Investing in Influence: How Minority Interests Can Prevail in a Democracy

by

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Abstract: How can the West's economic and political polarization be explained? We argue that persuasive lobbying at various levels of government leads to systematic deviations of policies from those desired by the majority. Implemented policies diverge from the majority position despite centripetal forces that induce interest groups to select positions closer to that majority position. Resources, organization, and cognitive biases can induce one-sided outcomes. When we allow for long-term lobbying infrastructure investments in a simplified tax-and-spend model, the deviations between majority desires and implemented policies are even larger than those in the absence of long-term investments.

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1 Introduction

Western democracies appear to have been in turmoil for some time now. Inequality has been increasing since the 1970s (Piketty, 2013, Milanovic, 2016). Even life expectancy had been decreasing in the US for three straight years (2015-17) before the onset of Covid-19, largely due to deaths of "despair" (Case and Deaton, 2020). Political polarization has also been increasing in the US over time (McCarty, Poole, and Rosenthal, 2006) and political strains have been building up in Europe as well, with Brexit being only one recent manifestation of those strains. Crouch (2004) describes this state of economic and political affairs as "post-democracy." Indeed, Gilens and Page (2014) find evidence that the policy positions actually implemented in the US differ systematically from those of the median voter, supporting earlier findings by Bartels (2008) which used different data and methods.

How can one understand systematic deviations of actual policies from those which the majority prefers in a modern representative democracy? In confronting this question, we focus on the effect of lobbying at various levels of government but with several characteristics that are not all present in previous treatments. We first provide an overview of the empirical evidence on how lobbying affects policy making at the executive, legislative, and judicial branches as well as in the media and public sphere discourse. We then examine a single-dimension spatial policy setting in which two competing lobbies choose first which policy position to advocate and then lobby for their respective positions. The majority's preferred policy position (henceforth called "majority position") imposes explicit constraints on the effects of lobbying; the further away from the majority position a lobbyist argues, the more difficult it is to argue their case. A decision maker (e.g., pivotal legislator, an agency leader, or a regulator) chooses the policy position on the basis of evidence and arguments put forward to them by the competing interest groups. Although the decision-maker can have biases - both in terms of priors as well as other psychological predispositions towards the evidence - that can influence the final outcome, there is no presumption of venal incentives. The outcome itself can be probabilistic, with the decision maker choosing one of the advocated positions, or a compromise position arrived at through bargaining under the threat of more extreme outcomes.[1]

We first show that, under a wide set of conditions, two lobbies with different most preferred positions will never choose to advocate for the same position. We then find conditions under which each of the lobbies will choose to advocate for their most preferred position and show how outcomes can systematically vary from the majority position depending on the biases of the decision maker, the costs of lobbying and feasibility of a compromise.

We next examine a simplified variation of the Meltzer and Richard (1981) model as an application of our model with the "rich" and "poor" as the two lobbies. Even though the poor (and the median voter) advocate for 100 percent taxation in the intentionally stark model we analyze, the rich can easily lower the compromise tax rate to a very low level. The actual tax rate implemented ends up being inversely related to the population of the poor relative to the rich. That is, the greater is the proportion of the population that is poor, the *lower* is the implemented tax rate.

We extend this analysis further by allowing each of the two lobbies to invest in infrastructure that can enhance their lobbying efforts. This is meant to capture the long-term effects of organizing industry associations and think-tanks, of investing in policy development specialists, in relationships and other human and physical capital that can help shape public debates to a lobby's advantage. We show how the rich enhance their advantage even more when investments in lobbying infrastructure are allowed.

Our work is related to Acemoglu and Robinson (2008) who have shown how elites can significantly influence outcomes away from what is in the interest of the majority of the population and even how these elites can "capture" democracy. Their model, however, does not identify a specific locus through which the elites' greater monetary resources influence political outcomes. We provide a specific set of mechanisms through persuasive lobbying embedded within the spatial policy framework. More-

[1] In a somewhat different but interesting inquiry, Jordan and Meirowitz (2012) incorporate interest group competition to influence post-election policy at three levels of government: agenda-setting, legislature and agency implementation. They show how biases of these three levels of government (in favor of either group) might interact to generate endogenous delegation of decision making to the government agency responsible for executing the legislation.

over, in our setting, legislators and bureaucrats can be well-meaning but influenced through skillful persuasion.

