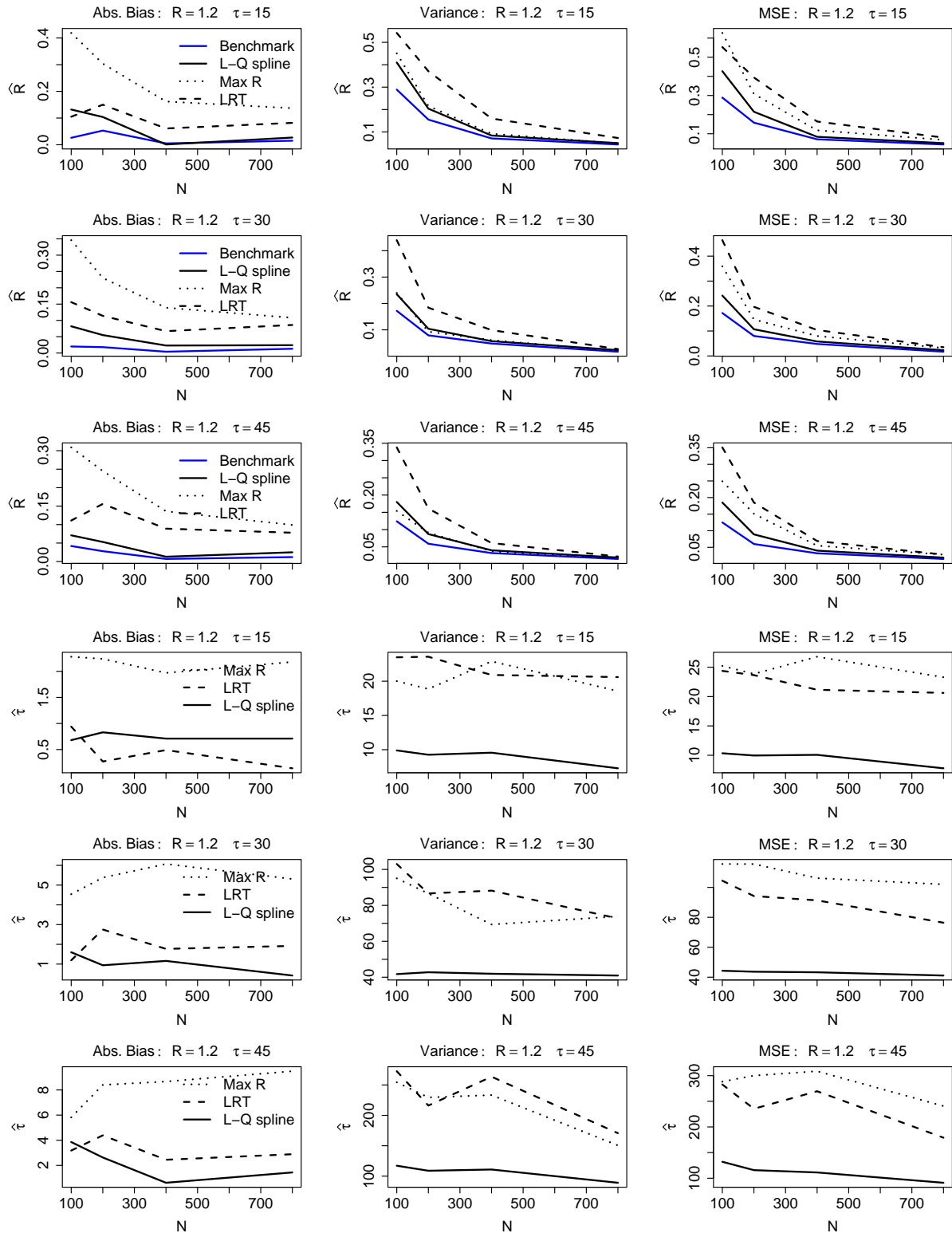


Figure 3: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 7: **Single Uniformly distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	1.572	0.072	0.398	0.404	1.723	0.223	0.601	0.651	2.05	0.55	0.67	0.973	1.882	0.382	0.83	0.976
15	200	1.583	0.083	0.218	0.225	1.63	0.13	0.244	0.261	1.865	0.365	0.268	0.401	1.762	0.262	0.319	0.388
15	400	1.489	0.011	0.084	0.084	1.534	0.034	0.113	0.114	1.676	0.176	0.103	0.134	1.61	0.11	0.124	0.136
15	800	1.501	0.001	0.049	0.049	1.532	0.032	0.056	0.057	1.621	0.121	0.065	0.079	1.579	0.079	0.055	0.061
30	100	1.476	0.024	0.228	0.228	1.54	0.04	0.281	0.282	1.786	0.286	0.302	0.384	1.634	0.134	0.494	0.512
30	200	1.517	0.017	0.105	0.105	1.564	0.064	0.126	0.13	1.735	0.235	0.13	0.185	1.676	0.176	0.16	0.191
30	400	1.529	0.029	0.057	0.057	1.559	0.059	0.075	0.079	1.68	0.18	0.074	0.107	1.647	0.147	0.067	0.088
30	800	1.501	0.001	0.026	0.026	1.526	0.026	0.029	0.029	1.588	0.088	0.031	0.039	1.556	0.056	0.029	0.032
45	100	1.509	0.009	0.176	0.177	1.561	0.061	0.238	0.242	1.807	0.307	0.214	0.308	1.735	0.235	0.28	0.335
45	200	1.491	0.009	0.073	0.073	1.534	0.034	0.088	0.089	1.676	0.176	0.089	0.12	1.634	0.134	0.093	0.111
45	400	1.514	0.014	0.038	0.038	1.543	0.043	0.045	0.047	1.644	0.144	0.048	0.069	1.607	0.107	0.042	0.054
45	800	1.493	0.007	0.021	0.021	1.507	0.007	0.026	0.026	1.578	0.078	0.028	0.034	1.543	0.043	0.023	0.025

Table 8: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.36	0.64	9.49	9.89	11.55	3.45	15.47	27.36	13.89	1.11	20.33	21.56
15	200	14.42	0.58	7.66	8	12.23	2.77	16.83	24.5	14.21	0.79	18.6	19.22
15	400	14.22	0.78	7.46	8.07	11.75	3.25	14.55	25.09	14.12	0.88	16.63	17.42
15	800	14.57	0.43	7.24	7.43	11.89	3.11	13.94	23.59	15.05	0.05	13.78	13.78
30	100	28.29	1.71	35.03	37.96	24.22	5.78	69.51	102.9	28.36	1.64	86.14	88.82
30	200	28.51	1.49	34.82	37.04	22.9	7.1	53.93	104.27	27.49	2.51	67.21	73.53
30	400	28.81	1.19	37.17	38.57	23.05	6.95	49.84	98.14	28.87	1.13	64.98	66.25
30	800	28.76	1.24	24.2	25.73	23.54	6.46	40.79	82.49	29.65	0.35	33.67	33.79
45	100	41.35	3.65	103.41	116.76	37.33	7.67	201.81	260.69	42.66	2.34	223.24	228.72
45	200	42.7	2.3	98.67	103.97	35.22	9.78	139.85	235.48	42.97	2.03	167.43	171.55
45	400	42.24	2.76	82.22	89.86	33.5	11.5	114.45	246.64	42.62	2.38	110.58	116.23
45	800	44.25	0.75	58.84	59.41	34.37	10.63	106.94	219.89	44.23	0.77	85.21	85.81

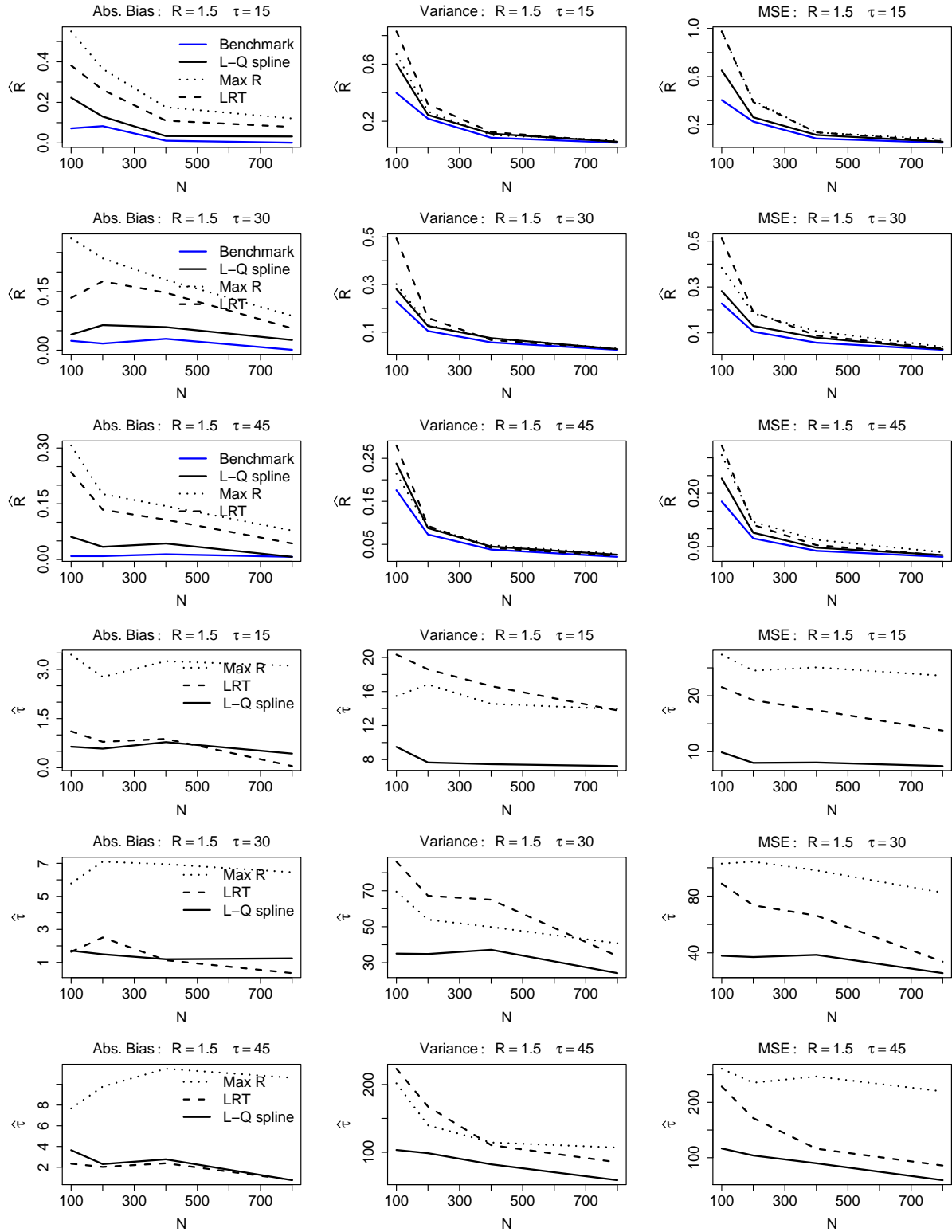


Figure 4: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 9: **Single Uniformly distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.965	0.035	0.649	0.651	2.116	0.116	0.804	0.818	2.405	0.405	0.906	1.07	2.263	0.263	1.031	1.1
15	200	2.047	0.047	0.252	0.255	2.11	0.11	0.306	0.318	2.314	0.314	0.336	0.435	2.237	0.237	0.332	0.388
15	400	2.004	0.004	0.127	0.127	2.037	0.037	0.159	0.16	2.179	0.179	0.181	0.213	2.102	0.102	0.141	0.151
15	800	2.033	0.033	0.074	0.075	2.028	0.028	0.081	0.081	2.156	0.156	0.098	0.122	2.085	0.085	0.086	0.093
30	100	1.98	0.02	0.306	0.307	2.036	0.036	0.416	0.417	2.344	0.344	0.418	0.536	2.24	0.24	0.461	0.518
30	200	2.02	0.02	0.161	0.162	2.079	0.079	0.173	0.18	2.23	0.23	0.182	0.235	2.176	0.176	0.182	0.213
30	400	2.037	0.037	0.078	0.08	2.063	0.063	0.083	0.087	2.172	0.172	0.096	0.125	2.122	0.122	0.088	0.103
30	800	1.982	0.018	0.038	0.039	1.987	0.013	0.043	0.043	2.053	0.053	0.044	0.047	2.006	0.006	0.04	0.04
45	100	2.006	0.006	0.216	0.216	2.102	0.102	0.279	0.289	2.283	0.283	0.27	0.35	2.216	0.216	0.27	0.316
45	200	2.021	0.021	0.112	0.112	2.053	0.053	0.132	0.135	2.221	0.221	0.134	0.183	2.158	0.158	0.123	0.148
45	400	2.012	0.012	0.063	0.063	2.021	0.021	0.065	0.065	2.121	0.121	0.07	0.084	2.074	0.074	0.063	0.069
45	800	1.998	0.002	0.029	0.029	2	0	0.028	0.028	2.053	0.053	0.029	0.032	2.019	0.019	0.028	0.029

Table 10: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	13.92	1.08	8.15	9.31	12.34	2.66	15.1	22.17	14.12	0.88	18.09	18.87	14.12	0.88	18.09	18.87
15	200	14.46	0.54	6.9	7.2	11.9	3.1	10.6	20.18	14.33	0.67	12.07	12.52	14.33	0.67	12.07	12.52
15	400	14.45	0.55	6.62	6.92	12.03	2.97	10.15	18.99	15.12	0.12	10.98	10.99	15.12	0.12	10.98	10.99
15	800	15.25	0.25	4.88	4.94	11.73	3.27	7.79	18.49	14.95	0.05	5.98	5.98	14.95	0.05	5.98	5.98
30	100	27.98	2.02	36.47	40.55	23.96	6.04	63.08	99.57	29.11	0.89	78.77	79.56	29.11	0.89	78.77	79.56
30	200	28.71	1.29	27.35	29.02	23.89	6.11	50.81	88.15	28.97	1.03	47.19	48.24	28.97	1.03	47.19	48.24
30	400	29.38	0.62	22.19	22.58	23.7	6.3	34.08	73.72	28.97	1.03	26.14	27.19	28.97	1.03	26.14	27.19
30	800	29.71	0.29	11.87	11.95	24.29	5.71	33.87	66.52	30.05	0.05	9.82	9.82	30.05	0.05	9.82	9.82
45	100	42.71	2.29	85.83	91.09	34.79	10.21	129.4	233.67	43.95	1.05	127.74	128.84	43.95	1.05	127.74	128.84
45	200	43.84	1.16	67.25	68.6	34.13	10.87	105.09	223.35	42.89	2.11	107.6	112.04	42.89	2.11	107.6	112.04
45	400	45.13	0.13	40.12	40.14	35.78	9.22	88.3	173.24	44.88	0.12	52.24	52.25	44.88	0.12	52.24	52.25
45	800	44.84	0.16	14.12	14.15	37.4	7.6	71.39	129.19	45.04	0.04	14.16	14.17	45.04	0.04	14.16	14.17

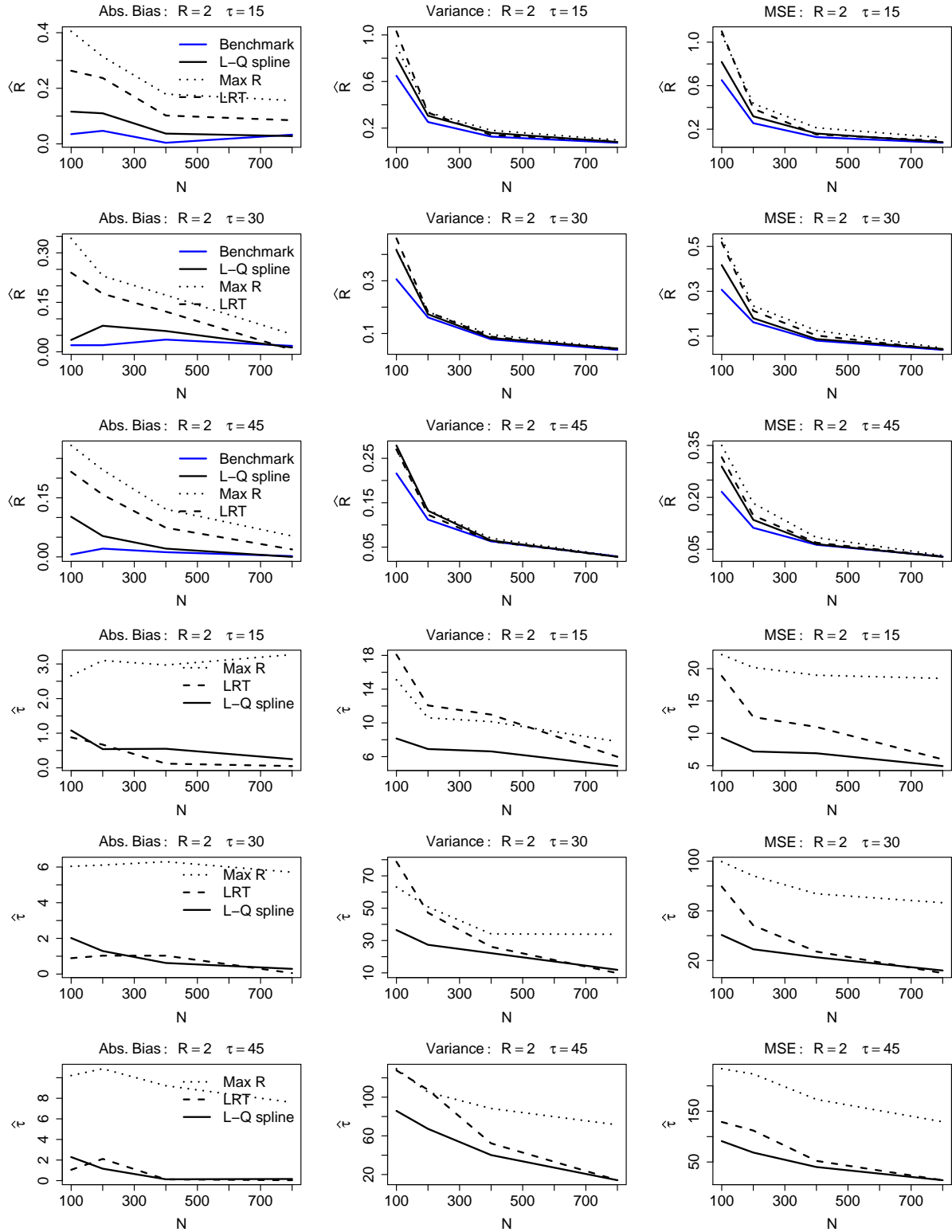


Figure 5: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 11: **Single Uniformly distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	4.094	0.094	1.179	1.188	4.292	0.292	1.324	1.409	4.592	0.592	1.383	1.734	4.238	0.238	1.458	1.514
15	200	4.035	0.035	0.634	0.635	4.118	0.118	0.698	0.711	4.366	0.366	0.822	0.957	4.189	0.189	0.73	0.765
15	400	4.061	0.061	0.33	0.334	4.083	0.083	0.355	0.362	4.285	0.285	0.346	0.427	4.14	0.14	0.343	0.363
15	800	3.998	0.002	0.156	0.156	4	0	0.163	0.163	4.126	0.126	0.176	0.192	4.014	0.014	0.158	0.158
30	100	4.234	0.234	0.948	1.003	4.296	0.296	0.986	1.073	4.62	0.62	1.047	1.431	4.454	0.454	0.989	1.195
30	200	4.129	0.129	0.397	0.414	4.146	0.146	0.413	0.434	4.336	0.336	0.445	0.557	4.195	0.195	0.411	0.45
30	400	4.012	0.012	0.196	0.196	4	0	0.205	0.205	4.099	0.099	0.2	0.21	4.025	0.025	0.206	0.207
30	800	3.998	0.002	0.112	0.112	3.976	0.024	0.112	0.113	4.036	0.036	0.111	0.112	4.006	0.006	0.113	0.113
45	100	4.093	0.093	0.799	0.808	4.137	0.137	0.826	0.844	4.342	0.342	0.826	0.943	4.27	0.27	0.814	0.886
45	200	4.051	0.051	0.381	0.384	4.034	0.034	0.365	0.366	4.161	0.161	0.36	0.386	4.114	0.114	0.36	0.373
45	400	4.051	0.051	0.158	0.161	4.02	0.02	0.161	0.161	4.107	0.107	0.156	0.168	4.075	0.075	0.163	0.169
45	800	4.001	0.001	0.081	0.081	3.974	0.026	0.08	0.081	4.021	0.021	0.083	0.083	4.01	0.01	0.081	0.081

