

# **Bias and Estimation under Misspecification of the Risk Period in Self-Controlled Case Series Studies**

**Supplemental Materials: Simulation Results for Models with Three  
Age Groups**

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## Additional simulation results

We provide results from additional simulations for the estimation of the relative incidence,  $\hat{R} = \exp(\hat{\beta})$ , and optimal risk length,  $\hat{\tau}$ , under different exposure models with two age effects.

Tables 1-2, 3-4, 5-6, 7-8, 9-10 and 11-12 show the results in estimation for the single Uniformly distributed exposure model and relative incidence of  $R = 0.7, 0.9, 1.2, 1.5, 2$  and 4 respectively. These also correspond respectively to Figures 1-6.

Tables 13-14, 15-16, 17-18, 19-20, 21-22 and 23-24 show the results in estimation for the multiple Uniformly distributed exposures model and relative incidence of  $R = 0.7, 0.9, 1.2, 1.5, 2$  and 4 respectively. These also correspond respectively to Figures 7-12.

Tables 25-26, 27-28, 29-30, 31-32, 33-34, 35-36 show the results in estimation for the single Normally distributed exposure model and relative incidence of  $R = 0.7, 0.9, 1.2, 1.5, 2$  and 4 respectively. These also correspond respectively to Figures 13-18.

Tables 37-38, 39-40, 41-42, 43-44, 45-46 and 47-48 show the results in estimation for the multiple Normally distributed exposures model and relative incidence of  $R = 0.7, 0.9, 1.2, 1.5, 2$  and 4 respectively. These also correspond respectively to Figures 19-24.

Table 1: **Single Uniformly distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.773	0.073	0.137	0.142	0.809	0.109	0.221	0.233	0.546	0.154	0.091	0.115	0.787	0.087	0.226	0.234
15	200	0.708	0.008	0.093	0.093	0.726	0.026	0.114	0.115	0.547	0.153	0.071	0.095	0.657	0.043	0.199	0.2
15	400	0.689	0.011	0.046	0.046	0.691	0.009	0.053	0.053	0.567	0.133	0.044	0.062	0.616	0.084	0.075	0.082
15	800	0.71	0.01	0.02	0.02	0.704	0.004	0.025	0.025	0.617	0.083	0.022	0.029	0.646	0.054	0.03	0.033
30	100	0.684	0.016	0.097	0.097	0.711	0.011	0.124	0.124	0.503	0.197	0.063	0.102	0.601	0.099	0.206	0.216
30	200	0.729	0.029	0.05	0.051	0.74	0.04	0.063	0.065	0.585	0.115	0.046	0.059	0.664	0.036	0.097	0.099
30	400	0.709	0.009	0.023	0.023	0.708	0.008	0.028	0.028	0.607	0.093	0.026	0.035	0.634	0.066	0.034	0.038
30	800	0.712	0.012	0.012	0.012	0.706	0.006	0.014	0.014	0.638	0.062	0.013	0.017	0.661	0.039	0.014	0.016
45	100	0.701	0.001	0.059	0.059	0.719	0.019	0.087	0.088	0.53	0.17	0.051	0.08	0.621	0.079	0.144	0.15
45	200	0.704	0.004	0.025	0.025	0.697	0.003	0.037	0.037	0.564	0.136	0.031	0.05	0.615	0.085	0.062	0.07
45	400	0.716	0.016	0.015	0.015	0.712	0.012	0.018	0.019	0.626	0.074	0.015	0.021	0.651	0.049	0.021	0.023
45	800	0.694	0.006	0.007	0.007	0.694	0.006	0.01	0.01	0.636	0.064	0.008	0.013	0.659	0.041	0.007	0.009

Table 2: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.49	0.49	10.6	10.84	13.83	1.17	18.29	19.65	15.91	0.91	21.4	22.23
15	200	15.47	0.47	9.56	9.78	13.64	1.36	19.84	21.68	15.02	0.02	21.27	21.27
15	400	15.07	0.07	8.9	8.9	12.4	2.6	16.89	23.67	15.09	0.09	18.92	18.92
15	800	14.72	0.28	8.25	8.33	12.35	2.65	13.84	20.88	15.12	0.12	18.44	18.45
30	100	29.95	0.05	36.92	36.92	25.91	4.09	79.75	96.48	29.48	0.52	90.52	90.79
30	200	29.88	0.12	39.58	39.59	23.72	6.28	68.42	107.88	29.24	0.76	86.66	87.23
30	400	29.57	0.43	37.95	38.14	23.59	6.41	63.54	104.65	28.83	1.17	76.94	78.3
30	800	29.78	0.22	34.97	35.02	23.2	6.8	59.83	106.06	30.24	0.24	67.36	67.42
45	100	43.79	1.21	108.67	110.14	38.06	6.94	187.32	235.55	44.56	0.44	210.85	211.04
45	200	43.17	1.83	100	103.35	34.98	10.02	183.91	284.31	43.55	1.45	209.97	212.06
45	400	44.37	0.63	88.37	88.77	35.77	9.23	183.78	268.99	43.7	1.3	170.72	172.4
45	800	44.46	0.54	82.5	82.79	33.53	11.47	116.01	247.51	46.01	1.01	118.12	119.13

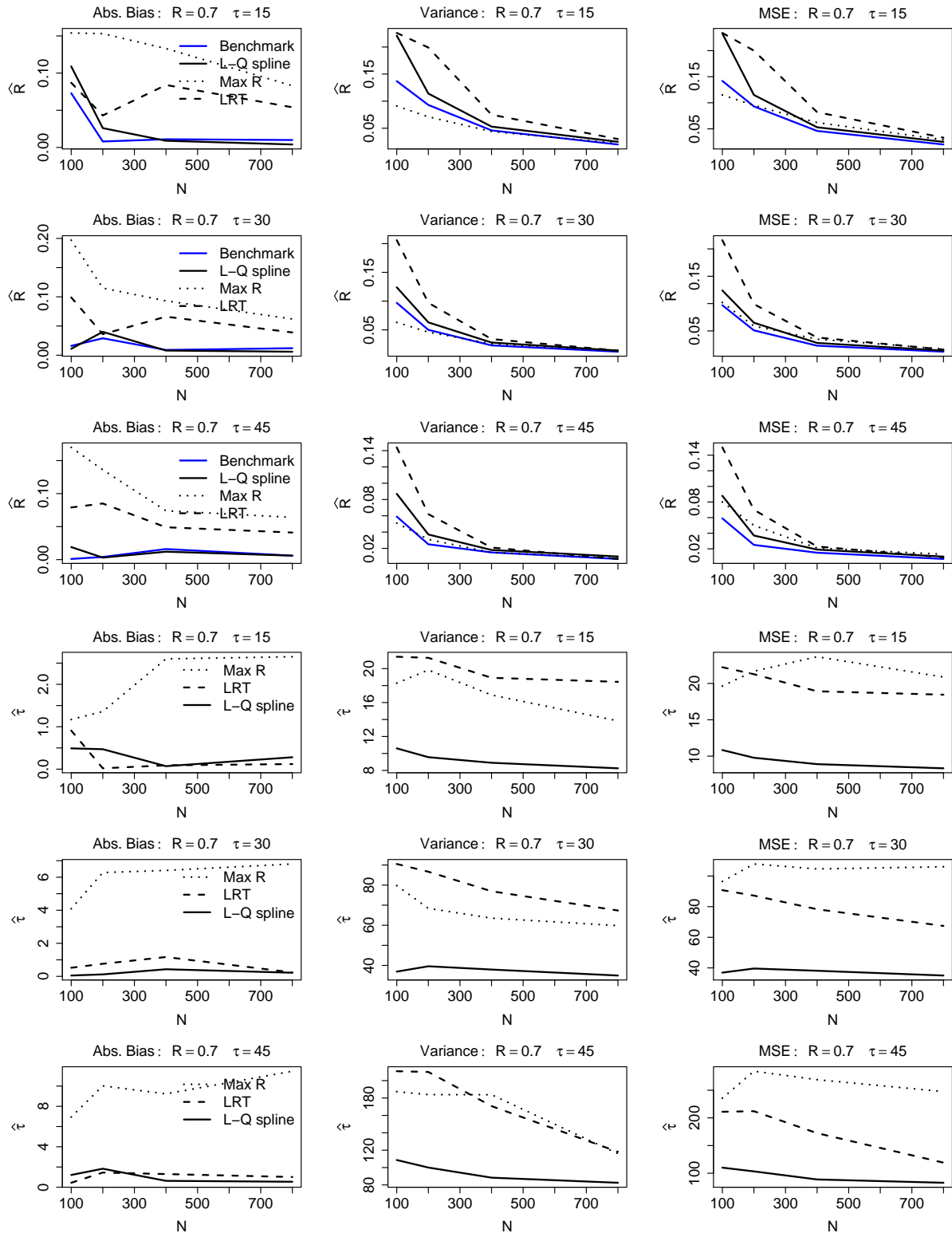


Figure 1: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.7$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.



Table 3: **Single Uniformly distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.923	0.023	0.224	0.224	1.004	0.104	0.284	0.295	0.657	0.243	0.131	0.19	0.909	0.009	0.492	0.492
15	200	0.906	0.006	0.101	0.101	0.949	0.049	0.131	0.134	0.693	0.207	0.085	0.128	0.886	0.014	0.27	0.27
15	400	0.915	0.015	0.051	0.051	0.929	0.029	0.069	0.07	0.743	0.157	0.05	0.075	0.858	0.042	0.135	0.136
15	800	0.901	0.001	0.025	0.025	0.906	0.006	0.03	0.03	0.798	0.102	0.025	0.035	0.885	0.015	0.065	0.066
30	100	0.925	0.025	0.129	0.13	0.971	0.071	0.169	0.174	0.673	0.227	0.079	0.131	0.932	0.032	0.323	0.324
30	200	0.884	0.016	0.061	0.061	0.884	0.016	0.075	0.076	0.699	0.201	0.053	0.094	0.827	0.073	0.147	0.152
30	400	0.895	0.005	0.025	0.025	0.901	0.001	0.031	0.031	0.767	0.133	0.026	0.044	0.856	0.044	0.075	0.077
30	800	0.912	0.012	0.014	0.014	0.909	0.009	0.017	0.017	0.829	0.071	0.016	0.021	0.885	0.015	0.034	0.034
45	100	0.906	0.006	0.083	0.083	0.936	0.036	0.126	0.127	0.676	0.224	0.064	0.114	0.897	0.003	0.28	0.28
45	200	0.897	0.003	0.029	0.029	0.895	0.005	0.041	0.041	0.724	0.176	0.033	0.064	0.828	0.072	0.097	0.102
45	400	0.905	0.005	0.016	0.016	0.907	0.007	0.024	0.024	0.79	0.11	0.018	0.03	0.87	0.03	0.054	0.055
45	800	0.912	0.012	0.011	0.011	0.915	0.015	0.016	0.017	0.828	0.072	0.012	0.017	0.878	0.022	0.03	0.03

Table 4: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.99	0.01	9.61	9.61	14.42	0.58	24.04	24.37	15.29	0.29	22.44	22.53
15	200	14.47	0.53	9.75	10.03	14.13	0.87	26.26	27.03	14.49	0.51	24.69	24.95
15	400	14.54	0.46	8.89	9.1	13.52	1.48	25.5	27.68	14.56	0.44	22.43	22.62
15	800	14.37	0.63	8.2	8.6	13.53	1.47	24.7	26.87	14.82	0.18	25.59	25.62
30	100	29.41	0.59	41.63	41.97	27.31	2.69	106.25	113.51	28.96	1.04	99.61	100.69
30	200	28.71	1.29	45.28	46.95	25.68	4.32	89.57	108.25	29.29	0.71	93.54	94.05
30	400	29.65	0.35	46.26	46.38	25.59	4.41	89.14	108.6	28.33	1.67	96.67	99.47
30	800	28.71	1.29	42.96	44.61	25.41	4.59	83.39	104.44	30.05	0.05	104.93	104.93
45	100	40.17	4.83	97.54	120.86	39.28	5.72	265.77	298.47	42.68	2.32	273.81	279.18
45	200	42.63	2.37	115.04	120.66	36.37	8.63	240.22	314.75	40.82	4.18	272.03	289.51
45	400	45.34	0.34	109.69	109.81	37.59	7.41	236.4	291.34	41.82	3.18	247.23	257.35
45	800	42.89	2.11	106	110.44	35.67	9.33	228.75	315.74	40.71	4.29	255.86	274.28

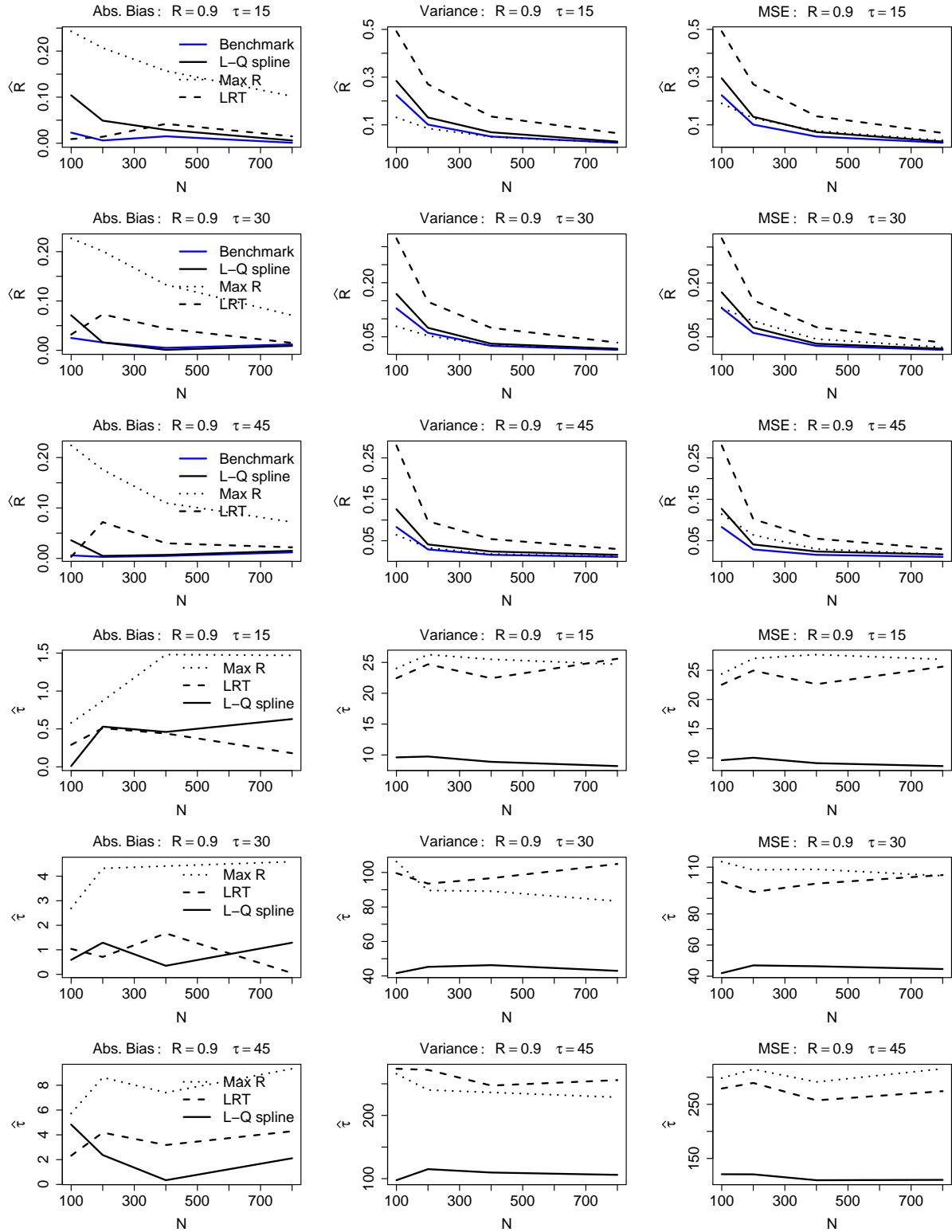


Figure 2: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.9$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 5: **Single Uniformly distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	1.171	0.029	0.291	0.292	1.265	0.065	0.359	0.363	1.546	0.346	0.415	0.535	1.319	0.119	0.666	0.681
15	200	1.173	0.027	0.132	0.133	1.214	0.014	0.169	0.169	1.413	0.213	0.158	0.203	1.224	0.024	0.317	0.317
15	400	1.192	0.008	0.078	0.078	1.196	0.004	0.102	0.102	1.358	0.158	0.095	0.12	1.252	0.052	0.171	0.174
15	800	1.193	0.007	0.044	0.044	1.208	0.008	0.052	0.052	1.312	0.112	0.055	0.068	1.254	0.054	0.088	0.091
30	100	1.209	0.009	0.184	0.184	1.28	0.08	0.268	0.274	1.522	0.322	0.245	0.349	1.34	0.14	0.447	0.466
30	200	1.203	0.003	0.092	0.092	1.236	0.036	0.126	0.127	1.423	0.223	0.115	0.165	1.294	0.094	0.222	0.231
30	400	1.211	0.011	0.051	0.052	1.228	0.028	0.06	0.061	1.342	0.142	0.058	0.078	1.279	0.079	0.096	0.102
30	800	1.196	0.004	0.023	0.023	1.21	0.01	0.031	0.031	1.292	0.092	0.027	0.035	1.256	0.056	0.037	0.04
45	100	1.216	0.016	0.121	0.121	1.264	0.064	0.163	0.167	1.501	0.301	0.156	0.246	1.344	0.144	0.315	0.335
45	200	1.194	0.006	0.058	0.058	1.225	0.025	0.082	0.082	1.395	0.195	0.073	0.111	1.283	0.083	0.156	0.162
45	400	1.202	0.002	0.03	0.03	1.217	0.017	0.04	0.04	1.336	0.136	0.034	0.052	1.276	0.076	0.065	0.071
45	800	1.195	0.005	0.013	0.013	1.21	0.01	0.02	0.02	1.285	0.085	0.016	0.023	1.259	0.059	0.022	0.025

Table 6: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.5	0.5	9.89	10.14	12.75	2.25	19.95	25	14.61	0.39	24.94	25.09
15	200	14.29	0.71	9.25	9.76	13.65	1.35	21.94	23.76	14.53	0.47	21.77	21.99
15	400	14.34	0.66	8.94	9.37	13.71	1.29	23.07	24.74	14.29	0.71	21.87	22.38
15	800	14.69	0.31	8.69	8.79	12.78	2.22	18.97	23.88	14.26	0.74	22.18	22.73
30	100	28.48	1.52	42.63	44.95	25.81	4.19	102.65	120.21	28.64	1.36	102.05	103.89
30	200	29.23	0.77	41.76	42.35	26.11	3.89	98.38	113.54	27.37	2.63	101.12	108.05
30	400	28.64	1.36	38.65	40.51	24.93	5.07	71.36	97.07	28.27	1.73	81.1	84.11
30	800	28.26	1.74	44.02	47.05	24.56	5.44	74.67	104.29	28.71	1.29	87.62	89.28
45	100	41.16	3.84	105.05	119.82	37.16	7.84	230.05	291.51	42.14	2.86	269.19	277.36
45	200	41.29	3.71	102.57	116.32	37.95	7.05	232.71	282.42	40.85	4.15	253.48	270.67
45	400	41.31	3.69	93.19	106.79	36.47	8.53	220.21	293.02	41.92	3.08	238.66	248.12
45	800	42.93	2.07	90.02	94.29	34.15	10.85	155.89	273.59	42.23	2.77	202	209.66

