Generic Discrete-Event Simulation Model of a Prison[1]

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Abstract

This essay describes a generic discrete-event simulation (DES) model of a prison system. The model tracks individual entities through a prison "career," beginning with admission and ending with release from custody. Career stage transitions are modeled as discrete events that occur in real time. With each career transition (or state-change) the model pauses to update the appropriate subpopulation databases. Entities carry information tags into and through the model. The tags can be used to create arbitrary subpopulations for forecasting or analysis. We demonstrate the model by forecasting the population of a large corrections system conditioned on two policy interventions.

Introduction

Correctional population modeling begins with Stollmack's (1973) attempt to predict the size of a jail population from admission and release rates. Where P_t is the population in year-t and where λ and μ are annual admission and release rates, this "mathematical-flow" model can be written as the autoregressive equation

$$P_t = P_{t-1}\, e^{-\mu} + (\lambda/\mu)(1-e^{-\mu}) \qquad (1)$$

This early model assumed that λ and μ are constant and that P_t is homogeneous. Violations of either assumption leads to biased forecasts. To address the problematic assumptions, Blumstein, Cohen, and Miller (1980) disaggregated P_t into subpopulations and allowed λ to vary with the at-risk population. Barnett (1987) further generalized the model by linking demographic change to the admission rate, λ. Lattimore and Baker (1997) incorporated feedback loops to account for recidivism. Despite these incremental improvements on (1), the performance of mathematical flow models has been disappointing.

A parallel approach, pioneered by Alfred Blumstein and colleagues (Blumstein and Larsen, 1969; Blumstein, Belkin and Glass, 1971) used Monte Carlo simulation models to project prison populations. To illustrate, a simulation model popularized by Auerhahn (2008b) can be written as the reduced differential equation,

$$P_t = P_{t-1} + \lambda N_t + \mu P_t + \gamma R_t \qquad (2)$$

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Here N_t is the general population at-risk, R_t is the population under parole supervision, and γ is a recidivism rate. Except that (2) distinguishes between *new* admissions and *re*admissions, the structure of this model is similar to Stollmack's original model (1). Since (2) can accommodate more variables and equations, it more accurately represents a high-dimensional, complex prison system. Model (2) has been used successfully to evaluate the impacts of "three strikes" laws (Auerhahn, 2008a), drug treatment programs (Auerhahn, 2004), probation reforms (Auerhahn, 2007), and sentencing reforms (Auerhahn, 2008b).

Though performing adequately for most purposes, the models used to simulate prison systems have their own shortcomings. Beginning with the earliest attempts (Blumstein and Larson, 1969; Blumstein, Belkin and Glass, 1971; Belkin, Blumstein, Glass and Lettre, 1972) and into the present (Auerhahn, 2002; 2004; 2007; 2008a; 2008b), virtually all of these models rely on the system dynamics simulation (SDS) models popularized by Forrester (1961).

In simple terms, change in an SDS model accrues gradually as individuals *flow* into and out of a set of *stocks*.[2] Since an SDS model uses real-time differential equations to represent the accrual process, the prisoners are treated as interchangeable units-of-analysis. This assumption can be relaxed to some extent by partitioning the population into component subpopulations but there is no simple way to track the movements of unique prisoners through an SDS model.

A more fundamental shortcoming of the SDS approach follows from the nature of the prison population. Although many dynamic phenomena can be adequately modeled as continuous-time flows into and out of a set of stocks, change in a prison population accrues in discrete steps as prisoners move through a sequence of discrete "career" stages.