Table 12: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	14.07	0.93	5.87	6.74	11.44	3.56	8.32	20.96	14.59	0.41	8.43	8.61	14.59	0.41	8.43	8.61
15	200	14.57	0.43	3.67	3.85	11.92	3.08	7.19	16.68	14.66	0.34	4.65	4.76	14.66	0.34	4.65	4.76
15	400	14.87	0.13	2.05	2.07	11.85	3.15	6.56	16.49	14.68	0.32	1.83	1.93	14.68	0.32	1.83	1.93
15	800	14.96	0.04	0.59	0.59	12.43	2.57	6.96	13.58	14.97	0.03	0.61	0.61	14.97	0.03	0.61	0.61
30	100	29.58	0.42	21.72	21.9	24.62	5.38	30.87	59.83	29.47	0.53	21.54	21.82	29.47	0.53	21.54	21.82
30	200	29.98	0.02	8.57	8.57	24.96	5.04	29.55	54.95	30.02	0.02	8.36	8.36	30.02	0.02	8.36	8.36
30	400	30.13	0.13	3.28	3.3	26.5	3.5	21.75	34.02	30.26	0.26	3.13	3.19	30.26	0.26	3.13	3.19
30	800	30.13	0.13	0.91	0.93	27.96	2.04	13.24	17.38	29.96	0.04	0.46	0.46	29.96	0.04	0.46	0.46
45	100	44.6	0.4	30.49	30.65	38.41	6.59	83.96	127.36	43.86	1.14	35.82	37.12	43.86	1.14	35.82	37.12
45	200	45.38	0.38	13.07	13.21	40.01	4.99	48.07	73.02	44.95	0.05	14.78	14.78	44.95	0.05	14.78	14.78
45	400	45.27	0.27	4.2	4.27	41.61	3.39	33.23	44.7	44.86	0.14	1.69	1.7	44.86	0.14	1.69	1.7
45	800	45.27	0.27	1.74	1.81	43.43	1.57	13.43	15.89	44.87	0.13	0.43	0.44	44.87	0.13	0.43	0.44

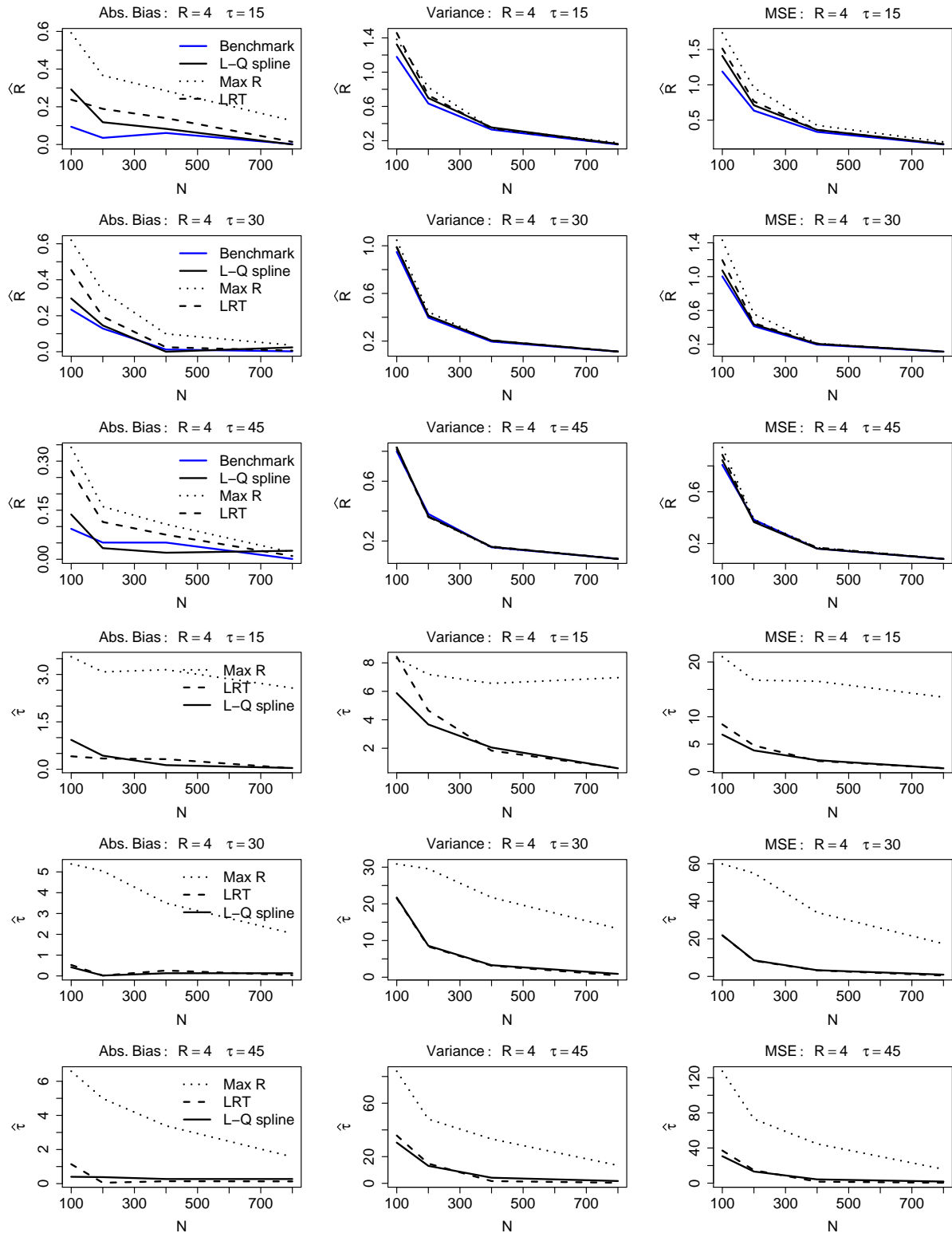


Figure 6: **Single Uniformly distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 13: **Multiple Uniformly distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.689	0.011	0.107	0.107	0.722	0.022	0.138	0.138	0.523	0.177	0.08	0.111	0.7	0	0.183	0.183
15	200	0.664	0.036	0.057	0.059	0.684	0.016	0.073	0.073	0.52	0.18	0.051	0.083	0.582	0.118	0.112	0.126
15	400	0.704	0.004	0.027	0.027	0.704	0.004	0.034	0.034	0.597	0.103	0.028	0.039	0.637	0.063	0.041	0.045
15	800	0.699	0.001	0.018	0.018	0.705	0.005	0.021	0.021	0.628	0.072	0.02	0.026	0.652	0.048	0.021	0.023
30	100	0.684	0.016	0.054	0.054	0.696	0.004	0.071	0.071	0.512	0.188	0.045	0.08	0.585	0.115	0.12	0.134
30	200	0.735	0.035	0.035	0.036	0.75	0.05	0.052	0.052	0.607	0.093	0.036	0.044	0.662	0.038	0.075	0.077
30	400	0.72	0.02	0.016	0.016	0.713	0.013	0.02	0.021	0.636	0.064	0.016	0.02	0.656	0.044	0.021	0.023
30	800	0.699	0.001	0.007	0.007	0.694	0.006	0.008	0.009	0.646	0.054	0.008	0.011	0.666	0.034	0.007	0.008
45	100	0.715	0.015	0.041	0.041	0.73	0.03	0.057	0.058	0.541	0.159	0.042	0.067	0.615	0.085	0.107	0.115
45	200	0.69	0.01	0.022	0.023	0.687	0.013	0.034	0.034	0.567	0.133	0.024	0.042	0.6	0.1	0.029	0.039
45	400	0.689	0.011	0.011	0.011	0.684	0.016	0.015	0.015	0.603	0.097	0.012	0.022	0.622	0.078	0.011	0.017
45	800	0.692	0.008	0.006	0.006	0.687	0.013	0.007	0.007	0.642	0.058	0.006	0.009	0.661	0.039	0.005	0.007

Table 14: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.95	0.05	9.48	9.48	14.13	0.87	21.23	21.99	16.04	1.04	23.43	24.5
15	200	15.02	0.02	8.99	8.99	13.2	1.8	17.91	21.16	15.02	0.02	20.31	20.31
15	400	14.94	0.06	7.64	7.65	12.9	2.1	18.7	23.12	15.41	0.41	18.01	18.18
15	800	14.97	0.03	7.37	7.37	12.1	2.9	13.05	21.45	15.01	0.01	14.58	14.58
30	100	30.29	0.29	44.43	44.51	26.08	3.92	74.66	90.07	29.71	0.29	95.52	95.61
30	200	29.61	0.39	38.65	38.8	25.34	4.66	83.04	104.78	29.13	0.87	92.41	93.18
30	400	29.84	0.16	34.09	34.12	24.85	5.15	68.95	95.48	29.89	0.11	66.68	66.69
30	800	29.56	0.44	27.56	27.76	23.35	6.65	44.55	88.81	30.07	0.07	42.77	42.78
45	100	43.23	1.77	117.33	120.48	37.02	7.98	205.68	269.44	44.26	0.74	226.92	227.48
45	200	43.39	1.61	107.92	110.52	33.52	11.48	182.18	313.91	44.11	0.89	215.25	216.05
45	400	43.77	1.23	88.96	90.48	34.62	10.38	166.25	274.03	43.8	1.2	165.69	167.13
45	800	44.65	0.35	66.63	66.75	33.57	11.43	117.71	248.29	44.7	0.3	92.75	92.84

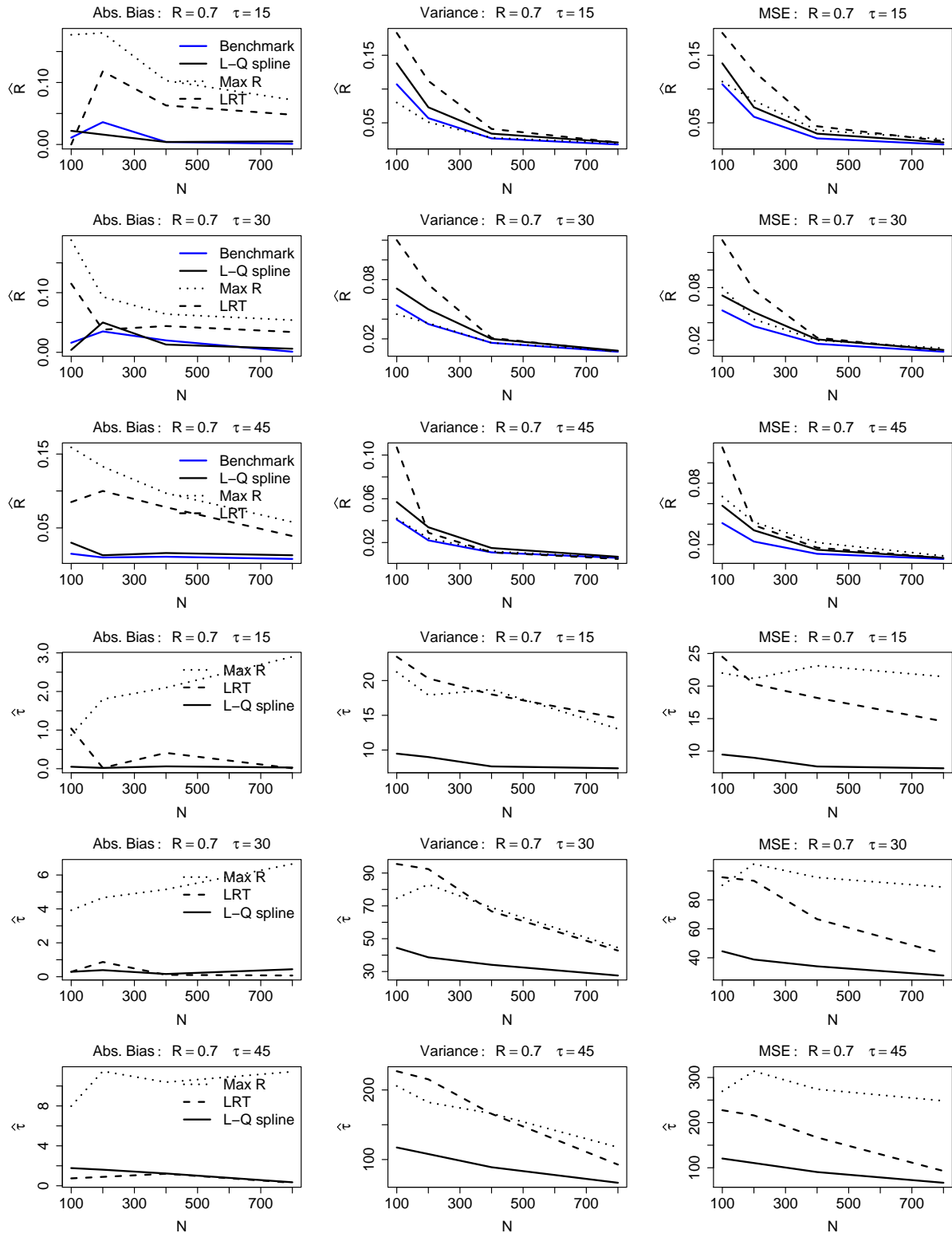


Figure 7: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 15: **Multiple Uniformly distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.941	0.041	0.15	0.151	0.979	0.079	0.202	0.209	0.696	0.204	0.108	0.149	0.958	0.058	0.402	0.406
15	200	0.905	0.005	0.087	0.087	0.927	0.027	0.104	0.105	0.707	0.193	0.084	0.121	0.887	0.013	0.241	0.241
15	400	0.887	0.013	0.04	0.041	0.904	0.004	0.056	0.056	0.755	0.145	0.041	0.062	0.841	0.059	0.093	0.096
15	800	0.914	0.014	0.023	0.023	0.922	0.022	0.029	0.03	0.813	0.087	0.021	0.029	0.875	0.025	0.047	0.048
30	100	0.903	0.003	0.079	0.079	0.945	0.045	0.119	0.121	0.68	0.22	0.056	0.104	0.89	0.01	0.231	0.231
30	200	0.931	0.031	0.048	0.049	0.936	0.036	0.074	0.075	0.772	0.128	0.049	0.065	0.934	0.034	0.157	0.158
30	400	0.917	0.017	0.023	0.024	0.925	0.025	0.033	0.034	0.808	0.092	0.022	0.03	0.897	0.003	0.061	0.061
30	800	0.887	0.013	0.01	0.01	0.885	0.015	0.013	0.013	0.81	0.09	0.01	0.018	0.842	0.058	0.02	0.023
45	100	0.913	0.013	0.072	0.072	0.936	0.036	0.104	0.106	0.7	0.2	0.053	0.093	0.908	0.008	0.23	0.231
45	200	0.919	0.019	0.035	0.036	0.925	0.025	0.046	0.046	0.757	0.143	0.031	0.051	0.879	0.021	0.101	0.101
45	400	0.918	0.018	0.018	0.018	0.915	0.015	0.024	0.024	0.809	0.091	0.015	0.023	0.889	0.011	0.05	0.05
45	800	0.904	0.004	0.008	0.008	0.904	0.004	0.012	0.012	0.834	0.066	0.008	0.013	0.874	0.026	0.021	0.022

Table 16: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.02	0.02	9.64	9.64	14.13	0.87	22.75	23.5	14.76	0.24	21.97	22.03
15	200	15.11	0.11	9.36	9.37	13.59	1.41	23.54	25.53	14.29	0.71	24.16	24.66
15	400	14.68	0.32	9.63	9.73	13.52	1.48	23.61	25.79	14.53	0.47	22.56	22.78
15	800	15	0	10.37	10.37	13.27	1.73	22.46	25.45	14.78	0.22	22.08	22.13
30	100	28.93	1.07	47.94	49.09	26.64	3.36	110.6	121.9	27.97	2.03	98.87	102.99
30	200	28.94	1.06	43.62	44.75	27.22	2.78	104.82	112.57	28.35	1.65	103.33	106.05
30	400	28.84	1.16	40.52	41.86	27.42	2.58	100	106.65	29.37	0.63	98.58	98.97
30	800	29.06	0.94	40.46	41.34	25.26	4.74	96.43	118.89	29.16	0.84	91.98	92.69
45	100	42.46	2.54	93.9	100.34	41.23	3.77	259.51	273.72	41.04	3.96	244.59	260.31
45	200	43	2	110.37	114.37	37.72	7.28	268.72	321.74	42.82	2.18	281.6	286.34
45	400	43.14	1.86	107.25	110.73	39.73	5.27	270.19	297.97	42.8	2.2	276.68	281.5
45	800	44.47	0.53	118.46	118.74	36.71	8.29	234.66	303.41	43.1	1.9	240.78	244.41