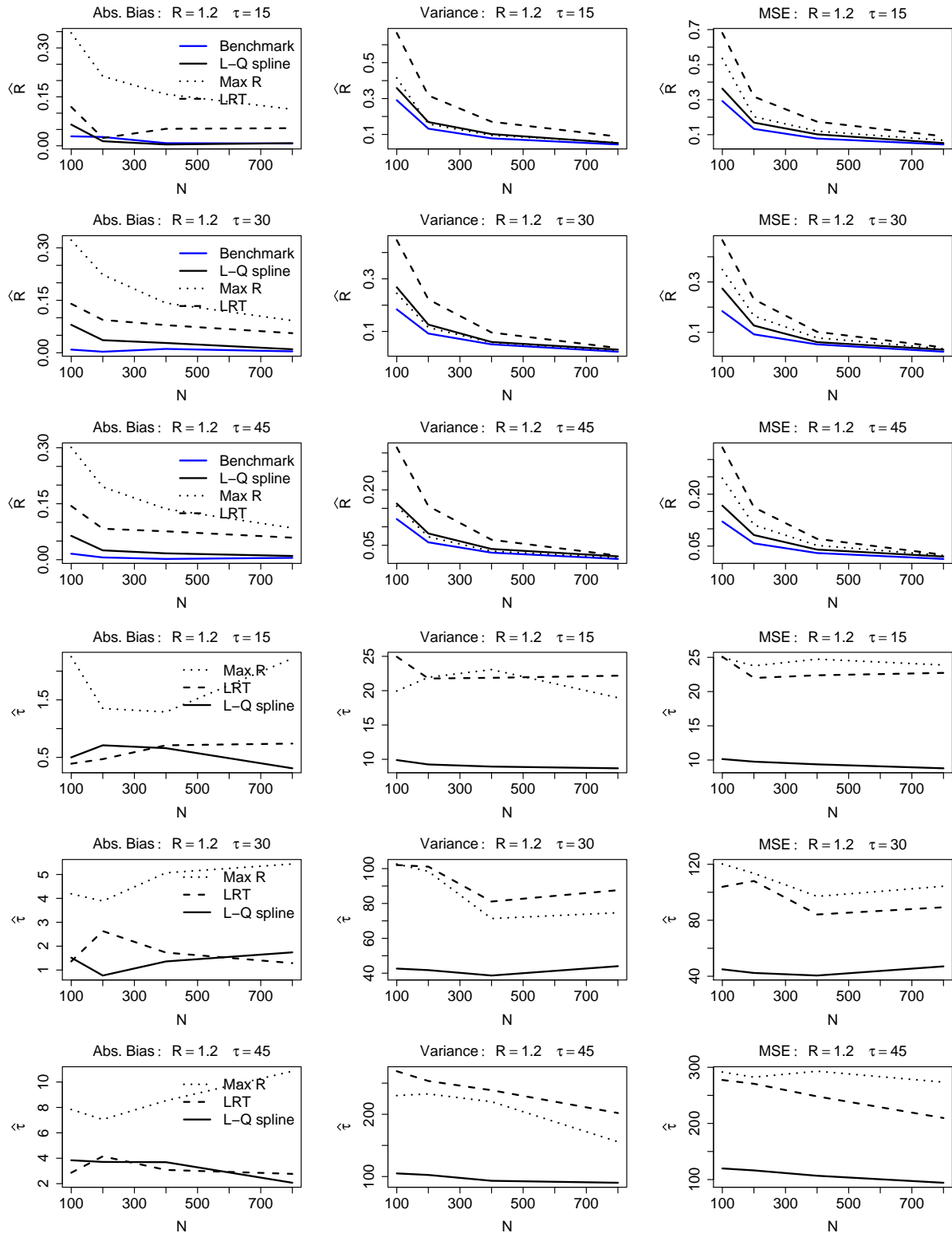


Figure 3: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 7: **Single Uniformly distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.503	0.003	0.416	0.416	1.6	0.1	0.555	0.565	1.916	0.416	0.562	0.735	1.762	0.262	0.814	0.883
15	200	1.506	0.006	0.203	0.203	1.553	0.053	0.235	0.238	1.755	0.255	0.237	0.302	1.658	0.158	0.297	0.322
15	400	1.472	0.028	0.091	0.092	1.511	0.011	0.105	0.105	1.664	0.164	0.123	0.15	1.621	0.121	0.13	0.145
15	800	1.493	0.007	0.054	0.054	1.519	0.019	0.061	0.062	1.601	0.101	0.067	0.077	1.557	0.057	0.072	0.075
30	100	1.56	0.06	0.243	0.247	1.669	0.169	0.382	0.411	1.91	0.41	0.34	0.507	1.784	0.284	0.484	0.564
30	200	1.549	0.049	0.117	0.12	1.611	0.111	0.144	0.156	1.764	0.264	0.131	0.2	1.704	0.204	0.153	0.195
30	400	1.504	0.004	0.058	0.058	1.527	0.027	0.072	0.073	1.648	0.148	0.07	0.092	1.605	0.105	0.065	0.076
30	800	1.511	0.011	0.028	0.029	1.523	0.023	0.03	0.03	1.598	0.098	0.034	0.043	1.565	0.065	0.028	0.032
45	100	1.539	0.039	0.16	0.161	1.586	0.086	0.203	0.211	1.829	0.329	0.207	0.315	1.735	0.235	0.311	0.366
45	200	1.512	0.012	0.069	0.069	1.551	0.051	0.091	0.093	1.71	0.21	0.095	0.139	1.652	0.152	0.097	0.12
45	400	1.498	0.002	0.035	0.035	1.519	0.019	0.038	0.038	1.629	0.129	0.047	0.063	1.593	0.093	0.044	0.052
45	800	1.504	0.004	0.017	0.017	1.516	0.016	0.019	0.019	1.588	0.088	0.023	0.03	1.553	0.053	0.021	0.024

Table 8: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.16	0.84	8.62	9.34	12.18	2.82	19.11	27.06	13.73	1.27	20.11	21.72
15	200	14.47	0.53	8.41	8.69	12.37	2.63	14.97	21.87	14.49	0.51	18.21	18.47
15	400	13.84	1.16	8.02	9.37	11.5	3.5	12.17	24.44	13.83	1.17	15.26	16.63
15	800	14.11	0.89	7.37	8.17	12.1	2.9	12.19	20.6	14.99	0.01	15.59	15.59
30	100	28.07	1.93	39.97	43.7	23.73	6.27	69.43	108.76	27.98	2.02	83.33	87.39
30	200	27.92	2.08	38.83	43.16	23.4	6.6	52.33	95.86	27.94	2.06	74.1	78.34
30	400	29.3	0.7	36.64	37.13	23.09	6.91	56.76	104.5	29.06	0.94	63.73	64.61
30	800	28.98	1.02	27.59	28.62	23.3	6.7	42.24	87.18	29.64	0.36	45.58	45.7
45	100	41.64	3.36	104.73	116	35.84	9.16	199.4	283.32	41.36	3.64	214.27	227.54
45	200	41.83	3.17	104.91	114.93	32.71	12.29	159.33	310.29	42.2	2.8	175.48	183.35
45	400	42.82	2.18	81.32	86.06	33.62	11.38	143.35	272.9	44.13	0.87	142.89	143.64
45	800	44.44	0.56	62.01	62.33	33.94	11.06	103.19	225.52	44.73	0.27	78.32	78.39

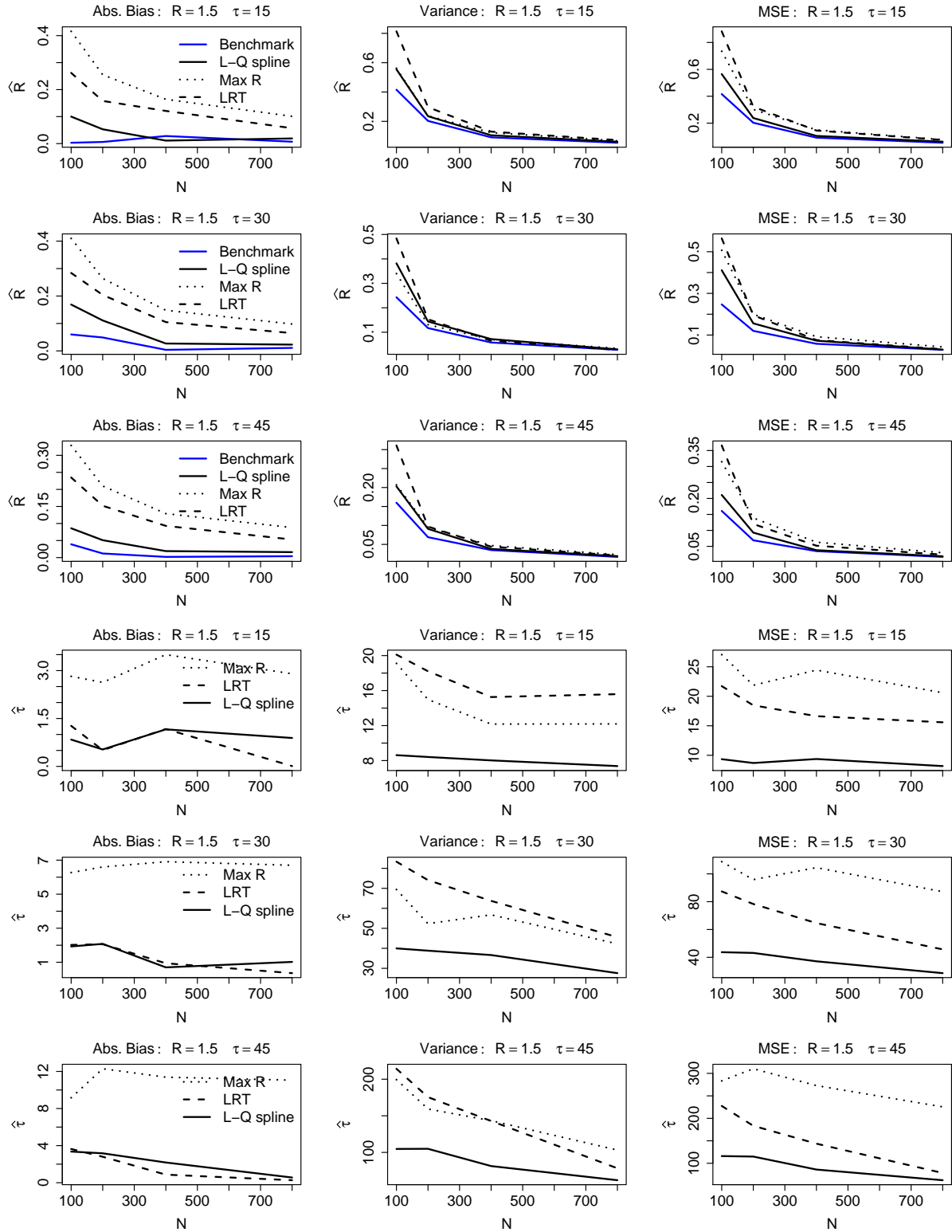


Figure 4: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.5$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 9: **Single Uniformly distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.011	0.011	0.641	0.641	2.144	0.144	0.767	0.788	2.449	0.449	0.854	1.056	2.305	0.305	0.939	1.031
15	200	1.976	0.024	0.252	0.252	2.074	0.074	0.335	0.34	2.288	0.288	0.36	0.443	2.191	0.191	0.334	0.371
15	400	2.02	0.02	0.123	0.123	2.068	0.068	0.141	0.146	2.224	0.224	0.172	0.222	2.143	0.143	0.146	0.167
15	800	1.99	0.01	0.068	0.068	2.009	0.009	0.076	0.076	2.114	0.114	0.082	0.095	2.042	0.042	0.072	0.074
30	100	2.038	0.038	0.254	0.256	2.126	0.126	0.344	0.36	2.373	0.373	0.377	0.516	2.293	0.293	0.357	0.442
30	200	2.004	0.004	0.16	0.16	2.039	0.039	0.18	0.182	2.194	0.194	0.157	0.194	2.122	0.122	0.162	0.177
30	400	2.004	0.004	0.073	0.073	2.033	0.033	0.081	0.082	2.149	0.149	0.091	0.113	2.082	0.082	0.083	0.09
30	800	2.016	0.016	0.039	0.039	2.026	0.026	0.04	0.041	2.099	0.099	0.042	0.052	2.045	0.045	0.038	0.041
45	100	2.111	0.111	0.287	0.299	2.196	0.196	0.303	0.342	2.46	0.46	0.342	0.554	2.389	0.389	0.323	0.474
45	200	1.99	0.01	0.119	0.119	2.027	0.027	0.136	0.136	2.179	0.179	0.143	0.175	2.123	0.123	0.135	0.15
45	400	2.035	0.035	0.065	0.066	2.049	0.049	0.067	0.069	2.153	0.153	0.072	0.095	2.098	0.098	0.06	0.069
45	800	2.001	0.001	0.027	0.027	2	0	0.026	0.026	2.059	0.059	0.029	0.033	2.02	0.02	0.026	0.027

Table 10: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.13	0.87	8.28	9.04	11.76	3.24	14.83	25.31	14.5	0.5	17.51	17.76
15	200	14.15	0.85	8.16	8.88	11.45	3.55	11.9	24.48	14.68	0.32	15.81	15.91
15	400	14.4	0.6	5.92	6.28	11.61	3.39	9.71	21.19	14.82	0.18	10.36	10.39
15	800	14.98	0.02	4.27	4.27	11.37	3.63	7.28	20.44	14.89	0.11	6.23	6.24
30	100	28	2	33.07	37.07	23.63	6.37	53.55	94.15	28.76	1.24	61.97	63.51
30	200	29.42	0.58	26.27	26.61	23.27	6.73	42.68	87.95	29.55	0.45	43.69	43.89
30	400	29.54	0.46	19.73	19.94	23.48	6.52	39.08	81.63	29.79	0.21	31.45	31.5
30	800	29.91	0.09	10.43	10.44	23.84	6.16	32.09	69.98	30.27	0.27	16.67	16.74
45	100	42.27	2.73	79.43	86.88	33.47	11.53	120.27	253.27	41.03	3.97	129.48	145.24
45	200	44.3	0.7	65.07	65.57	34.44	10.56	116.08	227.54	43.83	1.17	107.25	108.62
45	400	44.7	0.3	39.9	39.98	34.54	10.46	100.12	209.58	44.12	0.88	50.29	51.08
45	800	45.26	0.26	17.75	17.82	37.64	7.36	79.76	133.95	45.38	0.38	17.54	17.68

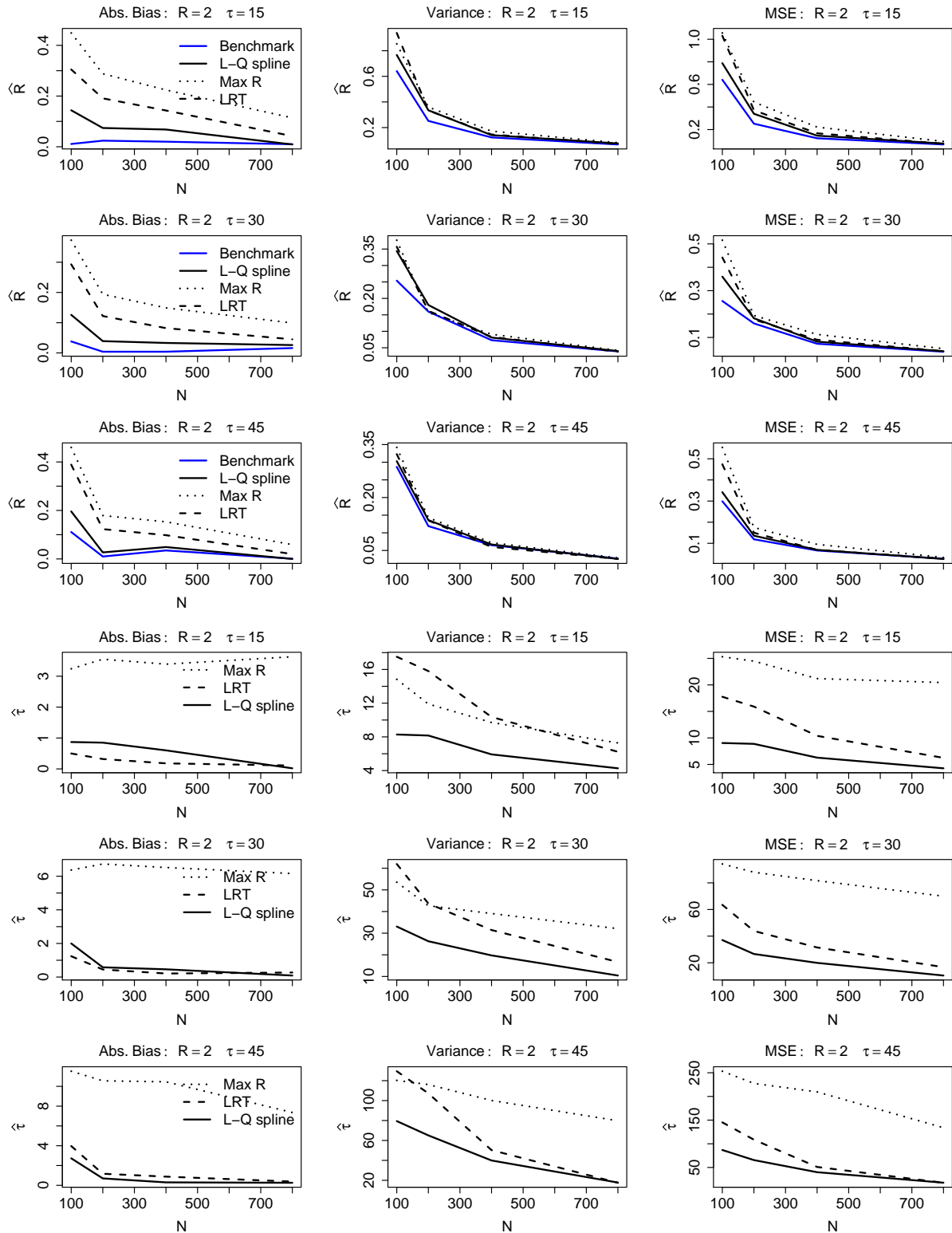


Figure 5: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.



Table 11: **Single Uniformly distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	4.146	0.146	1.203	1.224	4.314	0.314	1.381	1.48	4.567	0.567	1.287	1.609	4.482	0.482	1.398	1.631
15	200	3.97	0.03	0.671	0.672	4.046	0.046	0.75	0.752	4.31	0.31	0.804	0.901	4.115	0.115	0.777	0.79
15	400	3.962	0.038	0.359	0.36	3.989	0.011	0.387	0.387	4.186	0.186	0.422	0.457	4.024	0.024	0.39	0.39
15	800	3.975	0.025	0.156	0.156	3.976	0.024	0.164	0.165	4.097	0.097	0.172	0.181	3.989	0.011	0.163	0.163
30	100	4.013	0.013	0.756	0.756	4.11	0.11	0.853	0.865	4.367	0.367	0.849	0.984	4.209	0.209	0.875	0.918
30	200	4.017	0.017	0.462	0.462	4.013	0.013	0.472	0.472	4.209	0.209	0.437	0.481	4.074	0.074	0.455	0.461
30	400	4.011	0.011	0.232	0.232	3.996	0.004	0.237	0.237	4.11	0.11	0.243	0.255	4.023	0.023	0.231	0.232
30	800	4.013	0.013	0.106	0.106	3.992	0.008	0.109	0.109	4.048	0.048	0.108	0.11	4.015	0.015	0.107	0.108
45	100	4.08	0.08	0.67	0.677	4.129	0.129	0.64	0.657	4.341	0.341	0.676	0.792	4.263	0.263	0.666	0.735
45	200	4.137	0.137	0.457	0.476	4.133	0.133	0.465	0.482	4.256	0.256	0.467	0.533	4.21	0.21	0.47	0.514
45	400	3.995	0.005	0.193	0.193	3.972	0.028	0.196	0.197	4.049	0.049	0.197	0.199	4.022	0.022	0.199	0.2
45	800	3.982	0.018	0.105	0.105	3.949	0.051	0.098	0.101	4	0	0.101	0.101	3.99	0.01	0.102	0.102

Table 12: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT									
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	13.95	1.05	5.59	6.68	12.01	2.99	9.55	18.49	14.68	0.32	9.94	10.04	14.68	0.32	9.94	10.04
15	200	14.64	0.36	4.45	4.58	11.6	3.4	7.5	19.04	14.77	0.23	6.28	6.33	14.77	0.23	6.28	6.33
15	400	14.85	0.15	2.2	2.22	12.04	2.96	6.74	15.53	14.83	0.17	1.83	1.86	14.83	0.17	1.83	1.86
15	800	14.94	0.06	0.71	0.71	12.43	2.57	6.67	13.29	14.95	0.05	0.38	0.38	14.95	0.05	0.38	0.38
30	100	28.83	1.17	18.51	19.87	24.28	5.72	32.69	65.45	29.69	0.31	23.21	23.31	29.69	0.31	23.21	23.31
30	200	30.04	0.04	9.6	9.6	24.86	5.14	30.33	56.76	30.14	0.14	9.82	9.84	30.14	0.14	9.82	9.84
30	400	30.04	0.04	3.6	3.6	25.93	4.07	25.71	42.24	30.19	0.19	1.88	1.92	30.19	0.19	1.88	1.92
30	800	30.1	0.1	0.93	0.94	28.1	1.9	11.56	15.17	30.07	0.07	0.41	0.42	30.07	0.07	0.41	0.42
45	100	44.17	0.83	36.65	37.35	39.32	5.68	71.96	104.26	44.57	0.43	43.99	44.18	44.57	0.43	43.99	44.18
45	200	44.99	0.01	12.12	12.12	40.72	4.28	45.05	63.38	44.63	0.37	11.45	11.58	44.63	0.37	11.45	11.58
45	400	45.2	0.2	5.16	5.2	42.23	2.77	23.49	31.19	44.88	0.12	2.8	2.81	44.88	0.12	2.8	2.81
45	800	45.38	0.38	1.89	2.04	43.98	1.02	6.03	7.07	44.94	0.06	0.73	0.73	44.94	0.06	0.73	0.73

