

Shorter Notes: The Tangent Microbundle of a Suitable Manifold

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SHORTER NOTES

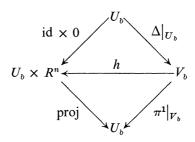
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THE TANGENT MICROBUNDLE OF A SUITABLE MANIFOLD

RONALD J. STERN¹

ABSTRACT. The purpose of this note is to generalize to the topological category the fact that a suitable differentiable manifold is parallelizable (Theorem 4 of [1]). This result has a "folk-theorem" status in some quarters, but I believe that in view of the recent interest in *H*-manifolds [2], it would be desirable to have the result on record.

Let M be an n-manifold. Define $\Delta: M \to M \times M$ to be the diagonal map, and π^1 , $\pi^2: M \times M \to M$ to be the projections on the first and second factor respectively. Milnor [3] calls the diagram $\Delta: M \rightleftharpoons M \times M: \pi^1$ the tangent microbundle of M, where for each point $b \in M$ there exists an open set U_b in M containing b, an open set V_b in $M \times M$ containing $\Delta(b)$, and a homeomorphism $h: V_b \to U_b \times R^n$ such that the following diagram commutes:



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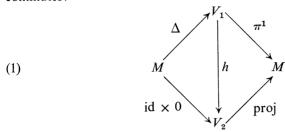
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An *n*-manifold M is topologically parallelizable if there exists an open set V_1 in $M \times M$ containing $\Delta(M)$, an open set V_2 in $M \times R^n$ containing $M \times 0$, and a homeomorphism $h: V_1 \rightarrow V_2$ so that the following diagram commutes:



Pick $e \in M$. M is suitable if there is a continuous map $\Phi: M \to G(M)$ such that $\Phi(x)(x) = e$ and $\Phi(e) = identity$, where G(M) is the group of all homeomorphisms of M onto itself with the compact-open topology. By Theorem 2 of [1], M is suitable iff there exists a $\theta \in G(M \times M)$ such that

$$\theta(M \times (M-e)) = \{(x, y) \in M \times M : x \neq y\}$$
 and $\pi^1 \theta = \pi^1$.

Note that a suitable manifold supports an H-space structure [1].

THEOREM. A suitable n-manifold M is topologically parallelizable.

PROOF. Let U_b and V_b be as in the definition of the tangent microbundle of M. Let W be an open set in M such that $e \in W \subset \operatorname{cl} W \subset U_e$. Choose the V_b 's so that $\pi^2 \theta^{-1}(x, y) \in W$ for $(x, y) \in V_b$. Let $k : U_e \to R^n$ be a co-ordinate map such that k(e) = 0. Define $\lambda : M \to [0, 1]$ so that λ is 1 on a neighborhood of $\operatorname{cl} W$ and 0 on a neighborhood of $M - U_e$.

Let $V = \bigcup_{b \in M} V_b$ and define $h: V \to M \times R^n$ by

$$h(x, y) = (x, \lambda(\pi^2\theta^{-1}(x, y))k(\pi^2\theta^{-1}(x, y))).$$

h is a local homeomorphism, i.e. for $b \in M$, $h: V_b \rightarrow \text{image } h|_{V_b}$ is a homeomorphism, for define $h': \text{image } h|_{V_b} \rightarrow V_b$ by

$$h'(x,r) = (x, \, \pi^2 \theta(x, \, k^{-1}(r))).$$

Then on V_b , $\pi^2\theta^{-1}(x, y)\in W$ so $h'h=\mathrm{id}$, and on image $h|_{V_b}$, $k^{-1}(r)\in W$ so $hh'=\mathrm{id}$.

However, $h:\Delta(M)\to M\times 0$ homeomorphically, so by Lemma 4.1 of [4], there is a neighborhood V_1 in $M\times M$ of $\Delta(M)$ and a neighborhood V_2 in $M\times R^n$ of $M\times 0$ such that $h:V_1\to V_2$ is a homeomorphism. As h commutes in (1) this proves our result.

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