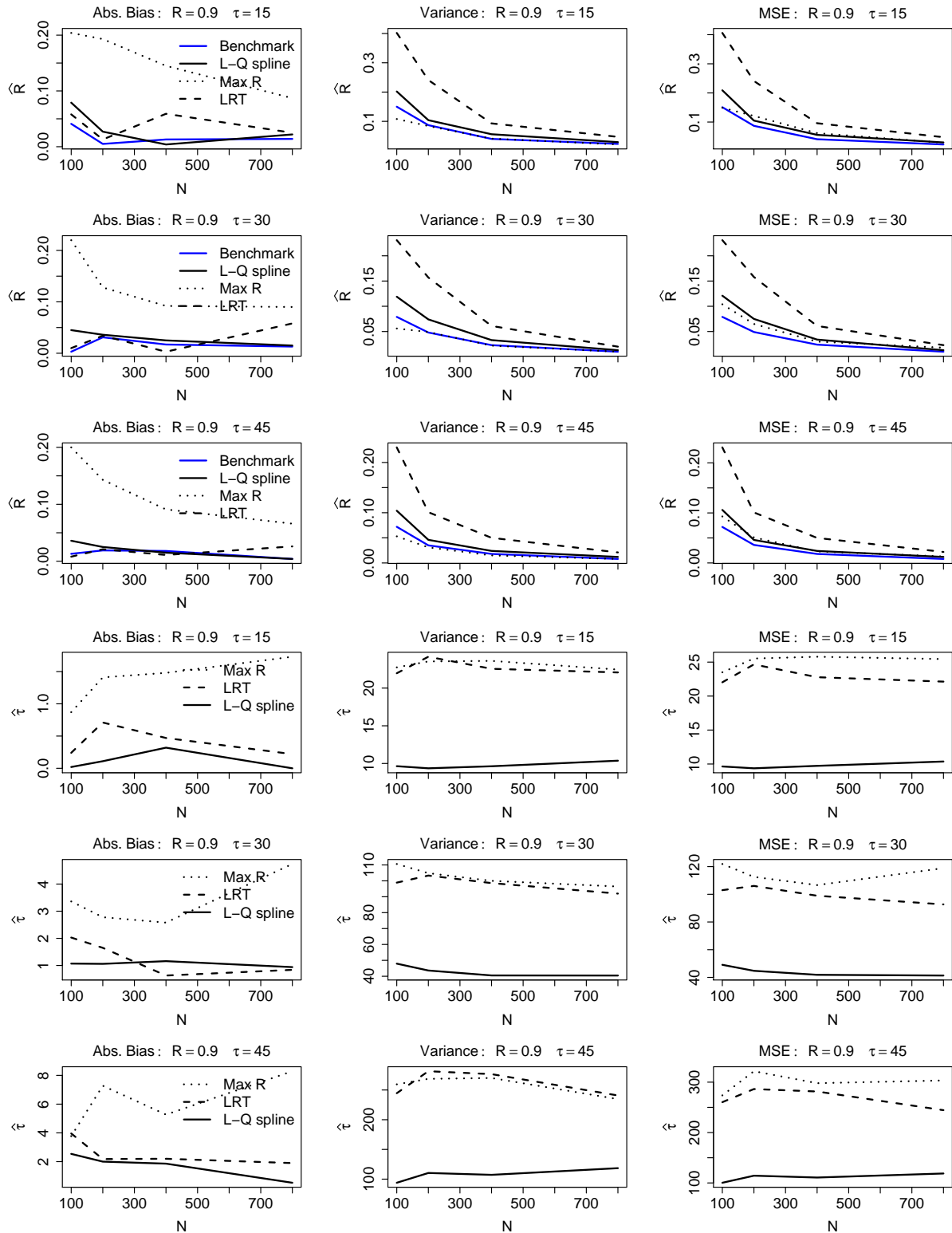


Figure 8: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 17: **Multiple Uniformly distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.167	0.033	0.218	0.219	1.247	0.047	0.289	0.292	1.492	0.292	0.255	0.34	1.276	0.076	0.491	0.496
15	200	1.258	0.058	0.122	0.125	1.282	0.082	0.16	0.166	1.496	0.296	0.167	0.254	1.364	0.164	0.284	0.311
15	400	1.229	0.029	0.056	0.057	1.254	0.054	0.079	0.082	1.39	0.19	0.083	0.119	1.324	0.124	0.122	0.137
15	800	1.217	0.017	0.024	0.024	1.229	0.029	0.033	0.034	1.311	0.111	0.029	0.041	1.281	0.081	0.041	0.048
30	100	1.188	0.012	0.124	0.124	1.228	0.028	0.185	0.186	1.483	0.283	0.153	0.233	1.322	0.122	0.31	0.325
30	200	1.236	0.036	0.072	0.073	1.281	0.081	0.093	0.099	1.434	0.234	0.094	0.148	1.339	0.139	0.165	0.184
30	400	1.229	0.029	0.033	0.034	1.251	0.051	0.044	0.047	1.359	0.159	0.036	0.061	1.318	0.118	0.053	0.067
30	800	1.192	0.008	0.014	0.014	1.203	0.003	0.017	0.018	1.275	0.075	0.018	0.023	1.253	0.053	0.022	0.025
45	100	1.21	0.01	0.104	0.104	1.258	0.058	0.141	0.144	1.485	0.285	0.135	0.217	1.348	0.148	0.252	0.274
45	200	1.207	0.007	0.058	0.058	1.237	0.037	0.078	0.079	1.371	0.171	0.065	0.095	1.269	0.069	0.133	0.137
45	400	1.21	0.01	0.027	0.027	1.225	0.025	0.033	0.033	1.326	0.126	0.035	0.051	1.285	0.085	0.052	0.059
45	800	1.197	0.003	0.011	0.011	1.212	0.012	0.014	0.014	1.273	0.073	0.012	0.017	1.256	0.056	0.014	0.017

Table 18: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.16	0.84	8.21	8.92	13.05	1.95	23.1	26.9	14.58	0.42	22.59	22.77
15	200	14.39	0.61	9.32	9.69	12.71	2.29	19.87	25.1	14.22	0.78	24.43	25.05
15	400	14.25	0.75	8.89	9.45	12.5	2.5	16.66	22.92	14.38	0.62	19.3	19.68
15	800	14.69	0.31	8.31	8.41	12.96	2.04	17.14	21.28	14.4	0.6	16.64	17.01
30	100	29.39	0.61	42.87	43.24	26.4	3.6	100.84	113.82	28.38	1.62	102.51	105.14
30	200	28.44	1.56	37.8	40.23	25.58	4.42	81.48	101.04	28.33	1.67	83.48	86.28
30	400	28.74	1.26	43.5	45.08	24.68	5.32	81.84	110.11	29.09	0.91	92.26	93.1
30	800	28.82	1.18	38.88	40.28	23.98	6.02	61.55	97.73	28.81	1.19	77.52	78.94
45	100	42.13	2.87	110.55	118.78	36.66	8.34	240.67	310.17	41.41	3.59	265.37	278.28
45	200	42.59	2.41	94.61	100.41	36.86	8.14	208.35	274.54	41.77	3.23	195.97	206.38
45	400	42.11	2.89	94.84	103.22	36.95	8.05	204.68	269.4	42.56	2.44	238.48	244.44
45	800	44.02	0.98	97.26	98.23	35.14	9.86	152.89	250.1	42.18	2.82	178.12	186.1

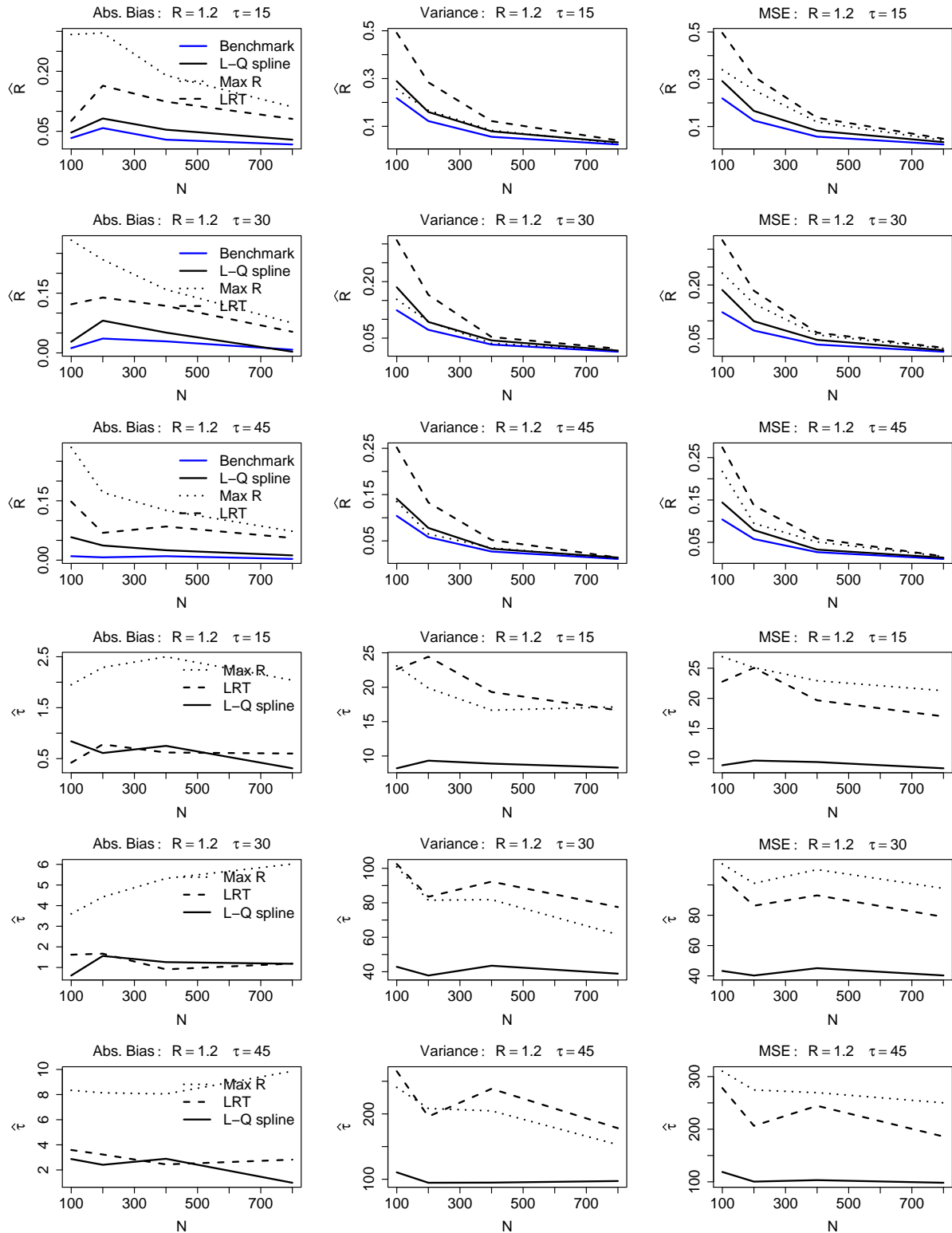


Figure 9: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 19: **Multiple Uniformly distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.537	0.037	0.39	0.391	1.611	0.111	0.484	0.497	1.886	0.386	0.507	0.656	1.728	0.228	0.729	0.781
15	200	1.52	0.02	0.139	0.139	1.551	0.051	0.162	0.164	1.74	0.24	0.186	0.243	1.678	0.178	0.214	0.246
15	400	1.503	0.003	0.067	0.067	1.548	0.048	0.092	0.094	1.662	0.162	0.097	0.123	1.612	0.112	0.098	0.11
15	800	1.508	0.008	0.038	0.039	1.518	0.018	0.046	0.046	1.603	0.103	0.043	0.054	1.569	0.069	0.036	0.041
30	100	1.562	0.062	0.14	0.144	1.617	0.117	0.194	0.208	1.844	0.344	0.207	0.325	1.789	0.289	0.265	0.348
30	200	1.468	0.032	0.072	0.073	1.508	0.008	0.092	0.092	1.647	0.147	0.086	0.107	1.603	0.103	0.105	0.116
30	400	1.508	0.008	0.041	0.041	1.54	0.04	0.051	0.052	1.624	0.124	0.049	0.064	1.595	0.095	0.048	0.057
30	800	1.509	0.009	0.016	0.016	1.529	0.029	0.018	0.018	1.576	0.076	0.019	0.025	1.552	0.052	0.017	0.02
45	100	1.551	0.051	0.169	0.171	1.616	0.116	0.231	0.244	1.827	0.327	0.217	0.324	1.757	0.257	0.302	0.369
45	200	1.509	0.009	0.069	0.069	1.543	0.043	0.081	0.083	1.688	0.188	0.085	0.12	1.649	0.149	0.097	0.12
45	400	1.509	0.009	0.037	0.037	1.541	0.041	0.047	0.049	1.616	0.116	0.044	0.058	1.593	0.093	0.04	0.049
45	800	1.485	0.015	0.018	0.019	1.494	0.006	0.019	0.019	1.545	0.045	0.018	0.02	1.522	0.022	0.017	0.018

Table 20: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \hat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \hat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.1	0.9	7.92	8.73	12.87	2.13	18.71	23.25	13.9	1.1	20.27	21.47
15	200	14.49	0.51	8.19	8.45	12.33	2.67	14.13	21.28	14.5	0.5	19.03	19.29
15	400	14.42	0.58	8.93	9.26	11.79	3.21	10.4	20.68	14.83	0.17	16.32	16.35
15	800	14.69	0.31	6.58	6.68	12.02	2.98	9.66	18.57	14.74	0.26	11.3	11.37
30	100	28.84	1.16	42.94	44.27	25.49	4.51	80.56	100.92	28.97	1.03	89.77	90.83
30	200	29.04	0.96	35.54	36.47	24.64	5.36	59.57	88.26	29.3	0.7	71.23	71.73
30	400	28.79	1.21	34.29	35.75	23.88	6.12	46.37	83.83	29.27	0.73	48.43	48.96
30	800	29.17	0.83	24.61	25.3	24.49	5.51	35.92	66.31	29.53	0.47	29.77	29.98
45	100	41.82	3.18	99.71	109.8	36.84	8.16	193.79	260.39	41.59	3.41	214.61	226.22
45	200	43.19	1.81	100.46	103.75	34.82	10.18	174.91	278.46	41.54	3.46	162.15	174.14
45	400	43.7	1.3	68.55	70.23	37.46	7.54	132.5	189.39	44.2	0.8	114.45	115.1
45	800	45.06	0.06	48.58	48.59	35.39	9.61	93.95	186.26	45.18	0.18	69.18	69.22

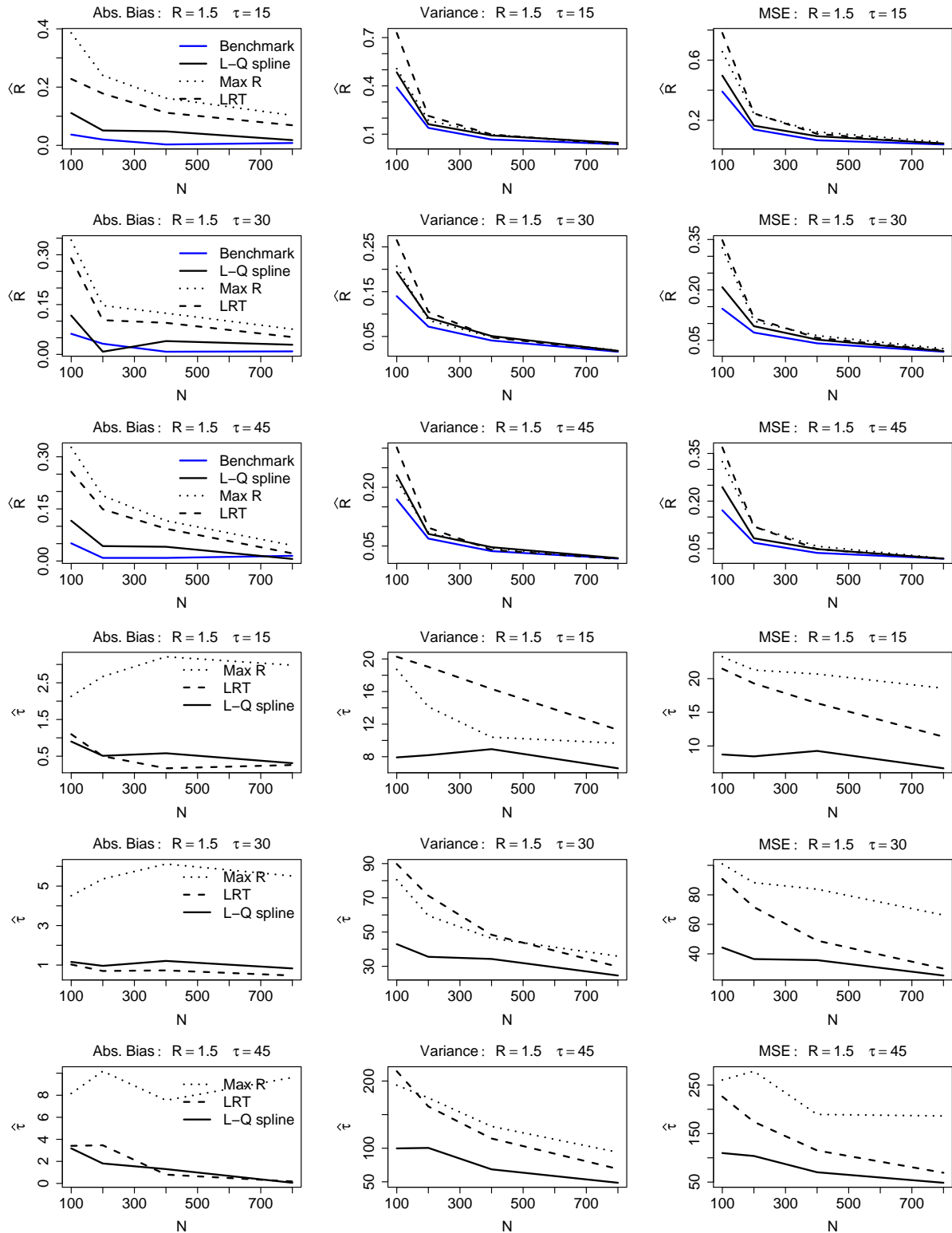


Figure 10: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 21: **Multiple Uniformly distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.022	0.022	0.432	0.433	2.145	0.145	0.576	0.597	2.385	0.385	0.627	0.775	2.31	0.31	0.66	0.756
15	200	1.966	0.034	0.207	0.208	2.027	0.027	0.243	0.243	2.227	0.227	0.269	0.321	2.147	0.147	0.281	0.303
15	400	1.986	0.014	0.105	0.105	2.016	0.016	0.127	0.128	2.139	0.139	0.136	0.156	2.079	0.079	0.12	0.126
15	800	2.001	0.001	0.059	0.059	2.006	0.006	0.061	0.061	2.096	0.096	0.065	0.075	2.033	0.033	0.059	0.06
30	100	2.112	0.112	0.256	0.268	2.184	0.184	0.279	0.313	2.399	0.399	0.302	0.462	2.345	0.345	0.306	0.426
30	200	2.035	0.035	0.113	0.114	2.081	0.081	0.131	0.138	2.203	0.203	0.131	0.172	2.144	0.144	0.125	0.145
30	400	2.041	0.041	0.062	0.063	2.067	0.067	0.068	0.072	2.146	0.146	0.068	0.089	2.099	0.099	0.066	0.076
30	800	1.995	0.005	0.029	0.029	1.993	0.007	0.032	0.032	2.055	0.055	0.033	0.036	2.016	0.016	0.031	0.031
45	100	2.017	0.017	0.23	0.23	2.062	0.062	0.247	0.251	2.259	0.259	0.255	0.322	2.21	0.21	0.243	0.287
45	200	2.063	0.063	0.126	0.131	2.093	0.093	0.138	0.147	2.211	0.211	0.135	0.179	2.173	0.173	0.135	0.164
45	400	1.988	0.012	0.053	0.054	1.992	0.008	0.058	0.058	2.065	0.065	0.054	0.058	2.034	0.034	0.054	0.055
45	800	2.023	0.023	0.027	0.027	2.014	0.014	0.027	0.027	2.061	0.061	0.028	0.031	2.045	0.045	0.027	0.029

Table 22: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	14.33	0.67	8.35	8.8	12.57	2.43	14.43	20.32	14.17	0.83	15.8	16.49	14.17	0.83	15.8	16.49
15	200	14.43	0.57	6.98	7.3	11.75	3.25	12.63	23.2	14.44	0.56	13.65	13.96	14.44	0.56	13.65	13.96
15	400	14.7	0.3	6.74	6.83	11.96	3.04	8.53	17.77	14.77	0.23	8.26	8.31	14.77	0.23	8.26	8.31
15	800	14.99	0.01	3.92	3.92	11.78	3.22	7.4	17.74	15.03	0.03	4.07	4.07	15.03	0.03	4.07	4.07
30	100	28.88	1.12	34.21	35.46	23.87	6.13	44.78	82.31	28.3	1.7	52.23	55.13	28.3	1.7	52.23	55.13
30	200	29.1	0.9	27.22	28.03	24.22	5.78	42.94	76.4	30.4	0.4	48	48.16	30.4	0.4	48	48.16
30	400	29.42	0.58	16.59	16.93	24.25	5.75	31.07	64.18	29.81	0.19	24.5	24.53	29.81	0.19	24.5	24.53
30	800	30.31	0.31	9.37	9.46	25.47	4.53	27.79	48.29	30.16	0.16	6.98	7	30.16	0.16	6.98	7
45	100	43.09	1.91	87.27	90.91	37.32	7.68	151.53	210.49	42.88	2.12	129.27	133.77	42.88	2.12	129.27	133.77
45	200	44.76	0.24	60.64	60.69	37.06	7.94	113.14	176.18	44.65	0.35	86.88	87	44.65	0.35	86.88	87
45	400	44.83	0.17	33.87	33.89	38.52	6.48	80.7	122.72	45.11	0.11	41.81	41.82	45.11	0.11	41.81	41.82
45	800	45.71	0.71	13.55	14.06	40.21	4.79	54.4	77.33	44.83	0.17	8.84	8.86	44.83	0.17	8.84	8.86