Table 1 – Career Transitions as Discrete Events

		T_1		T_2		T_3		T_4		T_5	
Individual 1	$\mathbf{S_1}$		$\mathbf{S_2}$		$\mathbf{S_3}$		$\mathbf{S_4}$				
Individual 2	$\mathbf{S_1}$				$\mathbf{S_2}$				$\mathbf{S_3}$		$\mathbf{S_4}$
Individual 3	$\mathbf{S_1}$		$\mathbf{S_2}$		$\mathbf{S_3}$				$\mathbf{S_4}$		
Individual 4	$\mathbf{S_1}$		$\mathbf{S_2}$		$\mathbf{S_3}$				$\mathbf{S_4}$		
Individual 5	$\mathbf{S_1}$				$\mathbf{S_2}$ $\mathbf{S_3}$		$\mathbf{S_4}$				

The five hypothetical career paths plotted in Table 1 illustrate a simple career process. Each career path consists of four stages, S_1, S_2, S_3 and S_4. Due to individual differences such as hard work and good fortune, careers progress at different rates. The simple model could be made more realistic by allowing backward moves, by allowing individuals to compete for advancement, and so forth. But a simple model is sufficient for demonstrating the basic DES

[2] To illustrate these terms, N_t, P_t, and R_t in (2) are *stocks*, λ, μ and γ, are *flows*. SDS models are known colloquially as "stock-and-flow" models.

modeling approach.

Although time runs continuously in the DES model, individual career moves are discrete events. Each row of Table 1 describes an individual career. At T_1, for example, all five individuals are in the first stage of their careers. By T_2, the first, third and fourth individuals have moved to the second stage. and can be applied to institutional scales ranging from one small jail to a large, complex correctional system. Individual actors can be distinguished by age, sex, prior history, sentence type or any other policy-relevant variable. Other than standard DES assumptions (Schriber and Brunner, 1997), the model's only requirement is that its prisoner-entities follow a career path.

The third and fourth individuals move quickly and have completed their careers before T_4, The second individual moves slowly, in contrast. The columns of Table 1 describe the *system-state* at any point in time. Given five individuals and four career stages, there are twenty state-changes in Table 1. Since the system runs in continuous time, each time-point is a "column" with no area. The probability of a state-change at any time-point then is zero.

Because a DES model of the system depicted in Table 1 requires only twenty evaluations, it consumes fewer resources than an SDS model. Computational efficiency was an crucial consideration when computing resources were rare but, now, has become a minor consideration. In the following section, we describe a generic DES model of a correctional system. The model is generic in the sense proposed by Harper (2002) and can be adapted to scales ranging from one jail to a several dozen institutions. Our decision to model the system as a discrete-event process -- and hence, to use a DES model -- reflects the close fit of this approach to the institutional career process.

Overview of the Generic DES Model

Figure 1 diagrams the Generic DES Model as a flow chart at the start of T_1, the time interval when the simulation experiment begins. Barrels in Figure 1 represent four databases, Admission, Reception, Prison, and Parole. Rectangles represent processors that manage the databases. When a prisoner exits Reception and enters Prison, for example, the Reception processor copies the prisoner's record from to the Prison database and erases the record from the Reception database. Because the prisoner's record includes the time and circumstances of past career moves, records grow longer with each move.

In addition to updating databases, the processors use data in prisoner records to keep prisoners on their proper career paths and to regulate their rates of progress. Table 2 lists some examples of the variables that might be found in a prisoner's record. *Path Router* variables assign prisoners to specific career paths. The *Path Router* variable "Probability of Discharge," for example, determines whether a prisoner will exit Reception and enter Prison or, in the case of prisoners with short sentences, whether the prisoner will move directly to Discharge. *Temporal Trigger* variables determine the prisoner's rate of progress along the career path. "Waiting Time in Reception," for example, tells the Reception processor when to send the prisoner to Prison or Discharge.

