Using Floer's exact triangle to compute Donaldson invariants

R. Fintushel[1] and R.J. Stern[2]

1 Introduction

In Floer's 1990 ICM talk ([F3]), he stated that «with luck, one may be able to analyse the change of the Donaldson polynomial under handle addition. The exact triangle for surgery on M may be considered as one step in this direction.» It is the purpose of this note to elaborate on this comment and to illustrate this circle of ideas by computing the 0-degree Donaldson invariant for the $K3$ surface. This invariant was first calculated by Donaldson [D3] using stable bundles. It was recently calculated by Kronheimer [K] who reduced the calculation to a count of representations of a related orbifold fundamental group. Our purpose here is to illustrate how Floer's exact triangle can be implemented to compute Donaldson invariants without resorting to any underlying complex structure. Such cut and paste techniques were also utilized in [FS4] where we constructed irreducible 4-manifolds not homotopy equivalent to any complex surface. The methods in this note complement those of [FS3] where it is shown that, for any 4-manifold X homotopy equivalent to the $K3$ surface and containing the Brieskorn homology 3-sphere $\Sigma(2,3,7)$, certain values of the degree 10 Donaldson polynomial invariant are odd.

2 A Mayer-Vietoris principle and Floer's exact triangle

Suppose we are given an oriented simply connected 4-manifold $X = W \cup_\Sigma V$ with $\partial W = \Sigma = -\partial V$, an integral homology 3-sphere. For simplicity we assume that the character variety $\chi(\Sigma) = \mathrm{Hom}(\pi_1(\Sigma), SO(3))/\text{conjugacy}$ consists of isolated points. (This assumption will hold for all the situations which we shall consider.) The general idea for calculating the Donaldson invariant of X is that if one stretches out a collar $\Sigma \times [-r, r]$ to have infinite length, the invariant of X can be computed as a sum of products $q_M[\lambda] \cdot q_B[\lambda]$ of relative invariants where $[\lambda]$ runs over the character variety $\chi(\Sigma)$. However this is not precisely correct and requires an understanding of the Floer homology of the homology 3-sphere Σ which we now review. (See [F1], [F2], [FS2], and [DFK] for more details.)

The Floer chain groups are free abelian groups, graded mod 8 and generated by $\chi(\Sigma) \setminus [\vartheta]$ (where ϑ denotes the trivial $SO(3)$ representation). A particular

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character $[\lambda]$ gives rise to a generator $\langle\lambda\rangle$ of the chain group C_n if n is the mod 8 index of the anti-self-duality operator on $\Sigma \times \mathbf{R}$ with asymptotic values $[\lambda]$ at $-\infty$ and $[\vartheta]$ at ∞. The boundary operator of the complex is defined by $\partial\langle\lambda\rangle = \sum n([\lambda],[\beta])\langle\beta\rangle$ where $[\beta]$ runs over the characters generating C_{n-1}, and $n([\lambda],[\beta])$ is a signed count of the connected components of the 1-dimensional moduli space $\mathcal{M}^1_{\Sigma\times\mathbf{R}}([\lambda],[\beta])$. Floer homology $HF_*(\Sigma)$ is the (mod 8 graded) homology of this complex. Orientation reversal from Σ to $-\Sigma$ causes a change in grading from i to $-3-i$ but, of course, no change in the character variety; so there is a canonical identification of $C_i(\Sigma)$ with $C_{-3-i}(-\Sigma)$. Here, for a manifold N with boundary, $\mathcal{M}^n_N[\lambda]$ denotes the n-dimensional component of the oriented moduli space of finite-action anti-self-dual $SO(3)$ connections on a bundle E (which will be clear from the context) over $N_+ = N \cup (\partial N \times \mathbf{R}^+)$ with a metric which is cylindrical on the end and limits asymptotically to a flat connection over ∂N corresponding to $[\lambda] \in \chi(\partial N)$.

If W is a simply connected 4-manifold with $\partial W = \Sigma$ and E is an $SO(3)$ bundle over W, there is a (degree 0) Donaldson invariant q_W with values in $HF_n(\Sigma)$ where $n = -2w_2(E)^2 - 3(1+b^+_W)$. It is defined by

$$q_W = \sum q_W([\lambda])\langle\lambda\rangle$$

where $[\lambda]$ runs over the characters generating C_n and $q_W([\lambda])$ is the signed count of points in $\mathcal{M}^0_W[\lambda]$, and where this moduli space is assumed to be compact. It is an illuminating exercise to see that q_W is a cycle, and so defines a class in $HF_n(\Sigma)$.