In the existing literature, the role of interest groups in policy determination has been studied primarily at the electoral level via campaign contributions to competing candidates in an election contest. Research that takes this approach includes Baron (1994) and Grossman and Helpman (2001). At the post-election level, interest group influence has been similarly examined via policy-contingent contribution schedules as in Baron and Hirsch (2012), Helpman and Persson (2001) and Grossman and Helpman (2001). In these models, policies are indirectly influenced by interest groups via campaign donations that are contingent on the policy positions chosen by the candidates. These approaches therefore predominantly focus on the venal impact of interest groups on policy - their ability to indirectly influence policy by satisfying the politicians' need for money to remain electorally competitive. However, this approach does not take account of competing interest groups' active advocacy of specific policy positions at the legislative and agency level and their spending considerable amounts of resources towards producing supporting information.

We model the impact of evidence and arguments presented by the competing interest groups on the decision-maker via a persuasion contest function based on Skaperdas and Vaidya (2012). This approach is different from the existing literature on informational lobbying through cheap talk (Krishna and Morgan (2001) and Crawford and Sobel (1982)) and costly information acquisition (Potters and van Winden (1992) and Lohmann (1993)).[2] Our key point of difference is that unlike the literature on informational lobbying, we assume that the decision-maker chooses the policy position purely on the basis of Bayesian inference using evidence and arguments that are taken at face value, because that is what they are supposed to do (as in a court of law or what is considered to be in the "public record") or because of limitations on knowing the whole universe of interactions. Some empirical evidence on behavior of specific audiences that are targets of persuasion seems to support such a view.[3] Similarly, Kwak (2013) alludes to the importance of such "cultural capture"

[2]See Grossman and Helpman (2001), chapters 4 and 5 for an excellent discussion of this literature.
[3]See DellaVigna and Gentzkow (2010), Malmendier and Shanthikumar (2007), De Franco et al.

by lobbyists of agency decision making where the industry to be regulated exerts an intellectual influence over the agency to dilute the regulation in its favor.

Our work is also related to the literature on policy determination through electoral competition. The early contributions by Wittman (1983) and Calvert (1985), emphasized policy convergence. These approaches take voter preferences as exogenous, and argue that any platform divergence from the median positions on issues salient to the voters will be limited and incremental. More recent approaches, such as Ashworth and Mesquita (2009), Balart, Casas and Troumpounis (2022) and Zakharov (2009) allow for voter preferences to be influenced through resource expenditures, either because of a voter concern for valence or due to existence of impressionable voters. In these papers, competing politicians first choose policy positions and then spend resources towards election campaigning to win voter support. Despite office motivated politicians, there is policy divergence and the incentive to differentiate arises out of a desire to reduce the intensity of costly resource competition. [4] In our setting, the competing lobby groups are policy motivated. We find that policy extremism can persist despite intensifying resource competition and the presence of a bias that confers an advantage to the lobby group adopting a more central position. Further, in contrast to these papers, the chosen policy positions influence win probabilities differently in our paper by impacting the degree of asymmetry between the candidates.

This paper is most closely related to Epstein and Nitzan (2004) where two competing lobbies choose their policy positions endogenously and compete by spending resources, but without the penalty we allow for moving further away from the majority position.[5] They find that such policy competition leads to policy non-convergence

(2007) and Cain et. al. (2005). See also Jacobs et. al. (2021) for the biases that emerge in economic reporting.

[4]In an alternative approach, Carrilo and Castanheira (2008) show that candidates may deviate from the median in an attempt to provide a credible signal about their quality to voters who also care about valence.