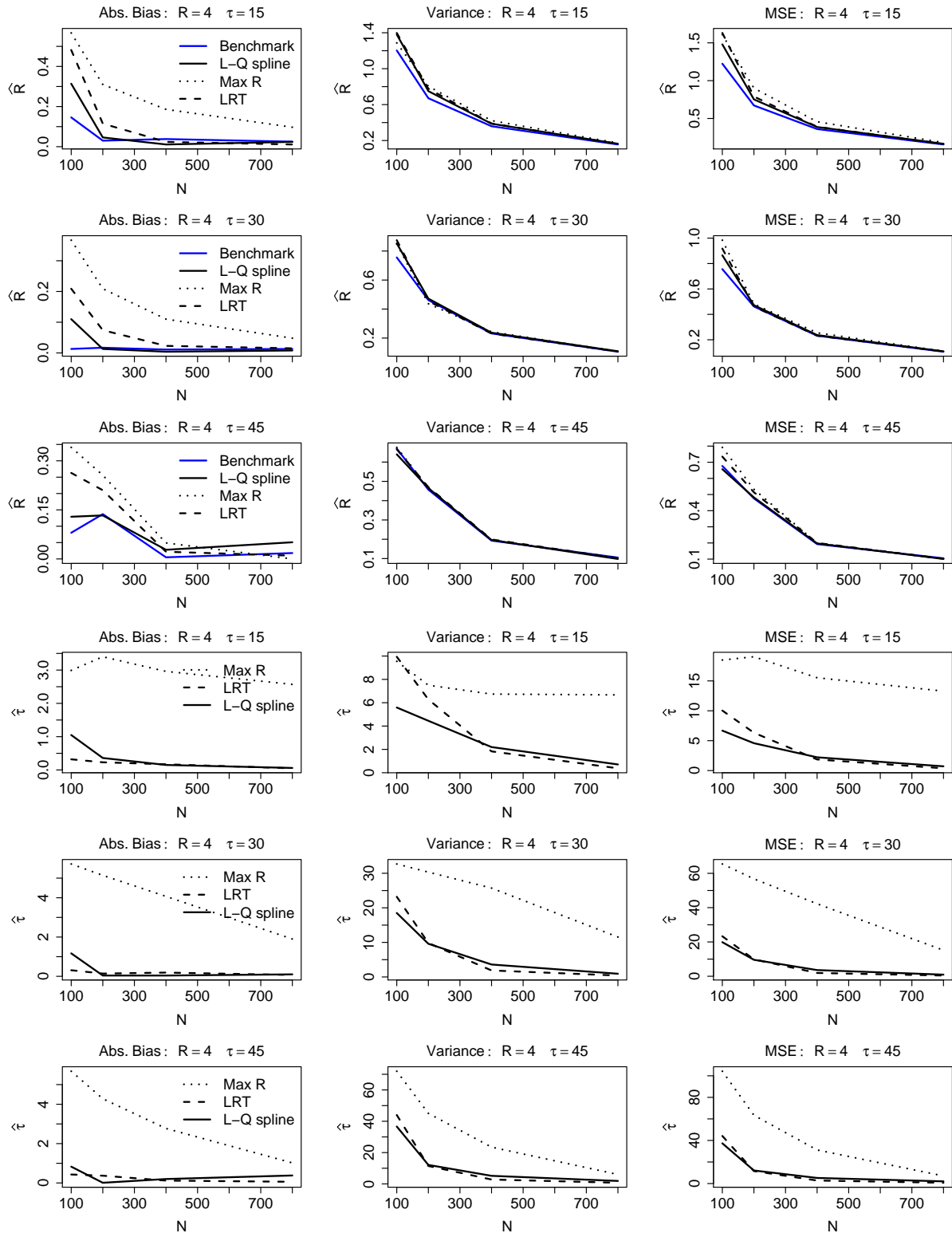


Figure 6: **Single Uniformly distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 4$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 13: **Multiple Uniformly distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.721	0.021	0.12	0.121	0.762	0.062	0.172	0.176	0.532	0.168	0.081	0.109	0.654	0.046	0.291	0.294
15	200	0.696	0.004	0.061	0.061	0.703	0.003	0.079	0.079	0.553	0.147	0.061	0.082	0.642	0.058	0.138	0.142
15	400	0.692	0.008	0.025	0.025	0.697	0.003	0.03	0.03	0.575	0.125	0.025	0.041	0.599	0.101	0.029	0.04
15	800	0.718	0.018	0.014	0.015	0.718	0.018	0.017	0.017	0.648	0.052	0.016	0.019	0.669	0.031	0.017	0.018
30	100	0.72	0.02	0.078	0.078	0.761	0.061	0.092	0.095	0.546	0.154	0.072	0.095	0.648	0.052	0.175	0.178
30	200	0.716	0.016	0.04	0.04	0.72	0.02	0.048	0.049	0.586	0.114	0.04	0.053	0.633	0.067	0.072	0.077
30	400	0.703	0.003	0.018	0.018	0.71	0.01	0.024	0.024	0.627	0.073	0.019	0.024	0.651	0.049	0.025	0.027
30	800	0.698	0.002	0.008	0.008	0.701	0.001	0.012	0.012	0.639	0.061	0.009	0.013	0.66	0.04	0.008	0.01
45	100	0.723	0.023	0.045	0.045	0.731	0.031	0.064	0.065	0.553	0.147	0.043	0.065	0.632	0.068	0.115	0.119
45	200	0.707	0.007	0.024	0.024	0.715	0.015	0.035	0.035	0.608	0.092	0.026	0.034	0.649	0.051	0.045	0.048
45	400	0.706	0.006	0.014	0.014	0.707	0.007	0.019	0.019	0.638	0.062	0.013	0.017	0.657	0.043	0.013	0.015
45	800	0.696	0.004	0.006	0.006	0.691	0.009	0.007	0.007	0.652	0.048	0.006	0.008	0.668	0.032	0.006	0.007

Table 14: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.15	0.15	10.05	10.08	13.5	1.5	19.49	21.74	15.03	0.03	19.87	19.87
15	200	15.24	0.24	8.66	8.71	13.4	1.6	20.2	22.77	15.29	0.29	22.34	22.42
15	400	15.25	0.25	8.64	8.7	12.34	2.66	14.69	21.78	14.56	0.44	18.27	18.46
15	800	15.31	0.31	8.27	8.37	12.46	2.54	15.24	21.71	15.33	0.33	16.4	16.51
30	100	29.88	0.12	40.59	40.61	26.4	3.6	88.49	101.44	30.01	0.01	89.95	89.95
30	200	29.87	0.13	37.78	37.8	24.54	5.46	68.46	98.3	29.24	0.76	78.89	79.46
30	400	30.24	0.24	41.96	42.02	24.9	5.1	56.99	82.95	30.59	0.59	63.57	63.91
30	800	30.48	0.48	32.9	33.14	23.28	6.72	53.47	98.67	30.63	0.63	54.44	54.84
45	100	43.32	1.68	117.26	120.1	37.75	7.25	237.51	290.02	43.47	1.53	232.95	235.29
45	200	43.74	1.26	97.74	99.32	36.65	8.35	192.19	261.85	45.94	0.94	174.73	175.61
45	400	43.92	1.08	84.56	85.72	36.74	8.26	147.26	215.42	45.81	0.81	132.2	132.86
45	800	45.1	0.1	70.95	70.96	35.56	9.44	131.4	220.46	45.79	0.79	96.89	97.51

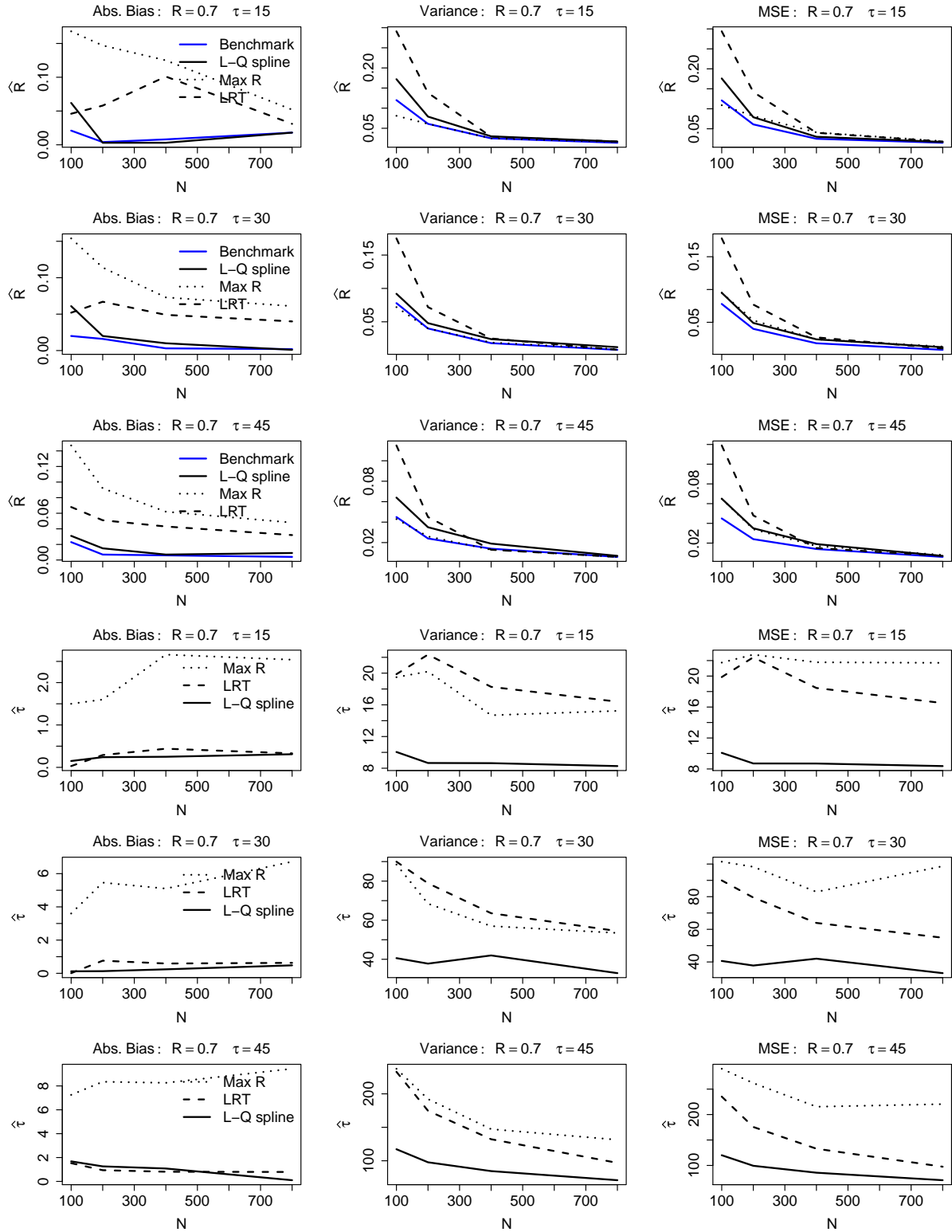


Figure 7: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.7$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 15: **Multiple Uniformly distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.918	0.018	0.173	0.174	0.955	0.055	0.2	0.203	0.682	0.218	0.136	0.184	0.938	0.038	0.464	0.466
15	200	0.902	0.002	0.075	0.075	0.919	0.019	0.098	0.099	0.699	0.201	0.067	0.107	0.85	0.05	0.183	0.185
15	400	0.906	0.006	0.043	0.043	0.914	0.014	0.051	0.051	0.771	0.129	0.038	0.054	0.882	0.018	0.102	0.103
15	800	0.89	0.01	0.021	0.021	0.886	0.014	0.029	0.029	0.791	0.109	0.023	0.035	0.844	0.056	0.048	0.051
30	100	0.899	0.001	0.101	0.101	0.941	0.041	0.138	0.14	0.676	0.224	0.075	0.125	0.881	0.019	0.26	0.26
30	200	0.909	0.009	0.055	0.055	0.927	0.027	0.065	0.066	0.738	0.162	0.048	0.074	0.872	0.028	0.135	0.135
30	400	0.924	0.024	0.023	0.024	0.926	0.026	0.031	0.032	0.816	0.084	0.019	0.026	0.905	0.005	0.06	0.06
30	800	0.888	0.012	0.012	0.012	0.882	0.018	0.015	0.015	0.814	0.086	0.012	0.019	0.852	0.048	0.025	0.027
45	100	0.887	0.013	0.059	0.059	0.916	0.016	0.085	0.085	0.679	0.221	0.038	0.086	0.843	0.057	0.168	0.172
45	200	0.921	0.021	0.036	0.037	0.93	0.03	0.051	0.052	0.76	0.14	0.033	0.053	0.879	0.021	0.108	0.108
45	400	0.913	0.013	0.013	0.013	0.915	0.015	0.02	0.02	0.812	0.088	0.014	0.021	0.886	0.014	0.045	0.045
45	800	0.905	0.005	0.007	0.007	0.901	0.001	0.01	0.01	0.83	0.07	0.009	0.013	0.864	0.036	0.019	0.021

Table 16: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.73	0.27	9.24	9.32	14.91	0.09	24.61	24.62	15.09	0.09	26.9	26.91
15	200	14.86	0.14	10.11	10.13	13.86	1.14	24.46	25.76	14.85	0.15	22.58	22.6
15	400	14.42	0.58	8.69	9.02	13.22	1.78	20.2	23.38	14.65	0.35	23.44	23.56
15	800	14.84	0.16	8.76	8.78	13.37	1.63	21.24	23.9	14.9	0.1	21.09	21.1
30	100	28.98	1.02	40.76	41.79	24.96	5.04	96.84	122.19	29.18	0.82	96.09	96.77
30	200	29.81	0.19	40.19	40.23	25.49	4.51	85.13	105.5	27.57	2.43	89.75	95.67
30	400	29.07	0.93	42.89	43.75	26.83	3.17	96.48	106.54	29.53	0.47	99.83	100.05
30	800	29.6	0.4	40.91	41.07	25.69	4.31	88.27	106.82	29.32	0.68	95.06	95.53
45	100	43.49	1.51	119.11	121.4	38.01	6.99	240.46	289.39	43.09	1.91	262.41	266.06
45	200	43.62	1.38	114.59	116.5	39.53	5.47	259.95	289.84	41.92	3.08	242.07	251.56
45	400	44.03	0.97	115.15	116.1	37.76	7.24	239.54	291.9	42.15	2.85	267.83	275.94
45	800	43.31	1.69	108.62	111.47	36.35	8.65	220.49	295.37	41.37	3.63	240.31	253.48

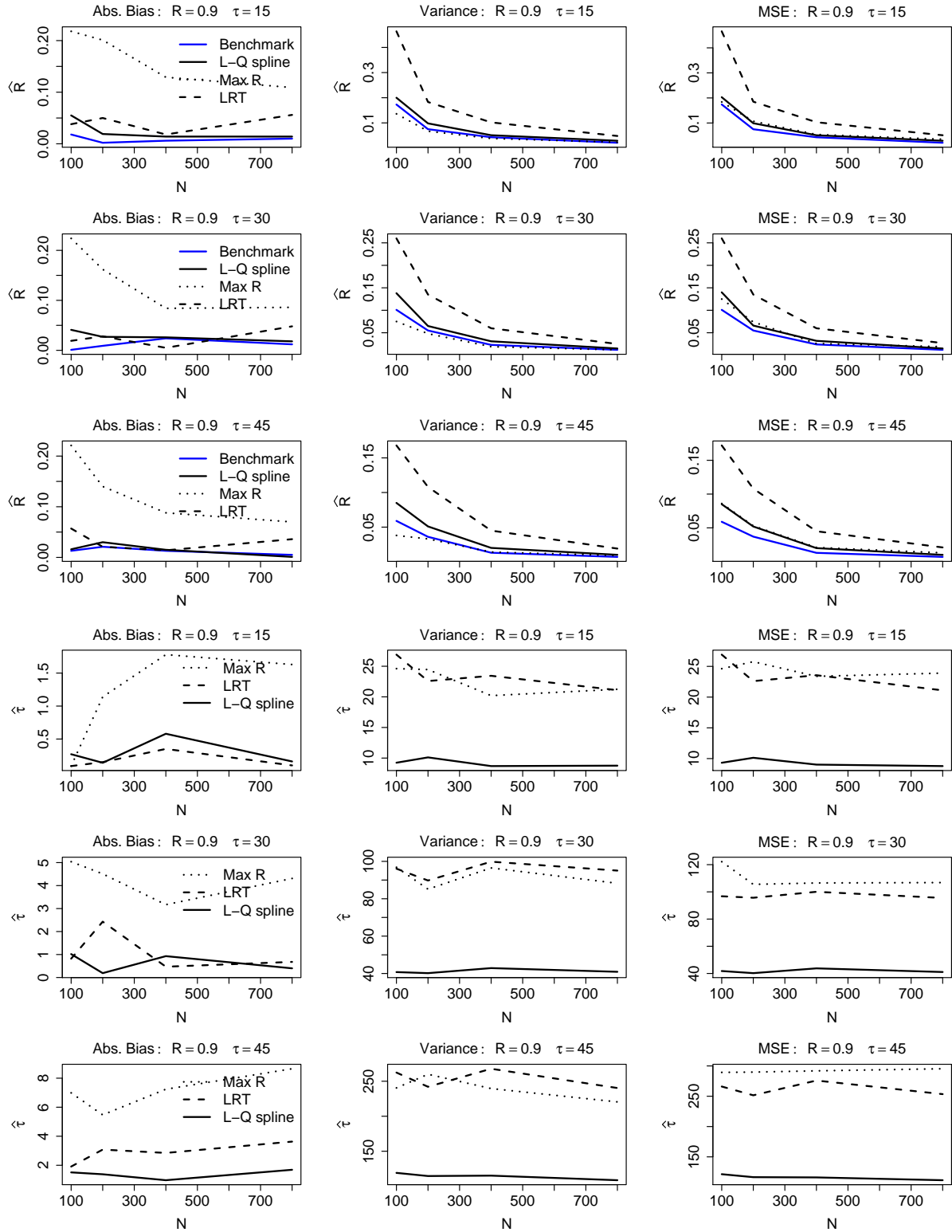


Figure 8: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.9$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 17: **Multiple Uniformly distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.202	0.002	0.211	0.211	1.286	0.086	0.284	0.291	1.524	0.324	0.296	0.401	1.285	0.085	0.535	0.542
15	200	1.197	0.003	0.103	0.103	1.226	0.026	0.144	0.145	1.43	0.23	0.131	0.184	1.278	0.078	0.262	0.268
15	400	1.209	0.009	0.066	0.066	1.233	0.033	0.081	0.083	1.365	0.165	0.079	0.107	1.291	0.091	0.127	0.136
15	800	1.203	0.003	0.033	0.033	1.223	0.023	0.04	0.04	1.305	0.105	0.043	0.054	1.264	0.064	0.055	0.06
30	100	1.209	0.009	0.121	0.121	1.255	0.055	0.161	0.164	1.467	0.267	0.144	0.215	1.314	0.114	0.293	0.306
30	200	1.196	0.004	0.061	0.061	1.22	0.02	0.079	0.08	1.355	0.155	0.074	0.098	1.247	0.047	0.145	0.147
30	400	1.21	0.01	0.035	0.035	1.231	0.031	0.044	0.045	1.33	0.13	0.037	0.054	1.281	0.081	0.061	0.067
30	800	1.214	0.014	0.019	0.019	1.224	0.024	0.025	0.026	1.3	0.1	0.024	0.034	1.277	0.077	0.027	0.033
45	100	1.185	0.015	0.086	0.086	1.228	0.028	0.139	0.14	1.463	0.263	0.133	0.202	1.304	0.104	0.273	0.284
45	200	1.179	0.021	0.043	0.044	1.212	0.012	0.06	0.061	1.352	0.152	0.059	0.082	1.27	0.07	0.113	0.118
45	400	1.222	0.022	0.021	0.022	1.245	0.045	0.031	0.033	1.341	0.141	0.027	0.047	1.311	0.111	0.038	0.05
45	800	1.195	0.005	0.012	0.012	1.211	0.011	0.014	0.014	1.273	0.073	0.014	0.019	1.255	0.055	0.019	0.022

Table 18: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.35	0.65	8.91	9.34	12.52	2.48	17.5	23.63	14.29	0.71	22.25	22.76
15	200	14.66	0.34	9.73	9.85	13.46	1.54	23.21	25.59	13.73	1.27	21.5	23.12
15	400	14.67	0.33	8.09	8.2	12.73	2.27	20.28	25.44	14.36	0.64	20.48	20.89
15	800	14.22	0.78	7.45	8.06	12.37	2.63	16.96	23.89	14.27	0.73	18.38	18.91
30	100	28.5	1.5	36.47	38.71	26.98	3.02	92.43	101.55	29.88	0.12	97.62	97.64
30	200	28.18	1.82	39.37	42.7	25.98	4.02	86.32	102.44	28.34	1.66	96.9	99.65
30	400	28.7	1.3	39.96	41.64	25.76	4.24	86.11	104.06	30.14	0.14	88.22	88.24
30	800	29.58	0.42	40.82	40.99	24.64	5.36	68.4	97.15	30.01	0.01	79.53	79.53
45	100	42.67	2.33	118.03	123.44	36.49	8.51	233	305.38	39.95	5.05	230.45	255.95
45	200	42.04	2.96	118.57	127.33	38.22	6.78	231.15	277.1	42.32	2.68	236.62	243.82
45	400	42.3	2.7	98.29	105.57	35.17	9.83	186.66	283.28	41.82	3.18	211.05	221.14
45	800	43.15	1.85	86.74	90.18	35.45	9.55	173.51	264.77	42.51	2.49	181.85	188.04

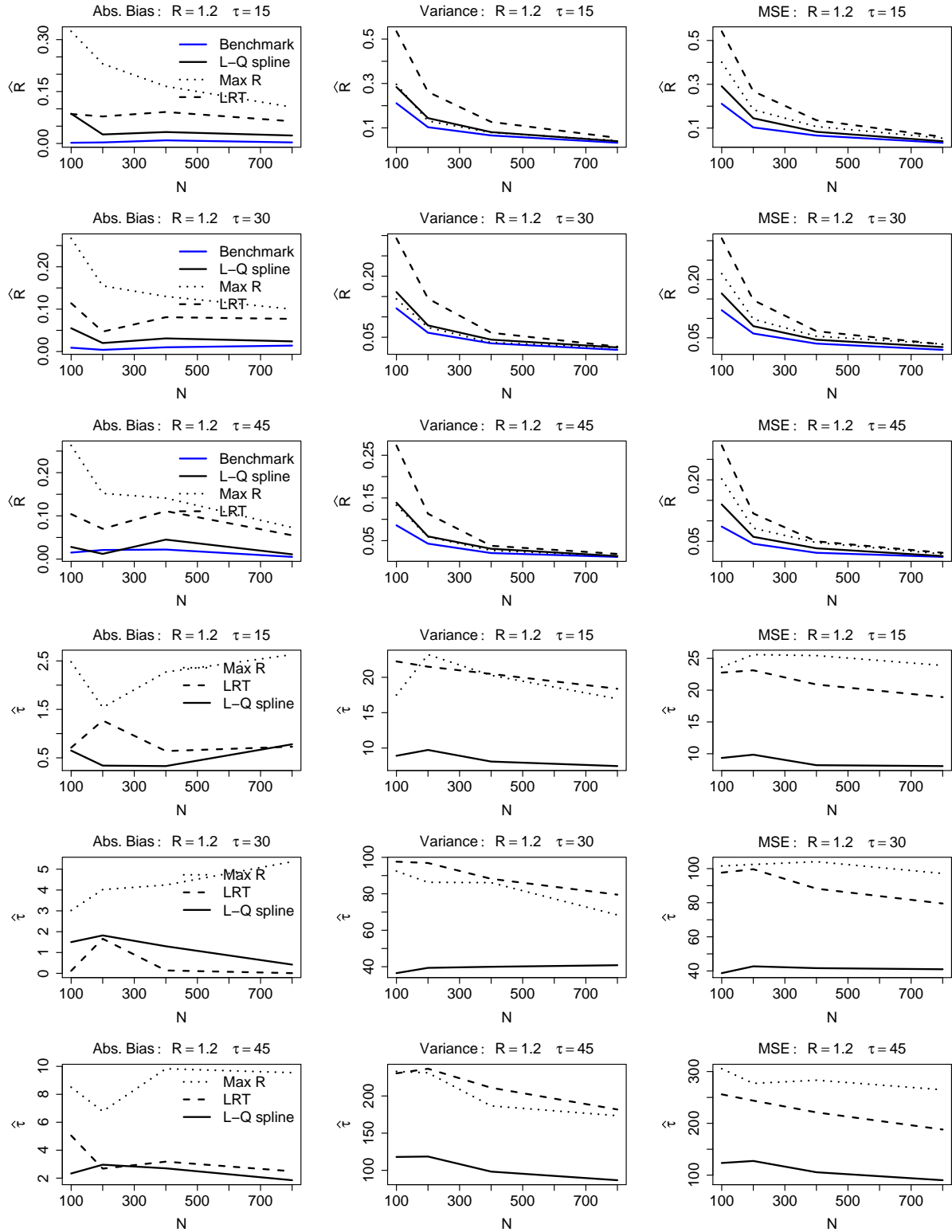


Figure 9: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.