2.1 Theorem *(Donaldson; cf. [A], [DFK]). Let $X = W \cup_\Sigma V$ be a simply connected 4-manifold with $\partial W = \Sigma$ and $\partial V = -\Sigma$. Suppose that W and V are also simply connected. Let q_X be a degree 0 Donaldson invariant corresponding to a bundle P over X. If*

(i) $b^+_W > 0$ *or* $w_2(P_W) \neq 0$, *and*

(ii) $b^+_V > 0$ *or* $w_2(P_V) \neq 0$

then

$$q_X = \langle q_W, q_V\rangle$$

where the pairing is the «Kronecker» pairing of $HF_*(\Sigma)$ with $HF_{-3-*}(-\Sigma)$.

The trivial representation ϑ fails to show up in the formula because the assumptions (i) and (ii) allow one to invoke the arguments of Donaldson's connected sum theorem [D3].

Floer homology can also be defined for 3-manifolds M which are homology equivalent to $S^2 \times S^1$ [F2]. This is done by taking, for C_*, the free chain complex generated by the characters $[\lambda] \in \chi(M)$ whose corresponding flat $SO(3)$ bundle V_λ is the nontrivial bundle over M, i.e. the bundle with $w_2(V_\lambda) \neq 0$. All such representations are irreducible, and so one cannot «compare» λ to ϑ to obtain an absolute grading. Instead there are only relative gradings. In either case, the ambiguity of the gradings arises from the formal dimensions of moduli spaces $\mathcal{M}_{M\times\mathbf{R}}([\lambda],[\lambda])$ which can be computed from the bundle over $M\times S^1$ obtained by identifying the ends. Since for a homology $S^2\times S^1$ we have $H^1(M;\mathbf{Z}_2)=\mathbf{Z}_2$, there are two such bundles. For each, $p_1\equiv(w_2)^2 \bmod 4$. But $(w_2)^2\equiv 0 \bmod 2$ since $M\times S^1$ has an even intersection form, and so $\dim\mathcal{M}_{M\times\mathbf{R}}([\lambda],[\lambda])\equiv 0 \bmod 4$. Thus the chain complex C_* is (relatively) graded mod 4 as are the resulting Floer homology groups $HF_*(M)$.

Let C be a simply connected cobordism with boundary $\partial C = Y_1 - Y_0$ where each Y_i is a homology sphere or homology $S^2\times S^1$, and suppose that E is an $SO(3)$ bundle over C which is nontrivial over each boundary component which is a homology $S^2\times S^1$. It is shown in [F1] that the Donaldson invariant of C which comes from the 0 dimensional components of the moduli space of anti-self-dual connections on E induces a homomorphism in Floer homology

$$C : HF_*(Y_0)\to HF_{*'}(Y_1)$$

where $*' = -2w_2(E)^2 - 3b_C^+ + *$.

Given a homology 3-sphere Σ and a knot K in Σ, let $S_r(\Sigma,K)$ denote the 3-manifold obtained from an r-framed surgery on the knot K. Then there is a natural cobordism B_K (the trace of the surgery) with negative definite intersection form (-1) from Σ to the homology 3-sphere $S_{-1}(\Sigma,K)$. Likewise, there is a cobordism C_K with intersection form (0) from $S_{-1}(\Sigma,K)$ to $S_0(\Sigma,K)$, a homology $S^2\times S^1$, and there is a cobordism D_K with intersection form (0) from $S_0(\Sigma,K)$ back to Σ. Using the trivial $SO(3)$ bundle over B_K and the nontrivial $SO(3)$ bundles over C_K and D_K, there are induced homomorphisms on Floer homology. In [F2] (cf. [BD], [F3]) it is shown that the following triangle is exact:

$$\begin{tikzcd}[column sep=small] & HF_*(S_0(\Sigma,K)) \arrow[dl, "D_K"'] & \\ HF_*(\Sigma) \arrow[rr, "B_K"] & & HF_*(S_{-1}(\Sigma,K)) \arrow[ul, "C_K"'] \end{tikzcd}$$