Figure 1 – The Generic Prison Model at T_1

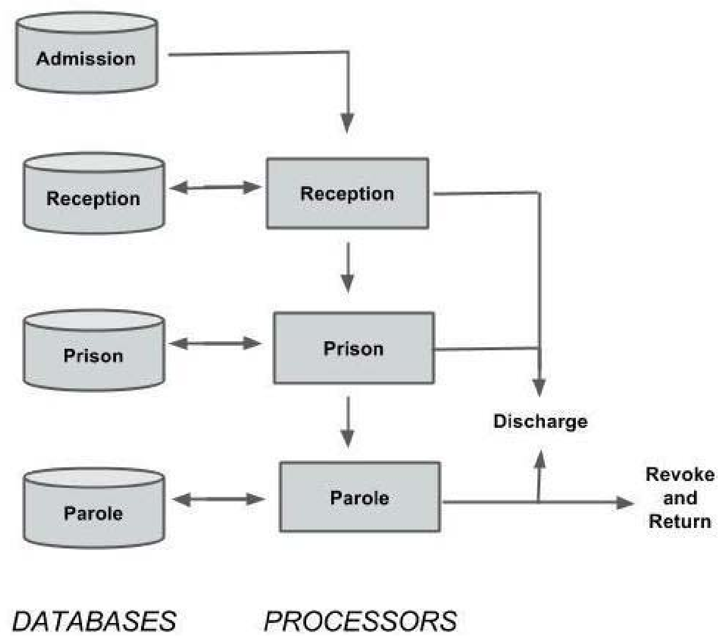


Table 2 – Example Variables from Prisoners' Records

Path Routers	*Temporal Triggers*	*Analytic Tags*
Probability of Death	Month of Admission (T_1, …, T_N)	Age at T_1
Probability of Discharge	Waiting Time in Reception	Gender
Probability of Parole	Waiting Time in Prison	Ethnicity
Probability of Revoke/Return	Good Time Credit Rate	County of Admission
	Waiting Time in Parole	Current Offense
		Prior Offenses

Finally, *Analytic Tag* variables describe the characteristics of a prisoner. These variables are superfluous to the Model's logic and operation. They are not referenced or used during an experiment. They are recorded in all four databases and can be used to sort prisoners into arbitrary subpopulations at the end of the experiment. We illustrate this use of *Analytic Tags* in the applications section below.

Design of Simulation Experiments

Prior to the onset of the experiment, at a time-interval designated T_0, the Reception, Prison and Parole databases are loaded into the Model. Rows of the Reception and Prison databases correspond to prisoners who are housed in reception center or prison beds at T_0. Rows of Parole corresponds to released prisoners who are under parole supervision at T_0. The columns of the Reception, Prison and Parole databases include the *Path Router* and *Temporal Trigger* variables required by the Model's logic as well as any *Analytic Tag* variables required to analyze the experimental results.

Since many *Temporal Trigger* variables depend on events that are unobserved or unknown at T_0, their values are imputed from available data. Variables in the Admission database are particularly problematic in that respect. These variables describe the prisoners who are expected to enter the system after the end of T_0, all of these values are imputed.

As T_1 begins, the Reception processor extracts a prisoner's record from Admission and writes it to the Reception database. During T_1, the Reception, Prison and Parole processors read and update their databases. If a prisoner is scheduled for a career move between T_1 and T_2, the processor extracts the prisoner from its own database and moves the prisoner to the destination database. As T_1 ends, all career moves have been recorded. Before T_2 begins, however, the Model requires two additional pieces of information.

Secular trend in admissions. If the simulation experiment runs for N months, the one-step ahead forecasts of the monthly populations $P_1, \ldots, P_N$ are written as

$$P_1 = P_0 + A_1 - R_1 \tag{3}$$

$$\vdots$$

$$P_N = P_{N-1} + A_N - R_N$$

Only P_0, the population at T_0, is known. Although the number of prisoners released in each of the N months is unknown, $R_1, \ldots, R_N$ are endogenous, so their values can be predicted accurately from the sentence-lengths of prisoners in the Reception and Prison databases at T_0. The number of prisoners admitted in each of the N months is wholly exogenous, on the other hand, and must be estimated from auxiliary data prior to T_1.

A survey of state corrections departments (McCleary and Alexander, 2007) identifies three methods for estimating $A_1, \ldots, A_N$. (1) The Delphi method estimates the mean of $A_1, \ldots, A_N$ from the consensus opinion of justice system experts (Day *et al.*, 2013). (2) The demographic trend method estimates $A_1, \ldots, A_N$ from trends in the at-risk population and, often, from crime trends (Olson, 1992; Oregon Department of Administrative Services, 2013). (3) The time series forecasting method uses formal time series models to forecast the values of $A_1, \ldots,$

A_N. At least 30 active models use this method and that is the method used in the applications section below.