Table 19: **Multiple Uniformly distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.535	0.035	0.358	0.359	1.604	0.104	0.438	0.449	1.868	0.368	0.455	0.591	1.701	0.201	0.676	0.716
15	200	1.558	0.058	0.152	0.155	1.589	0.089	0.191	0.199	1.79	0.29	0.208	0.293	1.705	0.205	0.261	0.303
15	400	1.505	0.005	0.062	0.062	1.525	0.025	0.08	0.081	1.653	0.153	0.089	0.112	1.618	0.118	0.088	0.101
15	800	1.477	0.023	0.04	0.041	1.493	0.007	0.046	0.046	1.578	0.078	0.05	0.056	1.544	0.044	0.046	0.048
30	100	1.48	0.02	0.152	0.153	1.54	0.04	0.207	0.209	1.784	0.284	0.207	0.288	1.71	0.21	0.268	0.312
30	200	1.518	0.018	0.09	0.09	1.569	0.069	0.107	0.112	1.703	0.203	0.111	0.152	1.648	0.148	0.13	0.152
30	400	1.483	0.017	0.045	0.046	1.505	0.005	0.05	0.05	1.602	0.102	0.055	0.066	1.571	0.071	0.052	0.057
30	800	1.512	0.012	0.024	0.025	1.519	0.019	0.029	0.029	1.576	0.076	0.028	0.033	1.549	0.049	0.025	0.027
45	100	1.522	0.022	0.143	0.144	1.579	0.079	0.19	0.197	1.793	0.293	0.188	0.274	1.717	0.217	0.28	0.327
45	200	1.501	0.001	0.064	0.064	1.544	0.044	0.078	0.08	1.661	0.161	0.079	0.105	1.63	0.13	0.087	0.104
45	400	1.503	0.003	0.038	0.038	1.518	0.018	0.044	0.044	1.594	0.094	0.042	0.051	1.572	0.072	0.042	0.047
45	800	1.51	0.01	0.016	0.016	1.517	0.017	0.016	0.017	1.56	0.06	0.017	0.02	1.542	0.042	0.015	0.017

Table 20: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.79	0.21	9.16	9.2	12.61	2.39	19.92	25.64	14.22	0.78	21.81	22.42
15	200	14.69	0.31	8.29	8.39	12.42	2.58	17.28	23.92	14.88	0.12	20.01	20.02
15	400	14.52	0.48	7.5	7.72	12.84	2.16	17.71	22.36	15.22	0.22	17.92	17.96
15	800	14.58	0.42	6.99	7.17	11.9	3.1	10.99	20.61	14.89	0.11	13.51	13.52
30	100	28.32	1.68	38.28	41.1	23.1	6.9	66.78	114.45	27.47	2.53	86.01	92.42
30	200	28.97	1.03	37.43	38.48	23.97	6.03	61.33	97.69	29	1	70.99	71.99
30	400	29.05	0.95	28.73	29.63	23.75	6.25	48.15	87.23	28.96	1.04	54.54	55.62
30	800	29.46	0.54	22.89	23.19	23.91	6.09	45.87	82.9	30.38	0.38	29.01	29.15
45	100	41.99	3.01	97.91	106.97	36.58	8.42	203.34	274.19	41.46	3.54	208.99	221.54
45	200	42.56	2.44	88.41	94.35	38.77	6.23	168.35	207.11	44.63	0.37	166.17	166.31
45	400	44.07	0.93	68.57	69.45	36.44	8.56	128.81	202.13	44.44	0.56	122.6	122.91
45	800	44.18	0.82	46.84	47.52	37.68	7.32	86.57	140.1	44.77	0.23	54.18	54.23

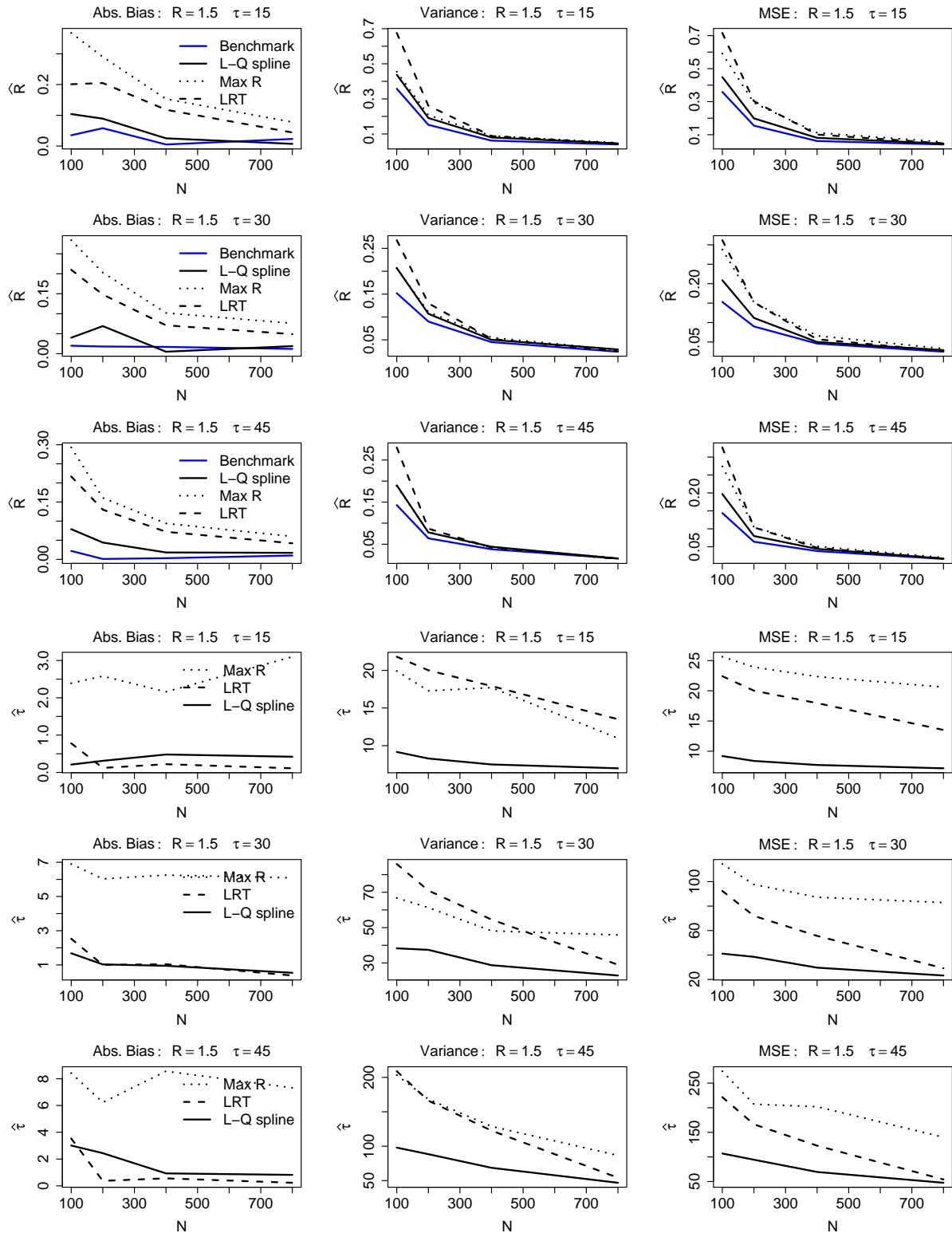


Figure 10: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.5$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 21: **Multiple Uniformly distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.093	0.093	0.427	0.436	2.167	0.167	0.514	0.542	2.449	0.449	0.573	0.774	2.366	0.366	0.633	0.767
15	200	2.089	0.089	0.233	0.241	2.145	0.145	0.301	0.322	2.329	0.329	0.288	0.396	2.254	0.254	0.275	0.34
15	400	1.975	0.025	0.105	0.106	1.999	0.001	0.125	0.125	2.145	0.145	0.136	0.157	2.074	0.074	0.122	0.127
15	800	2.029	0.029	0.052	0.053	2.043	0.043	0.055	0.057	2.116	0.116	0.059	0.072	2.055	0.055	0.053	0.056
30	100	2.048	0.048	0.261	0.264	2.126	0.126	0.303	0.319	2.332	0.332	0.337	0.448	2.269	0.269	0.341	0.413
30	200	2.024	0.024	0.124	0.125	2.078	0.078	0.137	0.143	2.194	0.194	0.146	0.183	2.136	0.136	0.134	0.152
30	400	2.001	0.001	0.073	0.073	2.011	0.011	0.077	0.077	2.086	0.086	0.076	0.083	2.046	0.046	0.075	0.077
30	800	1.988	0.012	0.032	0.032	1.994	0.006	0.034	0.034	2.041	0.041	0.034	0.035	2.011	0.011	0.033	0.033
45	100	1.997	0.003	0.189	0.189	2.073	0.073	0.213	0.218	2.251	0.251	0.231	0.294	2.218	0.218	0.218	0.266
45	200	2.067	0.067	0.101	0.105	2.09	0.09	0.106	0.114	2.191	0.191	0.105	0.141	2.152	0.152	0.097	0.12
45	400	2.011	0.011	0.054	0.054	2.021	0.021	0.055	0.055	2.09	0.09	0.059	0.067	2.055	0.055	0.056	0.059
45	800	2.009	0.009	0.027	0.027	2	0	0.028	0.028	2.039	0.039	0.026	0.027	2.023	0.023	0.027	0.027

Table 22: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.59	0.41	6.88	7.05	12.11	2.89	13.6	21.94	13.85	1.15	13.68	15
15	200	14.34	0.66	7.67	8.11	11.83	3.17	10.91	20.96	14.46	0.54	13.33	13.62
15	400	14.52	0.48	6.97	7.21	11.43	3.57	8.53	21.26	14.79	0.21	11.46	11.5
15	800	14.73	0.27	3.57	3.65	11.84	3.16	8.26	18.22	15.16	0.16	4.25	4.28
30	100	28.4	1.6	29.5	32.07	24.45	5.55	51.75	82.59	28.67	1.33	57.51	59.28
30	200	28.97	1.03	28.01	29.07	24.08	5.92	40.18	75.28	29.91	0.09	40.46	40.47
30	400	30.29	0.29	18.79	18.87	24.85	5.15	30.61	57.14	30.2	0.2	19.81	19.84
30	800	29.69	0.31	8.15	8.25	25.2	4.8	27.6	50.68	29.83	0.17	7.96	7.98
45	100	42.97	2.03	85.09	89.22	36.05	8.95	125.89	206.08	41.68	3.32	114.74	125.74
45	200	44.29	0.71	51.35	51.86	37.7	7.3	98.93	152.17	45.22	0.22	58.1	58.15
45	400	44.72	0.28	35.85	35.92	37.53	7.47	80.09	135.85	44.78	0.22	35.26	35.31
45	800	45.64	0.64	14.35	14.76	40.68	4.32	54.04	72.71	45.47	0.47	13.78	13.99

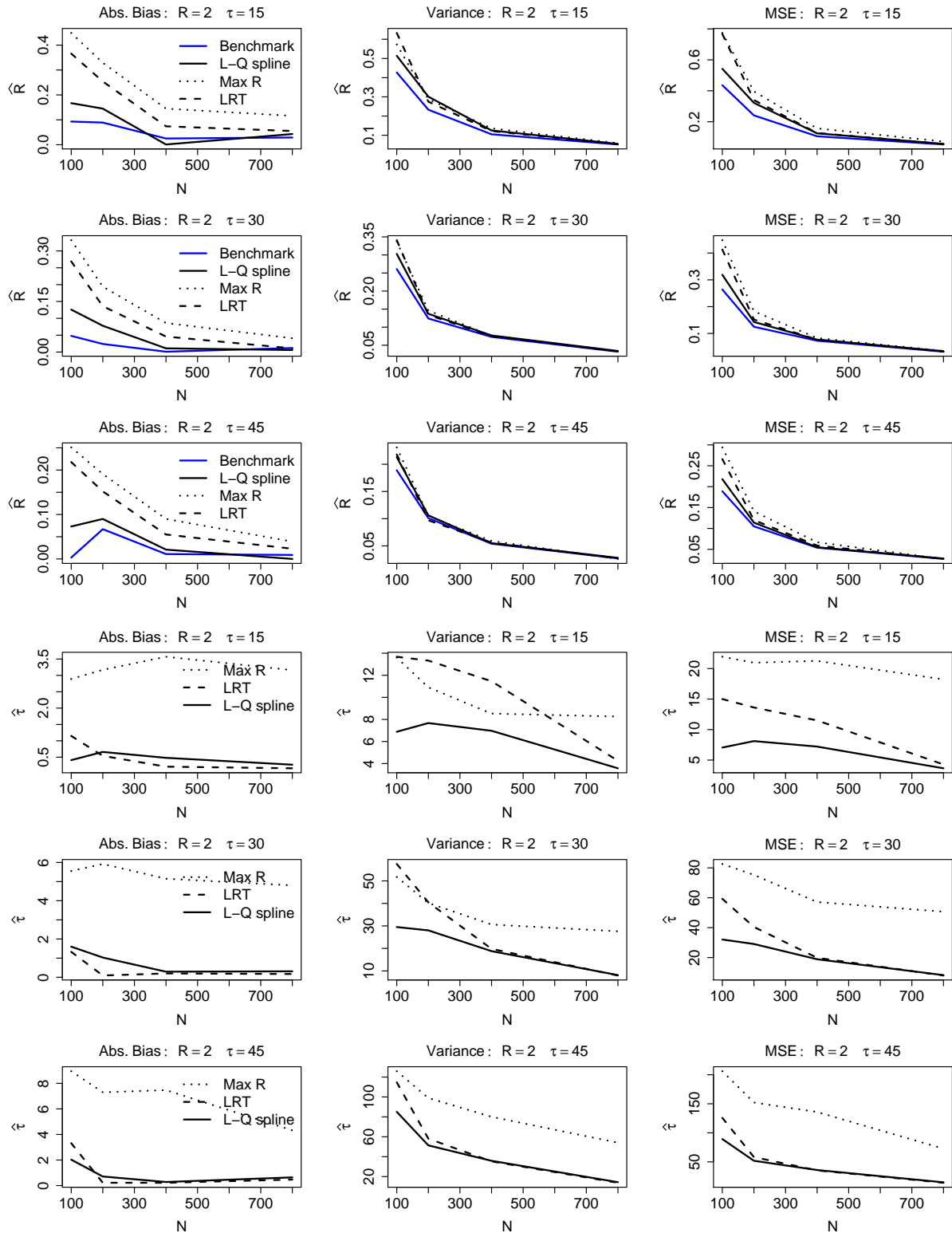


Figure 11: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 23: **Multiple Uniformly distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	4.094	0.094	0.974	0.982	4.214	0.214	1.107	1.153	4.458	0.458	1.079	1.289	4.293	0.293	1.031	1.117
15	200	4.068	0.068	0.457	0.461	4.117	0.117	0.528	0.541	4.311	0.311	0.545	0.642	4.164	0.164	0.5	0.527
15	400	4.045	0.045	0.237	0.239	4.07	0.07	0.272	0.277	4.193	0.193	0.245	0.282	4.083	0.083	0.243	0.25
15	800	3.989	0.011	0.153	0.153	3.988	0.012	0.16	0.16	4.053	0.053	0.159	0.162	4	0	0.153	0.153
30	100	4.085	0.085	0.681	0.688	4.115	0.115	0.703	0.716	4.326	0.326	0.656	0.762	4.251	0.251	0.672	0.736
30	200	4.049	0.049	0.512	0.514	4.055	0.055	0.539	0.542	4.197	0.197	0.498	0.536	4.134	0.134	0.502	0.52
30	400	4.068	0.068	0.213	0.218	4.029	0.029	0.214	0.215	4.115	0.115	0.206	0.219	4.088	0.088	0.211	0.219
30	800	4.009	0.009	0.098	0.098	3.988	0.012	0.098	0.098	4.025	0.025	0.096	0.097	4.015	0.015	0.098	0.098
45	100	4.075	0.075	0.618	0.623	4.09	0.09	0.636	0.644	4.273	0.273	0.646	0.721	4.234	0.234	0.658	0.713
45	200	4.077	0.077	0.347	0.353	4.05	0.05	0.339	0.341	4.158	0.158	0.347	0.372	4.135	0.135	0.358	0.376
45	400	4.042	0.042	0.176	0.178	4.004	0.004	0.171	0.171	4.071	0.071	0.173	0.178	4.062	0.062	0.175	0.179
45	800	4.043	0.043	0.092	0.094	4	0	0.092	0.092	4.051	0.051	0.091	0.094	4.049	0.049	0.091	0.094

Table 24: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.47	0.53	4.67	4.95	12.11	2.89	8.45	16.8	14.86	0.14	6.14	6.16
15	200	14.86	0.14	3.36	3.38	12.32	2.68	7.15	14.35	14.89	0.11	2.88	2.89
15	400	14.89	0.11	1.75	1.76	12.86	2.14	6.23	10.79	14.94	0.06	1.36	1.37
15	800	14.92	0.08	0.44	0.44	13.41	1.59	5.15	7.68	14.94	0.06	0.27	0.27
30	100	29.61	0.39	13.6	13.75	26.29	3.71	30.14	43.93	29.83	0.17	22.59	22.62
30	200	29.79	0.21	7.42	7.46	26.25	3.75	22.88	36.97	29.41	0.59	7.7	8.05
30	400	30.31	0.31	1.71	1.81	27.97	2.03	13.31	17.41	29.91	0.09	1.85	1.85
30	800	30.15	0.15	0.92	0.94	29.27	0.73	3.09	3.63	29.95	0.05	0.31	0.32
45	100	45.32	0.32	28.01	28.11	40.72	4.28	61.26	79.59	44.5	0.5	28.64	28.89
45	200	45.62	0.62	12.81	13.19	42.27	2.73	26.29	33.73	44.72	0.28	8.8	8.87
45	400	45.64	0.64	3.55	3.96	44.01	0.99	5.87	6.86	45.01	0.01	1.9	1.9
45	800	45.67	0.67	1.11	1.55	44.78	0.22	0.55	0.6	45.01	0.01	0.5	0.5

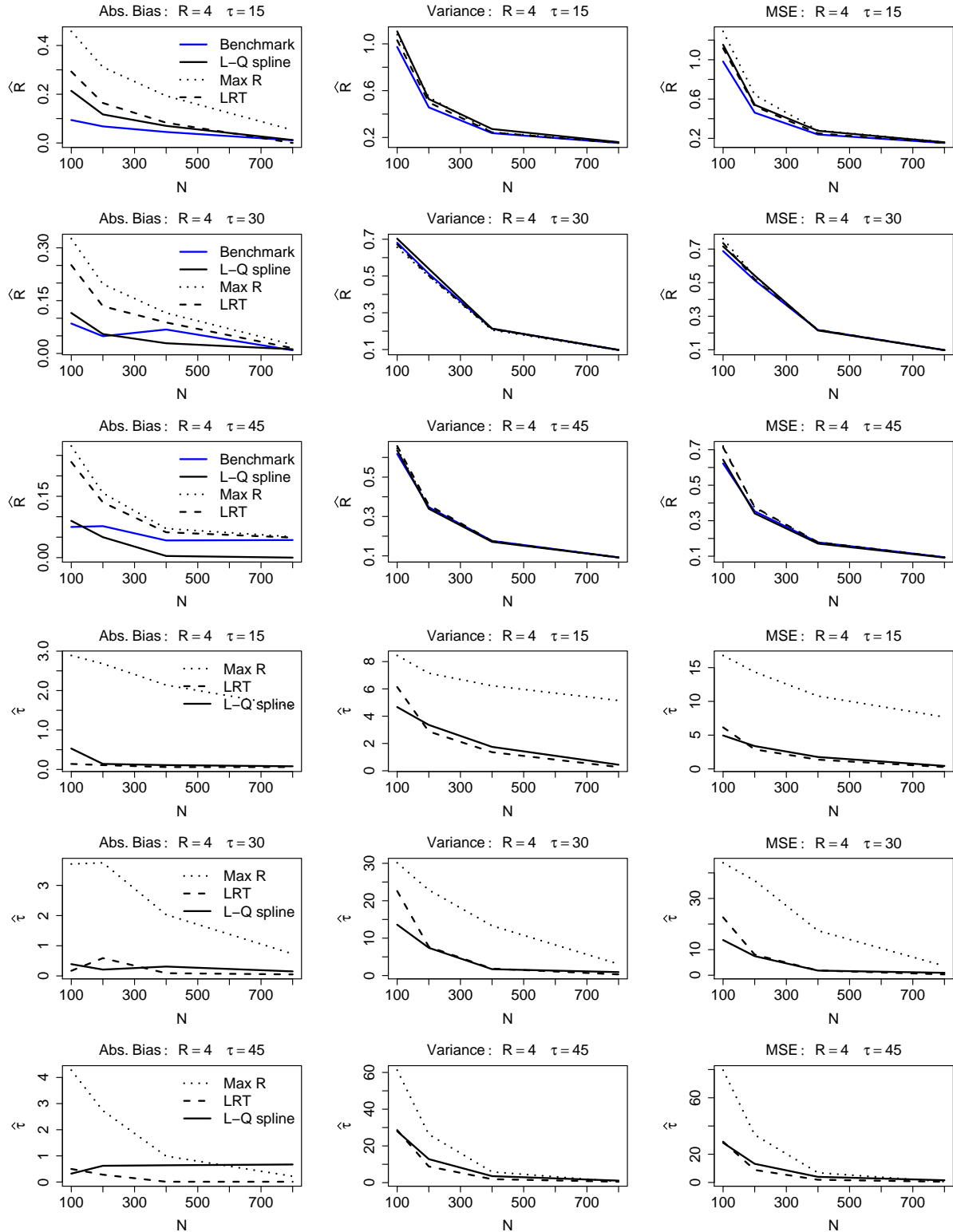


Figure 12: **Multiple Uniformly distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 4$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 25: **Single Normally distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.72	0.02	0.146	0.147	0.764	0.064	0.208	0.212	0.532	0.168	0.098	0.126	0.642	0.058	0.334	0.337
15	200	0.75	0.05	0.102	0.105	0.788	0.088	0.134	0.142	0.551	0.149	0.089	0.111	0.676	0.024	0.226	0.226
15	400	0.703	0.003	0.039	0.039	0.706	0.006	0.044	0.044	0.569	0.131	0.035	0.052	0.608	0.092	0.059	0.067
15	800	0.712	0.012	0.023	0.023	0.716	0.016	0.029	0.029	0.62	0.08	0.023	0.03	0.65	0.05	0.032	0.035
30	100	0.707	0.007	0.1	0.1	0.738	0.038	0.133	0.135	0.5	0.2	0.066	0.106	0.616	0.084	0.212	0.219
30	200	0.729	0.029	0.049	0.05	0.726	0.026	0.066	0.067	0.584	0.116	0.044	0.058	0.655	0.045	0.103	0.105
30	400	0.7	0	0.026	0.026	0.698	0.002	0.035	0.035	0.598	0.102	0.027	0.037	0.624	0.076	0.033	0.039
30	800	0.705	0.005	0.009	0.009	0.702	0.002	0.014	0.014	0.637	0.063	0.012	0.016	0.659	0.041	0.012	0.014
45	100	0.73	0.03	0.062	0.063	0.761	0.061	0.104	0.108	0.537	0.163	0.047	0.073	0.637	0.063	0.154	0.158
45	200	0.707	0.007	0.03	0.03	0.712	0.012	0.043	0.043	0.563	0.137	0.03	0.048	0.608	0.092	0.054	0.063
45	400	0.718	0.018	0.015	0.015	0.719	0.019	0.019	0.019	0.62	0.08	0.016	0.023	0.65	0.05	0.022	0.025
45	800	0.701	0.001	0.008	0.008	0.697	0.003	0.01	0.01	0.645	0.055	0.009	0.012	0.667	0.033	0.009	0.01