This is known as Floer's exact triangle. Note that the gradings of both $HF_*(\Sigma)$ and $HF_*(S_{-1}(\Sigma,K))$ are well-defined, while the relative grading for $HF_*(S_0(\Sigma,K))$ is fixed in this diagram by the requirement that the homomorphism C_K preserve the grading. Because $H^1(S_0(\Sigma,K);\mathbf{Z}_2)=\mathbf{Z}_2$, there are two ways to glue together the bundles over C_K and D_K to form one over $C_K\cup D_K$. Thus there are two distinct $SO(3)$ bundles over the union $B_K\cup C_K\cup D_K$ which restrict to the correct bundles over each of the cobordisms B_K, C_K, and D_K. These bundles induce endomorphisms of $HF_*(\Sigma)$ of degrees -1 and -5. It is common practice to use the bundle which induces the degree -1 endomorphism. Then it follows that D_K carries $HF_*(\Sigma_{K_0})$ to $HF_{*-1}(S_{-1}(\Sigma,K))$. Thus, the Floer exact triangle induces a long exact sequence of Floer homology groups where the connecting homomorphism has degree -1.

3 Floer's exact triangle and 0-degree Donaldson invariants

Let X be an oriented simply connected 4-manifold and consider an $SO(3)$ bundle E over X with $p_1(E) = -\frac{3}{2}(1+b^+)$. Then the formal dimension of the moduli space $\mathcal{M}_E$ of anti-self-dual connections is 0. For a generic metric on X, the Donaldson invariant $q_E \in \mathbf{Z}$ counts (with sign) the number of points in $\mathcal{M}_E$. Recall that the Pontryagin number p_1 does not completely determine E; the missing data is the second Stiefel-Whitney class $w_2(E) \in H^2(X;\mathbf{Z}_2)$ with $w_2{}^2 \equiv p_1 \bmod 4$. The $SO(3)$ bundles over X with $p_1(E) = -\frac{3}{2}(1+b^+)$ define degree 0 Donaldson invariants, which give a function

$$q_X : \mathcal{C}_X \longrightarrow \mathbf{Z}$$

where $\mathcal{C}_X = \{\eta \in H^2(X;\mathbf{Z}_2) \mid \eta \neq 0,\ \eta^2 \equiv -\frac{3}{2}(1+b^+) \bmod 4\}$.

Now suppose that the intersection form Q_X for X^4 decomposes as $Q_X = D \oplus r\mathbf{H} \oplus E$ where $\mathbf{H}$ denotes a hyperbolic pair and D and E are negative definite forms not diagonalizable over $\mathbf{Z}$. This algebraic decomposition can be realized topologically as follows. According to [FT] there are simply connected smooth 4-manifolds W_1, W_2, and W_3 with $M = W_1 \cup_{\Sigma_1} W_2 \cup_{\Sigma_{r+1}} W_3$ where the Σ_i are homology 3-spheres with $\partial W_1 = \Sigma_1$, $\partial W_2 = -\Sigma_1 \coprod \Sigma_{r+1}$, and $\partial W_3 = \Sigma_{r+1}$. Furthermore, the intersection forms Q_{W_i} for the bounded manifolds W_i satisfy $Q_{W_1} = D$, $Q_{W_2} = r\mathbf{H}$, and $Q_{W_3} = E$. An ambitious goal would be to compute the relative Donaldson invariants for the W_i, and then paste them together as modeled in [A] and [DFK] (see (2.1)) to recover the Donaldson invariants for M.

As we shall see, the techniques of [FS1] can be implemented to show that certain 0-degree relative Donaldson polynomials for W_1 and W_3 are ± 1. To see how the Floer exact triangle can be used to compute the relative Donaldson invariants for W_2, further decompose W_2 as $W_2 = H_1 \cup_{\Sigma_2} H_2 \cup \cdots \cup_{\Sigma_r} H_r$ where the Σ_j's are homology spheres with $\partial H_j = -\Sigma_j \coprod \Sigma_{j+1}$ and with $Q_{H_j} = \mathbf{H}$. Further, suppose that each H_j is composed of two 2-handles $h_{j,1}$ and $h_{j,2}$ attached to $\Sigma_j \times I$ with the new boundary of $H_{j,1} = \Sigma_j \times I \cup h_{j,1}$ a homology $S^2 \times S^1$ and $H_j = H_{j,1} \cup H_{j,2}$. Then each $H_{j,1}$ induces a homomorphism C_K in some Floer exact sequence for which this data completely determines Σ, D_K and B_K. (See §4 below.) Similarly, each $H_{j,2}$ induces a homomorphism D_K in another Floer exact sequence for which $S_{-1}(\Sigma, K)$, B_K and C_K are determined. Thus, an understanding of these Floer exact sequences and their homomorphisms would yield a computation of $q_X : \mathcal{C}_X \longrightarrow \mathbf{Z}$ for certain elements of $\mathcal{C}_X$.