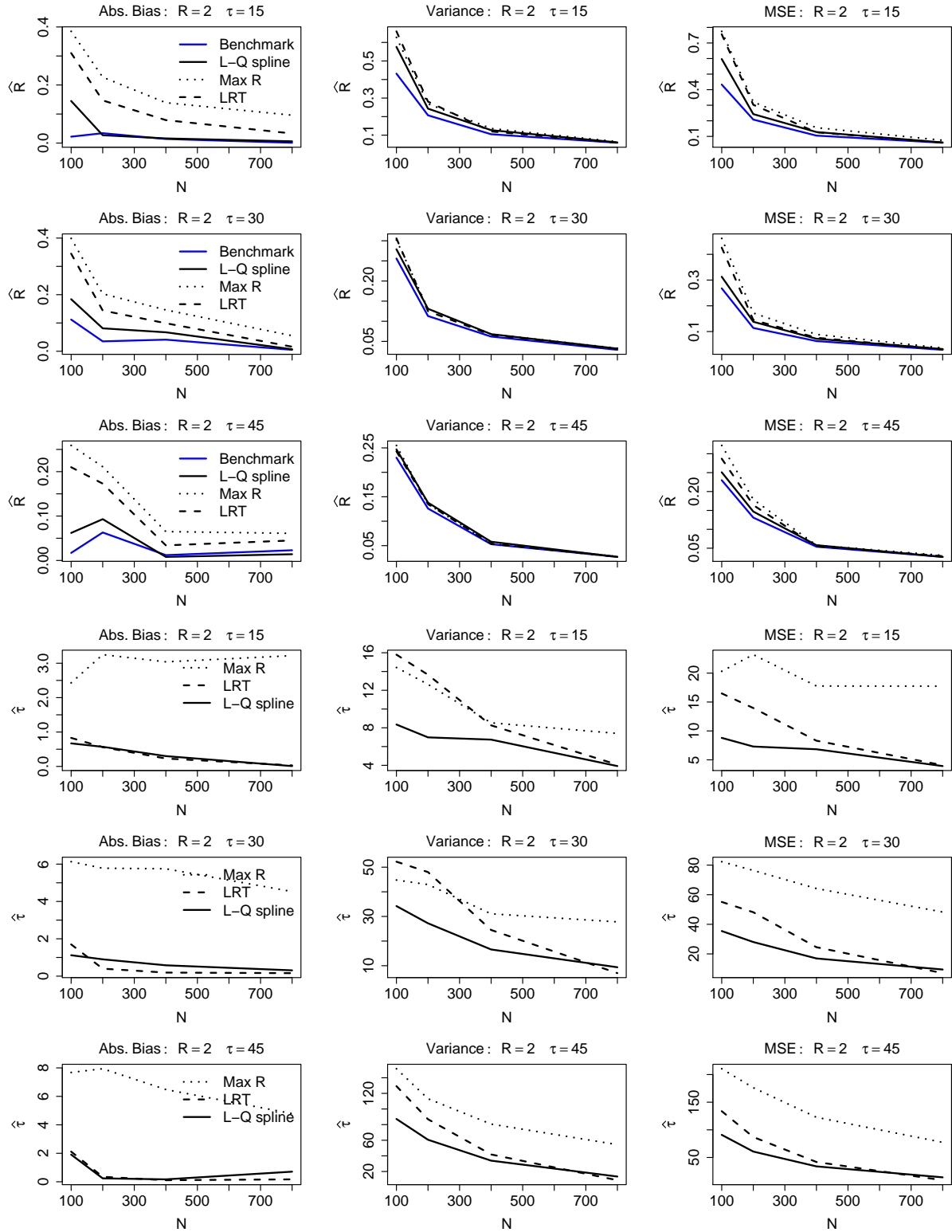


Figure 11: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 23: **Multiple Uniformly distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	3.906	0.094	0.928	0.937	3.985	0.015	1.081	1.081	4.299	0.299	1.111	1.201	4.118	0.118	0.981	0.995
15	200	4.001	0.001	0.52	0.52	4.05	0.05	0.585	0.587	4.198	0.198	0.588	0.627	4.093	0.093	0.587	0.596
15	400	3.977	0.023	0.255	0.256	3.985	0.015	0.278	0.278	4.11	0.11	0.266	0.278	4	0	0.259	0.259
15	800	4.005	0.005	0.129	0.129	3.997	0.003	0.134	0.134	4.062	0.062	0.126	0.129	4.006	0.006	0.129	0.129
30	100	4.097	0.097	0.976	0.986	4.172	0.172	1.058	1.087	4.36	0.36	1.065	1.195	4.275	0.275	1.052	1.128
30	200	4.01	0.01	0.385	0.386	4.028	0.028	0.391	0.391	4.135	0.135	0.383	0.402	4.089	0.089	0.4	0.408
30	400	4.052	0.052	0.188	0.19	4.027	0.027	0.194	0.194	4.099	0.099	0.185	0.195	4.076	0.076	0.189	0.195
30	800	4.036	0.036	0.095	0.096	4.014	0.014	0.096	0.096	4.053	0.053	0.094	0.096	4.043	0.043	0.095	0.097
45	100	4.05	0.05	0.6	0.602	4.068	0.068	0.62	0.624	4.265	0.265	0.598	0.668	4.219	0.219	0.622	0.67
45	200	4.005	0.005	0.361	0.361	3.969	0.031	0.354	0.355	4.082	0.082	0.36	0.367	4.053	0.053	0.363	0.366
45	400	4.041	0.041	0.166	0.167	4.004	0.004	0.162	0.162	4.069	0.069	0.162	0.166	4.061	0.061	0.163	0.167
45	800	4.021	0.021	0.093	0.094	3.979	0.021	0.089	0.09	4.025	0.025	0.093	0.094	4.022	0.022	0.093	0.094

Table 24: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.52	0.48	5.3	5.53	12.15	2.85	8.35	16.49	14.89	0.11	7.66	7.67
15	200	14.76	0.24	2.76	2.82	12.7	2.3	6.6	11.87	14.92	0.08	3.1	3.11
15	400	14.85	0.15	1.57	1.59	12.7	2.3	6.2	11.5	15	0	0.96	0.96
15	800	15	0	0.31	0.31	13.27	1.73	5.24	8.23	15.04	0.04	0.15	0.16
30	100	29.48	0.52	14.97	15.24	26.15	3.85	27.71	42.57	29.81	0.19	18.83	18.87
30	200	29.69	0.31	6.76	6.85	26.75	3.25	20.08	30.65	29.46	0.54	5.95	6.24
30	400	30.21	0.21	2.11	2.15	28.39	1.61	7.27	9.86	29.9	0.1	1.11	1.12
30	800	30.04	0.04	0.81	0.81	29.18	0.82	3.79	4.46	29.92	0.08	0.37	0.37
45	100	44.66	0.34	36.93	37.04	40.19	4.81	61.34	84.47	44.61	0.39	44.78	44.93
45	200	45.48	0.48	10.9	11.14	42.15	2.85	29.18	37.33	44.9	0.1	10.63	10.64
45	400	45.53	0.53	2.77	3.06	43.98	1.02	9.48	10.51	45	0	1.44	1.44
45	800	45.62	0.62	1.22	1.61	44.78	0.22	0.8	0.84	45.1	0.1	0.25	0.26

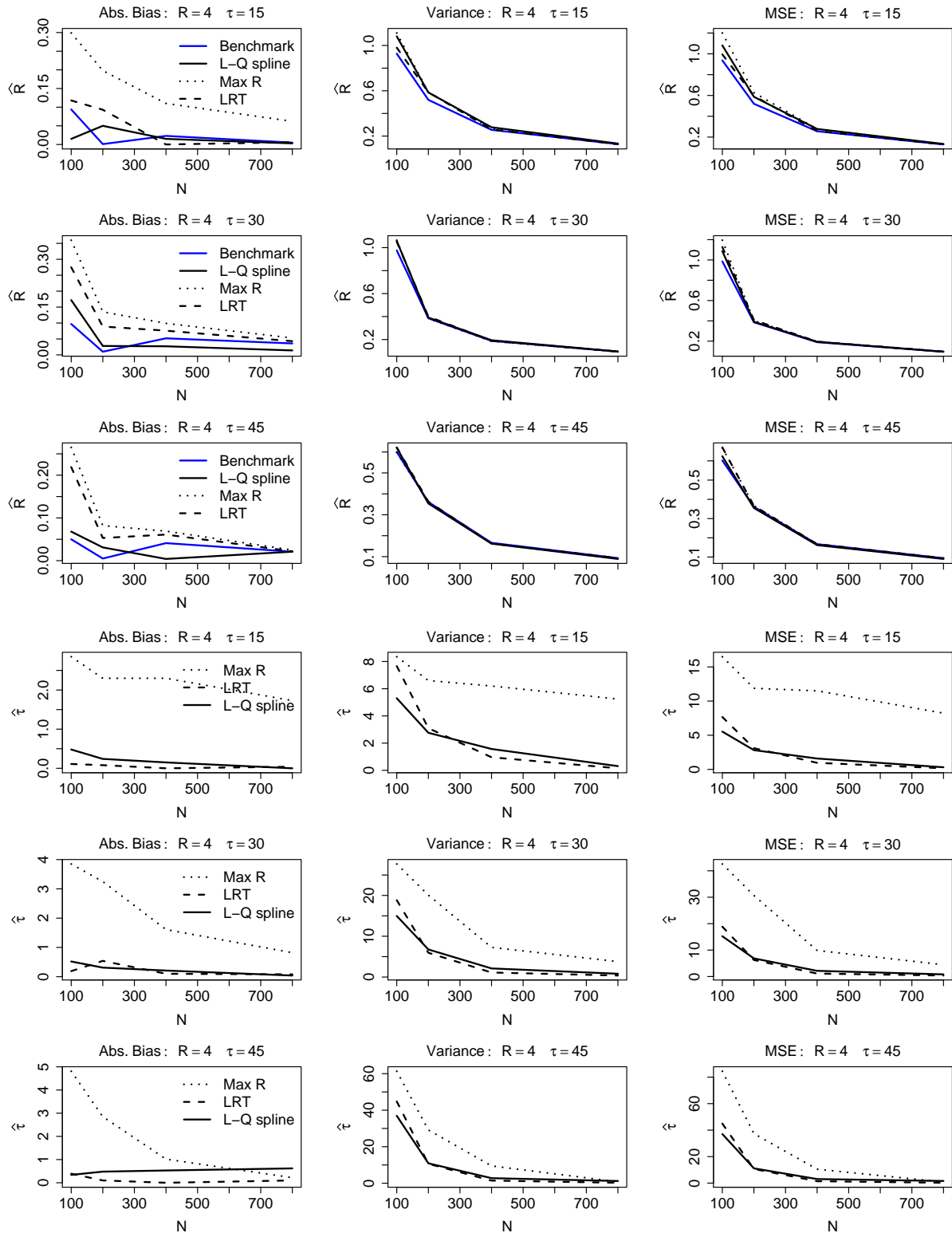


Figure 12: **Multiple Uniformly distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 25: **Single Normally distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	0.731	0.031	0.15	0.15	0.756	0.056	0.2	0.203	0.552	0.148	0.084	0.106	0.728	0.028	0.287	0.288
15	200	0.729	0.029	0.078	0.079	0.749	0.049	0.089	0.092	0.554	0.146	0.064	0.085	0.633	0.067	0.154	0.158
15	400	0.695	0.005	0.042	0.042	0.703	0.003	0.053	0.053	0.573	0.127	0.04	0.057	0.62	0.08	0.076	0.083
15	800	0.707	0.007	0.019	0.019	0.704	0.004	0.024	0.024	0.625	0.075	0.021	0.026	0.65	0.05	0.024	0.026
30	100	0.718	0.018	0.08	0.081	0.757	0.057	0.115	0.118	0.517	0.183	0.063	0.097	0.746	0.046	0.084	0.086
30	200	0.686	0.014	0.041	0.041	0.689	0.011	0.059	0.059	0.536	0.164	0.044	0.071	0.592	0.108	0.088	0.1
30	400	0.69	0.01	0.02	0.021	0.692	0.008	0.025	0.025	0.595	0.105	0.023	0.034	0.628	0.072	0.031	0.036
30	800	0.703	0.003	0.013	0.013	0.693	0.007	0.014	0.015	0.634	0.066	0.015	0.019	0.657	0.043	0.017	0.018
45	100	0.715	0.015	0.059	0.059	0.714	0.014	0.075	0.075	0.532	0.168	0.046	0.075	0.613	0.087	0.117	0.125
45	200	0.718	0.018	0.03	0.03	0.709	0.009	0.042	0.042	0.581	0.119	0.03	0.044	0.63	0.07	0.059	0.064
45	400	0.701	0.001	0.016	0.016	0.693	0.007	0.02	0.021	0.615	0.085	0.017	0.024	0.64	0.06	0.019	0.023
45	800	0.712	0.012	0.008	0.008	0.706	0.006	0.009	0.009	0.65	0.05	0.009	0.011	0.674	0.026	0.008	0.008

Table 26: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.48	0.48	9.15	9.38	14.7	0.3	20.32	20.41	15.74	0.74	21.51	22.07
15	200	15.05	0.05	9.81	9.81	13.22	1.78	20.99	24.18	15.08	0.08	23.04	23.05
15	400	15.41	0.41	9.94	10.11	13.3	1.7	18.96	21.85	14.89	0.11	19.71	19.72
15	800	14.65	0.35	8.64	8.76	13.11	1.89	18.23	21.81	15.56	0.56	17.26	17.58
30	100	29.03	0.97	41.34	42.28	26.24	3.76	87.1	101.22	28.7	1.3	93.21	94.91
30	200	29.47	0.53	42.37	42.65	25.34	4.66	67.96	89.71	29.43	0.57	89.32	89.64
30	400	30.59	0.59	33.5	33.84	23.73	6.27	61	100.33	29.6	0.4	75.32	75.48
30	800	29.94	0.06	27.43	27.44	22.88	7.12	48.35	98.98	29.72	0.28	58.42	58.5
45	100	43.4	1.6	100.88	103.45	37.35	7.65	221.22	279.79	45.87	0.87	243.47	244.23
45	200	43.91	1.09	99.1	100.29	35.81	9.19	190.78	275.25	44.11	0.89	219.33	220.13
45	400	43.75	1.25	94.22	95.77	34.84	10.16	154.89	258.03	44.17	0.83	174.08	174.77
45	800	45.07	0.07	88.57	88.57	33.43	11.57	128.57	262.38	45.38	0.38	124.87	125.01

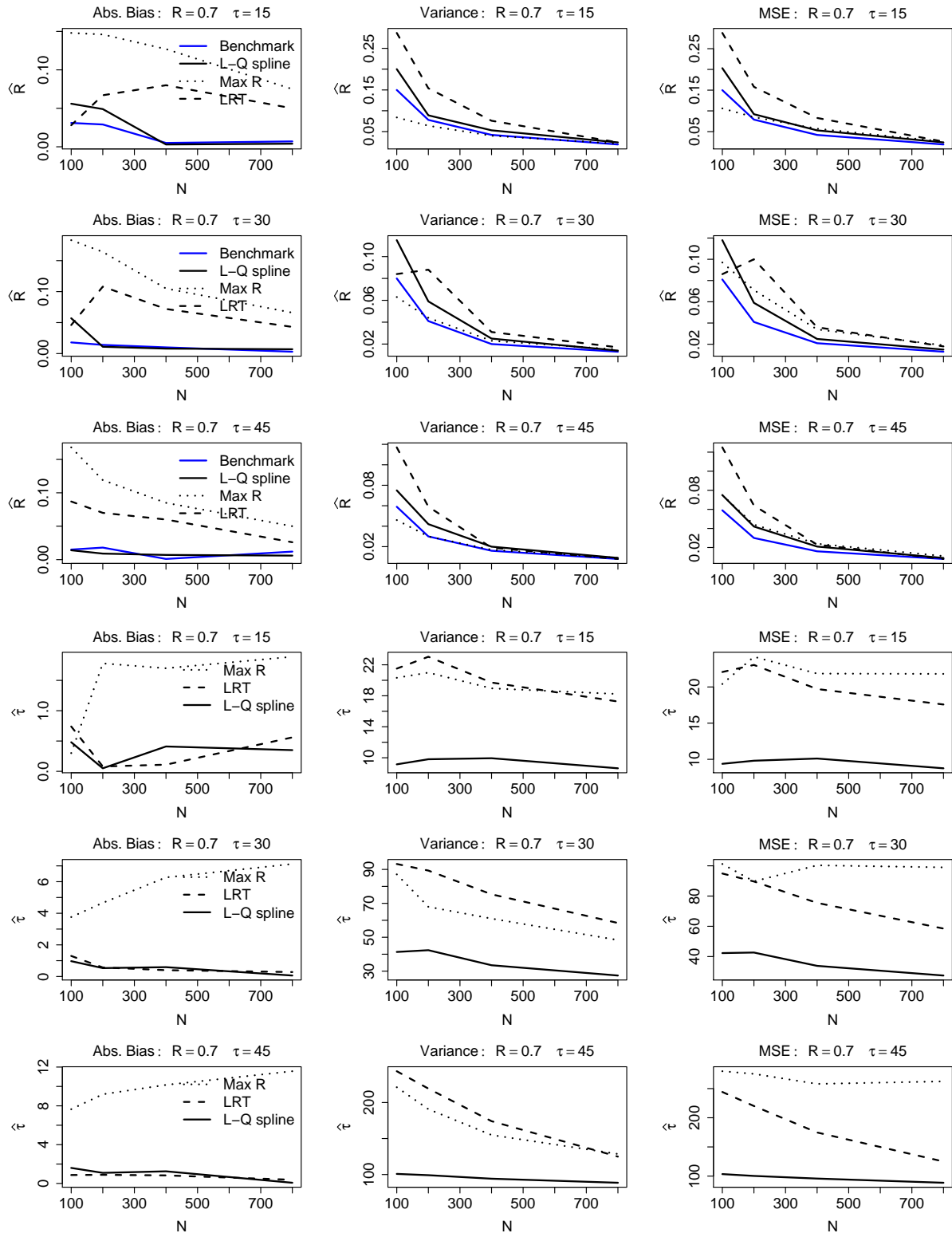


Figure 13: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 27: **Single Normally distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	0.973	0.073	0.227	0.233	1.049	0.149	0.287	0.309	0.701	0.199	0.147	0.187	0.989	0.089	0.348	0.356
15	200	0.894	0.006	0.114	0.114	0.92	0.02	0.15	0.151	0.68	0.22	0.094	0.142	0.856	0.044	0.265	0.267
15	400	0.895	0.005	0.05	0.05	0.911	0.011	0.063	0.064	0.746	0.154	0.05	0.073	0.874	0.026	0.139	0.14
15	800	0.908	0.008	0.027	0.027	0.905	0.005	0.031	0.031	0.795	0.105	0.025	0.036	0.876	0.024	0.061	0.062
30	100	0.916	0.016	0.107	0.107	0.99	0.09	0.159	0.167	0.667	0.233	0.09	0.144	0.904	0.004	0.319	0.319
30	200	0.914	0.014	0.051	0.051	0.931	0.031	0.071	0.072	0.728	0.172	0.049	0.078	0.885	0.015	0.15	0.15
30	400	0.908	0.008	0.028	0.028	0.918	0.018	0.04	0.04	0.781	0.119	0.033	0.047	0.878	0.022	0.082	0.083
30	800	0.897	0.003	0.014	0.014	0.894	0.006	0.019	0.019	0.806	0.094	0.015	0.024	0.851	0.049	0.033	0.035
45	100	0.887	0.013	0.073	0.073	0.91	0.01	0.1	0.1	0.67	0.23	0.06	0.113	0.888	0.012	0.254	0.255
45	200	0.913	0.013	0.042	0.042	0.93	0.03	0.063	0.063	0.751	0.149	0.041	0.063	0.89	0.01	0.129	0.13
45	400	0.919	0.019	0.018	0.018	0.918	0.018	0.026	0.026	0.796	0.104	0.017	0.027	0.883	0.017	0.057	0.058
45	800	0.895	0.005	0.01	0.01	0.892	0.008	0.014	0.014	0.813	0.087	0.009	0.017	0.855	0.045	0.024	0.026