[5]Formally, our paper is also related to Hirsch and Shotts (2015) and Munster (2006) who - instead of using probablistic contests - examine all-pay auctions (or perfectly discriminating contests). A crucial substantive difference of Hirsch and Schotts (2015) from our approach is that the efforts of their policy developers - formally equivalent to our lobbyists - are productive in the sense that they improve the quality of the final outcome. Moreover, their results, because of the mixed-strategy

which we show to persist even with the penalty we allow in our model. However, contrary to Epstein and Nitzan, we find that the competing lobbies need not exhibit strategic restraint under plausible alternative single-peaked preferences of the lobbies. In our setting both interest groups can advocate their most-preferred positions. We find that only a sufficiently strong bias in decision making in favor of one of the lobbies induces the disadvantaged group to adopt a moderate position that is closer to the majority position. Whereas Epstein and Nitzan assume that the derivative around a lobby's most preferred position is always zero, in our specification moving away from one's most preferred position induces a loss; as we will discuss further this is apparently the key reason for the lower strategic restraint in our setting.

We begin in Section 2 with an overview of how lobbying and influence take place in the three branches of government and in the public sphere; the discussion is meant to establish the empirical relevance of the model that follows. We then develop in Section 3 the basic framework, show that the lobbies will not advocate the same policy position, and then using specific functional forms, characterize equilibria with and without the possibility of compromise. In Section 4 we develop the "rich-and-poor" model that allows for lobbying infrastructure investments. Section 5 concludes.

2 Locating Lobbying and Influence

While what we call the *majority* position constrains the actions of the lobbies that we examine, we focus on the effect of influence and lobbying and the ways the position that eventually prevails differs from the majority position. Before introducing the model, we discuss the different levels of politics in which persuasive lobbying and influence matter. They include lobbying the three traditional branches of government to influence public debates.

Lobbying the executive branch of government

equilibria of their model (due their employment of the all-pay auction) are not directly comparable with those of our model. Munster (2005) finds complete convergence of policies and zero lobbying in the second stage of the game.

Presidents, Prime Ministers, Cabinet secretaries or ministers, officials of government agencies, and the agencies themselves all have formal procedures for lobbying as well as informal avenues for interested parties to lobby and provide information to the executive branch of government. Laws passed by legislatures are typically incomplete contracts that still require further specification and clarification. Governments and their agencies can often have large discretion in interpreting the law. Interested parties that have access to informal contacts and formal procedures (such as allowing for written public submissions) can play a critical role in the implementation of a law.

An example of the extent to which laws can be modified is the Volcker Rule, part of the Dodd-Frank Act that was enacted in the US in July 2010. Named after the former Federal Reserve Chair, the Volcker Rule called primarily for the separation of trading by financial institutions on their own account (typically called "proprietary" trading) from trading on behalf of their customers. After the enactment of the legislation, the input provided to the Securities and Exchange Commission (SEC) and other concerned regulatory agencies came overwhelmingly from financial institutions. The rule first came into effect in July 2015, five years after the law was enacted, but there have been many exemptions and continual modifications made since then. The current version of the rule runs 430 pages and became effective in July 2020 (Securities and Exchange Commission, 2020).

The length and complexity of the law's interpretation runs the risk that it is very far from - and to an extent nullifies - the intent of the original legislation. As chair Volcker commented: "I'd write a much simpler bill. I'd love to see a four-page bill that bans proprietary trading and makes the board and chief executive responsible for compliance. And I'd have strong regulators. If the banks didn't comply with the spirit of the bill, they'd go after them" (Stewart, 2011).

The Dodd-Frank Act was a reaction to the type of regulation and its enforcement that precipitated the Great Financial Crisis of 2008. As Johnson and Kwak (2010) and others have argued, part of the problem was that many regulators adopted the regulatory worldview of their regulatees in what can be called "cognitive" capture. Similarly, Kwak (2013) alludes to the importance of "cultural" capture by lobbyists of agency decision making where the industry to be regulated exerts an intellectual influence over the agency to dilute the regulation in its favor. To quote Carpenter (2013), who studies interest group influence on FDA (Food and Drug Administration of USA), "Perhaps the most plausible mechanism for capture to have occurred is that of cultural capture...in so far as some of the FDA's most long-serving members (Robert Temple, Janet Woodcock, John Jenkins) have become, in recent years, more receptive to some industry arguments about [the trade-off between] drug innovation and the regulation of safety..the political organizations of the global pharmaceuticals industry have come to shape the conversation of how drugs ought to be regulated."[6]