Table 26: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.41	0.41	10.62	10.79	14.23	0.77	21.57	22.17	16.29	1.29	20.14	21.8
15	200	14.74	0.26	9.78	9.84	13.83	1.17	22.31	23.67	15.16	0.16	24.26	24.29
15	400	15.1	0.1	8.35	8.36	12.72	2.28	17.16	22.36	14.45	0.55	20.04	20.34
15	800	14.85	0.15	7.18	7.21	12.31	2.69	15.15	22.4	14.7	0.3	17.01	17.11
30	100	29.82	0.18	47.69	47.72	26.46	3.54	94.95	107.5	30.03	0.03	100.47	100.47
30	200	29.69	0.31	39.17	39.27	24.91	5.09	66.4	92.32	29.63	0.37	77.47	77.61
30	400	29.49	0.51	38.83	39.1	24.38	5.62	71.32	102.88	29.46	0.54	71.38	71.67
30	800	30.09	0.09	36.42	36.43	23.7	6.3	50.92	90.63	30.94	0.94	67	67.88
45	100	44.4	0.6	122.19	122.55	38.1	6.9	197.68	245.28	44.69	0.31	215.23	215.33
45	200	44.41	0.59	96.89	97.23	36.47	8.53	197.67	270.47	43.93	1.07	214.98	216.13
45	400	45.53	0.53	82.8	83.09	34.13	10.87	154.47	272.72	44.42	0.58	196.92	197.26
45	800	44.94	0.06	71.32	71.32	35.24	9.76	145.23	240.46	45.86	0.86	132.43	133.18

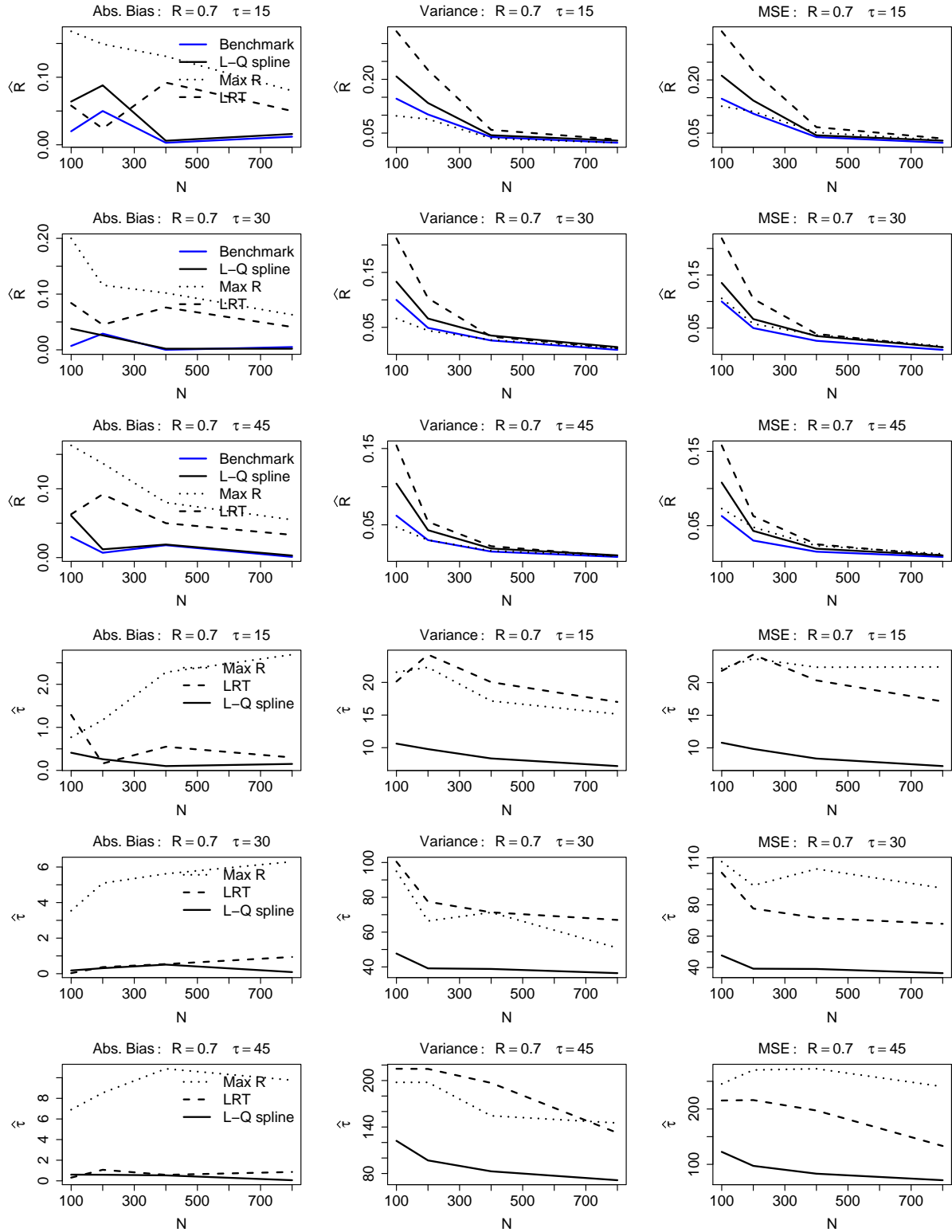


Figure 13: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.7$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.



Table 27: **Single Normally distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	0.884	0.016	0.23	0.23	0.924	0.024	0.286	0.287	0.627	0.273	0.123	0.198	0.926	0.026	0.461	0.462
15	200	0.955	0.055	0.131	0.134	0.989	0.089	0.172	0.18	0.717	0.183	0.102	0.136	0.932	0.032	0.32	0.321
15	400	0.879	0.021	0.048	0.049	0.886	0.014	0.059	0.059	0.739	0.161	0.043	0.069	0.858	0.042	0.133	0.135
15	800	0.909	0.009	0.025	0.025	0.922	0.022	0.032	0.032	0.788	0.112	0.024	0.037	0.854	0.046	0.058	0.061
30	100	0.925	0.025	0.118	0.119	0.98	0.08	0.174	0.181	0.667	0.233	0.096	0.151	0.922	0.022	0.384	0.384
30	200	0.915	0.015	0.059	0.059	0.924	0.024	0.086	0.087	0.722	0.178	0.047	0.078	0.899	0.001	0.178	0.178
30	400	0.899	0.001	0.03	0.03	0.905	0.005	0.046	0.046	0.767	0.133	0.026	0.044	0.858	0.042	0.072	0.074
30	800	0.894	0.006	0.014	0.014	0.889	0.011	0.019	0.019	0.806	0.094	0.016	0.025	0.856	0.044	0.036	0.038
45	100	0.894	0.006	0.081	0.081	0.915	0.015	0.114	0.114	0.633	0.267	0.054	0.125	0.828	0.072	0.252	0.257
45	200	0.899	0.001	0.037	0.037	0.913	0.013	0.054	0.054	0.727	0.173	0.038	0.068	0.872	0.028	0.135	0.136
45	400	0.887	0.013	0.019	0.019	0.889	0.011	0.029	0.029	0.772	0.128	0.018	0.034	0.846	0.054	0.053	0.056
45	800	0.911	0.011	0.011	0.011	0.911	0.011	0.016	0.016	0.833	0.067	0.012	0.017	0.881	0.019	0.029	0.029

Table 28: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.36	0.36	8.61	8.74	14.3	0.7	21.96	22.45	15.35	0.35	23.38	23.5
15	200	14.88	0.12	8.62	8.64	14.2	0.8	24.97	25.62	14.35	0.65	22.46	22.88
15	400	14.8	0.2	10.04	10.08	13.37	1.63	21.2	23.85	14.23	0.77	22.17	22.76
15	800	14.84	0.16	9.87	9.9	13.09	1.91	22.3	25.95	14.35	0.65	23.62	24.05
30	100	28.48	1.52	52.58	54.9	27.81	2.19	118.38	123.18	29.14	0.86	111.12	111.85
30	200	28.61	1.39	41.63	43.56	26.76	3.24	98.22	108.69	27.92	2.08	113.37	117.67
30	400	29.03	0.97	40.7	41.64	24.79	5.21	99.68	126.78	29.27	0.73	96.8	97.33
30	800	29.22	0.78	43.59	44.2	24.58	5.42	86.18	115.53	27.55	2.45	85.67	91.66
45	100	42.62	2.38	114.24	119.91	36.56	8.44	244.38	315.56	42.91	2.09	277.48	281.82
45	200	41.87	3.13	94.96	104.76	38.94	6.06	246.34	283.01	42.21	2.79	269.95	277.76
45	400	43.81	1.19	104.83	106.24	39.11	5.89	239.81	274.56	45.08	0.08	244.13	244.14
45	800	42.4	2.6	112.4	119.18	38.43	6.57	237.24	280.44	45.65	0.65	244.51	244.93

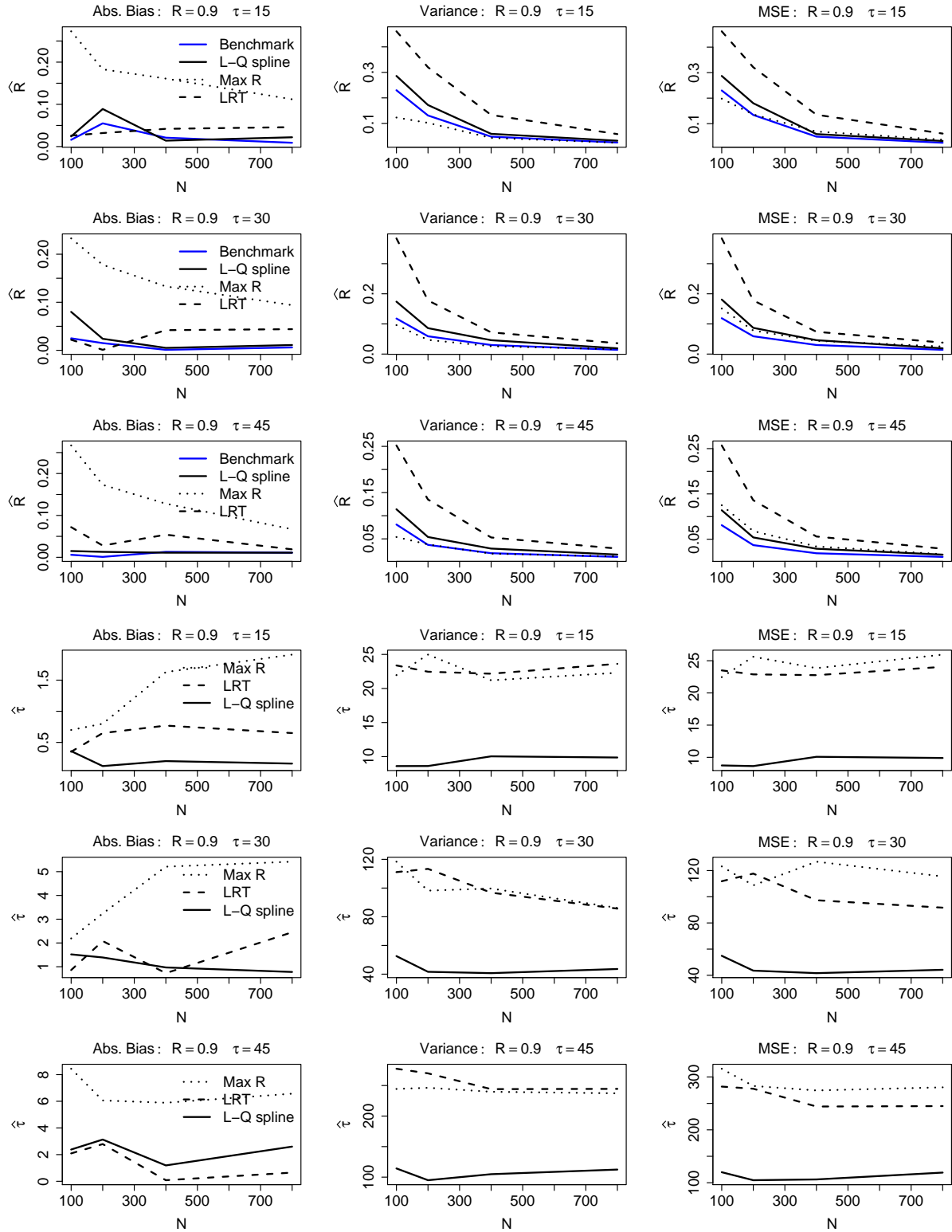


Figure 14: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.9$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 29: **Single Normally distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.211	0.011	0.351	0.351	1.314	0.114	0.435	0.448	1.618	0.418	0.475	0.65	1.38	0.18	0.762	0.794
15	200	1.252	0.052	0.147	0.15	1.296	0.096	0.178	0.187	1.53	0.33	0.217	0.326	1.373	0.173	0.369	0.399
15	400	1.194	0.006	0.083	0.083	1.21	0.01	0.098	0.099	1.355	0.155	0.102	0.126	1.249	0.049	0.176	0.178
15	800	1.2	0	0.04	0.04	1.218	0.018	0.055	0.055	1.333	0.133	0.051	0.068	1.256	0.056	0.092	0.095
30	100	1.205	0.005	0.154	0.154	1.257	0.057	0.252	0.256	1.541	0.341	0.25	0.366	1.324	0.124	0.486	0.502
30	200	1.192	0.008	0.069	0.069	1.221	0.021	0.103	0.103	1.409	0.209	0.095	0.138	1.304	0.104	0.179	0.19
30	400	1.223	0.023	0.051	0.051	1.256	0.056	0.075	0.078	1.374	0.174	0.066	0.097	1.31	0.11	0.109	0.121
30	800	1.181	0.019	0.019	0.019	1.196	0.004	0.024	0.024	1.281	0.081	0.023	0.029	1.243	0.043	0.035	0.037
45	100	1.188	0.012	0.12	0.12	1.24	0.04	0.185	0.187	1.485	0.285	0.168	0.249	1.273	0.073	0.349	0.354
45	200	1.223	0.023	0.066	0.066	1.236	0.036	0.081	0.083	1.428	0.228	0.087	0.138	1.305	0.105	0.169	0.18
45	400	1.207	0.007	0.029	0.029	1.232	0.032	0.04	0.041	1.355	0.155	0.04	0.064	1.311	0.111	0.061	0.074
45	800	1.197	0.003	0.014	0.014	1.207	0.007	0.018	0.018	1.283	0.083	0.015	0.022	1.265	0.065	0.019	0.023

Table 30: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.46	0.54	8.88	9.17	12.9	2.1	20.78	25.2	14.31	0.69	22.83	23.3
15	200	14.94	0.06	10.22	10.23	12.71	2.29	20.31	25.54	14.26	0.74	23.25	23.8
15	400	14.59	0.41	8.22	8.39	12.97	2.03	18.73	22.85	14.86	0.14	21.86	21.87
15	800	14.31	0.69	8.92	9.39	12.22	2.78	18.33	26.06	14.24	0.76	22.65	23.24
30	100	29.08	0.92	43.74	44.59	26.51	3.49	98.71	110.91	26.5	3.5	96.4	108.67
30	200	28.43	1.57	39.35	41.81	26.47	3.53	91.09	103.54	29.39	0.61	97.61	97.99
30	400	29.12	0.88	43.83	44.61	24.91	5.09	82.07	107.98	27.95	2.05	93.98	98.18
30	800	29.41	0.59	37.16	37.51	24.89	5.11	74.49	100.55	28.97	1.03	84.36	85.42
45	100	42.96	2.04	111.93	116.08	38.34	6.66	248.42	292.75	42.41	2.59	255.5	262.2
45	200	43.09	1.91	107.26	110.9	36.4	8.6	214.62	288.63	42.83	2.17	264.22	268.91
45	400	42.28	2.72	107.15	114.54	36.96	8.04	229.33	293.97	42.39	2.61	236.85	243.68
45	800	42.38	2.62	109.8	116.66	36.57	8.43	188.44	259.54	44.7	0.3	196.36	196.45

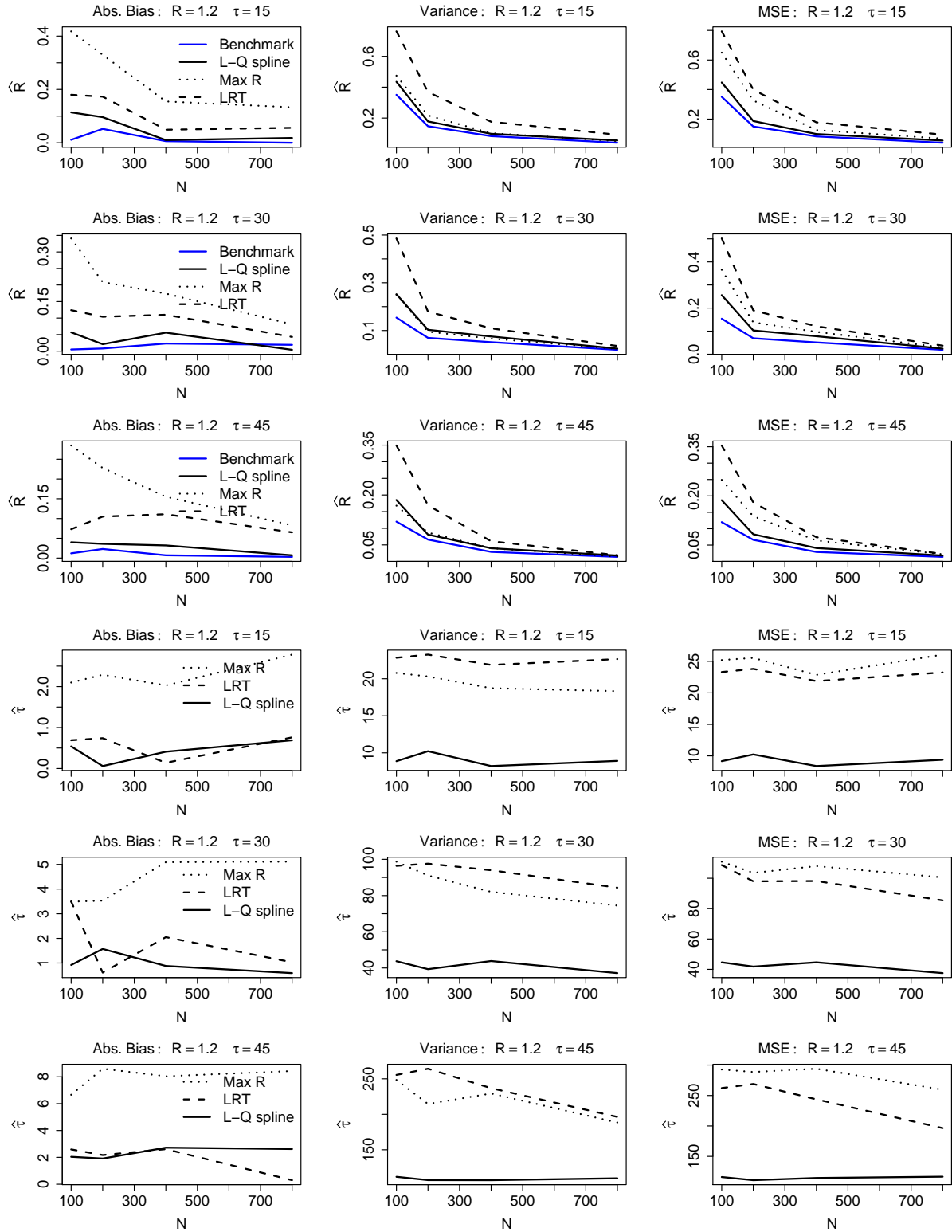


Figure 15: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 31: **Single Normally distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.488	0.012	0.393	0.394	1.579	0.079	0.499	0.505	1.883	0.383	0.643	0.789	1.724	0.224	0.831	0.881
15	200	1.533	0.033	0.199	0.2	1.625	0.125	0.3	0.315	1.813	0.313	0.283	0.381	1.708	0.208	0.377	0.42
15	400	1.463	0.037	0.092	0.093	1.471	0.029	0.111	0.111	1.629	0.129	0.1	0.116	1.585	0.085	0.111	0.118
15	800	1.517	0.017	0.047	0.047	1.544	0.044	0.052	0.054	1.639	0.139	0.053	0.072	1.597	0.097	0.051	0.061
30	100	1.508	0.008	0.221	0.221	1.575	0.075	0.308	0.314	1.81	0.31	0.265	0.361	1.699	0.199	0.398	0.437
30	200	1.532	0.032	0.1	0.101	1.57	0.07	0.117	0.121	1.748	0.248	0.134	0.195	1.698	0.198	0.147	0.186
30	400	1.492	0.008	0.054	0.054	1.526	0.026	0.077	0.078	1.642	0.142	0.072	0.092	1.605	0.105	0.067	0.078
30	800	1.49	0.01	0.024	0.024	1.501	0.001	0.027	0.027	1.577	0.077	0.029	0.035	1.542	0.042	0.024	0.026
45	100	1.56	0.06	0.213	0.217	1.633	0.133	0.273	0.291	1.845	0.345	0.247	0.366	1.747	0.247	0.365	0.426
45	200	1.563	0.063	0.098	0.102	1.629	0.129	0.123	0.14	1.762	0.262	0.118	0.187	1.712	0.212	0.135	0.18
45	400	1.492	0.008	0.034	0.034	1.517	0.017	0.038	0.039	1.619	0.119	0.044	0.058	1.592	0.092	0.038	0.046
45	800	1.511	0.011	0.023	0.023	1.514	0.014	0.026	0.026	1.579	0.079	0.027	0.033	1.552	0.052	0.024	0.027