Table 28: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.74	0.26	9.4	9.46	14.66	0.34	25.23	25.34	15.16	0.16	20.84	20.87
15	200	14.45	0.55	8.38	8.68	13.96	1.04	23.72	24.79	14.77	0.23	22.9	22.95
15	400	14.53	0.47	8.68	8.9	12.99	2.01	21.41	25.45	14.17	0.83	23.03	23.71
15	800	14.7	0.3	9.77	9.85	12.73	2.27	18.77	23.93	14.9	0.1	22.66	22.67
30	100	28.57	1.43	43.64	45.69	27.42	2.58	112.67	119.31	29.22	0.78	110.72	111.33
30	200	28.2	1.8	42.1	45.35	26.1	3.9	93.09	108.29	28.48	1.52	97.34	99.64
30	400	29.45	0.55	42.58	42.88	25.89	4.11	94.79	111.64	29.91	0.09	99.63	99.63
30	800	29.37	0.63	48.73	49.14	25.65	4.35	90.69	109.63	28.86	1.14	100.55	101.85
45	100	42.86	2.14	124.7	129.26	39.51	5.49	265.92	296.09	41.54	3.46	262.36	274.31
45	200	41.15	3.85	104.82	119.64	37.97	7.03	240.04	289.46	41.61	3.39	262.41	273.91
45	400	43.74	1.26	102.21	103.8	38.18	6.82	248.02	294.52	41.63	3.37	247.81	259.18
45	800	42.86	2.14	114.82	119.38	36.89	8.11	228.54	294.24	42.36	2.64	253.27	260.23

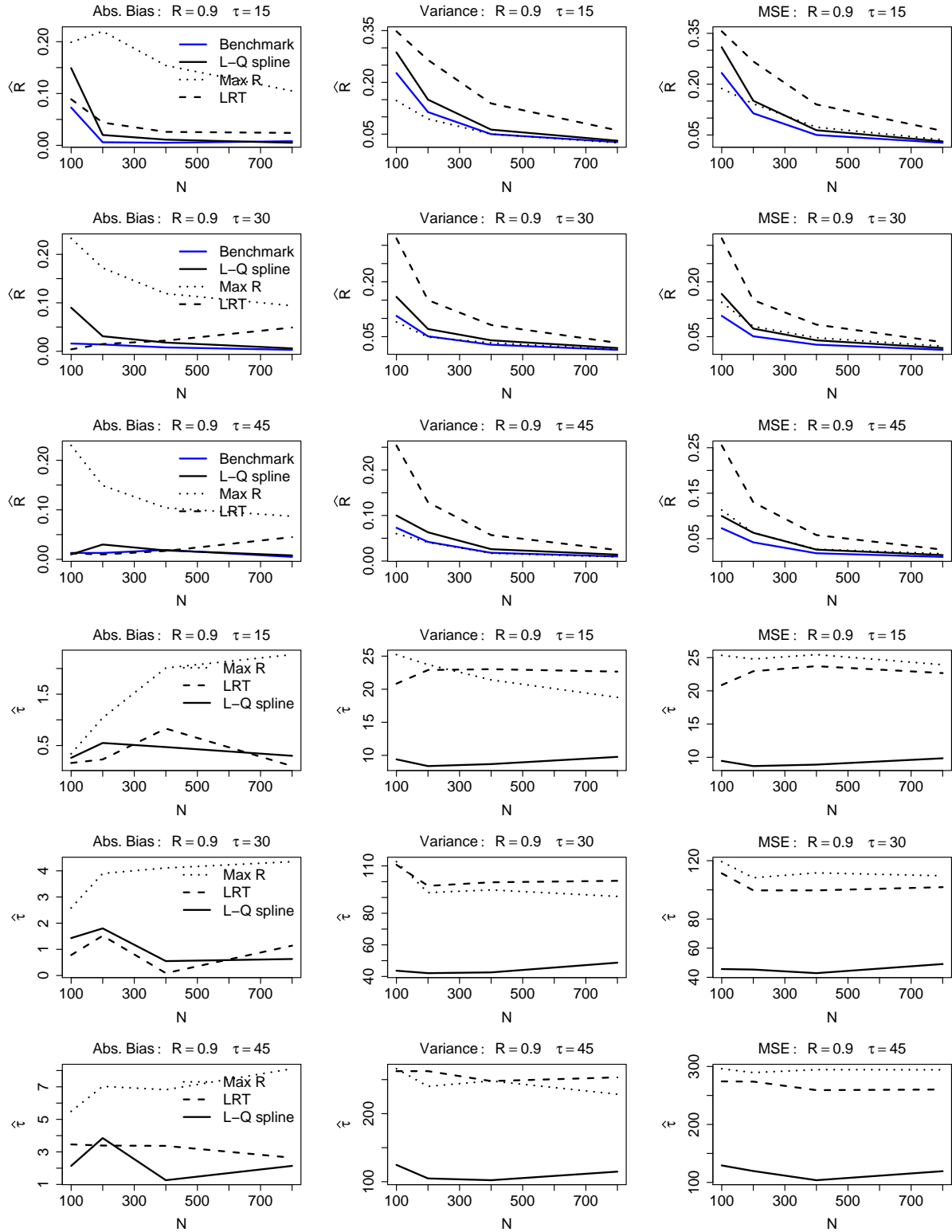


Figure 14: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 29: **Single Normally distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	1.219	0.019	0.339	0.34	1.311	0.111	0.511	0.523	1.691	0.491	0.595	0.835	1.44	0.24	0.903	0.961
15	200	1.179	0.021	0.132	0.132	1.238	0.038	0.176	0.178	1.468	0.268	0.179	0.251	1.246	0.046	0.369	0.371
15	400	1.216	0.016	0.082	0.082	1.245	0.045	0.095	0.097	1.389	0.189	0.092	0.128	1.29	0.09	0.164	0.172
15	800	1.21	0.01	0.033	0.033	1.226	0.026	0.04	0.041	1.327	0.127	0.039	0.055	1.278	0.078	0.065	0.071
30	100	1.223	0.023	0.172	0.173	1.286	0.086	0.244	0.252	1.541	0.341	0.226	0.343	1.355	0.155	0.439	0.463
30	200	1.207	0.007	0.073	0.073	1.235	0.035	0.097	0.098	1.433	0.233	0.1	0.154	1.313	0.113	0.193	0.206
30	400	1.223	0.023	0.04	0.04	1.244	0.044	0.052	0.054	1.369	0.169	0.05	0.079	1.305	0.105	0.084	0.095
30	800	1.183	0.017	0.017	0.018	1.202	0.002	0.023	0.023	1.275	0.075	0.022	0.027	1.244	0.044	0.031	0.033
45	100	1.174	0.026	0.099	0.099	1.226	0.026	0.128	0.128	1.492	0.292	0.15	0.235	1.298	0.098	0.301	0.311
45	200	1.198	0.002	0.072	0.072	1.23	0.03	0.09	0.091	1.398	0.198	0.09	0.129	1.3	0.1	0.163	0.173
45	400	1.209	0.009	0.032	0.032	1.225	0.025	0.042	0.042	1.335	0.135	0.04	0.058	1.275	0.075	0.073	0.078
45	800	1.195	0.005	0.013	0.013	1.211	0.011	0.018	0.018	1.283	0.083	0.017	0.024	1.254	0.054	0.024	0.027

Table 30: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.59	0.41	9.2	9.37	12.67	2.33	22.87	28.31	14.15	0.85	24.18	24.9
15	200	14.44	0.56	9.98	10.29	12.38	2.62	21.41	28.29	13.95	1.05	23.35	24.44
15	400	14.39	0.61	9.49	9.86	12.79	2.21	19.84	24.73	13.78	1.22	21.17	22.66
15	800	14.55	0.45	8.55	8.75	12.61	2.39	16.88	22.58	14.64	0.36	19.28	19.41
30	100	28.08	1.92	38.28	41.98	25.9	4.1	88.31	105.13	27.46	2.54	90.45	96.91
30	200	28.78	1.22	45.1	46.59	23.54	6.46	72.28	114.04	27.61	2.39	96.2	101.89
30	400	28.54	1.46	40.05	42.17	23.94	6.06	65.21	101.87	28.89	1.11	91.93	93.16
30	800	28.97	1.03	39.06	40.13	24.59	5.41	71.54	100.77	29.82	0.18	85.07	85.11
45	100	42.79	2.21	123.21	128.1	36.34	8.66	244.65	319.7	41.23	3.77	268.97	283.18
45	200	42.51	2.49	120.68	126.87	36.84	8.16	216.34	282.86	40.85	4.15	254.03	271.22
45	400	42.6	2.4	89.24	94.99	37.36	7.64	220.78	279.13	42.89	2.11	225.2	229.66
45	800	41.92	3.08	95.47	104.92	35.09	9.91	169.32	267.52	42.79	2.21	185.75	190.64

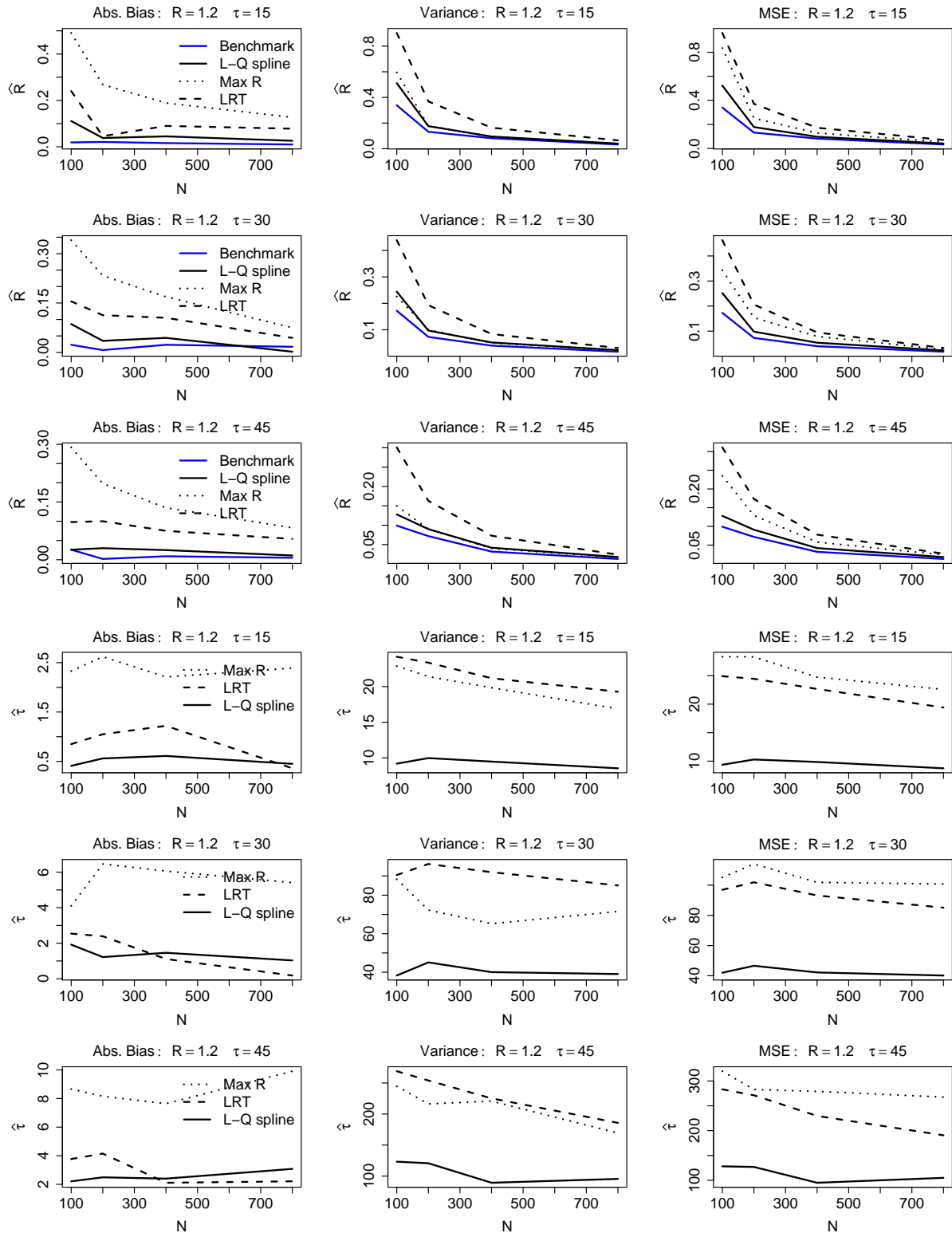


Figure 15: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 31: **Single Normally distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	1.553	0.053	0.336	0.339	1.644	0.144	0.475	0.496	1.936	0.436	0.495	0.685	1.774	0.274	0.716	0.792
15	200	1.519	0.019	0.163	0.163	1.592	0.092	0.221	0.229	1.785	0.285	0.207	0.288	1.689	0.189	0.279	0.315
15	400	1.517	0.017	0.105	0.105	1.56	0.06	0.137	0.141	1.703	0.203	0.138	0.18	1.651	0.151	0.163	0.186
15	800	1.516	0.016	0.046	0.046	1.541	0.041	0.054	0.056	1.638	0.138	0.059	0.078	1.597	0.097	0.054	0.063
30	100	1.524	0.024	0.2	0.201	1.586	0.086	0.247	0.254	1.82	0.32	0.243	0.345	1.718	0.218	0.341	0.388
30	200	1.536	0.036	0.118	0.119	1.593	0.093	0.148	0.157	1.749	0.249	0.134	0.196	1.676	0.176	0.169	0.2
30	400	1.477	0.023	0.055	0.055	1.5	0	0.07	0.07	1.615	0.115	0.071	0.084	1.579	0.079	0.066	0.073
30	800	1.49	0.01	0.022	0.022	1.515	0.015	0.026	0.026	1.584	0.084	0.025	0.032	1.546	0.046	0.022	0.024
45	100	1.49	0.01	0.155	0.155	1.568	0.068	0.231	0.236	1.807	0.307	0.2	0.294	1.702	0.202	0.302	0.343
45	200	1.479	0.021	0.066	0.066	1.517	0.017	0.084	0.084	1.651	0.151	0.082	0.105	1.604	0.104	0.098	0.109
45	400	1.512	0.012	0.037	0.037	1.536	0.036	0.042	0.043	1.628	0.128	0.042	0.059	1.59	0.09	0.039	0.047
45	800	1.521	0.021	0.021	0.021	1.536	0.036	0.023	0.024	1.595	0.095	0.025	0.034	1.562	0.062	0.022	0.026

Table 32: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.25	0.75	8.14	8.7	12.8	2.2	18.19	23.04	14.31	0.69	20.29	20.77
15	200	14.09	0.91	8.79	9.61	12.59	2.41	16.15	21.97	14.34	0.66	18.78	19.22
15	400	14.41	0.59	8.54	8.89	12.06	2.94	13.54	22.19	14.51	0.49	15.6	15.83
15	800	14.24	0.76	7.06	7.63	11.79	3.21	10.31	20.62	14.8	0.2	13.54	13.58
30	100	28.4	1.6	40.83	43.39	24.59	5.41	71.98	101.22	29.23	0.77	81.5	82.1
30	200	28.55	1.45	31.69	33.79	24.01	5.99	60.85	96.79	29.18	0.82	73.29	73.96
30	400	28.99	1.01	34.68	35.69	24.04	5.96	58.95	94.47	30.09	0.09	63.58	63.59
30	800	28.51	1.49	28.52	30.75	22.69	7.31	37.16	90.62	29.75	0.25	48.05	48.11
45	100	41.33	3.67	118.39	131.85	35.25	9.75	189.71	284.74	40.62	4.38	211.64	230.84
45	200	41.4	3.6	87.41	100.35	37.36	7.64	189.33	247.67	44.73	0.27	185.67	185.74
45	400	42.78	2.22	76.9	81.83	33.66	11.34	115.61	244.13	44.88	0.12	120.42	120.44
45	800	43.75	1.25	45.1	46.67	35.1	9.9	94.17	192.27	44.66	0.34	55.58	55.7

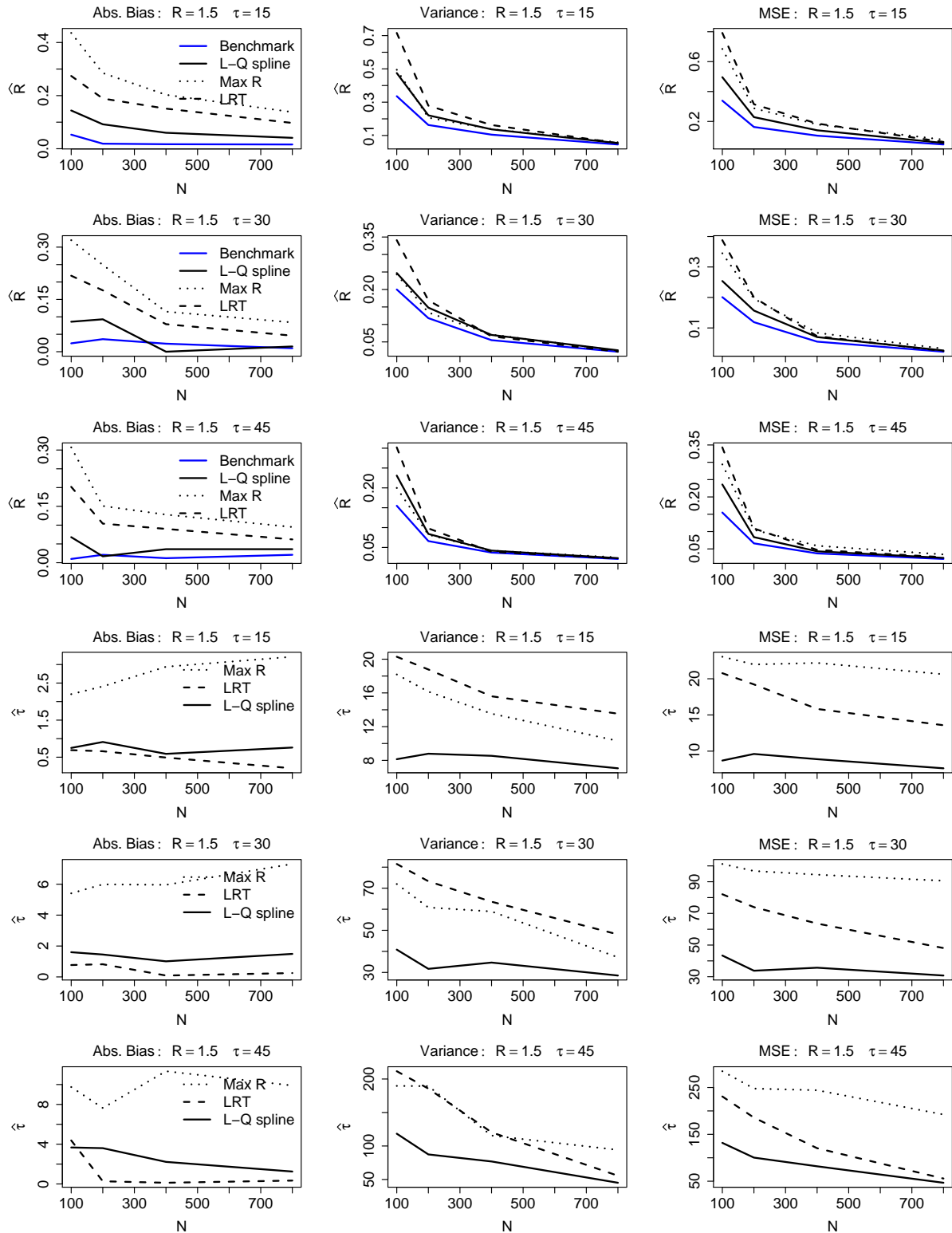


Figure 16: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 33: **Single Normally distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.032	0.032	0.457	0.458	2.195	0.195	0.594	0.632	2.454	0.454	0.646	0.852	2.35	0.35	0.762	0.885
15	200	1.941	0.059	0.264	0.268	2.029	0.029	0.333	0.334	2.221	0.221	0.365	0.414	2.142	0.142	0.34	0.36
15	400	1.998	0.002	0.133	0.133	2.021	0.021	0.149	0.149	2.211	0.211	0.176	0.221	2.124	0.124	0.15	0.165
15	800	2.026	0.026	0.07	0.071	2.034	0.034	0.077	0.078	2.146	0.146	0.092	0.114	2.081	0.081	0.08	0.086
30	100	2.005	0.005	0.342	0.342	2.112	0.112	0.433	0.446	2.354	0.354	0.448	0.573	2.259	0.259	0.485	0.552
30	200	2.064	0.064	0.165	0.169	2.128	0.128	0.201	0.217	2.265	0.265	0.199	0.27	2.201	0.201	0.184	0.225
30	400	2.025	0.025	0.081	0.082	2.044	0.044	0.086	0.088	2.161	0.161	0.105	0.131	2.097	0.097	0.091	0.101
30	800	2.022	0.022	0.042	0.042	2.026	0.026	0.044	0.044	2.099	0.099	0.05	0.06	2.043	0.043	0.044	0.045
45	100	2.079	0.079	0.265	0.271	2.189	0.189	0.342	0.377	2.384	0.384	0.321	0.468	2.308	0.308	0.3	0.395
45	200	2.01	0.01	0.119	0.12	2.063	0.063	0.151	0.155	2.204	0.204	0.145	0.187	2.142	0.142	0.135	0.155
45	400	2.016	0.016	0.05	0.05	2.026	0.026	0.052	0.053	2.124	0.124	0.058	0.074	2.081	0.081	0.052	0.059
45	800	2.007	0.007	0.024	0.024	1.997	0.003	0.026	0.026	2.061	0.061	0.027	0.031	2.029	0.029	0.025	0.026

