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A μ-INVARIANT ONE HOMOLOGY 3-SPHERE THAT BOUNDS AN ORIENTABLE RATIONAL BALL

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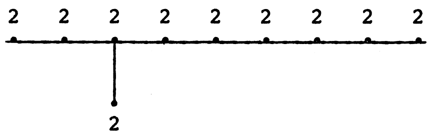
In this note we show that the Brieskorn homology sphere $\Sigma(2,3,7)$ bounds an orientable rational ball Q. It is known that the μ-invariant of $\Sigma(2,3,7)$ is one as it bounds the plumbed 4-manifold W^4



Note that W^4 has an even intersection form with signature $\sigma(W^4) = 8$ and rank 10. Thus $M^4 = Q \cup_\Sigma W^4$ is a closed orientable 4-manifold with even intersection form of signature 8 and rank 10. (Note that M^4 cannot be a spin 4-manifold.) As a corollary we have the following recent theorem of N. Habegger [1]:

COROLLARY. *Every even unimodular symmetric bilinear form* F *with* $\left|{}^{\mathrm{rank}(F)}/_{\sigma(F)}\right| \geq 5/4$ *can be realized as the intersection form of a closed orientable 4-manifold.*

THEOREM. $\Sigma(2,3,7)$ *bounds an orientable rational ball* Q^4.

PROOF. First we attach a 1-handle and a 2-handle to $\Sigma(2,3,7) \times I$ to obtain a rational homology cobordism W_1 between $\Sigma(2,3,7)$ and a 3-manifold K^3 which has the integral homology of $L(4,-1)$. Then we describe an integral homology cobordism W_2 between K^3 and $L(4,-1)$. Since $L(4,-1)$ bounds a rational ball W_3, we let $Q = W_1 \cup W_2 \cup W_3$. This is done as follows.

It is well known that $\Sigma(2,3,7)$ is obtained by $+1$ surgery on the figure eight knot. Attach a 1-handle to $\Sigma(2,3,7) \times I$ to obtain a cobordism from $\Sigma(2,3,7)$ to $\Sigma(2,3,7) \# S^2 \times S^1$:

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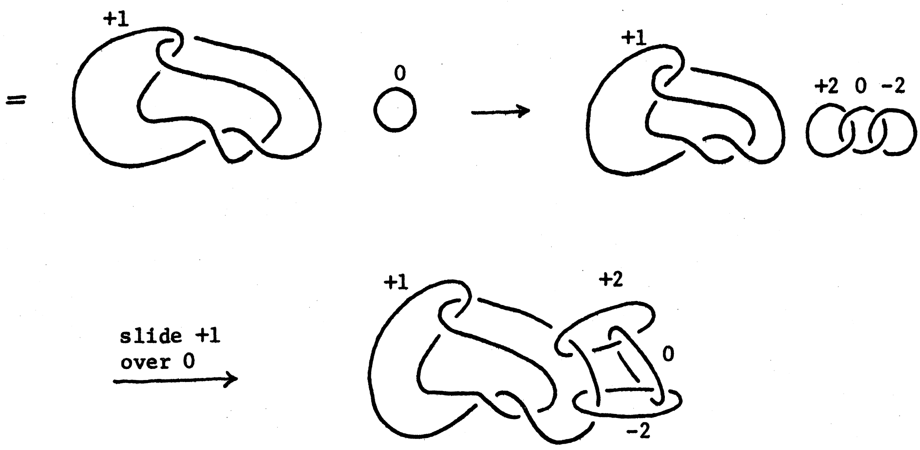
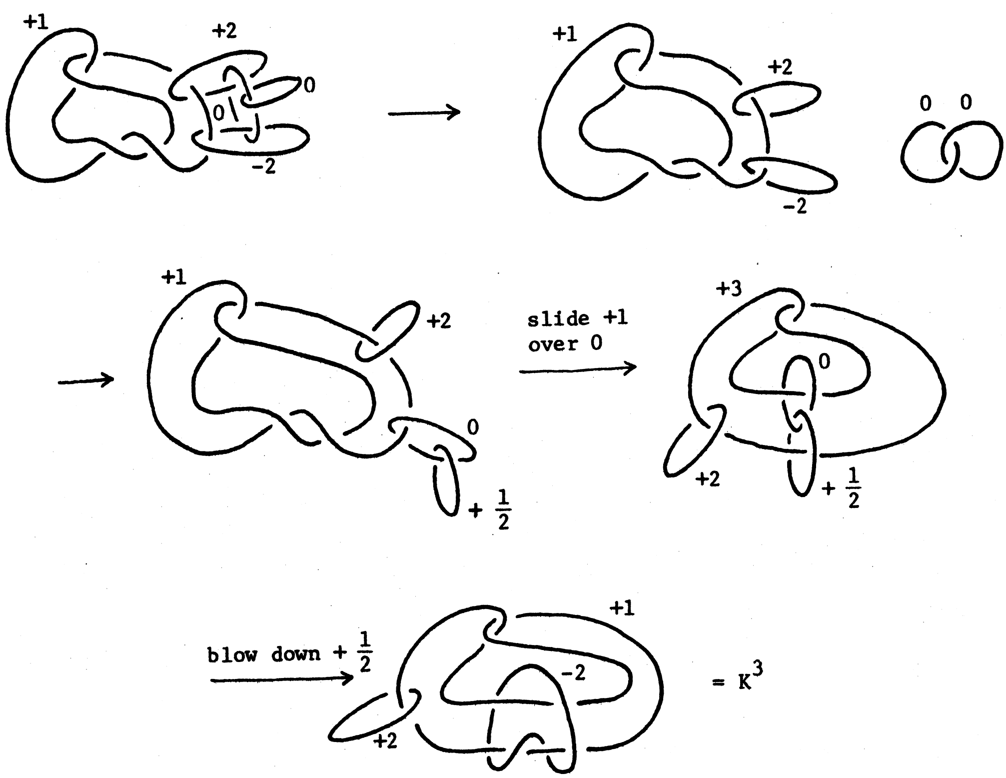
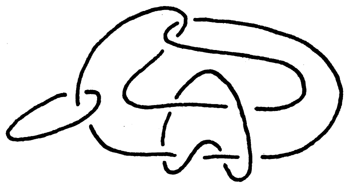


Now attach a 2-handle



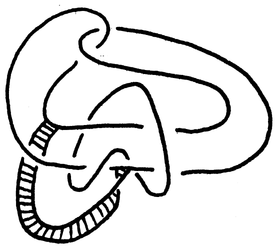
This describes the cobordism W_1. To see that it is a rational homology cobordism note that the attached 2-handle kills 4 times the generator of H_1 which was introduced by the 1-handle.

Now the link

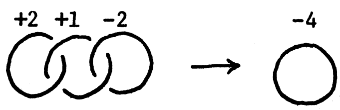


is ribbon concordant to the link

by means of the ribbon

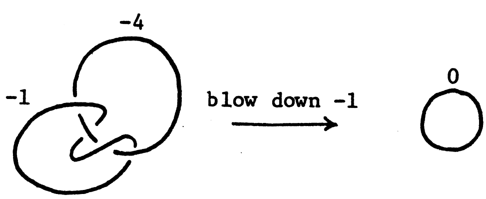


Thus K^3 is integral homology cobordant to



i.e. to L(4,-1). Hence we have W_2.

Finally L(4,-1) bounds a rational ball W_3. To see this attach the following 2-handle to L(4,-1) to obtain $S^2 \times S^1$:



QUESTION. Does there exist a closed orientable 4-manifold with definite even intersection pairing and signature 8?

BIBLIOGRAPHY

1. N. Habegger, Une variete dimension 4 avec form d'intersection paire et signature -8, Comment. Math. Helv. 57 (1982), 22-24.

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