We illustrate these ideas by computing the 0-degree Donaldson invariant for the $K3$ surface. Here, the self-diffeomorphism group of $K3$ acts transitively on the elements of $\mathcal{C}_{K3}$ [M] so that the Donaldson invariant is constant on $\mathcal{C}_{K3}$. Our goal is to show that $q_{K3} \equiv \pm 1$.

4 The decomposition of $K3$

The intersection form Q_{K3} for the $K3$ surface decomposes as $Q_{K3} = E_8 \oplus 3H \oplus E_8$. We realize this decomposition topologically as follows. First, let B be the Milnor fiber for the $(2,3,5)$ Brieskorn singularity. It is a plumbing manifold whose intersection form is E_8 (negative definite) and whose boundary $\partial B = P$ is diffeomorphic to the Poincaré homology 3-sphere. We realize the above algebraic decomposition Q_{K3} by $K3 = B \cup C \cup B$ where C is obtained by attaching six 2-handles h_i, $1 \leq i \leq 6$, to P. To more explicitly describe C view P as the boundary of the handlebody of Figure 1.

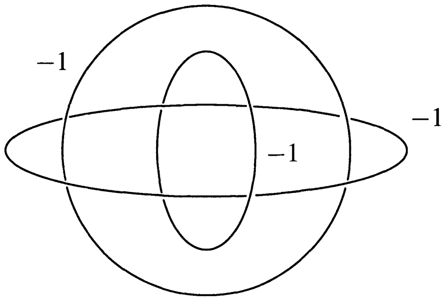

Fig. 1

(See [Hr],[HKK], and [G] for a more detailed discussion of such pictures.) Let $M\{p,q,r\}$ denote the result of p, q, r surgery on the 3 components of the Borromean rings so that $P = M\{-1,-1,-1\}$. Since the Borromean rings have an obvious 3-fold symmetry, $M\{p,q,r\}$ does not depend on the ordering of the surgery coefficients.

To obtain C we attach six 2-handles to P as in Figure 2.

It is shown in [FS4] that $K3 = B \cup C \cup B$. Note that if $P_0 = P$, $M_k = \partial_+ C_k$, and $P_k = \partial_+ D_k$ where $C_k = P_{k-1} \cup h_{2k-1}$ and $D_k = M_k \cup h_{2k}$, $1 \leq k \leq 3$, then each

$$Q_{C_k \cup D_k} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$$

which is equivalent to $\mathbf{H}$. Furthermore the manifolds $M_1 = M\{0,-1,-1\}$, $M_2 = M\{1,0,-1\}$, and $M_3 = M\{1,1,0\}$ have the integral homology of $S^2 \times S^1$, while $P_1 = M\{1,-1,-1\}$, $P_2 = M\{1,1,-1\}$, and $P_3 = M\{1,1,1\} = -P$ are integral homology 3-spheres. Let $\mathscr{L}$ denote the left-handed trefoil knot, $\mathscr{E}$ the figure-eight knot, and $\mathscr{T}$ the right-handed trefoil knot. By blowing down appropriate ± 1 curves in the surgery descriptions of the $M\{p,q,r\}$, we get the following simple surgery descriptions of these 3-manifolds:

$P_0 = P$ is the result of -1-surgery on $\mathscr{L}$,

P_1 is the result of both a $+1$-surgery on $\mathscr{L}$ and a -1 surgery on $\mathscr{E}$,

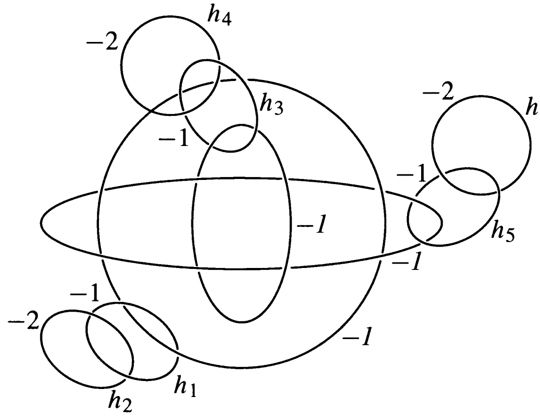

Fig. 2

P_2 is the result of both a $+1$-surgery on $\mathscr{C}$ and a -1-surgery on $\mathscr{T}$,
M_1 is the result of 0-framed surgery on $\mathscr{L}$,
M_2 is the result of 0-framed surgery on $\mathscr{C}$, and
M_3 is the result of 0-framed surgery on $\mathscr{T}$.