Table 34: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	13.64	1.36	6.58	8.42	12.16	2.84	13.87	21.93	13.83	1.17	15.22	16.59	13.83	1.17	15.22	16.59
15	200	13.97	1.03	6.58	7.64	11.6	3.4	11.87	23.41	14.43	0.57	13.61	13.93	14.43	0.57	13.61	13.93
15	400	14.83	0.17	6.54	6.57	11.44	3.56	8.36	21.05	14.51	0.49	11.65	11.89	14.51	0.49	11.65	11.89
15	800	14.77	0.23	5.67	5.72	11.52	3.48	7.22	19.32	14.82	0.18	5.97	6.01	14.82	0.18	5.97	6.01
30	100	28.49	1.51	34.07	36.36	23.76	6.24	61.51	100.46	28.26	1.74	62.97	65.99	28.26	1.74	62.97	65.99
30	200	28.65	1.35	26.13	27.94	23.6	6.4	44.32	85.24	29.48	0.52	39.84	40.11	29.48	0.52	39.84	40.11
30	400	29.62	0.38	16.94	17.09	23.6	6.4	36.76	77.68	29.34	0.66	19.74	20.18	29.34	0.66	19.74	20.18
30	800	30.05	0.05	9.98	9.98	23.96	6.04	31.21	67.7	30.16	0.16	9.47	9.5	30.16	0.16	9.47	9.5
45	100	42.22	2.78	92.07	99.79	35.14	9.86	127.26	224.57	44.1	0.9	141.69	142.51	44.1	0.9	141.69	142.51
45	200	42.71	2.29	69.31	74.56	34.84	10.16	114.11	217.25	43.53	1.47	90.09	92.26	43.53	1.47	90.09	92.26
45	400	45.35	0.35	41.18	41.3	35.88	9.12	94.33	177.43	43.87	1.13	42.98	44.26	43.87	1.13	42.98	44.26
45	800	45.9	0.9	13.8	14.61	37.21	7.79	77.73	138.4	44.79	0.21	14.42	14.46	44.79	0.21	14.42	14.46

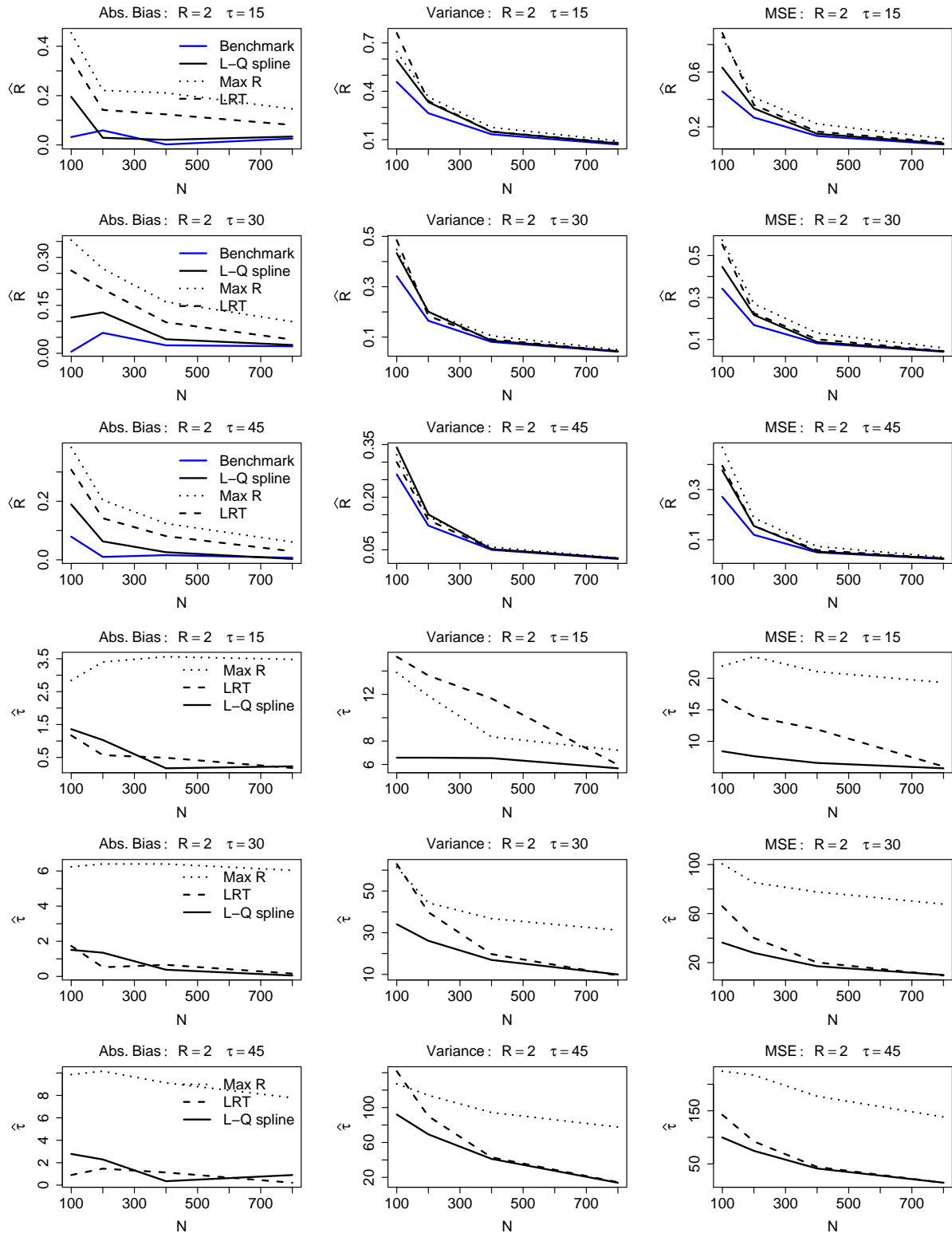


Figure 17: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 35: **Single Normally distributed exposure with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark				(b) Linear-quadratic spline				(c) \widehat{R}_{\max} approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	4.1	0.1	1.153	1.163	4.252	0.252	1.201	1.264	4.542	0.542	1.123	1.416	4.382	0.382	1.137	1.283
15	200	3.994	0.006	0.676	0.676	4.063	0.063	0.652	0.656	4.312	0.312	0.76	0.857	4.164	0.164	0.715	0.741
15	400	4.009	0.009	0.286	0.286	4.003	0.003	0.3	0.3	4.19	0.19	0.368	0.405	4.047	0.047	0.303	0.305
15	800	3.956	0.044	0.172	0.174	3.953	0.047	0.175	0.178	4.076	0.076	0.201	0.207	3.972	0.028	0.175	0.176
30	100	4.228	0.228	0.771	0.823	4.327	0.327	0.868	0.975	4.587	0.587	0.882	1.227	4.486	0.486	1.011	1.246
30	200	4.043	0.043	0.443	0.445	4.061	0.061	0.463	0.467	4.238	0.238	0.466	0.523	4.128	0.128	0.476	0.492
30	400	3.963	0.037	0.204	0.206	3.954	0.046	0.205	0.207	4.051	0.051	0.206	0.209	3.998	0.002	0.212	0.212
30	800	3.979	0.021	0.106	0.106	3.952	0.048	0.106	0.108	4.02	0.02	0.113	0.113	3.983	0.017	0.106	0.107
45	100	4.064	0.064	0.746	0.75	4.065	0.065	0.751	0.756	4.315	0.315	0.743	0.842	4.225	0.225	0.765	0.816
45	200	4.052	0.052	0.317	0.32	4.032	0.032	0.301	0.302	4.161	0.161	0.314	0.34	4.095	0.095	0.324	0.333
45	400	4.055	0.055	0.212	0.215	4.01	0.01	0.219	0.219	4.098	0.098	0.212	0.221	4.073	0.073	0.216	0.221
45	800	4.004	0.004	0.078	0.078	3.974	0.026	0.082	0.083	4.015	0.015	0.079	0.08	4.008	0.008	0.079	0.079

Table 36: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline				(c) \widehat{R}_{\max} approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.09	0.91	5.67	6.5	11.7	3.3	7.79	18.65	14.49	0.51	6.42	6.68
15	200	14.64	0.36	2.94	3.08	11.99	3.01	6.99	16.02	14.43	0.57	3.37	3.7
15	400	14.97	0.03	1.61	1.61	12.22	2.78	6.79	14.54	14.93	0.07	0.93	0.93
15	800	14.97	0.03	0.67	0.67	12.46	2.54	6.38	12.82	14.94	0.06	0.25	0.25
30	100	29.23	0.77	20.07	20.67	24.58	5.42	31.96	61.36	29.39	0.61	20.33	20.7
30	200	29.53	0.47	6.73	6.95	24.93	5.07	29.69	55.35	29.39	0.61	6.44	6.81
30	400	30.11	0.11	2.72	2.73	26.85	3.15	20.42	30.35	29.79	0.21	2.2	2.24
30	800	30.12	0.12	0.75	0.76	27.36	2.64	18	24.98	29.97	0.03	0.26	0.26
45	100	45.4	0.4	36.8	36.96	38.42	6.58	75.97	119.24	44.67	0.33	28.72	28.83
45	200	45.09	0.09	13.03	13.04	40.58	4.42	49.24	68.79	45.22	0.22	9.8	9.84
45	400	45.37	0.37	4.44	4.58	42.31	2.69	30.59	37.85	45.09	0.09	2.68	2.69
45	800	45.36	0.36	1.2	1.33	44.39	0.61	1.59	1.95	45.01	0.01	0.43	0.43

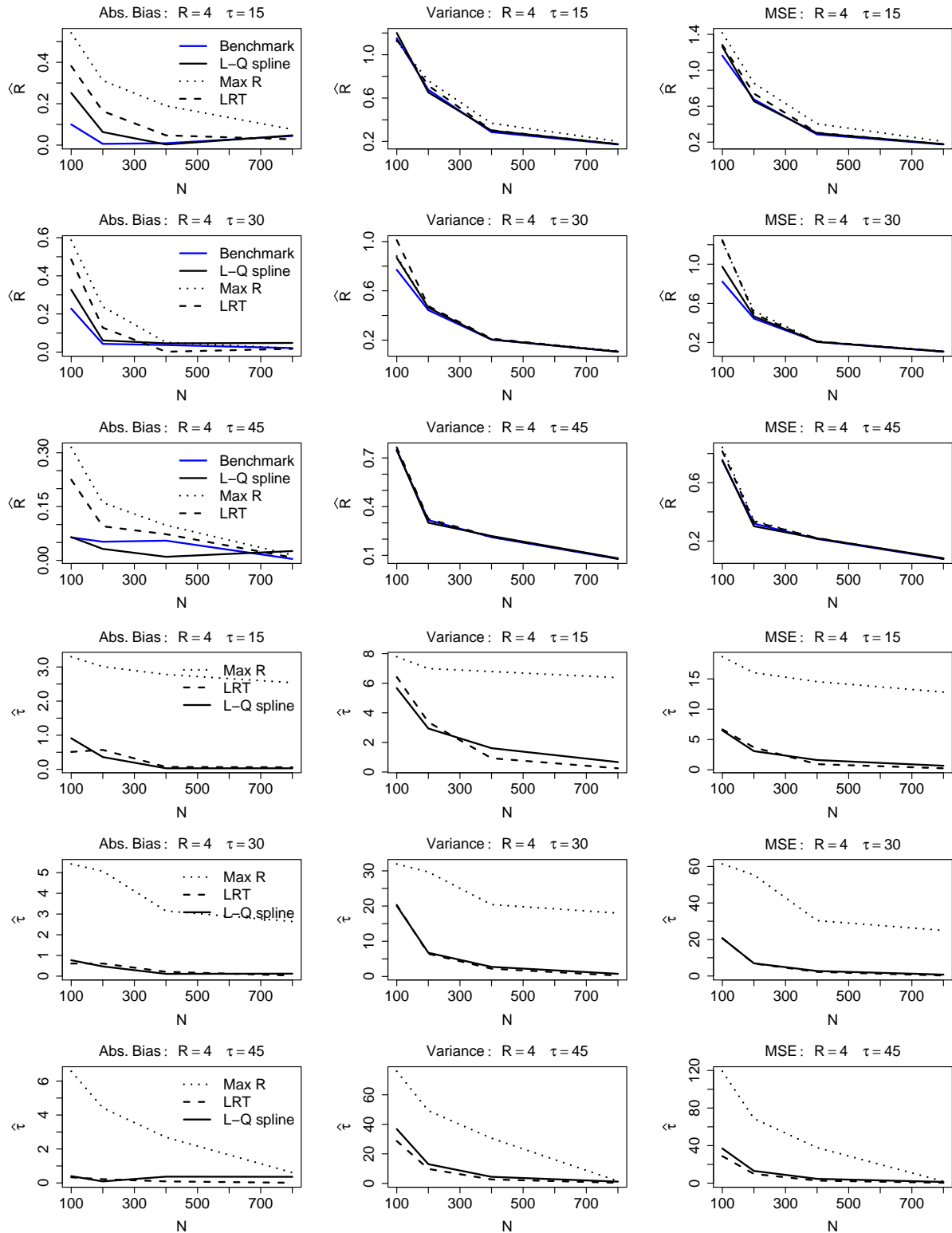


Figure 18: **Single Normally distributed exposure with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 37: **Multiple Normally distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.748	0.048	0.139	0.141	0.801	0.101	0.191	0.201	0.553	0.147	0.107	0.128	0.712	0.012	0.299	0.3
15	200	0.699	0.001	0.062	0.062	0.716	0.016	0.08	0.08	0.546	0.154	0.054	0.078	0.619	0.081	0.123	0.129
15	400	0.707	0.007	0.037	0.037	0.716	0.016	0.047	0.047	0.607	0.093	0.038	0.047	0.644	0.056	0.058	0.061
15	800	0.716	0.016	0.016	0.016	0.714	0.014	0.019	0.019	0.643	0.057	0.018	0.021	0.664	0.036	0.021	0.022
30	100	0.714	0.014	0.074	0.074	0.742	0.042	0.099	0.101	0.545	0.155	0.059	0.083	0.641	0.059	0.163	0.167
30	200	0.681	0.019	0.026	0.027	0.677	0.023	0.035	0.035	0.564	0.136	0.028	0.047	0.611	0.089	0.048	0.056
30	400	0.697	0.003	0.013	0.013	0.703	0.003	0.019	0.019	0.618	0.082	0.013	0.02	0.636	0.064	0.014	0.018
30	800	0.697	0.003	0.007	0.007	0.692	0.008	0.009	0.009	0.64	0.06	0.008	0.012	0.659	0.041	0.007	0.009
45	100	0.708	0.008	0.046	0.046	0.725	0.025	0.064	0.065	0.549	0.151	0.038	0.061	0.629	0.071	0.109	0.114
45	200	0.715	0.015	0.02	0.02	0.711	0.011	0.029	0.029	0.622	0.078	0.021	0.028	0.662	0.038	0.041	0.042
45	400	0.715	0.015	0.011	0.012	0.715	0.015	0.014	0.014	0.65	0.05	0.013	0.015	0.67	0.03	0.016	0.017
45	800	0.702	0.002	0.006	0.006	0.7	0	0.007	0.007	0.66	0.04	0.006	0.008	0.675	0.025	0.006	0.006

Table 38: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.7$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	15.1	0.1	9.73	9.74	13.54	1.46	19.95	22.08	15.02	0.02	20.03	20.03	20.03	0.02	20.03	20.03
15	200	14.74	0.26	8.68	8.75	12.89	2.11	17.15	21.58	14.62	0.38	19.14	19.29	19.29	0.38	19.14	19.29
15	400	15.13	0.13	7.92	7.94	12.66	2.34	14.97	20.43	14.67	0.33	17.64	17.74	17.74	0.33	17.64	17.74
15	800	15.19	0.19	7.27	7.3	12.23	2.77	12.2	19.89	15.03	0.03	15.74	15.74	15.74	0.03	15.74	15.74
30	100	29.08	0.92	42.01	42.86	26.5	3.5	86.1	98.37	29.72	0.28	90.84	90.91	90.91	0.28	90.84	90.91
30	200	29.96	0.04	45.8	45.8	25.16	4.84	76.86	100.32	30.85	0.85	86.6	87.32	87.32	0.85	86.6	87.32
30	400	30.14	0.14	33.36	33.38	24.61	5.39	65.28	94.3	30.29	0.29	69.66	69.74	69.74	0.29	69.66	69.74
30	800	30.42	0.42	29.16	29.33	23.55	6.45	47.82	89.45	29.8	0.2	42.9	42.94	42.94	0.2	42.9	42.94
45	100	43.56	1.44	109.96	112.03	36.66	8.34	187.98	257.57	43.42	1.58	220.85	223.34	223.34	1.58	220.85	223.34
45	200	45.26	0.26	96.22	96.29	37.9	7.1	181.69	232.04	45.7	0.7	168.64	169.13	169.13	0.7	168.64	169.13
45	400	44.29	0.71	83.16	83.66	37.03	7.97	156.89	220.41	46.09	1.09	142.58	143.76	143.76	1.09	142.58	143.76
45	800	44.96	0.04	64.07	64.07	35.78	9.22	119.8	204.73	45.29	0.29	99.74	99.83	99.83	0.29	99.74	99.83