The main point we are trying to illustrate is that the implementation of a law can be different from the intent of the original legislation as the intervening bureaucratic process is complex, subject to the formal and informal persuasive influence of powerful actors, and government officials can favor some actors without themselves being corrupt or consciously aware of actually granting a favor.

Lobbying the legislative branch of government

The processes for lobbying legislators have similarities to those for the executive branch. Lobbyists can provide information, suggest legislation, and even draft bills through formal and informal connections with legislators and their staffers. Legislators and legislative staff - the latter often poorly paid in the US - can sometimes expect to be hired by the lobbying and law firms that are directly involved in lobbying them or by corporations that might have directly benefited from the laws that they have helped pass due to the so-called "revolving" door (see Blanes i Vidal et. al., 2012, for a clever method of assessing the monetary revolving-door rewards of staffers of key US Congressional Committees). The revolving door also holds for high officials of the executive branch and the military.

Another reality for members of legislatures and political parties is that they need

[6] Carpenter (2013) summarizes Jim Dickinson's view on FDA capture as follows: "In January 2010, Jim Dickinson, editor of FDA Webview and a long-time columnist and consultant on matters in the pharmaceutical sphere, remarked that the FDA was more "pro-industry" than at any time in the previous 35 years, and that new Obama Administration appointees Margaret Hamburg (Commissioner) and Joshua Sharfstein (then Principal Deputy Commissioner) could do little to change this fact, because the mechanism was embedded in agency culture."

funding to finance their campaigns and other political activities. The average member of the US Congress spends between thirty and seventy percent of their time fundraising (Ferguson, 2013). In other Western countries with parliamentary systems, political parties may receive a larger share of political contributions than they do in the US. The extent of influence that funders can have varies across countries (McMenamin, 2013), and it can be disputed what exactly the political contributors receive in return. At a minimum, large contributors benefit from personal access to legislators. This "access" itself - the ability to have one's voice heard, even if not heeded - is relevant in winnowing down the range of available voices. This increases the likelihood that contributor's preferences will eventually be incorporated into legislation even though legislators and their staff can be true believers in their intent to make impartial decisions on what bills to introduce and how to vote. As Page and Gilens note, "In 2012, for example, a tiny sliver of the US population - just *one-tenth of one-tenth of 1 percent* of Americans - provided *almost half* of all the money spent on Federal elections" (Page and Gilens, 2017, p.7, emphasis in the original).[7]

Litigation and Legal Coding

Litigation is expensive and money can be an important factor in court verdicts. Beyond the possible distributional effects of a particular verdict on the litigants themselves, some verdicts can have long-lasting effects that set landmark precedents in English common law based systems. That is, law can be made through litigation. Sometimes not even much litigation is needed to essentially create a new law according to Pistor's (2019) insightful analysis. The proliferation of financial instruments and tax-haven-based trusts over the past few decades and their legal acceptance (first in New York and London, and then in much of the rest of the world) were the result of "legal coding" by expert lawyers. Even though these instruments, pioneered in Wall Street or the City of London, created enormous regulatory and legal issues (and, among other things, precipitated the Great Financial Crisis of 2008), they became

[7] Ferguson et. al. (2020, p.2) found that "For every $100,000 that Democratic representatives [who had already voted for the Dodd-Frank Act] received from finance, the odds they would break with their party's majority support for the Dodd-Frank legislation increased by 13.9 percent. Democratic representatives who voted in favor of finance often received $200,000–$300,000 from that sector, which raised the odds of switching by 25–40 percent."

essentially ratified by both courts and governments, as no alternatives to them were perceived to exist.