Table 32: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.18	0.82	9.5	10.18	12.66	2.34	19.52	24.98	14.3	0.7	21.46	21.96
15	200	13.77	1.23	8.21	9.72	12.56	2.44	15.89	21.86	14.61	0.39	19.31	19.46
15	400	14.84	0.16	9.47	9.5	12.27	2.73	14.2	21.67	14.39	0.61	17.69	18.07
15	800	14.37	0.63	6.5	6.9	11.94	3.06	12.81	22.18	14.72	0.28	15.23	15.3
30	100	28.43	1.57	37.8	40.26	25.15	4.85	70.25	93.81	28.66	1.34	88.77	90.57
30	200	29.5	0.5	35.28	35.53	23.97	6.03	59.39	95.75	28.44	1.56	67.51	69.95
30	400	28.1	1.9	34.15	37.76	23.34	6.66	53.3	97.7	29.32	0.68	64.2	64.66
30	800	29.76	0.24	28.04	28.1	23.87	6.13	43.15	80.73	30.48	0.48	42.09	42.32
45	100	41.86	3.14	100.08	109.91	36.7	8.3	154.1	223.02	40.88	4.12	192.54	209.52
45	200	42.23	2.77	84.52	92.19	34.71	10.29	145.91	251.72	43.64	1.36	178.88	180.72
45	400	43.69	1.31	84.75	86.45	35.2	9.8	121.73	217.75	43.32	1.68	134.45	137.27
45	800	45.58	0.58	57.18	57.51	35.91	9.09	99.72	182.26	44.92	0.08	67.84	67.85

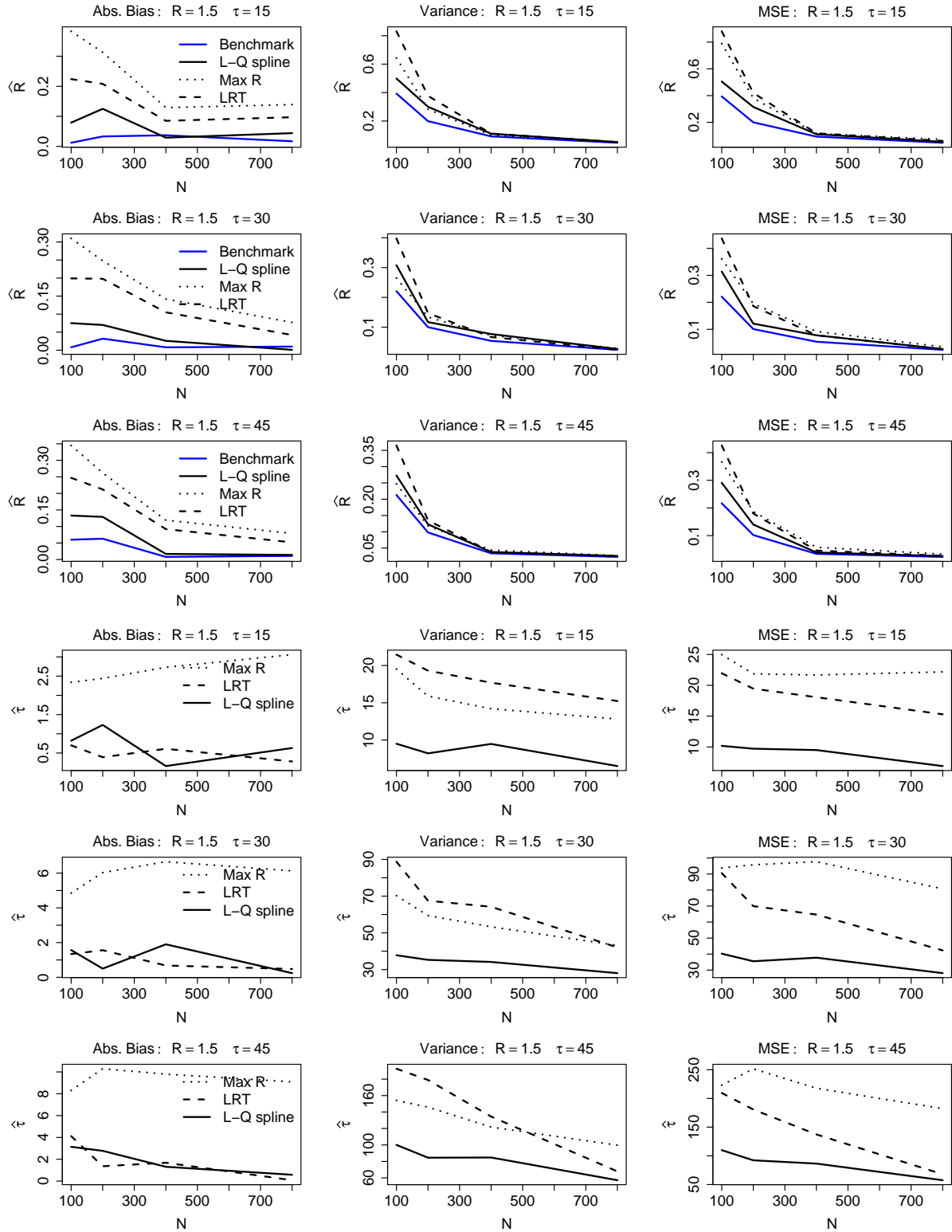


Figure 16: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.5$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 33: **Single Normally distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	2.073	0.073	0.496	0.501	2.213	0.213	0.611	0.657	2.533	0.533	0.768	1.052	2.429	0.429	0.802	0.986
15	200	1.997	0.003	0.263	0.263	2.089	0.089	0.354	0.361	2.288	0.288	0.403	0.485	2.216	0.216	0.374	0.421
15	400	2.018	0.018	0.149	0.149	2.045	0.045	0.179	0.181	2.221	0.221	0.184	0.233	2.136	0.136	0.165	0.184
15	800	1.998	0.002	0.078	0.078	2.012	0.012	0.077	0.077	2.104	0.104	0.085	0.096	2.044	0.044	0.078	0.08
30	100	2.056	0.056	0.378	0.381	2.169	0.169	0.467	0.495	2.377	0.377	0.446	0.588	2.298	0.298	0.466	0.555
30	200	2.012	0.012	0.147	0.147	2.053	0.053	0.179	0.181	2.205	0.205	0.177	0.22	2.148	0.148	0.158	0.18
30	400	2.02	0.02	0.075	0.075	2.049	0.049	0.087	0.09	2.149	0.149	0.09	0.112	2.092	0.092	0.081	0.089
30	800	1.998	0.002	0.042	0.042	1.998	0.002	0.044	0.044	2.068	0.068	0.042	0.047	2.023	0.023	0.042	0.042
45	100	2.036	0.036	0.235	0.236	2.094	0.094	0.321	0.33	2.337	0.337	0.355	0.469	2.279	0.279	0.371	0.449
45	200	2.062	0.062	0.15	0.154	2.087	0.087	0.161	0.168	2.223	0.223	0.165	0.214	2.181	0.181	0.158	0.19
45	400	1.984	0.016	0.072	0.072	1.993	0.007	0.077	0.077	2.078	0.078	0.077	0.083	2.032	0.032	0.075	0.076
45	800	2.016	0.016	0.032	0.033	2.015	0.015	0.033	0.033	2.07	0.07	0.032	0.037	2.038	0.038	0.031	0.033

Table 34: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.01	0.99	8.71	9.7	12.08	2.92	13.55	22.1	13.89	1.11	16.46	17.68
15	200	14.12	0.88	7.43	8.21	12.05	2.95	11.39	20.1	14.62	0.38	13.08	13.23
15	400	14.63	0.37	7.13	7.26	11.22	3.78	8.27	22.55	14.63	0.37	10.88	11.01
15	800	14.68	0.32	4.64	4.74	12.08	2.92	7.9	16.45	14.98	0.02	6.14	6.14
30	100	27.57	2.43	33.58	39.5	24.03	5.97	43.8	79.5	28.61	1.39	61.16	63.09
30	200	29.03	0.97	28.73	29.67	24.01	5.99	45.36	81.3	29.36	0.64	40.72	41.13
30	400	29.42	0.58	16.86	17.19	24.11	5.89	35.44	70.12	29.78	0.22	25.97	26.02
30	800	30.24	0.24	7.09	7.15	24.14	5.86	32.04	66.37	30.08	0.08	10.32	10.32
45	100	42.88	2.12	90.31	94.79	37.43	7.57	142.85	200.2	44.35	0.65	140.86	141.28
45	200	44.49	0.51	52.99	53.26	36.87	8.13	101.47	167.49	44.45	0.55	75.3	75.61
45	400	45.69	0.69	38.08	38.56	37.11	7.89	85.87	148.11	45.84	0.84	42.35	43.07
45	800	45.21	0.21	13.08	13.12	38.58	6.42	77.05	118.23	44.92	0.08	10.46	10.46

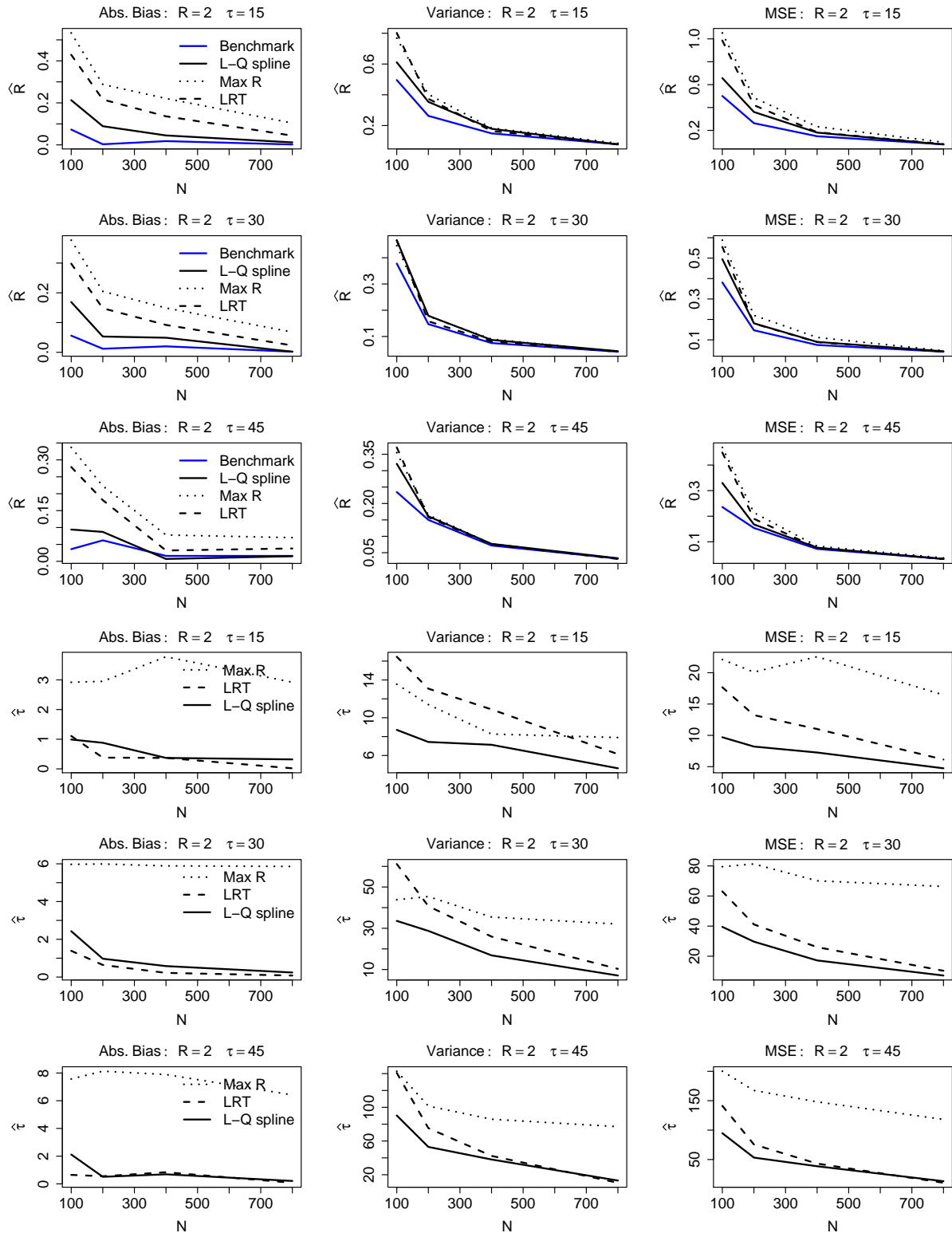


Figure 17: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.



Table 35: **Single Normally distributed exposure with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	4.009	0.009	1.323	1.323	4.11	0.11	1.428	1.44	4.388	0.388	1.411	1.562	4.241	0.241	1.253	1.311
15	200	3.881	0.119	0.65	0.664	3.96	0.04	0.652	0.654	4.154	0.154	0.675	0.699	4.031	0.031	0.68	0.681
15	400	4.033	0.033	0.344	0.345	4.057	0.057	0.365	0.368	4.235	0.235	0.439	0.494	4.093	0.093	0.378	0.387
15	800	4.012	0.012	0.224	0.224	4.017	0.017	0.218	0.218	4.098	0.098	0.216	0.226	4.022	0.022	0.219	0.219
30	100	4.002	0.002	1.068	1.068	4.112	0.112	1.057	1.069	4.324	0.324	1.061	1.166	4.222	0.222	1.086	1.135
30	200	4.086	0.086	0.365	0.372	4.097	0.097	0.359	0.368	4.254	0.254	0.372	0.437	4.178	0.178	0.404	0.435
30	400	4.005	0.005	0.219	0.219	3.982	0.018	0.224	0.225	4.085	0.085	0.226	0.234	4.042	0.042	0.226	0.228
30	800	4.021	0.021	0.116	0.117	3.993	0.007	0.118	0.118	4.045	0.045	0.113	0.115	4.026	0.026	0.116	0.116
45	100	4.028	0.028	0.798	0.799	4.056	0.056	0.845	0.848	4.252	0.252	0.823	0.886	4.197	0.197	0.846	0.885
45	200	3.952	0.048	0.373	0.375	3.936	0.064	0.378	0.382	4.054	0.054	0.365	0.368	4.024	0.024	0.366	0.367
45	400	4.056	0.056	0.219	0.223	4.021	0.021	0.213	0.214	4.086	0.086	0.217	0.225	4.073	0.073	0.216	0.221
45	800	4.01	0.01	0.098	0.098	3.969	0.031	0.096	0.097	4.02	0.02	0.099	0.099	4.014	0.014	0.098	0.098

Table 36: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.5	0.5	6.63	6.88	12.31	2.69	8.69	15.91	14.82	0.18	7.57	7.6
15	200	14.74	0.26	3.27	3.33	12.22	2.78	7	14.72	14.68	0.32	4.46	4.56
15	400	14.92	0.08	1.65	1.66	12.41	2.59	7.33	14.05	14.93	0.07	2.21	2.22
15	800	14.92	0.08	0.51	0.51	13.18	1.82	4.96	8.27	15.02	0.02	0.55	0.55
30	100	29.07	0.93	17.29	18.16	25.42	4.58	30.12	51.08	29.19	0.81	20.15	20.81
30	200	29.75	0.25	7.59	7.65	26.55	3.45	21.98	33.89	29.73	0.27	7.79	7.86
30	400	30.06	0.06	3.26	3.27	27.17	2.83	17.18	25.22	29.75	0.25	2.58	2.64
30	800	30.19	0.19	1	1.04	28.69	1.31	7.38	9.1	29.98	0.02	0.29	0.29
45	100	44.82	0.18	22.99	23.03	39.66	5.34	67.17	95.65	43.92	1.08	27.26	28.41
45	200	45.48	0.48	10.38	10.61	41.76	3.24	36.12	46.59	44.56	0.44	12.22	12.41
45	400	45.39	0.39	2.61	2.76	43.97	1.03	6.48	7.53	45.08	0.08	1.53	1.53
45	800	45.45	0.45	1.35	1.55	44.49	0.51	2.04	2.3	45.04	0.04	0.42	0.42

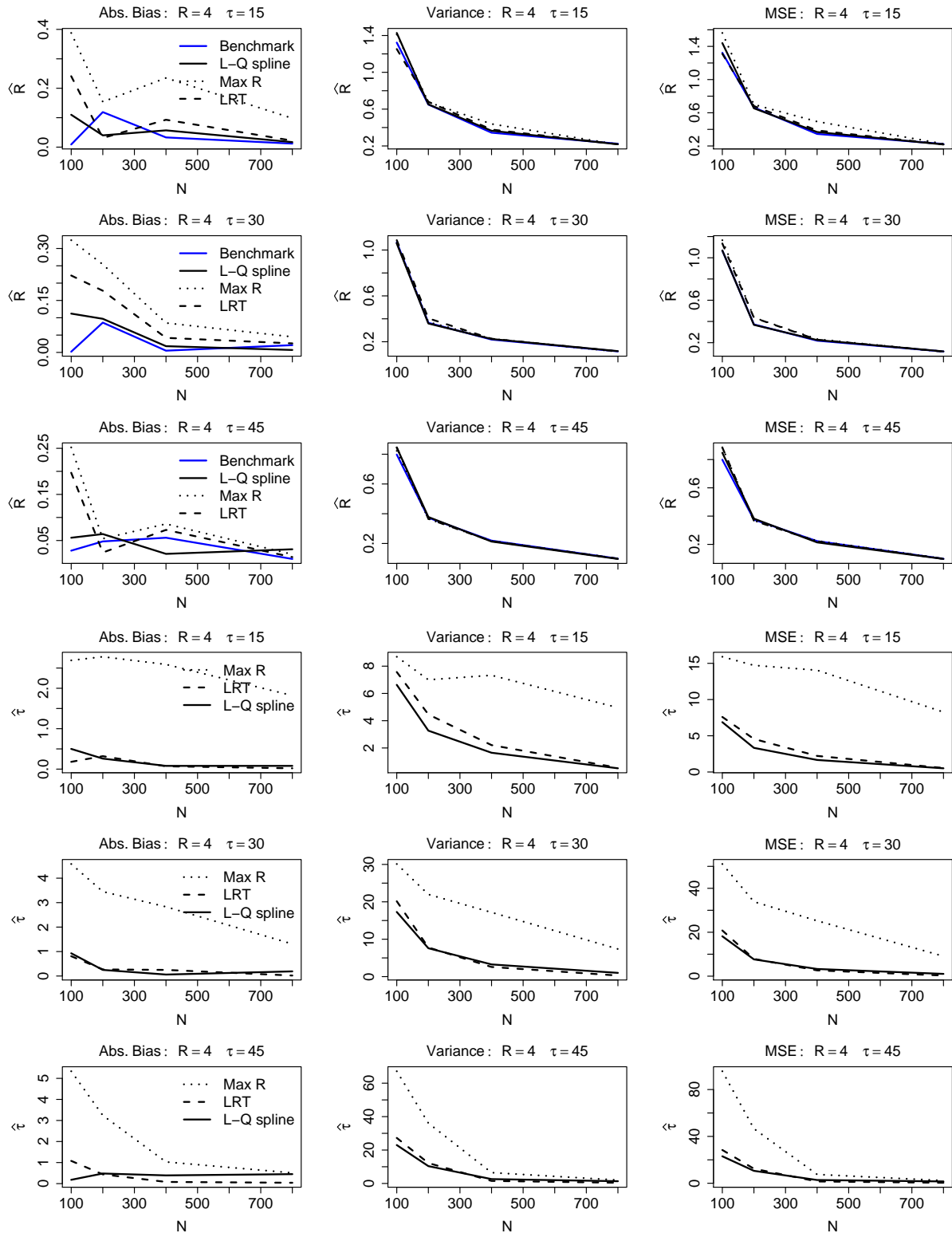


Figure 18: **Single Normally distributed exposure with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 4$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 37: **Multiple Normally distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.731	0.031	0.09	0.091	0.751	0.051	0.127	0.13	0.532	0.168	0.076	0.104	0.662	0.038	0.266	0.267
15	200	0.718	0.018	0.07	0.07	0.743	0.043	0.098	0.1	0.57	0.13	0.062	0.079	0.67	0.03	0.165	0.166
15	400	0.704	0.004	0.029	0.029	0.718	0.018	0.038	0.039	0.603	0.097	0.029	0.039	0.632	0.068	0.04	0.045
15	800	0.686	0.014	0.013	0.013	0.691	0.009	0.018	0.019	0.619	0.081	0.014	0.021	0.637	0.063	0.015	0.019
30	100	0.719	0.019	0.078	0.078	0.736	0.036	0.108	0.11	0.534	0.166	0.059	0.087	0.657	0.043	0.202	0.204
30	200	0.688	0.012	0.033	0.034	0.697	0.003	0.042	0.042	0.564	0.136	0.033	0.052	0.622	0.078	0.065	0.071
30	400	0.69	0.01	0.017	0.017	0.687	0.013	0.02	0.02	0.611	0.089	0.015	0.022	0.632	0.068	0.02	0.025
30	800	0.701	0.001	0.008	0.008	0.695	0.005	0.009	0.009	0.647	0.053	0.009	0.012	0.665	0.035	0.008	0.01
45	100	0.701	0.001	0.038	0.038	0.704	0.004	0.051	0.051	0.526	0.174	0.03	0.061	0.604	0.096	0.093	0.102
45	200	0.709	0.009	0.02	0.02	0.701	0.001	0.028	0.028	0.601	0.099	0.02	0.03	0.643	0.057	0.04	0.044
45	400	0.71	0.01	0.011	0.011	0.705	0.005	0.015	0.015	0.635	0.065	0.01	0.014	0.649	0.051	0.009	0.011
45	800	0.701	0.001	0.006	0.006	0.699	0.001	0.006	0.006	0.658	0.042	0.006	0.008	0.675	0.025	0.005	0.006