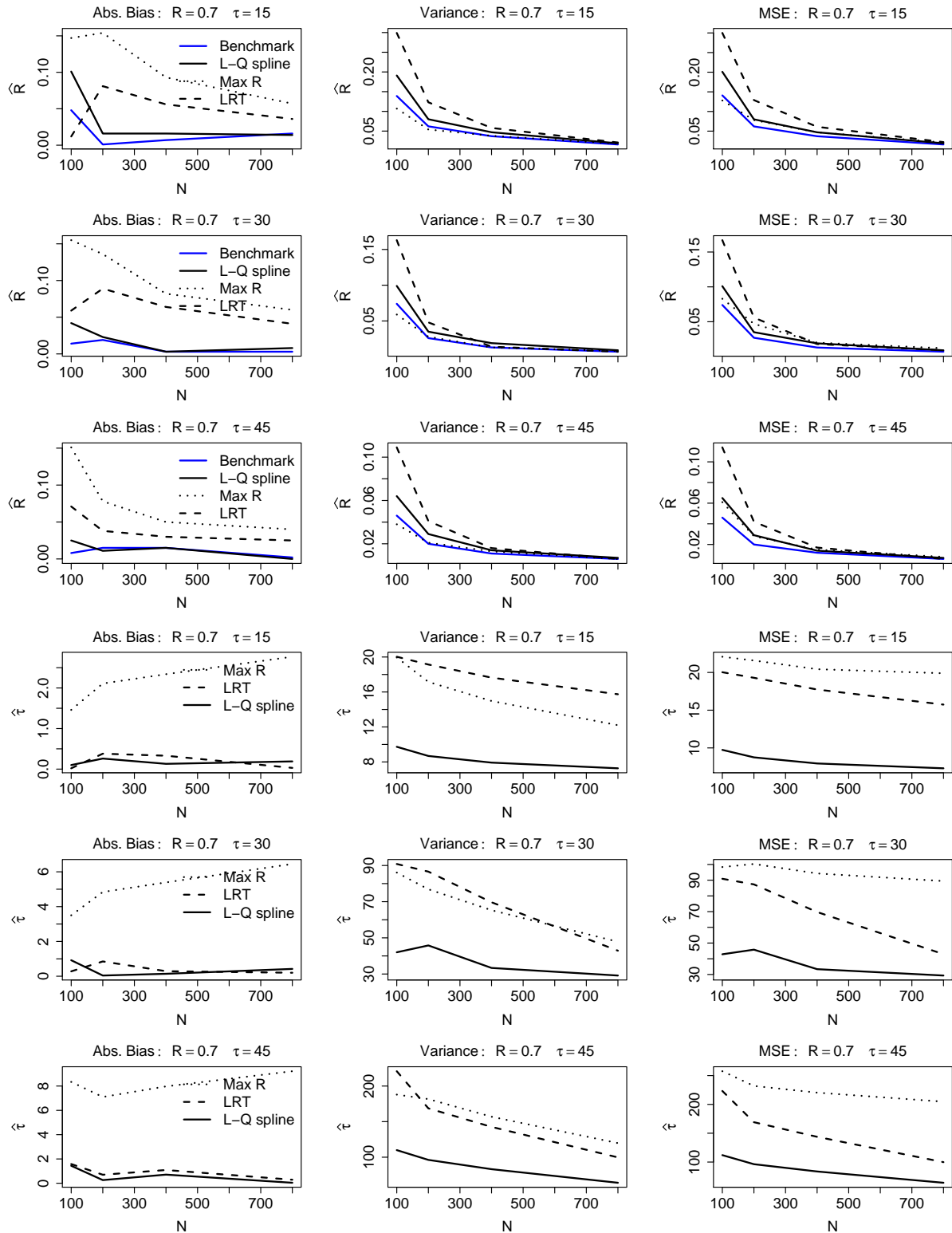


Figure 19: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.7$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 39: **Multiple Normally distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.945	0.045	0.152	0.154	0.987	0.087	0.182	0.189	0.697	0.203	0.103	0.144	0.964	0.064	0.43	0.434
15	200	0.909	0.009	0.087	0.087	0.93	0.03	0.107	0.108	0.719	0.181	0.078	0.111	0.872	0.028	0.199	0.2
15	400	0.886	0.014	0.04	0.04	0.899	0.001	0.051	0.051	0.76	0.14	0.038	0.057	0.864	0.036	0.097	0.098
15	800	0.897	0.003	0.021	0.021	0.894	0.006	0.025	0.025	0.803	0.097	0.019	0.029	0.855	0.045	0.042	0.044
30	100	0.916	0.016	0.096	0.096	0.937	0.037	0.122	0.124	0.68	0.22	0.065	0.114	0.887	0.013	0.242	0.242
30	200	0.901	0.001	0.048	0.048	0.91	0.01	0.068	0.068	0.749	0.151	0.046	0.069	0.891	0.009	0.128	0.128
30	400	0.902	0.002	0.022	0.022	0.897	0.003	0.03	0.03	0.786	0.114	0.022	0.035	0.867	0.033	0.062	0.063
30	800	0.893	0.007	0.011	0.011	0.886	0.014	0.015	0.015	0.821	0.079	0.011	0.017	0.858	0.042	0.023	0.024
45	100	0.897	0.003	0.059	0.059	0.917	0.017	0.089	0.089	0.686	0.214	0.047	0.093	0.859	0.041	0.193	0.195
45	200	0.893	0.007	0.028	0.028	0.899	0.001	0.038	0.038	0.743	0.157	0.027	0.052	0.838	0.062	0.084	0.088
45	400	0.904	0.004	0.018	0.018	0.908	0.008	0.022	0.022	0.801	0.099	0.018	0.027	0.858	0.042	0.045	0.047
45	800	0.913	0.013	0.009	0.009	0.912	0.012	0.012	0.012	0.839	0.061	0.009	0.012	0.877	0.023	0.021	0.021

Table 40: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 0.9$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.01	0.01	10.05	10.05	13.49	1.51	22.42	24.69	14.22	0.78	22.62	23.24
15	200	14.82	0.18	8.81	8.84	13.74	1.26	21.04	22.63	14.64	0.36	23.59	23.72
15	400	15.08	0.08	9.36	9.36	13.52	1.48	21.42	23.6	14.26	0.74	21.63	22.18
15	800	15.01	0.01	10.03	10.03	13.2	1.8	22.12	25.36	15.04	0.04	24.22	24.22
30	100	29.77	0.23	45.58	45.64	25.84	4.16	93.97	111.24	27.79	2.21	90.95	95.84
30	200	29.52	0.48	41.22	41.46	26.57	3.43	103.59	115.34	29.42	0.58	95.54	95.88
30	400	29.08	0.92	45.02	45.88	25.02	4.98	81.31	106.16	27.95	2.05	94.7	98.9
30	800	29.15	0.85	43.81	44.54	26.48	3.52	94.41	106.82	29.75	0.25	92.83	92.89
45	100	43.85	1.15	121.14	122.47	38.94	6.06	276.83	313.56	42.8	2.2	249.75	254.57
45	200	44.1	0.9	110.01	110.82	38.02	6.98	231.31	280.03	42.69	2.31	238.9	244.22
45	400	44.37	0.63	95.2	95.59	36.93	8.07	223.85	288.9	40.48	4.52	219.77	240.22
45	800	43.73	1.27	103.2	104.8	35.56	9.44	225.35	314.41	42.41	2.59	236.26	242.96

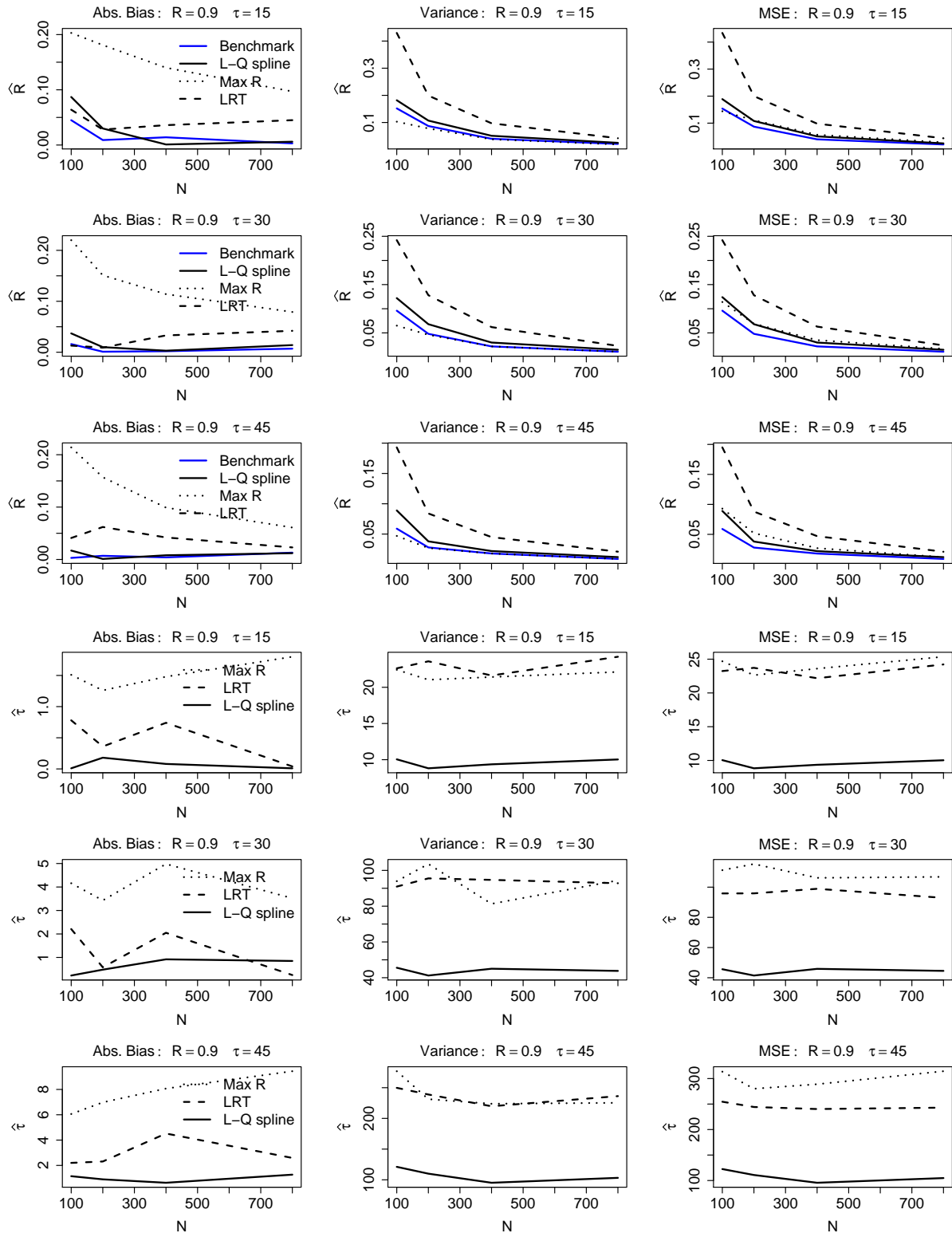


Figure 20: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 0.9$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 41: **Multiple Normally distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.205	0.005	0.219	0.219	1.263	0.063	0.294	0.298	1.524	0.324	0.298	0.403	1.317	0.117	0.518	0.531
15	200	1.215	0.015	0.117	0.117	1.238	0.038	0.153	0.154	1.443	0.243	0.145	0.204	1.325	0.125	0.25	0.265
15	400	1.221	0.021	0.064	0.065	1.235	0.035	0.071	0.072	1.349	0.149	0.074	0.096	1.266	0.066	0.123	0.127
15	800	1.202	0.002	0.028	0.028	1.217	0.017	0.033	0.033	1.309	0.109	0.035	0.047	1.272	0.072	0.052	0.057
30	100	1.204	0.004	0.121	0.121	1.251	0.051	0.137	0.139	1.477	0.277	0.147	0.224	1.298	0.098	0.317	0.327
30	200	1.23	0.03	0.061	0.062	1.258	0.058	0.083	0.086	1.408	0.208	0.067	0.11	1.323	0.123	0.122	0.137
30	400	1.214	0.014	0.029	0.03	1.229	0.029	0.034	0.035	1.337	0.137	0.036	0.055	1.29	0.09	0.057	0.065
30	800	1.211	0.011	0.012	0.012	1.227	0.027	0.017	0.018	1.293	0.093	0.015	0.023	1.266	0.066	0.02	0.024
45	100	1.245	0.045	0.114	0.116	1.265	0.065	0.157	0.161	1.508	0.308	0.143	0.237	1.351	0.151	0.286	0.309
45	200	1.189	0.011	0.051	0.051	1.199	0.001	0.063	0.063	1.367	0.167	0.071	0.099	1.28	0.08	0.122	0.129
45	400	1.192	0.008	0.025	0.025	1.206	0.006	0.032	0.032	1.3	0.1	0.03	0.04	1.255	0.055	0.05	0.053
45	800	1.207	0.007	0.013	0.013	1.215	0.015	0.016	0.017	1.286	0.086	0.015	0.022	1.265	0.065	0.018	0.022

Table 42: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.67	0.33	9.12	9.23	13.17	1.83	21.02	24.39	14.6	0.4	22.55	22.71
15	200	14.75	0.25	9.28	9.34	13.48	1.52	23.02	25.34	14.51	0.49	21.49	21.73
15	400	14.65	0.35	8.19	8.31	13.46	1.54	18.73	21.1	14.65	0.35	19.83	19.95
15	800	14.34	0.66	7.93	8.37	12.62	2.38	17.08	22.73	13.83	1.17	17.59	18.96
30	100	29.7	0.3	44.72	44.81	26.58	3.42	95.76	107.47	28.85	1.15	97.57	98.88
30	200	28.32	1.68	39.63	42.44	24.66	5.34	83	111.53	28.39	1.61	93.8	96.38
30	400	29.77	0.23	38.05	38.1	24.76	5.24	71.96	99.43	28.45	1.55	73.96	76.35
30	800	28.35	1.65	39.01	41.73	24.02	5.98	56.31	92.07	29.97	0.03	70.01	70.02
45	100	44.16	0.84	117.44	118.15	36.45	8.55	242.58	315.64	41	4	250.52	266.52
45	200	42.35	2.65	108.91	115.92	36.63	8.37	235.56	305.65	43.35	1.65	258.38	261.1
45	400	44.2	0.8	101.26	101.91	37.89	7.11	219.49	269.98	44.19	0.81	209.84	210.5
45	800	43.33	1.67	88.44	91.24	34.98	10.02	165.83	266.14	42.81	2.19	189.92	194.7

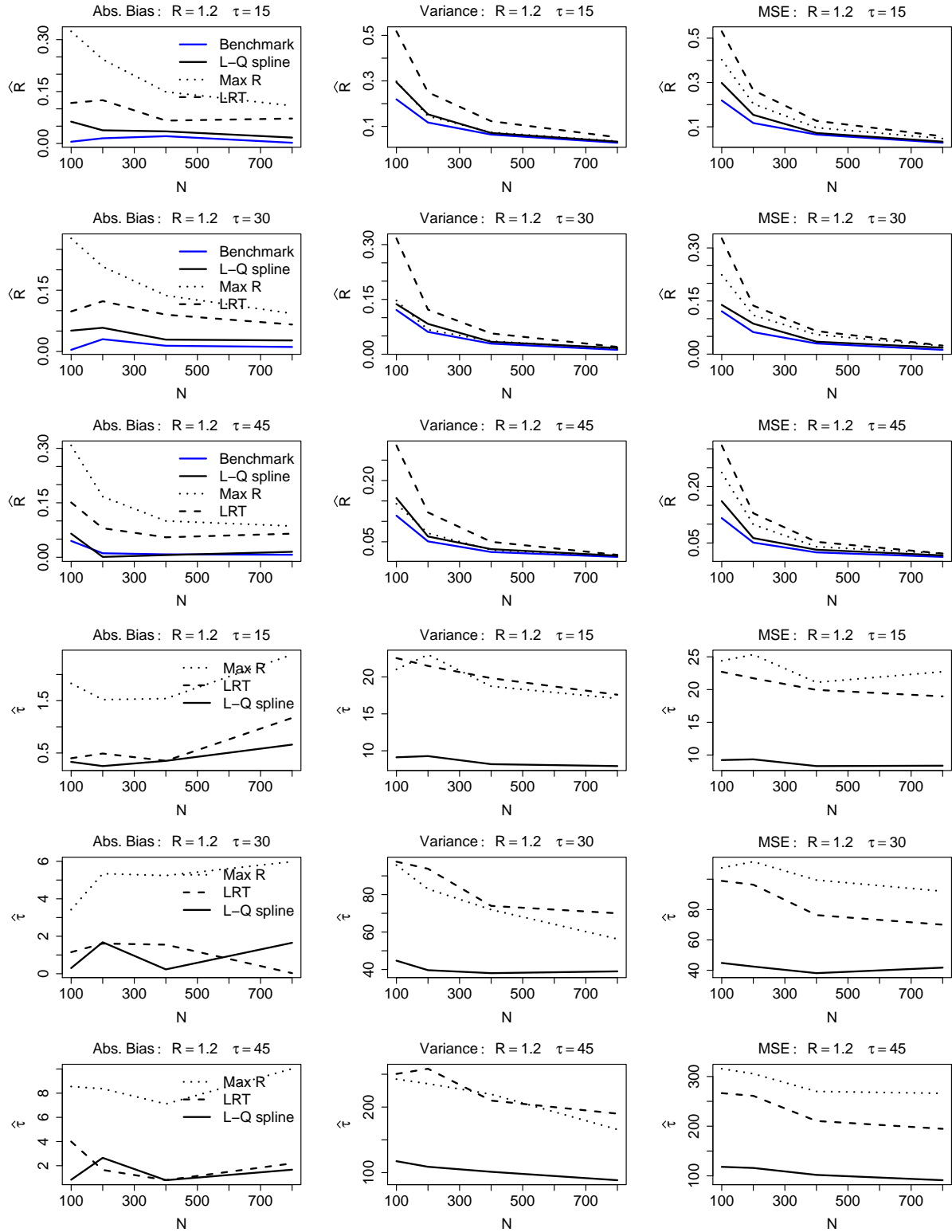


Figure 21: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 43: **Multiple Normally distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.57	0.07	0.327	0.331	1.688	0.188	0.428	0.463	1.897	0.397	0.444	0.602	1.758	0.258	0.612	0.679
15	200	1.522	0.022	0.146	0.146	1.57	0.07	0.185	0.19	1.757	0.257	0.207	0.274	1.686	0.186	0.255	0.29
15	400	1.503	0.003	0.081	0.081	1.523	0.023	0.101	0.102	1.658	0.158	0.102	0.127	1.611	0.111	0.114	0.126
15	800	1.5	0	0.034	0.034	1.519	0.019	0.04	0.04	1.6	0.1	0.043	0.053	1.565	0.065	0.039	0.043
30	100	1.525	0.025	0.152	0.152	1.596	0.096	0.198	0.207	1.8	0.3	0.204	0.294	1.738	0.238	0.267	0.324
30	200	1.531	0.031	0.077	0.078	1.579	0.079	0.095	0.101	1.711	0.211	0.089	0.134	1.67	0.17	0.108	0.137
30	400	1.505	0.005	0.039	0.039	1.515	0.015	0.048	0.048	1.614	0.114	0.046	0.059	1.588	0.088	0.044	0.052
30	800	1.497	0.003	0.021	0.021	1.511	0.011	0.025	0.025	1.57	0.07	0.025	0.03	1.542	0.042	0.023	0.025
45	100	1.531	0.031	0.111	0.112	1.573	0.073	0.172	0.178	1.794	0.294	0.162	0.248	1.746	0.246	0.198	0.258
45	200	1.504	0.004	0.064	0.064	1.534	0.034	0.079	0.08	1.665	0.165	0.085	0.113	1.631	0.131	0.084	0.101
45	400	1.527	0.027	0.04	0.041	1.545	0.045	0.043	0.045	1.612	0.112	0.04	0.053	1.593	0.093	0.039	0.047
45	800	1.497	0.003	0.013	0.013	1.51	0.01	0.015	0.015	1.552	0.052	0.014	0.017	1.533	0.033	0.013	0.014