Characteristics of the new admissions. With A_1, …, A_N estimated, the experimenter must estimate the values of the *Path Router*, *Temporal Trigger*, and *Analytic Tag* variables for the newly admitted prisoners. In principle, these values can be simulated with parameters estimated from the records of prisoners in the Reception, Prison and Parole databases at T_0. That approach leads to overfitting, however, and more important, is inflexible..

A more flexible approach uses *synthetic admission cohorts* sampled from the population of prisoners admitted to the correctional system in the preceding years. The synthetic admission cohort approach assume that the characteristics of newly admitted prisoners change slowly over time. Our research demonstrates that this assumption is plausible for horizons of 60 months or less. For longer horizons, inverse-time weighted sampling is advised. When the size of synthetic cohorts is more than five percent of the population, sampling with replacement is advised. The current version of the Model used a 36 month horizon with a sampling population of 75,000 new admissions.

Applications

In 2007, 36 state corrections agencies and the U.S. Bureau of Prisons used simulation models to forecast the size of key subpopulations and to evaluate the impacts of new policy interventions (Pew Charitable Trusts, 2007). The generic DES Model is well suited to both applications and, especially, to the second application. The Model evaluates the exogenous impact of a new policy intervention by comparing the population expected under the *status quo* condition to the population expected under the novel condition. We use two recent California policy interventions to demonstrate this application.

The first intervention, known as Assembly Bill 109, or AB 109, was a legislative response to a 2011 U.S. Court ruling on prison overcrowding. Although AB 109 was a complex legislative package, its central provisions diverted low-level offenders from state prisons to county jails (Schlanger, 2013; Taylor, 2012). The second intervention, Proposition 47, was a ballot initiative approved by California voters in the 2014 general election. Proposition 47 redefined several low-level offenses from felonies, which could result in a state prison sentence, to misdemeanors which could not (Males and Buchen, 2014). The results of Proposition 47 were unpredictable.

AB 109. Although AB 109 affected a large reduction in California’s prison population, the effect was not evenly distributed across subpopulations. One unanticipated consequence of AB 109 was a slight rise in the number of “second-strikers” admitted to state prisons. Since sentences for second-strikers served were twice as long as the sentences for other prisoners, a modest increases in second-strikers admissions can have a large impact on total population. If the large impact accrues gradually, however, it can go undetected until it reaches crisis proportions.

By 2013, growth in the second-striker subpopulation threatened to reverse the salutary effects of AB 109. To counteract this trend, in February, 2014, a three-judge panel of the U.S. District Court ordered an increase in the good-time credit earning rate for second-strikers. Figure 2 shows the projected impact of the order. In terms of "person-year" savings, the court-ordered change in good-time credits amounts to a reduction of approximately ten percent in the total population.

Figure 2 – California Male Second-Strikers

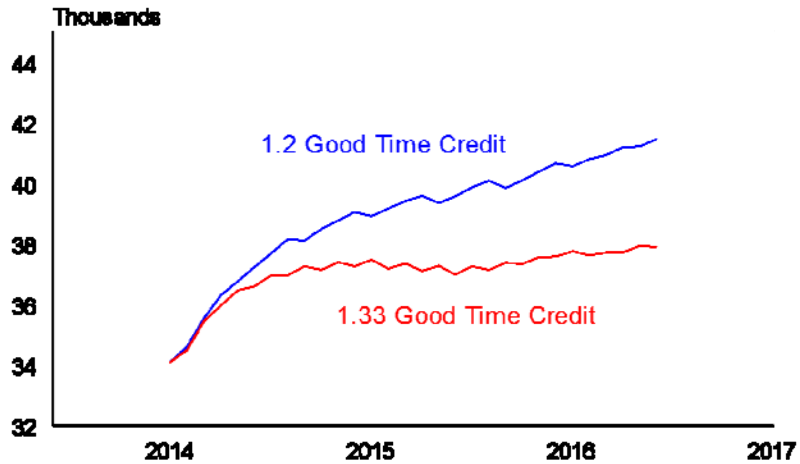
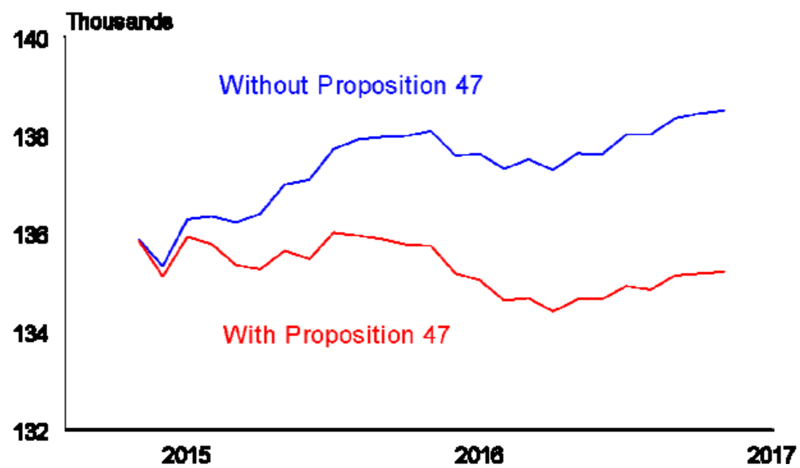