Legal coding by expert lawyers that then becomes law extends beyond finance proper:

"Global capitalism as we know it ... is built around two domestic legal systems, the laws of England and those of New York State, complemented by a few international treaties, and an extensive network of bilateral trade and investments regimes, which themselves are centered around a handful of advanced economies." (Pistor, 2019, 132)

Along the same lines, for nearly two centuries the nature and extent of legal personhood for corporations has been litigated in US courts, with the 2010 landmark case of *Citizens United vs Federal Election Commission* providing "free speech" rights to corporate entities (and their right for unlimited political campaigning). Klumpp et. al. (2016) have found that the Citizens United ruling has had the effect of increasing Republicans' election probabilities in state house races by about 4 percentage points overall and 10 or more percentage points in several states.

Therefore, resources expended on litigation and legal coding can have important effects on politics and the direction that economic and other policies take.

Overall, we see that lobbying and money influence decision-making on policy formation and implementation in all three branches of government. This can lead to the implemented policy deviating systematically from what is favored by the majority of citizens. Money matters in different ways: in obtaining the right to be heard, either through campaign contributions or hiring firms that specialize in gaining such access; in hiring the professionals who will make the arguments to politicians, judges, or government officials; and in having access to an infrastructure of policy think tanks and professionals who specialize in providing off-the-shelf arguments that can be refined and adapted by the actual lobbyists. Whereas venality and corruption could be a factor in how government officials, legislators, and even judges make their decisions, there is no need to invoke them. Persuasion, the weighing of evidence in view of prevailing attitudes and constraints could well explain the deviations as we will show in the remainder of this paper.

3 The basic framework

Two interest groups, each denoted by i =A, B, have preferences $V_i(t)$ over a one-dimensional policy variable t that are single-peaked at $\hat{t}_i$ (which denotes group i's "ideal" position) so that the further away is t from a group's ideal position, the lower is the group's utility. For ease of exposition we normalize the most-preferred positions at $\hat{t}_A = 0$ and $\hat{t}_B = 1$, with t taking values in the interval $[0, 1]$. Each interest group can attempt to influence the implemented level of t by costly persuasive lobbying to a decision-maker. The decision-maker can be: a pivotal member in the relevant legislative body, a judge, or an agency head responsible for design and implementation of the policy. We assume that the decision-maker is predisposed towards implementing the *majority* position $\widetilde{t}$ unless she is persuaded to do otherwise in light of the arguments presented by the interest groups.

The sequence of moves in our basic framework is as follows:

1. The two lobbies simultaneously choose the policy positions they will advocate, t_A and t_B.

2. The two lobbies simultaneously choose lobbying expenditures, R_A and R_B.

3. The probabilities of winning for the two lobbies are then determined by (1) and one of the advocated positions will be implemented.

If $R_A = R_B = 0$, so that neither party invests any resources towards arguing for its advocated position, we assume that the majority position prevails and neither group wins. The payoff to each group is then $V_i(\widetilde{t})$. However, as long as R_i or $R_j \neq 0$, the probability of winning for group $i = A, B$ is given by the persuasion function P_i:

$$P_i(t_i, t_j; R_i, R_j) = \frac{\lambda_i f_i(R_i)}{\lambda_i f_i(R_i) + \lambda_j f_j(R_j)} \text{for } i, j = A, B, i \neq j \tag{1}$$

The win-probability specification in (1) is motivated by Skaperdas and Vaidya (2012), who derive such a function in a persuasion setting as an outcome of Bayesian reasoning by a decision-maker who is affected by the evidence presented by either side via their assessment of the likelihood ratio of the correctness of each side's position and where evidence production is deterministically and positively related to resources invested by either party.[8] The impact of resources on evidence is captured through $f_i(R_i)$ which is differentiable in R_i with $f_i(0) = 0$, $f_i'(R_i) > 0$, and $f_i''(R_i) \leq 0$ for $R_i \geq 0$ for $i = A, B$ - these are typical assumptions in the contests literature (see Konrad, 2009, for an overview and Cornes and Hartley, 2005, for an analysis of equilibria in common contest games).