Table 38: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.7$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	15.39	0.39	9.08	9.24	13.97	1.03	19.67	20.73	15.52	0.52	21.51	21.77
15	200	14.93	0.07	8.13	8.14	12.6	2.4	15.89	21.63	14.56	0.44	20.8	20.99
15	400	14.96	0.04	9.75	9.75	12.89	2.11	18.11	22.55	15.34	0.34	19.66	19.77
15	800	15.26	0.26	8.96	9.03	12.31	2.69	13.02	20.25	14.96	0.04	13.94	13.94
30	100	30.52	0.52	41.5	41.76	25.86	4.14	86.01	103.16	29.92	0.08	93.24	93.24
30	200	30.39	0.39	42.06	42.21	24.71	5.29	71.9	99.85	29.81	0.19	78.47	78.51
30	400	30.29	0.29	33.33	33.42	24.94	5.06	63.56	89.17	30.64	0.64	62.8	63.22
30	800	29.89	0.11	27.58	27.59	23.94	6.06	53.09	89.82	30.63	0.63	50.09	50.49
45	100	43.46	1.54	103.43	105.81	37.61	7.39	200.12	254.76	45.66	0.66	224.8	225.24
45	200	46.36	1.36	94.09	95.93	36.52	8.48	175.37	247.32	44.35	0.65	179.27	179.69
45	400	44.67	0.33	81.23	81.34	35.24	9.76	125.57	220.9	43.55	1.45	121.79	123.89
45	800	44.4	0.6	58.3	58.66	35.12	9.88	118.54	216.25	46.69	1.69	92.29	95.15

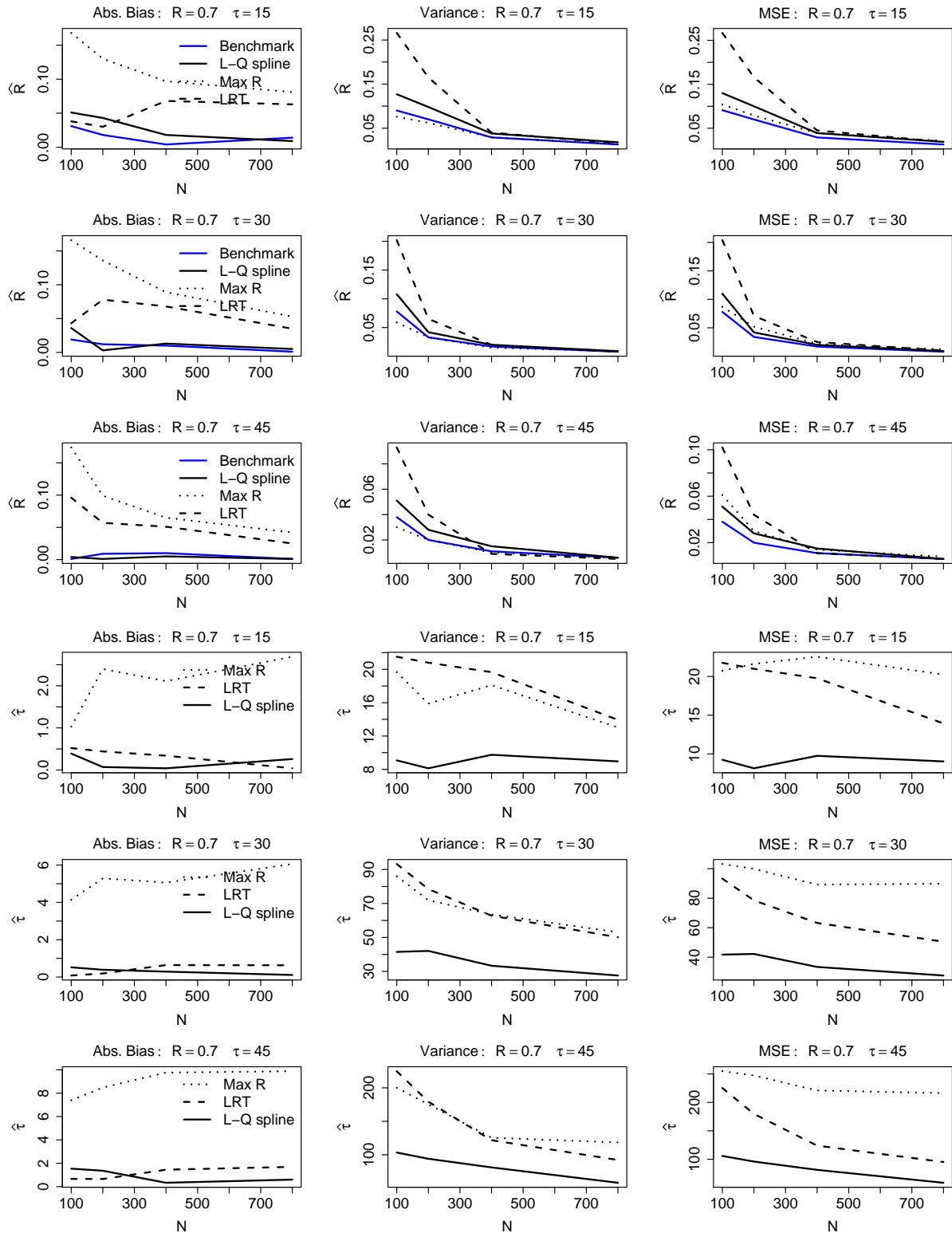


Figure 19: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.7$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 39: **Multiple Normally distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	0.9	0	0.168	0.168	0.961	0.061	0.231	0.235	0.638	0.262	0.119	0.188	0.876	0.024	0.425	0.426
15	200	0.901	0.001	0.073	0.073	0.918	0.018	0.104	0.104	0.698	0.202	0.061	0.102	0.86	0.04	0.197	0.198
15	400	0.912	0.012	0.042	0.042	0.93	0.03	0.054	0.055	0.778	0.122	0.039	0.054	0.888	0.012	0.11	0.11
15	800	0.935	0.035	0.021	0.023	0.938	0.038	0.027	0.029	0.835	0.065	0.02	0.024	0.914	0.014	0.052	0.052
30	100	0.877	0.023	0.087	0.088	0.894	0.006	0.125	0.125	0.648	0.252	0.062	0.125	0.82	0.08	0.238	0.244
30	200	0.892	0.008	0.044	0.044	0.892	0.008	0.057	0.057	0.728	0.172	0.035	0.065	0.828	0.072	0.109	0.114
30	400	0.917	0.017	0.024	0.024	0.916	0.016	0.032	0.033	0.797	0.103	0.023	0.034	0.883	0.017	0.064	0.064
30	800	0.899	0.001	0.011	0.011	0.901	0.001	0.015	0.015	0.819	0.081	0.011	0.017	0.858	0.042	0.025	0.027
45	100	0.911	0.011	0.069	0.069	0.938	0.038	0.1	0.102	0.697	0.203	0.048	0.089	0.926	0.026	0.227	0.227
45	200	0.91	0.01	0.026	0.026	0.924	0.024	0.04	0.041	0.754	0.146	0.023	0.044	0.894	0.006	0.102	0.102
45	400	0.899	0.001	0.016	0.016	0.905	0.005	0.022	0.022	0.799	0.101	0.014	0.025	0.87	0.03	0.045	0.046
45	800	0.904	0.004	0.009	0.009	0.906	0.006	0.011	0.011	0.834	0.066	0.008	0.012	0.866	0.034	0.019	0.02

Table 40: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 0.9$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.69	0.31	10.29	10.39	14.01	0.99	23.6	24.59	14.52	0.48	21.39	21.62
15	200	14.46	0.54	9.15	9.45	13.4	1.6	21.95	24.52	14.66	0.34	22.69	22.8
15	400	14.61	0.39	9.4	9.56	14.15	0.85	22.37	23.1	14.34	0.66	22.97	23.4
15	800	14.61	0.39	8.18	8.33	13.84	1.16	24.37	25.71	14.62	0.38	23.53	23.68
30	100	29.92	0.08	43.9	43.9	27.85	2.15	96.63	101.26	29.18	0.82	105.67	106.34
30	200	29.1	0.9	40.34	41.16	25.69	4.31	93.93	112.48	29.06	0.94	89.96	90.84
30	400	29.4	0.6	42.32	42.68	26.69	3.31	90.47	101.43	29.37	0.63	99.68	100.08
30	800	29.54	0.46	33.9	34.11	26.03	3.97	86.92	102.72	28.84	1.16	94.38	95.72
45	100	41.36	3.64	106.81	120.05	38.58	6.42	246.8	287.97	41.5	3.5	258.7	270.93
45	200	42.96	2.04	108.64	112.8	38.97	6.03	270.74	307.04	43.81	1.19	290.55	291.96
45	400	44.59	0.41	106.29	106.46	38.21	6.79	247.61	293.7	43.76	1.24	258.41	259.94
45	800	43.72	1.28	106.43	108.06	37.93	7.07	241.56	291.55	44.12	0.88	238.17	238.95

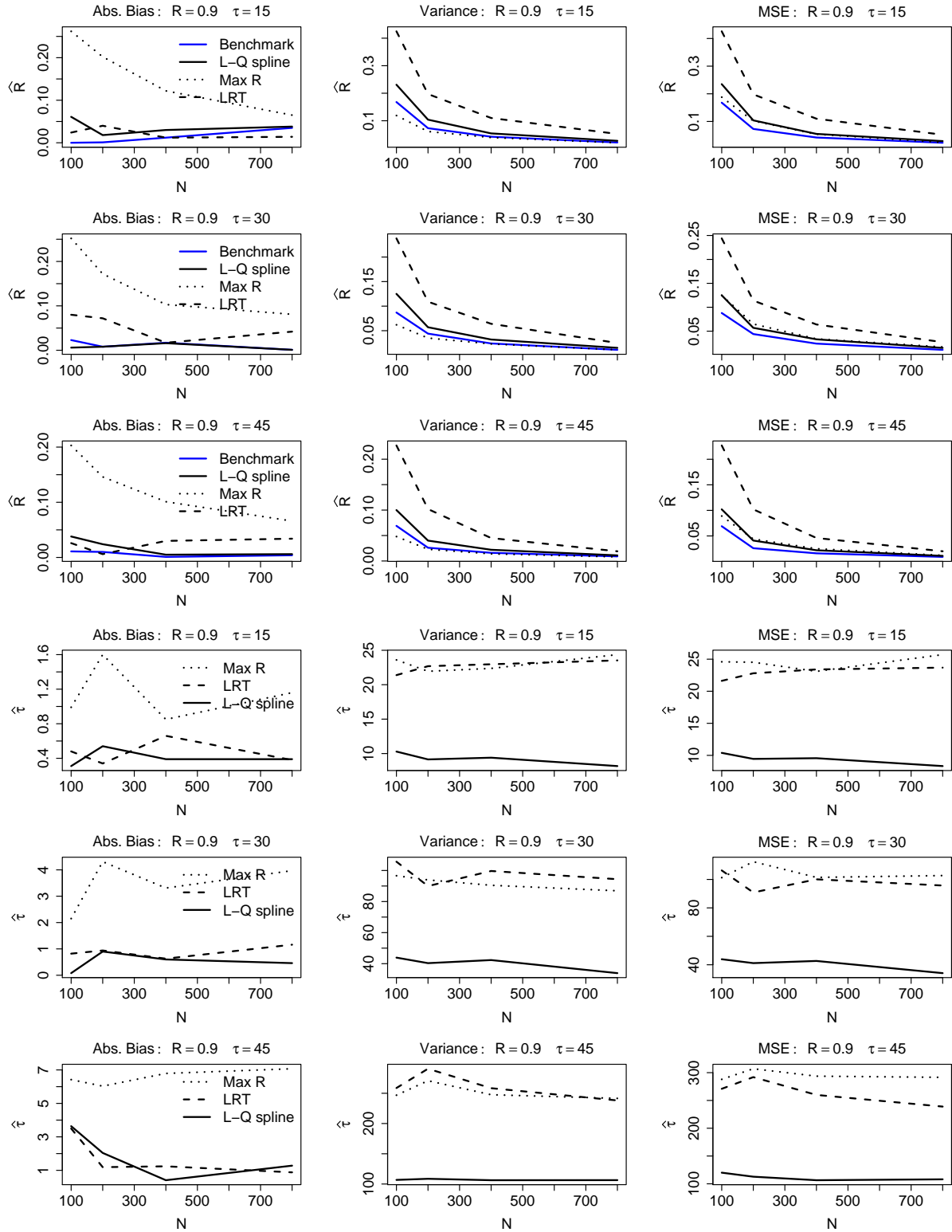


Figure 20: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 0.9$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 41: **Multiple Normally distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.247	0.047	0.214	0.217	1.306	0.106	0.262	0.274	1.566	0.366	0.308	0.442	1.36	0.16	0.518	0.543
15	200	1.172	0.028	0.088	0.089	1.199	0.001	0.112	0.112	1.387	0.187	0.107	0.142	1.255	0.055	0.2	0.203
15	400	1.185	0.015	0.052	0.052	1.184	0.016	0.057	0.057	1.336	0.136	0.072	0.09	1.261	0.061	0.118	0.121
15	800	1.197	0.003	0.022	0.022	1.206	0.006	0.028	0.028	1.289	0.089	0.024	0.032	1.249	0.049	0.04	0.043
30	100	1.227	0.027	0.115	0.116	1.28	0.08	0.162	0.168	1.526	0.326	0.171	0.277	1.364	0.164	0.331	0.358
30	200	1.217	0.017	0.078	0.079	1.248	0.048	0.094	0.096	1.395	0.195	0.076	0.114	1.297	0.097	0.145	0.155
30	400	1.239	0.039	0.032	0.033	1.263	0.063	0.044	0.048	1.371	0.171	0.035	0.064	1.333	0.133	0.055	0.073
30	800	1.196	0.004	0.016	0.016	1.204	0.004	0.022	0.022	1.277	0.077	0.019	0.024	1.254	0.054	0.026	0.029
45	100	1.218	0.018	0.103	0.104	1.249	0.049	0.145	0.147	1.479	0.279	0.145	0.223	1.32	0.12	0.301	0.316
45	200	1.21	0.01	0.045	0.046	1.229	0.029	0.064	0.064	1.391	0.191	0.054	0.091	1.316	0.116	0.106	0.12
45	400	1.214	0.014	0.026	0.026	1.224	0.024	0.034	0.035	1.336	0.136	0.026	0.045	1.307	0.107	0.041	0.053
45	800	1.199	0.001	0.013	0.013	1.219	0.019	0.017	0.018	1.276	0.076	0.015	0.02	1.261	0.061	0.019	0.023

Table 42: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.44	0.56	9.45	9.77	12.92	2.08	21.21	25.51	14.42	0.58	22.52	22.86
15	200	14.45	0.55	8.73	9.03	13.27	1.73	21.72	24.73	14.58	0.42	22.19	22.37
15	400	14.9	0.1	8.1	8.11	12.83	2.17	19.77	24.48	14.68	0.32	23.35	23.45
15	800	14.51	0.49	8.23	8.47	12.83	2.17	18.37	23.06	15.01	0.01	19.56	19.56
30	100	28.25	1.75	43.62	46.67	26.27	3.73	96.27	110.21	27.46	2.54	98.8	105.27
30	200	28.85	1.15	37.18	38.51	26.46	3.54	92.61	105.13	29.13	0.87	90.37	91.14
30	400	29.19	0.81	42.54	43.19	25.5	4.5	77.82	98.05	28.99	1.01	80.31	81.32
30	800	28.86	1.14	44.37	45.67	25.47	4.53	77.34	97.89	28.46	1.54	75.68	78.04
45	100	43.75	1.25	100.9	102.47	40.14	4.86	241.26	264.87	41.64	3.36	245.2	256.47
45	200	43.88	1.12	111.97	113.22	39	6	246.27	282.27	43.34	1.66	240.61	243.37
45	400	44.27	0.73	111.35	111.89	36.91	8.09	189.94	255.31	41.81	3.19	204.96	215.11
45	800	43.69	1.31	98.27	99.99	38.42	6.58	181.14	224.41	45.2	0.2	194.26	194.3

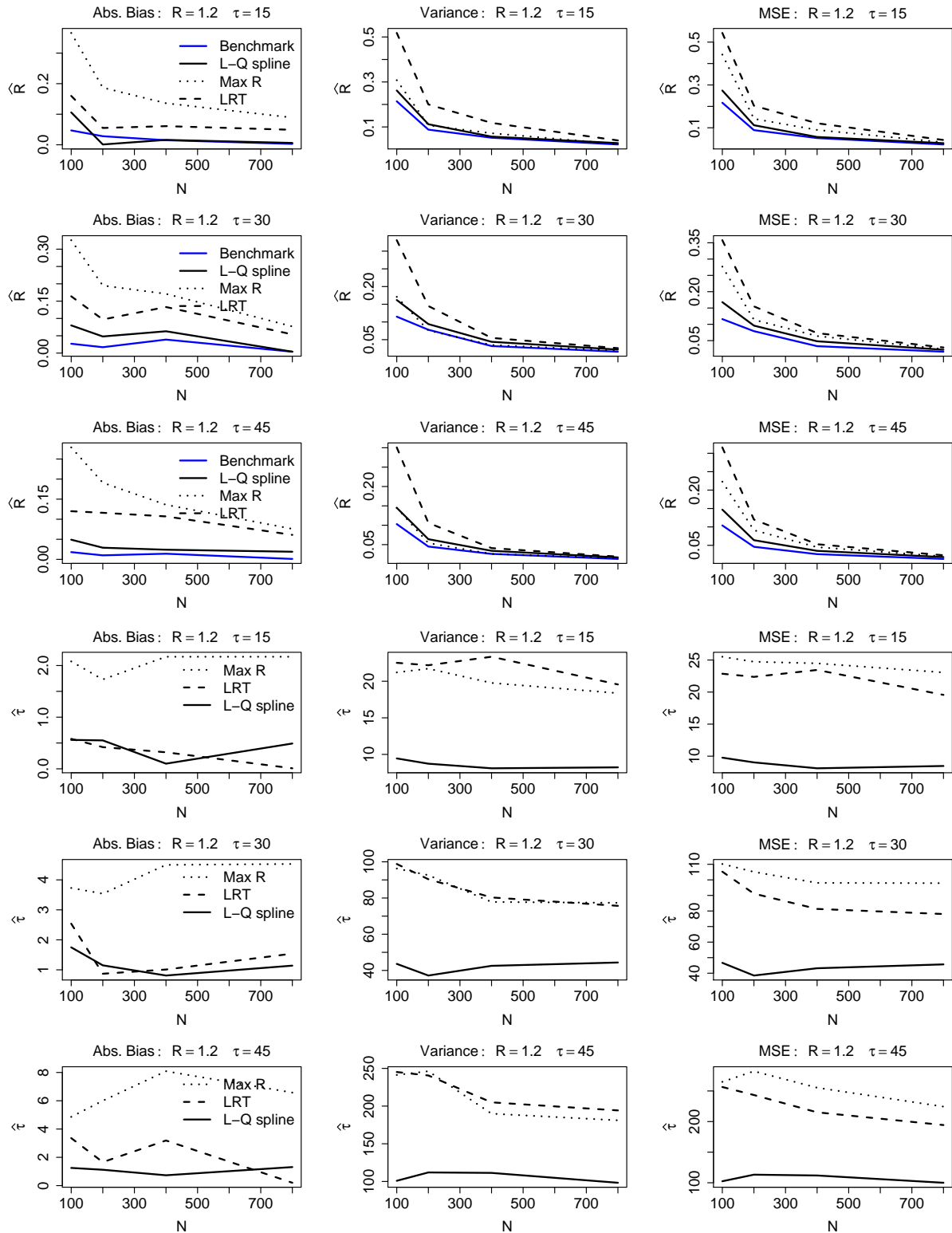


Figure 21: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.