Note that if Σ is a homology 3-sphere then

$$\Sigma = S_{-1}(S_1(\Sigma, K), K')$$

where K' is the belt circle of the 2-handle attached to Σ. Furthermore, it is easily checked that

$$S_0(\Sigma, K) = S_0(S_1(\Sigma, K), K')$$

Thus we may rewrite our surgery descriptions of the 3-manifolds P_i and M_j as follows:

$$P_0 = S_{-1}(S^3, \mathscr{L})$$

$$P_1 = S_{-1}(S^3, \mathscr{C}) \text{ and } S^3 = S_{-1}(P_1, \mathscr{L}')$$

$$P_2 = S_{-1}(S^3, \mathscr{T}) \text{ and } S^3 = S_{-1}(P_2, \mathscr{C}')$$

$$M_1 = S_0(S^3, \mathscr{L}) = S_0(P_1, \mathscr{L}')$$

$$M_2 = S_0(S^3, \mathscr{C}) = S_0(P_2, \mathscr{C}')$$

$$M_3 = S_0(S^3, \mathscr{T}) = S_0(P_3, \mathscr{T}')$$

5 The relative Donaldson invariants for the definite manifold B

The relative Donaldson invariants of B take their values in the Floer homology groups of its boundary, P. It is shown in [FS2] that (with our orientation conventions)

$$HF_i(P) = \begin{cases} \mathbf{Z} & i = 1,5 \\ 0 & i \neq 1,5 \end{cases}$$

Let $\langle\alpha\rangle \in HF_1(P)$ be a generator. Our goal in this section is to show that $q_B = \pm\langle\alpha\rangle$. But this follows almost immediately from the proof of the main theorem of [FS1]. For, let $e \in H^2(B;\mathbf{Z})$ be a cohomology class with square $Q_B(e,e) = -2$ and let $E = L \oplus \mathbf{R}$ be the reducible $SO(3)$-bundle over B such that the Euler class of L is e. The moduli space $\mathcal{M}_B(\vartheta)$ of anti-self-dual connections in E with trivial asymptotic condition is a 1-manifold whose connected components are circles, open arcs J_i, half-open arcs K_j, and closed arcs L_k. The endpoints of both the K_j and L_k correspond to reducible connections. Since there is a unique equivalence class of reductions of E (cf. [FS1]), there are no L_k and exactly one K_j. Each of the open ends of the J_i and K_1 corresponds to popping off a one dimensional moduli space $\mathcal{M}^1_{P\times\mathbf{R}}([\alpha],[\vartheta])$ and leaving a point moduli space $\mathcal{M}^0_B([\alpha])$. The number of these ends (counted with orientation) is thus

$$\pm 1 = \#\mathcal{M}^0_B([\alpha]) \cdot \#\mathcal{M}^1_{P\times\mathbf{R}}([\alpha],[\vartheta]),$$

where «#» denotes a count with signs.

$$q_B = \#\mathcal{M}^0_B([\alpha])\,\langle\alpha\rangle = \pm\langle\alpha\rangle \tag{5.1}$$

6 The relative Donaldson invariants for the indefinite manifold C

In §4 we decomposed C as $C = C_1 \cup D_1 \cup C_2 \cup D_2 \cup C_3 \cup D_3$. Each of the cobordisms C_i and D_i induce homomorphisms

$$\begin{aligned} C_i &: HF_*(P_{i-1}) \to HF_*(M_i) \\ D_i &: HF_*(M_i) \to HF_{*-1}(P_i) \end{aligned}$$

which (recalling the surgery descriptions given at the end of §4) fit into the following six Floer triangles.

