



Shorter Notes: The Tangent Microbundle of a Suitable Manifold

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Proceedings of the American Mathematical Society, Vol. 32, No. 1 (Mar., 1972),
324-326.

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SHORTER NOTES

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THE TANGENT MICROBUNDLE OF A SUITABLE MANIFOLD

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ABSTRACT. The purpose of this note is to generalize to the topological category the fact that a suitable differentiable manifold is parallelizable (Theorem 4 of [1]). This result has a "folk-theorem" status in some quarters, but I believe that in view of the recent interest in H -manifolds [2], it would be desirable to have the result on record.

Let M be an n -manifold. Define $\Delta: M \rightarrow M \times M$ to be the diagonal map, and $\pi^1, \pi^2: M \times M \rightarrow M$ to be the projections on the first and second factor respectively. Milnor [3] calls the diagram $\Delta: M \rightrightarrows M \times M: \pi^1$ the *tangent microbundle* of M , where for each point $b \in M$ there exists an open set U_b in M containing b , an open set V_b in $M \times M$ containing $\Delta(b)$, and a homeomorphism $h: V_b \rightarrow U_b \times R^n$ such that the following diagram commutes:

$$\begin{array}{ccccc}
 & & U_b & & \\
 & \swarrow \text{id} \times 0 & & \searrow \Delta|_{U_b} & \\
 U_b \times R^n & & & & V_b \\
 & \swarrow \text{proj} & & \searrow \pi^1|_{V_b} & \\
 & & U_b & &
 \end{array}$$

Received by the editors May 18, 1971.

AMS 1970 subject classifications. Primary 57A55.

Key words and phrases. Tangent microbundle, topological parallelizability, suitable manifolds, H -spaces.

¹ Work done while author was on an NSF Traineeship at the University of California, Los Angeles.

An n -manifold M is *topologically parallelizable* if there exists an open set V_1 in $M \times M$ containing $\Delta(M)$, an open set V_2 in $M \times R^n$ containing $M \times 0$, and a homeomorphism $h: V_1 \rightarrow V_2$ so that the following diagram commutes:

$$(1) \quad \begin{array}{ccc} & V_1 & \\ \Delta \nearrow & \downarrow h & \searrow \pi^1 \\ M & & M \\ \text{id} \times 0 \searrow & & \nearrow \text{proj} \\ & V_2 & \end{array}$$

Pick $e \in M$. M is *suitable* if there is a continuous map $\Phi: M \rightarrow G(M)$ such that $\Phi(x)(x) = e$ and $\Phi(e) = \text{identity}$, where $G(M)$ is the group of all homeomorphisms of M onto itself with the compact-open topology. By Theorem 2 of [1], M is suitable iff there exists a $\theta \in G(M \times M)$ such that

$$\theta(M \times (M - e)) = \{(x, y) \in M \times M : x \neq y\} \quad \text{and} \quad \pi^1 \theta = \pi^1.$$

Note that a suitable manifold supports an H -space structure [1].

THEOREM. *A suitable n -manifold M is topologically parallelizable.*

PROOF. Let U_e and V_b be as in the definition of the tangent microbundle of M . Let W be an open set in M such that $e \in W \subset \text{cl } W \subset U_e$. Choose the V_b 's so that $\pi^2 \theta^{-1}(x, y) \in W$ for $(x, y) \in V_b$. Let $k: U_e \rightarrow R^n$ be a co-ordinate map such that $k(e) = 0$. Define $\lambda: M \rightarrow [0, 1]$ so that λ is 1 on a neighborhood of $\text{cl } W$ and 0 on a neighborhood of $M - U_e$.

Let $V = \bigcup_{b \in M} V_b$ and define $h: V \rightarrow M \times R^n$ by

$$h(x, y) = (x, \lambda(\pi^2 \theta^{-1}(x, y))k(\pi^2 \theta^{-1}(x, y))).$$

h is a local homeomorphism, i.e. for $b \in M$, $h: V_b \rightarrow \text{image } h|_{V_b}$ is a homeomorphism, for define $h': \text{image } h|_{V_b} \rightarrow V_b$ by

$$h'(x, r) = (x, \pi^2 \theta(x, k^{-1}(r))).$$

Then on V_b , $\pi^2 \theta^{-1}(x, y) \in W$ so $h'h = \text{id}$, and on $\text{image } h|_{V_b}$, $k^{-1}(r) \in W$ so $hh' = \text{id}$.

However, $h: \Delta(M) \rightarrow M \times 0$ homeomorphically, so by Lemma 4.1 of [4], there is a neighborhood V_1 in $M \times M$ of $\Delta(M)$ and a neighborhood V_2 in $M \times R^n$ of $M \times 0$ such that $h: V_1 \rightarrow V_2$ is a homeomorphism. As h commutes in (1) this proves our result.

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