Figure 3 – California Total Prisoners



The projections plotted in Figure 2 were constructed by first running the Model under the *status quo* good time credit rate of 1.2 and, second, under the court-ordered rate of 1.33. Like all useful experimental results, this one has non-obvious elements. The court-ordered rate counteracts the unanticipated consequences of AB 109 almost exactly. Over the three-year horizon, the second-striker subpopulation is nearly flat.

Proposition 47. California voters approved Proposition 47 by a wide margin in the November, 2014 general election. Otherwise known as *The Safe Neighborhoods and Schools Act*, Proposition 47 reclassified low-level drug and property offenses from felonies, which called for a state prison sentence, to misdemeanors. The intended effect on prison populations

Figure 3 shows the impact of Proposition 47 on the total adult population. While Proposition 47 diverts all new admissions for drug possession to county-level custody starting in 2015, it is unclear at this moment what proportion of qualifying inmates will have their sentences commuted or at what rate. The impact analysis summarized in Figure 3 is based on the expected reduction in new admissions due to Proposition 47.

Conclusion

The Generic DES Model described here is a generalization of earlier models by Austin, Cuvelier and McVey (1992) and Scalia (2004). Like our Generic Model. these earlier models follow actors as they enter an institution and move along “career” paths, encountering delays, competing for resources, and satisfying conditions. Whereas the earlier models processed complete careers from start to finish, however, our Generic Model processes discrete events. This DES approach offers many practical advantages, especially scalability and flexibility.

Although the career concept originates with Weber’s analysis of the Prussian bureaucracy,[3] it is well suited to discrete-stage institutional processes. A hospital career, for example, moves from triage, to admission, to treatment, to discharge. Although hospitals and prisons have similar structures, there are salient differences. Prisons and hospitals operate on different natural time scales, for example. This difference is reflected in the problems the two models address. Patient-flow models are used to locate bottle-necks (Marshall, Christos and El-Darzi, 2005), to schedule staff (Zhu *et al.*, 2012), and to allocate beds (Gorunescu, McClean and Millard, 2002). Prisoner-flow models are used for population forecasting (Rich and Barnett, 1985) and evaluating the effects of policy interventions (Auerhahn, 2008).

The Generic DES Model’s most appealing application is policy impact analysis. Policy impacts are ordinarily evaluated by changing one or more of a model’s internal parameters. Since the Generic Model has no internal parameters, policy impacts must be evaluated with two different tag values. Once a *status quo* projection is produced, the experimenter simply changes the attribute values for a subpopulation affected by the policy change to reflect their values under the new policy. As demonstrated in the above applications, the difference between the *status quo* and policy-change population projections can be interpreted as the impact of the policy-change

[3] “The official is set for a 'career' within the hierarchical order of the public service. He moves from the lower, less important, and lower paid, to the higher positions.” (Gerth and Mills, 1948, p. 203).

on the projected subpopulation. This type of analysis would serve a multitude of uses for corrections department planning and policy evaluation.

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