$$\begin{tikzcd} & HF_*(M_1 = S_0(S^3,\mathcal{L})) \arrow[dl] & \\ 0 = HF_*(S^3) \arrow[rr] & & HF_*(P = S_{-1}(S^3,\mathcal{L})) \arrow[ul, "C_1"'] \end{tikzcd} \tag{6.1}$$

$$\begin{tikzcd} & HF_*(M_1 = S_0(P_1,\mathcal{L}')) \arrow[dl, "D_1"'] & \\ HF_*(P_1) \arrow[rr] & & HF_*(S^3 = S_{-1}(P_1,\mathcal{L}')) = 0 \arrow[ul] \end{tikzcd} \tag{6.2}$$

$$\begin{tikzcd} & HF_*(M_2 = S_0(S^3,\mathcal{C})) \arrow[dl] & \\ 0 = HF_*(S^3) \arrow[rr] & & HF_*(P_1 = S_{-1}(S^3,\mathcal{C})) \arrow[ul, "C_2"'] \end{tikzcd} \tag{6.3}$$

$$\begin{tikzcd} & HF_*(M_2 = S_0(P_2,\mathcal{C}')) \arrow[dl, "D_2"'] & \\ HF_*(P_2) \arrow[rr] & & HF_*(S^3 = S_{-1}(P_2,\mathcal{C}')) = 0 \arrow[ul] \end{tikzcd} \tag{6.4}$$

$$\begin{tikzcd} & HF_*(M_3 = S_0(S^3,\mathcal{T})) \arrow[dl] & \\ 0 = HF_*(S^3) \arrow[rr] & & HF_*(P_2 = S_{-1}(S^3,\mathcal{T})) \arrow[ul, "C_3"'] \end{tikzcd} \tag{6.5}$$

$$\begin{tikzcd} & HF_*(M_3 = S_0(P_3,\mathcal{T}')) \arrow[dl, "D_3"'] & \\ HF_*(P_3 = -P) \arrow[rr] & & HF_*(S^3) = 0 \arrow[ul] \end{tikzcd} \tag{6.6}$$

By the exactness of these triangles, all the homomorphisms C_1, D_1, C_2, D_2, C_3 and D_3 are isomorphisms. Keep in mind that the gradings of the $HF_*(M_i)$ are relative and only fixed by differing conventions determined by each exact triangle. Pasting these cobordisms together (using the analysis already present in [F1]) we have that

$$C : HF_*(P) \to HF_{*'}(-P)$$

is an isomomorphism. It remains to sort out what bundles we have and to determine the relationship between the gradings $*$ and $*'$.

Consider the cobordism $C_1 \cup D_1$ between P and P_1. As we pointed out in §3, there are two ways to glue together the bundles over C_1 and D_1 arising in the Floer exact triangles. Denote them by E_1 and E_2. Since $w_2(E_i) \neq 0$ on $H_2(M_1; \mathbf{Z}_2)$, which is generated by the core of the 2-handle h_1, these bundles are characterized by the conditions

$$\langle w_2(E_1), [h_2] \rangle = 0$$
$$\langle w_2(E_2), [h_2] \rangle \neq 0$$

Thus the Poincaré dual of $w_2(E_1)$ is $[h_2]$ so that $w_2(E_1)^2 \equiv 0 \bmod 4$, and the Poincaré dual of $w_2(E_2)$ is $[h_1] + [h_2]$ so that $w_2(E_2)^2 \equiv 2 \bmod 4$. The same is true for the other cobordisms. Thus, there are several w_2's that can be utilized for our computations. Fortunately, we only need one, so choose a bundle E' over C with $w_2(E'|_{M_i}) \neq 0$ and with $w_2(E')^2 \equiv 2 \bmod 4$. Then C induces the isomorphism

$$C : HF_*(P) \to HF_{*-5}(-P) \tag{6.7}$$

Pasting E' over C with two copies of the bundle over B from section §5 we get an $SO(3)$-bundle E over $K3$ with $w_2(E)^2 \equiv 2 \bmod 4$. Applying (2.1) together with the calculations (5.1) and (6.7), we have that

$$q_{K3}(w_2(E)) = \langle C(q_B), q_B \rangle = \pm \langle\, \langle \alpha \rangle, \langle \alpha \rangle \,\rangle = \pm 1.$$

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Department of Mathematics,
Michigan State University
East Lansing,
Michigan 48824, USA

Department of Mathematics,
University of California
Irvine,
California 92717, USA