Table 44: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 1.5$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	13.63	1.37	8.22	10.11	12.53	2.47	15.17	21.29	14.25	0.75	19.77	20.34
15	200	14.12	0.88	8.56	9.35	12.28	2.72	15.76	23.18	14.32	0.68	18.61	19.08
15	400	14.7	0.3	8.06	8.15	11.91	3.09	14.62	24.18	14.75	0.25	17.98	18.04
15	800	14.7	0.3	6.22	6.31	11.61	3.39	9.84	21.31	14.57	0.43	11.09	11.28
30	100	28.2	1.8	45.06	48.29	24.81	5.19	71.03	97.92	28.17	1.83	85.81	89.16
30	200	28.9	1.1	32.77	33.98	23.3	6.7	57.3	102.17	28.74	1.26	68	69.59
30	400	29.13	0.87	35.59	36.35	23.43	6.57	45.43	88.57	28.99	1.01	53.09	54.1
30	800	28.99	1.01	28.44	29.46	24.01	5.99	46.62	82.5	29.75	0.25	34.38	34.44
45	100	42.83	2.17	105.97	110.66	38.62	6.38	211.4	252.07	44.35	0.65	206.07	206.49
45	200	44.26	0.74	90.58	91.13	36.03	8.97	164.16	244.7	44.47	0.53	152.08	152.36
45	400	43.27	1.73	64.31	67.3	36.56	8.44	113.02	184.21	43.86	1.14	97.56	98.85
45	800	44.77	0.23	46.76	46.81	37.62	7.38	89.7	144.12	44.89	0.11	56.48	56.49

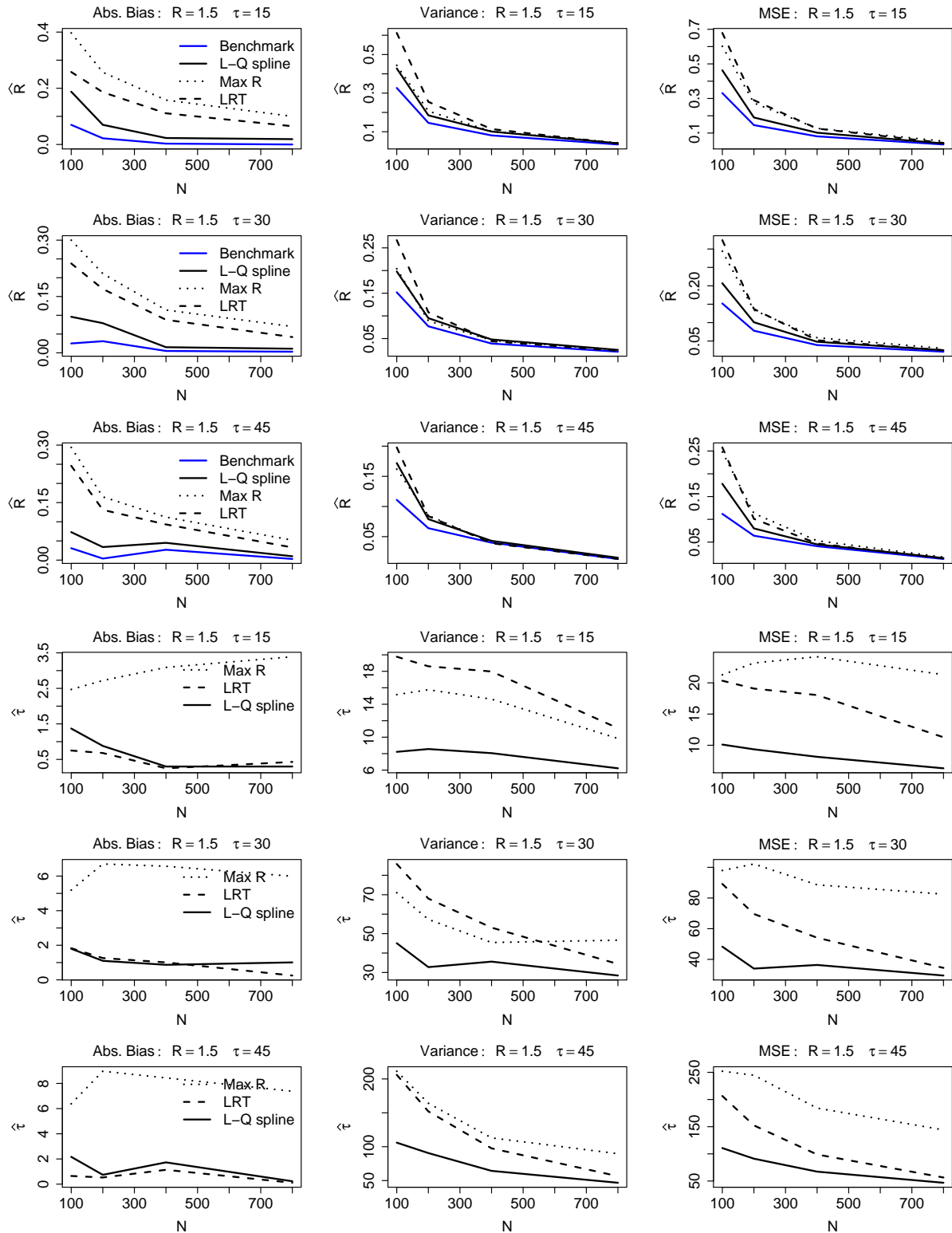


Figure 22: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 1.5$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 45: **Multiple Normally distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	2.042	0.042	0.394	0.396	2.158	0.158	0.48	0.505	2.403	0.403	0.486	0.648	2.329	0.329	0.532	0.64
15	200	2.018	0.018	0.214	0.215	2.054	0.054	0.246	0.249	2.272	0.272	0.276	0.35	2.198	0.198	0.268	0.307
15	400	2.006	0.006	0.109	0.109	2.048	0.048	0.113	0.115	2.163	0.163	0.131	0.158	2.095	0.095	0.117	0.126
15	800	1.982	0.018	0.055	0.055	1.994	0.006	0.061	0.061	2.073	0.073	0.062	0.068	2.014	0.014	0.055	0.055
30	100	2.036	0.036	0.244	0.245	2.123	0.123	0.333	0.348	2.346	0.346	0.31	0.43	2.286	0.286	0.297	0.379
30	200	2.003	0.003	0.127	0.127	2.037	0.037	0.139	0.141	2.165	0.165	0.139	0.167	2.116	0.116	0.134	0.147
30	400	1.997	0.003	0.065	0.065	2.013	0.013	0.07	0.07	2.088	0.088	0.068	0.076	2.049	0.049	0.066	0.069
30	800	2.01	0.01	0.037	0.037	2.014	0.014	0.039	0.039	2.063	0.063	0.04	0.044	2.03	0.03	0.039	0.04
45	100	2.008	0.008	0.202	0.202	2.051	0.051	0.253	0.256	2.254	0.254	0.243	0.308	2.216	0.216	0.239	0.286
45	200	1.99	0.01	0.101	0.101	2.009	0.009	0.11	0.11	2.13	0.13	0.109	0.126	2.093	0.093	0.096	0.105
45	400	2.038	0.038	0.058	0.06	2.049	0.049	0.061	0.064	2.109	0.109	0.058	0.07	2.082	0.082	0.057	0.063
45	800	2.007	0.007	0.026	0.026	1.995	0.005	0.026	0.026	2.036	0.036	0.024	0.025	2.022	0.022	0.025	0.025

Table 46: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 2$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.11	0.89	8.36	9.16	12.32	2.68	12.75	19.92	14.13	0.87	15.86	16.62
15	200	14.52	0.48	7.34	7.57	11.91	3.09	10.62	20.18	14.72	0.28	13.18	13.26
15	400	14.35	0.65	5.32	5.74	11.67	3.33	8.58	19.65	14.62	0.38	9.13	9.28
15	800	14.97	0.03	3.22	3.22	11.65	3.35	7.7	18.9	14.97	0.03	3.69	3.69
30	100	28.59	1.41	38.64	40.62	23.46	6.54	53.85	96.65	28.45	1.55	62.72	65.13
30	200	29.72	0.28	24.83	24.91	24.05	5.95	37.26	72.72	29.42	0.58	39.9	40.24
30	400	29.56	0.44	19.64	19.84	24.95	5.05	29.91	55.37	29.74	0.26	19.25	19.32
30	800	29.93	0.07	7.11	7.12	25.05	4.95	27.71	52.26	30.08	0.08	7.11	7.12
45	100	43.94	1.06	87.53	88.64	37.87	7.13	155.77	206.62	44.3	0.7	135.61	136.1
45	200	44.35	0.65	60.56	60.98	36.89	8.11	112.13	177.91	43.79	1.21	77.56	79.02
45	400	44.64	0.36	37.72	37.85	39.26	5.74	76.31	109.3	44.9	0.1	28.31	28.32
45	800	45.45	0.45	12.08	12.28	40.51	4.49	57.17	77.31	44.76	0.24	6.66	6.71

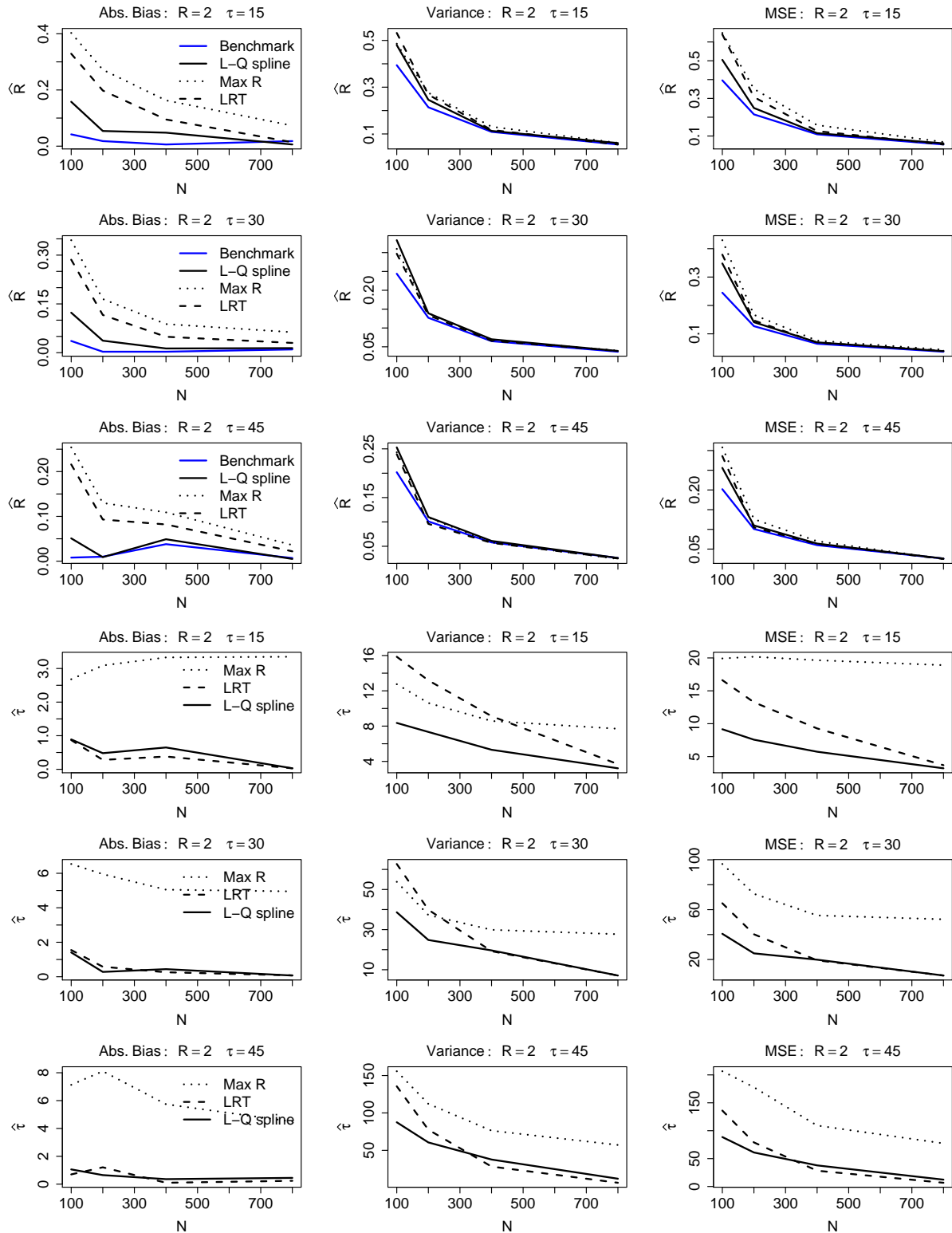


Figure 23: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 2$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 47: **Multiple Normally distributed exposures with one age effect.** Relative incidence estimation of $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(a) Benchmark			(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	4.023	0.023	1.024	1.025	4.133	0.133	1.082	1.1	4.399	0.399	1.186	1.345	4.222	0.222	1.024	1.073
15	200	4.059	0.059	0.495	0.498	4.072	0.072	0.516	0.521	4.284	0.284	0.535	0.615	4.127	0.127	0.506	0.522
15	400	3.989	0.011	0.239	0.24	3.983	0.017	0.254	0.255	4.109	0.109	0.251	0.263	4.023	0.023	0.244	0.245
15	800	4.005	0.005	0.098	0.098	3.99	0.01	0.102	0.102	4.058	0.058	0.106	0.11	4.002	0.002	0.1	0.1
30	100	3.971	0.029	0.815	0.816	4.024	0.024	0.821	0.821	4.206	0.206	0.818	0.86	4.134	0.134	0.83	0.848
30	200	4.016	0.016	0.413	0.413	4.021	0.021	0.408	0.408	4.137	0.137	0.403	0.422	4.082	0.082	0.4	0.406
30	400	3.961	0.039	0.168	0.17	3.943	0.057	0.167	0.171	4.011	0.011	0.174	0.174	3.98	0.02	0.173	0.173
30	800	3.991	0.009	0.1	0.1	3.965	0.035	0.103	0.104	4.005	0.005	0.103	0.103	3.995	0.005	0.103	0.103
45	100	4.097	0.097	0.816	0.825	4.106	0.106	0.852	0.864	4.274	0.274	0.873	0.948	4.253	0.253	0.894	0.958
45	200	4.055	0.055	0.348	0.351	4.015	0.015	0.352	0.352	4.123	0.123	0.341	0.356	4.114	0.114	0.345	0.358
45	400	3.995	0.005	0.161	0.161	3.948	0.052	0.164	0.167	4.019	0.019	0.158	0.159	4.013	0.013	0.159	0.159
45	800	3.986	0.014	0.087	0.087	3.94	0.06	0.087	0.09	3.992	0.008	0.087	0.087	3.991	0.009	0.087	0.087

Table 48: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , for $R = \exp(\beta) = 4$ based on (a) SCCS benchmark where the true risk period τ is used; (b) proposed linear-quadratic spline fit; (c) \widehat{R}_{\max} approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

τ	N	(b) Linear-quadratic spline			(c) \widehat{R}_{\max} approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.59	0.41	4.75	4.92	12.2	2.8	8.85	16.69	14.81	0.19	5.55	5.58
15	200	14.95	0.05	2.71	2.71	12.38	2.62	7.16	14.04	15.01	0.01	2.6	2.6
15	400	15.04	0.04	1.41	1.41	13.09	1.91	5.28	8.95	14.92	0.08	1.34	1.35
15	800	15.06	0.06	0.35	0.35	13.5	1.5	5.2	7.46	15.05	0.05	0.13	0.13
30	100	29.42	0.58	17.02	17.36	26.15	3.85	28.69	43.51	29.4	0.6	16.5	16.86
30	200	29.69	0.31	6.94	7.04	26.88	3.12	23.76	33.47	29.71	0.29	5.02	5.1
30	400	30.08	0.08	1.72	1.72	28.22	1.78	11.56	14.72	29.98	0.02	1.16	1.16
30	800	30.11	0.11	0.98	0.99	29.32	0.68	3.13	3.59	30	0	0.53	0.53
45	100	45.3	0.3	26.95	27.04	42.03	2.97	48.73	57.58	44.68	0.32	23.97	24.07
45	200	45.89	0.89	8.32	9.12	43.3	1.7	20.97	23.87	44.44	0.56	6.44	6.75
45	400	45.54	0.54	3.31	3.61	44.02	0.98	7.3	8.27	44.79	0.21	1.07	1.12
45	800	45.65	0.65	1.38	1.8	44.76	0.24	0.62	0.68	44.97	0.03	0.37	0.37

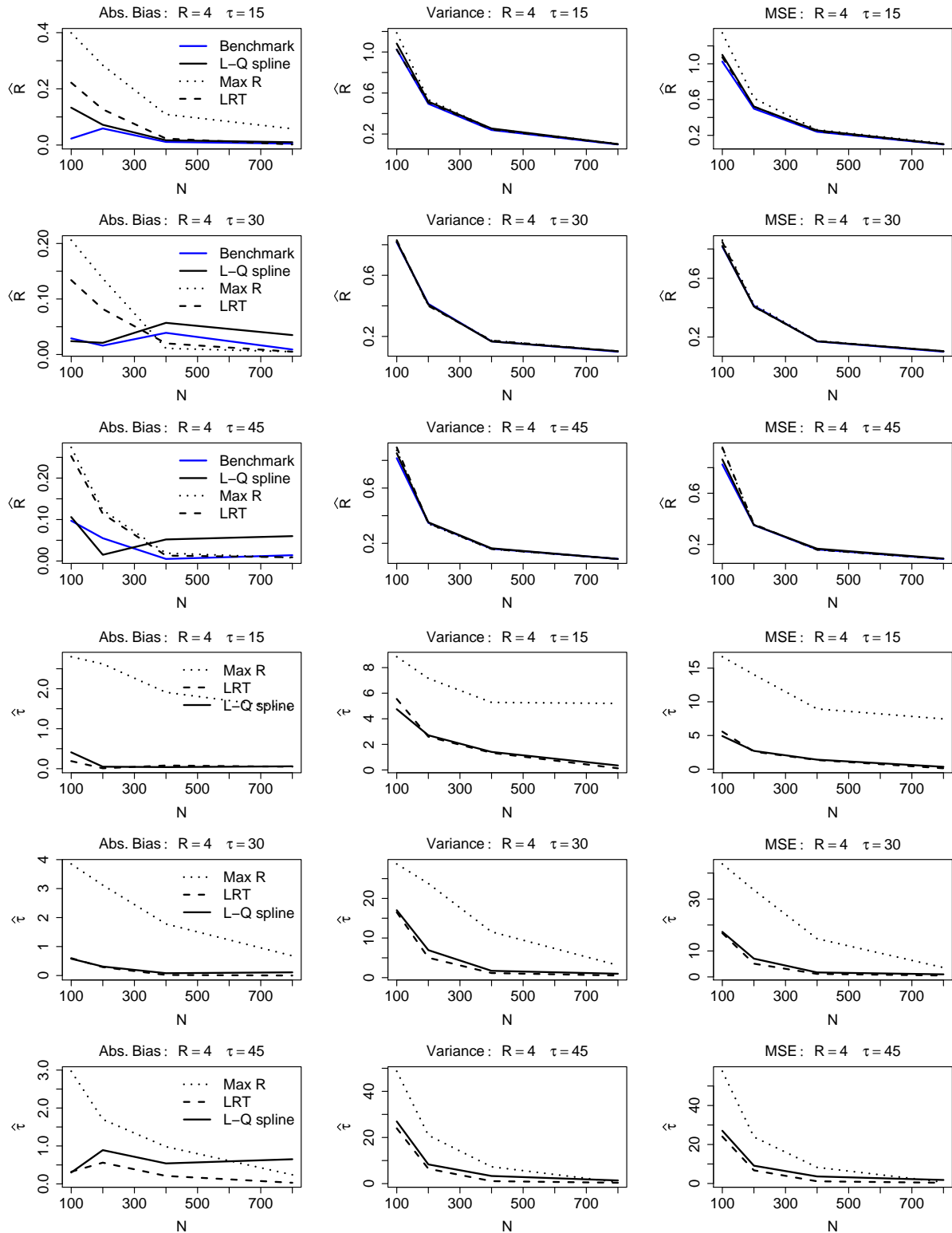


Figure 24: **Multiple Normally distributed exposures with one age effect.** Estimation of optimal (true) risk period length, τ , and relative incidence $R = \exp(\beta) = 4$. Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.