Goals of risk management:

- Understanding the risk profile of the entire portfolio for better risk/return positioning (typically, a large-scale problem)
- This requires identification and measurement of risk across many sources, including market, credit, and op risk

Risk tools:

- VAR is a measure of the "worst" loss over the horizon that will not be exceeded at a specified confidence level
- The Basel 1998 rules require commercial banks to hold capital to cover basically 3 times the average of the daily VAR at the 99% confidence level over a 10-day horizon (plus specific risk)
- Risk measures should be backtested using exception tests, counting the fraction of days losses exceeds VAR
- For well-diversified portfolios, risk goes down with the number of assets; for large and well-diversified portfolios, the average correlation becomes the main driver of risk; when the correlation is zero, portfolio risk goes down to zero.
- VAR tools include marginal VAR, which is the increase in VAR for a unit increase in the risk factor exposure, and component VAR, which is an additive decomposition of VAR; this is also marginal VAR times the size of the position
- Incremental VAR is the actual change in VAR if the position is dropped (which requires recomputing VAR)
- Volatility can be forecast well over a horizon of 1 day using GARCH models; GARCH models allow for persistence in squared returns, and have mean reversion to a long run value
- The exponentially weighted moving average (EWMA) is a special case of GARCH with no mean reversion
- Correlations are more difficult to model due to dimensionality problems and the need to keep the covariance matrix positive definite; EWMA has a very simple structure for correlations when the decay is the same for all assets
- Factor models try to simplify the covariance matrix by reducing the number of independent dimensions; principal component analysis extracts these factors from the actual correlation matrix; portfolio risk can then be expressed in terms of exposures on the main factors and their risks; in well-diversified portfolios, residual risk goes to zero
- Joint distributions can be described by their marginal distribution as well as a copula; most widely used is the normal copula; this implies, however, weak dependencies in the tails

Market risk models:

- Systems require (1) position measurement, (2) modeling of risk factors, and (3) risk engine that bring these together
- First step is pricing, or marking to market (V_0)
- Positions cannot be modeled individually; rather, they are mapped on risk factors
- The easiest approach includes <u>local</u> valuation using first derivatives $\partial V/\partial S$ and second derivatives $\partial^2 V/\partial S^2$
- <u>Full</u> valuation reprices all instruments, which is more precise but also slower
- Linear VAR is given by VAR1=$\Delta\times$VAR(dS); quadratic VAR is VAR1 $-$ 0.5 Γ VAR(dS)^2
- Long positions in options have positive gamma, and hence lower VAR than from the linear model
- VAR method 1: variance/covariance or delta/normal: Using linear mapping on the factors, compute portf. variance $x'\Sigma\, x$, from which VAR is computed, using a normal distribution; main defects are lack of fat tails and non-linearities
- VAR method 2: historical simulation, where the vectors of historical changes in risk factors are applied to current value. The portfolio is subject to full revaluation; main defect is short window
- VAR method 3: based on an analytical model of risk factors. Run Monte Carlo simulations with full portfolio valuation; main defect is sampling variability and model risk
- Stress tests must be used as a complement to VAR, to consider events not in the VAR window or that have not happened yet; main issue is how to build relevant scenarios that lead to consistent joint movements in the risk factors; historical correlations can be used but assume stationary relationships

Credit risk models:

- Credit risk involves probability of default (or change in credit rating), loss given default, exposure, and default correl.
- Probabilities of default can be estimated from default rates for different credit ratings
- Loss given default can be roughly estimated from traded prices of bonds right after default
- PD can also be estimated from market prices of bonds or stocks; these, however, lead to risk-neutral estimates
- Credit spreads include the RN PD times the LGD plus a premium for risk (liquidity, equity risk as proxied by beta)
- Structural, Merton-type models model stock prices as a call option on the value of the firm; distance to default depends on the market value of the firm, liabilities, and the volatility of firm values
- Exposure is the amount at risk, or claim on the counterparty upon default if positive
  - This is the positive value of a random variable whose distribution evolves over time
  - For bonds and loans, this is basically the notional amount
  - For int.rate swaps, initial exposure is zero, increases due to a dispersion effect, then decreases due to duration
  - For currency swaps, exposure increase until maturity due to the exchange of principals
- Exposure can be controlled by marking to market, or netting across contracts with the same counterparty
- The joint default process is usually modeled using latent variables for the asset values that have multivariate normal densities; credit migration or default is modeled using cutoff points for various transition probabilities; correlations are derived from equity correlations; tails are very sensitive to correlation and copula assumptions
- The Basel II rules impose a credit risk charge either a (1) ratings-based, (2) foundation or (3) advanced internal ratings model. The charge roughly covers unexpected credit loss at a 99.9% confidence level over 1 year

Major Formulas

Derivatives:

- Valuation of an outstanding forward contract: V = S exp(-y τ) – K exp(-r τ)
- Valuation of receive-fixed interest rate swap: V = B(r; notional, coupon, τ) – FRN
- Valuation of receive-fixed foreign currency swap: V = S($/FC) B*(r*; notional*, coupon*, τ) – B(r; notional, coupon, τ)

Market risk:

- Cross-FX rate risk: If $\ln S_3 = \ln S_1 - \ln S_2$, then $\sigma_3^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2$
- Fixed-income risk using (modified) duration: $\sigma(\Delta P / P) = |D^*|\,\sigma(\Delta y)$
- Optimal number of futures contracts to sell for minimum-variance hedge $N^* = -\rho_{s,f}\dfrac{\sigma_s}{\sigma_f}\dfrac{QS}{MF} = -\beta_{s,f}\dfrac{QS}{MF}$
- Optimal number of contracts for duration hedge $N^* = -\dfrac{QD_S^* S}{MD_F^* F}$
- Option partial derivatives
$$df = \frac{\partial f}{\partial S}dS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}dS^2 + \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial r^*}dr^* + \frac{\partial f}{\partial \sigma}d\sigma + \frac{\partial f}{\partial \tau}d\tau$$
$$df = \Delta dS + 0.5\ \Gamma dS^2 + \rho dr + \rho^* dr^* + \Lambda d\sigma + \Theta d\tau$$
- Delta is positive for calls, negative for puts; about 0.5 for ATM calls
- Gamma same for European calls and puts, highest for ATM short-term options
- Vega same for European calls and puts, highest for ATM long-term options
- |Theta| is highest for ATM short-term options
- Using a linear approximation, a long position in an option is equivalent of delta times the underlying asset with debt
- Long option positions have positive gamma and a shorter left tail, or lower probability of large losses
- GARCH model for conditional variance ht: $h_t = \alpha_0 + \alpha_1(R_{t-1} - \mu)^2 + \beta h_{t-1}$
- EWMA (exponentially weighted) $h_t = (1-\lambda)(R_{t-1} - \mu)^2 + \lambda h_{t-1}$
- The distribution of exceptions is binomial, with a normal approximation of $z = (N - pT) / \sqrt{[p(1-p)T]} \rightarrow N(0,1)$

Portfolio tools

- Marginal risk $\Delta\text{VAR}_i = (\text{VAR/W}) \times \beta_i = \alpha\sigma_p \times \beta_i = \alpha \times \sigma_i \rho_{i,p}$
- Component risk $\text{CVAR}_i = \text{VAR}_p \times \beta_i w_i = \Delta\text{VAR}_i \times x_i$ $\qquad \text{VAR}_p = \sum_{i=1}^{N} \text{CVAR}_i$

Portfolio credit risk

- Default-mode portfolio credit loss: $\text{CL} = \sum_{i=1}^{N} b_i \times \text{EAD}_i \times \text{LGD}_i$
- Joint default event (advanced): $E[b_A b_B] = \rho\sqrt{p_A(1-p_A)}\sqrt{p_B(1-p_B)} + p(A)p(B)$
- Cumulative default rate $C_N(R) = d_1 + (1-d_1)d_2 + ... + [\Pi_{i=1}^{N-1}(1-d_i)]d_N$
- Implied PD from credit spread $y^* - y \approx \pi(1-f)$
- Merton structural model: Stock = Call(Value of Firm, Strike=Debt Amount)
- Long corporate bond = long Treasury bond + short credit default swap
- Volatility of firm value V derived from $\sigma_S = N(d_1)(V/S)\sigma_V$
- KMV distance to default: DD = [Value of firm assets – (ST debt+0.5*LT debt)] / σ_V
- 1-factor model for asset values V: $R_{it} = \sqrt{\rho}M_t + \sqrt{1-\rho}\ \varepsilon_{it}$
- Without netting, gross exposure = Σ_i Max(Vi, 0), with netting, net exposure = Max(Σ_i Vi, 0)

Operational risk

- "risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events"
- Loss distribution approach combines:
  - Loss frequency, or number of losses per year
  - Loss severity, or size of loss

  into a distribution of dollar losses per year, using convolution methods

Regulatory requirements

- Basel I Credit Risk Charge (CRC) $\text{CRC} = \sum_i K_i \times \text{EAD}_i = 8\%\left(\sum_i \text{RW}_i \times \text{EAD}_i\right) = 8\%(\text{CRWA})$
- Basel I and II capital ratio $\dfrac{\text{Total Risk Capital}}{\text{Credit Risk+Market Risk+Op.Risk}} = \text{Capital Ratio} > 8\%$
$$\text{Total Risk Capital} > \text{CRC+MRC+ORC} = 8\%(\text{RWA})$$
- Basel exposure for derivatives $\text{CE} = \text{NRV} + \text{AddOnFactor} \times N \times (0.4 + 0.6 \times NGR)$
- Basel Market Risk Charge (MRC) for trading book = General RC + Specific RC + Incremental RC
  - General RC = 3 times 10-day VAR at 99% confidence level
  - Basel III added a StressVAR and higher IRC, which increases the total MRC sharply
- Operational Risk Charge (Basel 2)
  - Basic Indicator Approach $K_{BIA} = EI \times \alpha$
  - Standardized Approach $K_{SA} = \sum_i EI_i \times \beta_i$
  - Advanced Measurement Approach $K_{AMA} = \text{UL} = \text{VAR}(1-\text{year}, 99.9\%) - \text{EL}$