Table 43: **Multiple Normally distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.568	0.068	0.311	0.316	1.637	0.137	0.368	0.387	1.931	0.431	0.392	0.577	1.805	0.305	0.509	0.602
15	200	1.542	0.042	0.151	0.153	1.585	0.085	0.171	0.178	1.767	0.267	0.206	0.277	1.69	0.19	0.265	0.301
15	400	1.494	0.006	0.077	0.077	1.528	0.028	0.098	0.099	1.655	0.155	0.098	0.122	1.617	0.117	0.102	0.116
15	800	1.496	0.004	0.036	0.036	1.517	0.017	0.042	0.043	1.588	0.088	0.045	0.053	1.561	0.061	0.043	0.047
30	100	1.506	0.006	0.183	0.184	1.558	0.058	0.254	0.258	1.784	0.284	0.257	0.338	1.688	0.188	0.371	0.407
30	200	1.505	0.005	0.077	0.077	1.519	0.019	0.096	0.096	1.677	0.177	0.088	0.119	1.65	0.15	0.095	0.117
30	400	1.495	0.005	0.04	0.04	1.52	0.02	0.047	0.047	1.592	0.092	0.046	0.055	1.564	0.064	0.046	0.05
30	800	1.503	0.003	0.026	0.026	1.514	0.014	0.029	0.029	1.561	0.061	0.026	0.03	1.542	0.042	0.025	0.027
45	100	1.52	0.02	0.165	0.166	1.561	0.061	0.216	0.22	1.777	0.277	0.207	0.284	1.694	0.194	0.288	0.325
45	200	1.5	0	0.07	0.07	1.518	0.018	0.097	0.097	1.651	0.151	0.078	0.101	1.626	0.126	0.09	0.106
45	400	1.488	0.012	0.032	0.032	1.51	0.01	0.035	0.035	1.579	0.079	0.034	0.041	1.562	0.062	0.033	0.037
45	800	1.497	0.003	0.015	0.015	1.506	0.006	0.017	0.017	1.549	0.049	0.017	0.02	1.534	0.034	0.016	0.017

Table 44: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 1.5$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\hat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\hat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.65	0.35	10	10.12	12.75	2.25	18.48	23.52	14.32	0.68	21.87	22.34
15	200	14.46	0.54	8.63	8.92	12.29	2.71	15.28	22.64	14.18	0.82	17.09	17.76
15	400	14.49	0.51	8.31	8.57	12.14	2.86	12.45	20.65	14.17	0.83	18.02	18.71
15	800	14.75	0.25	5.98	6.05	12.17	2.83	10.26	18.27	14.9	0.1	10.23	10.24
30	100	28.32	1.68	39.68	42.49	26.43	3.57	85.72	98.45	28.86	1.14	85.22	86.52
30	200	29.53	0.47	35.97	36.19	25.75	4.25	62.35	80.38	29.73	0.27	66.29	66.36
30	400	28.66	1.34	33.51	35.3	25.2	4.8	49.76	72.83	30.38	0.38	46.11	46.26
30	800	29.25	0.75	18.75	19.32	25.48	4.52	35.91	56.32	30.3	0.3	34.58	34.67
45	100	42.44	2.56	82.72	89.26	37.24	7.76	199.29	259.56	42.54	2.46	227.46	233.5
45	200	44.88	0.12	80.54	80.56	38.46	6.54	170.17	212.91	45.25	0.25	165.02	165.09
45	400	44.68	0.32	73.24	73.34	37.41	7.59	112.1	169.68	44.13	0.87	96.89	97.64
45	800	44.37	0.63	52.8	53.19	37.77	7.23	94.45	146.66	44.46	0.54	47.93	48.22

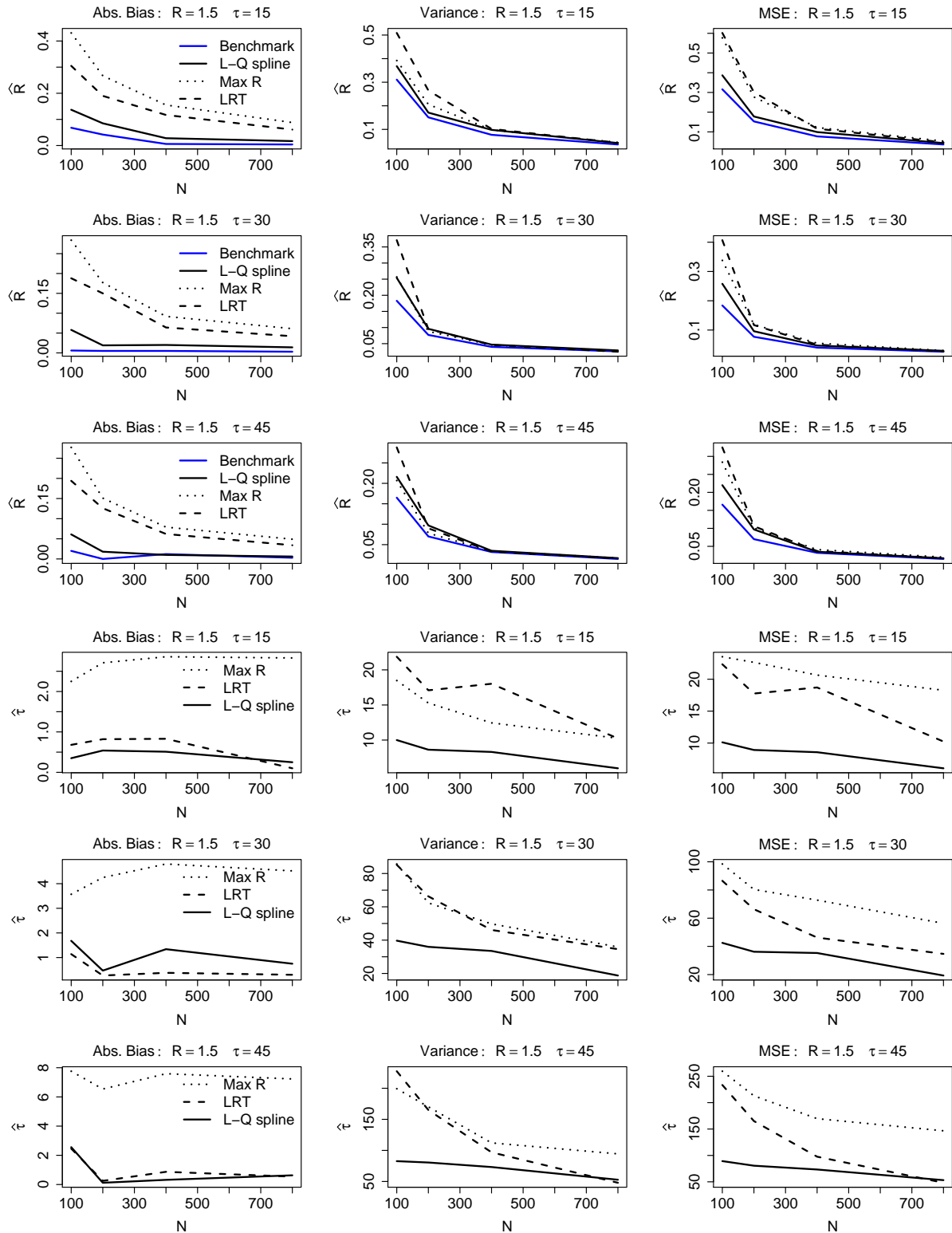


Figure 22: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 1.5$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 45: **Multiple Normally distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark			(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT						
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE				
15	100	1.993	0.007	0.443	0.443	2.116	0.116	0.579	0.592	2.357	0.357	0.605	0.732	2.278	0.278	0.667	0.744
15	200	2.016	0.016	0.24	0.24	2.082	0.082	0.302	0.308	2.284	0.284	0.339	0.42	2.222	0.222	0.337	0.386
15	400	2.001	0.001	0.102	0.102	2.048	0.048	0.109	0.111	2.152	0.152	0.129	0.152	2.092	0.092	0.12	0.128
15	800	2.003	0.003	0.063	0.063	2.018	0.018	0.07	0.07	2.081	0.081	0.064	0.071	2.035	0.035	0.063	0.064
30	100	1.967	0.033	0.217	0.218	2.061	0.061	0.282	0.286	2.27	0.27	0.295	0.368	2.215	0.215	0.321	0.367
30	200	1.982	0.018	0.135	0.135	2.023	0.023	0.144	0.144	2.139	0.139	0.145	0.164	2.098	0.098	0.142	0.152
30	400	2.032	0.032	0.07	0.072	2.043	0.043	0.076	0.078	2.126	0.126	0.076	0.092	2.09	0.09	0.076	0.084
30	800	2.009	0.009	0.033	0.033	2.011	0.011	0.034	0.034	2.051	0.051	0.037	0.04	2.027	0.027	0.034	0.035
45	100	2.049	0.049	0.227	0.229	2.112	0.112	0.294	0.307	2.315	0.315	0.263	0.362	2.28	0.28	0.26	0.339
45	200	2.033	0.033	0.102	0.103	2.047	0.047	0.114	0.116	2.154	0.154	0.117	0.14	2.129	0.129	0.113	0.129
45	400	2.011	0.011	0.065	0.065	2.014	0.014	0.068	0.069	2.07	0.07	0.07	0.075	2.056	0.056	0.068	0.071
45	800	1.989	0.011	0.029	0.029	1.986	0.014	0.03	0.031	2.018	0.018	0.029	0.029	2.01	0.01	0.03	0.03

Table 46: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 2$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline			(c) $\widehat{R}_{\max}$ approach			(d) Scan LRT					
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.08	0.92	7.84	8.69	12.48	2.52	13.07	19.41	13.93	1.07	15.23	16.36
15	200	14.45	0.55	7.37	7.67	11.77	3.23	11.69	22.13	14.15	0.85	13.88	14.61
15	400	14.39	0.61	5.01	5.39	12.28	2.72	9.57	16.96	15.04	0.04	8.78	8.78
15	800	14.86	0.14	3.9	3.91	12.51	2.49	7.46	13.67	15.04	0.04	3.85	3.85
30	100	29.06	0.94	35.01	35.89	24.52	5.48	56.33	86.33	28.24	1.76	58.53	61.64
30	200	29.19	0.81	23.09	23.74	25.12	4.88	38.86	62.67	29.28	0.72	34.29	34.81
30	400	30.07	0.07	17.12	17.12	25.24	4.76	34.47	57.17	29.8	0.2	18.09	18.13
30	800	30.08	0.08	4.03	4.04	26.66	3.34	22.91	34.04	29.98	0.02	3.38	3.38
45	100	44.22	0.78	87.68	88.29	37.45	7.55	152.66	209.7	43.98	1.02	135.44	136.48
45	200	44.72	0.28	45.16	45.24	39.39	5.61	96.71	128.22	45.11	0.11	69.54	69.55
45	400	44.96	0.04	27.9	27.9	41.17	3.83	63.93	78.59	45.05	0.05	39.87	39.87
45	800	45.08	0.08	8.71	8.71	42.1	2.9	35.98	44.39	44.68	0.32	9.23	9.34

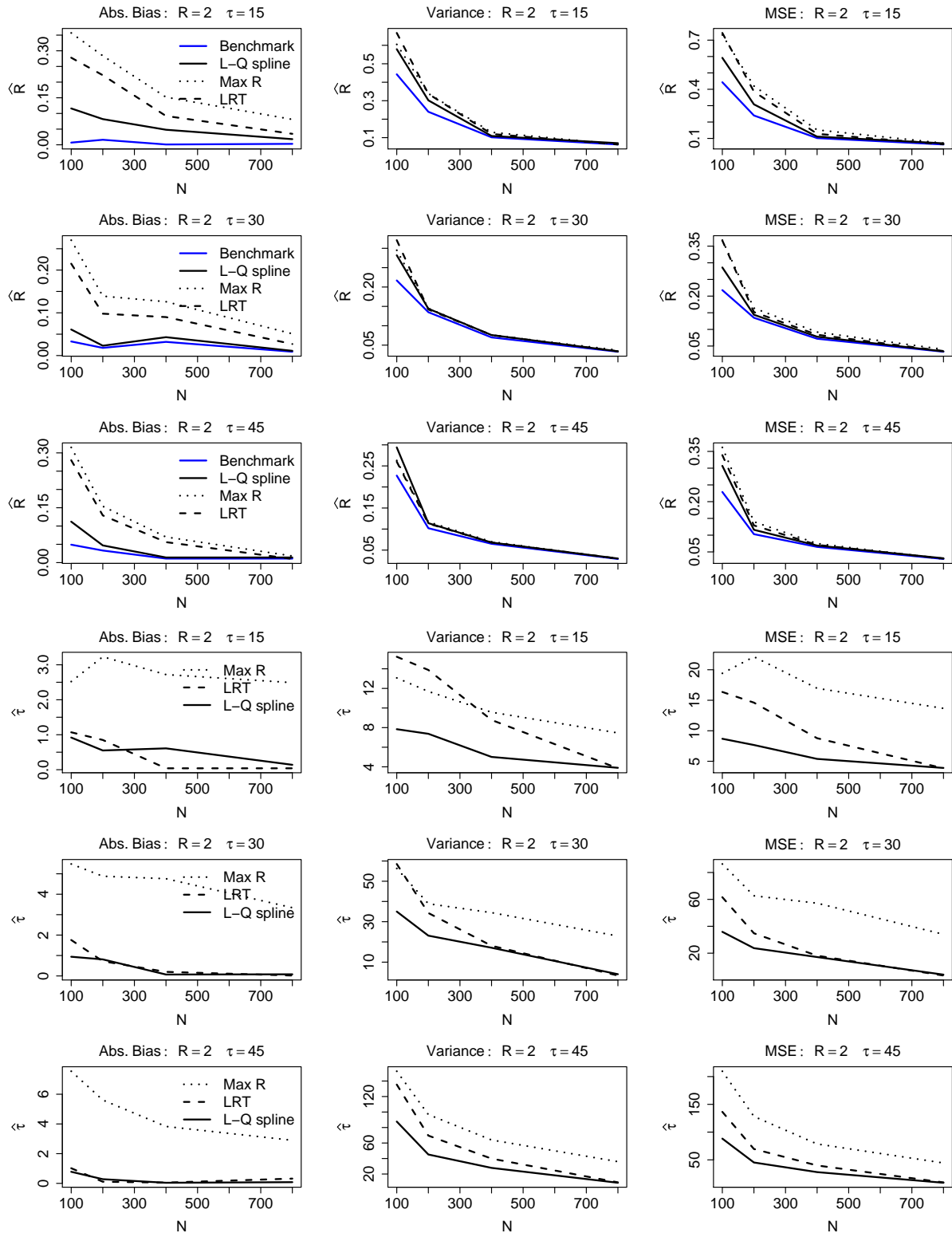


Figure 23: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 2$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

Table 47: **Multiple Normally distributed exposures with two age effects.** Relative incidence estimation of  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(a) Benchmark				(b) Linear-quadratic spline				(c) $\widehat{R}_{\max}$ approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	4.204	0.204	1.402	1.443	4.322	0.322	1.537	1.641	4.501	0.501	1.521	1.772	4.408	0.408	1.512	1.678
15	200	3.914	0.086	0.547	0.554	3.942	0.058	0.571	0.575	4.116	0.116	0.549	0.563	4.012	0.012	0.524	0.524
15	400	4.034	0.034	0.289	0.29	4.036	0.036	0.297	0.299	4.119	0.119	0.283	0.297	4.047	0.047	0.294	0.296
15	800	4.018	0.018	0.151	0.151	4.009	0.009	0.153	0.153	4.059	0.059	0.154	0.158	4.021	0.021	0.153	0.153
30	100	4.165	0.165	0.933	0.96	4.214	0.214	0.925	0.971	4.416	0.416	0.935	1.108	4.354	0.354	0.929	1.054
30	200	4.039	0.039	0.405	0.406	4.052	0.052	0.41	0.413	4.149	0.149	0.397	0.419	4.119	0.119	0.404	0.418
30	400	4.017	0.017	0.225	0.226	3.992	0.008	0.221	0.221	4.051	0.051	0.221	0.224	4.037	0.037	0.224	0.225
30	800	4.002	0.002	0.111	0.111	3.97	0.03	0.113	0.114	4.009	0.009	0.112	0.112	4.005	0.005	0.112	0.112
45	100	4.025	0.025	0.816	0.817	4.019	0.019	0.861	0.861	4.195	0.195	0.843	0.881	4.177	0.177	0.866	0.898
45	200	4.054	0.054	0.511	0.514	4.019	0.019	0.467	0.468	4.113	0.113	0.5	0.512	4.11	0.11	0.499	0.511
45	400	4.045	0.045	0.204	0.206	3.993	0.007	0.206	0.206	4.069	0.069	0.211	0.216	4.067	0.067	0.212	0.216
45	800	3.993	0.007	0.097	0.097	3.939	0.061	0.097	0.101	3.998	0.002	0.097	0.097	3.998	0.002	0.097	0.097

Table 48: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , for  $R = \exp(\beta) = 4$  based on (a) SCCS benchmark where the true risk period  $\tau$  is used; (b) proposed linear-quadratic spline fit; (c)  $\widehat{R}_{\max}$  approach; and (d) scan likelihood ratio test (LRT) statistic approach. Given are mean estimate (Est.), absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.

$\tau$	$N$	(b) Linear-quadratic spline				(c) $\widehat{R}_{\max}$ approach				(d) Scan LRT			
		Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE	Est.	Bias	Var.	MSE
15	100	14.63	0.37	4.17	4.31	12.86	2.14	6.61	11.2	14.85	0.15	4.18	4.21
15	200	14.81	0.19	3.08	3.12	12.77	2.23	6.44	11.42	14.8	0.2	3.2	3.24
15	400	14.99	0.01	1.18	1.18	13.44	1.56	5.17	7.6	15.17	0.17	0.99	1.02
15	800	15.01	0.01	0.38	0.38	14.05	0.95	2.53	3.43	15.01	0.01	0.15	0.15
30	100	30.2	0.2	12.91	12.95	26.6	3.4	32.94	44.52	29.79	0.21	13.72	13.77
30	200	29.72	0.28	5.59	5.67	27.69	2.31	14.39	19.73	29.66	0.34	6.23	6.35
30	400	30.24	0.24	1.92	1.98	28.79	1.21	6.09	7.55	29.94	0.06	1.03	1.03
30	800	30.28	0.28	0.75	0.83	29.7	0.3	0.67	0.76	30.04	0.04	0.25	0.25
45	100	45.15	0.15	25.78	25.8	42.71	2.29	60.59	65.81	44.7	0.3	34.25	34.34
45	200	45.45	0.45	9.88	10.08	43.89	1.11	9.22	10.44	44.47	0.53	4.94	5.22
45	400	45.76	0.76	2.96	3.53	44.57	0.43	3.82	4.01	44.91	0.09	2.61	2.62
45	800	45.8	0.8	1.09	1.72	44.93	0.07	0.3	0.3	45.01	0.01	0.32	0.32

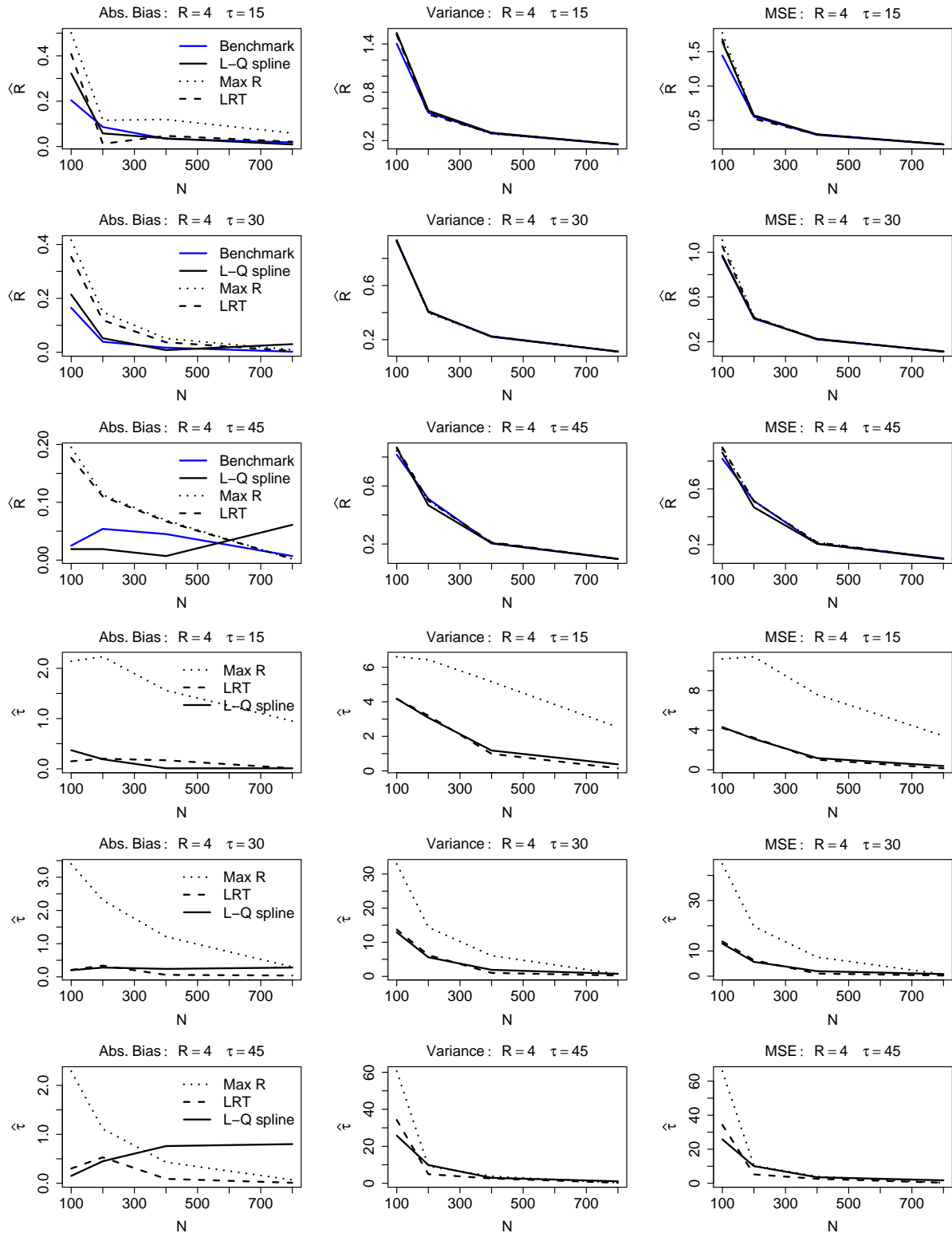


Figure 24: **Multiple Normally distributed exposures with two age effects.** Estimation of optimal (true) risk period length,  $\tau$ , and relative incidence  $R = \exp(\beta) = 4$ . Given are absolute bias (Bias), variance (Var.), and mean square error (MSE) over 200 simulated datasets.