The terms $\lambda_i \in (0, \infty)$, $i = A, B$ are strictly positive and capture the inherent biases in the decision-making that modulate the effect of resources invested. Hence it is feasible for $P_A > P_B$ even when A and B's resource investments are such that $f_A(R_A) = f_B(R_B)$, provided that the decision-maker has a bias in favor of A (and $\lambda_A > \lambda_B$). These bias parameters can be affected by the advocated positions of the two lobbies as well as an inherent bias the decision-maker may have in favor of one of the lobbies. We postulate $\lambda_A = \lambda_A(\lambda, \tilde{t}, t_A)$, $\lambda \in (0, \infty)$ and $\lambda_B = \lambda_B(\tilde{t}, t_B)$. $\tilde{t}$ and t_i impact λ_i as follows: λ_i attains its maximum at $t_i = \tilde{t}$. The farther is t_i from $\tilde{t}$, the lower is λ_i. This captures a bias in the decision-making towards the majority position $\tilde{t}$ as alluded to earlier. An example of these functions that we will employ in most of the paper is

$$\lambda_A(t_A) = \frac{\lambda}{\mid t_A - \tilde{t} \mid +1}, \lambda > 0 \qquad (2)$$
$$\lambda_B(t_B) = \frac{1}{\mid t_B - \tilde{t} \mid +1}$$

The parameter $\lambda \in (0, \infty)$ captures a cognitive or cultural bias in the decision making towards (or against) the arguments presented by A relative to B that is independent of the advocated positions. We assume that λ_A is an increasing function of λ so that a sufficiently large (or low) λ can represent a cognitive bias in favor of

[8] The holding of a public office creates an imperative for decisions to be justifiable in light of the information made available in public arena which in turn is produced by the advocates of the competing positions. A regulatory agency also needs to consider whether its interpretation can withstand a legal challenge in a court which by design must reach a verdict basis the evidence presented to it by the adversarial parties.

(or against) group A. Such a bias could be due to the party affiliation or ideological pre-disposition of the relevant decision maker in the legislative context. When the decision maker is an agency head, such a bias could be due to “cultural capture”, where factors such as group identification, status or relationship networks can lead to the regulator’s decision making being swayed towards one of the interest groups. The bias in agency decision making may also arise via the appointments made within the bureau and the mandatory procedures an agency is required to follow in shaping the regulation. These can open agency decision-making to scrutiny by favored constituencies and “stack the deck” in the direction intended by the legislature or the president as noted in McCubbins, Noll and Weingast (1987).

The marginal cost of R_i is $c_i > 0$ and is allowed to vary across the two lobbies. The cost can be thought of as inversely related to the lobbying ”infrastructure” of each lobby, which can consist of the office and internet presence of the lobby, the accumulated human capital and connections of its employees and ready access to research and other related investments that have been inherited from the past. We shall postpone examining the effect and determination of such lobbying infrastructure until the next section of the paper.

These properties of the lobbying process taken together imply that the win probability of each group is influenced by following factors: (i) the location of the majority position; (ii) the location of each group’s advocated position relative to the majority position; (iii) the amount of resources invested by the competing groups towards evidence production; and (iv) the degree of bias exhibited by the decision-maker.

We will later allow for a “compromise” position to be implemented based on bargaining given the probabilities of winning established from the choices made in stages 1 and 2. That will not change the equilibrium advocated positions and lobbying expenditures but, partly for expositional purposes, we will be primarily employing the probabilistic interpretation of the model.

We next present the equilibrium characteristics of the model and examine how the above discussed parameters influence the choice of policy positions and their relationship with the majority position.

3.1 Non-convergence in advocated policies

To explore the characteristics of the subgame perfect equilibria, we start with the second stage choices of (R_A, R_B) where (t_A, t_B) are given. As long as at least one group incurs a positive expenditure, the expected payoff functions of the two groups are as follows:

$$\pi_A(R_A, R_B; t_A, t_B) = \frac{\lambda_A f_A(R_A)}{\lambda_A f_A(R_A)+\lambda_B f_B(R_B)} V_A(t_A) + \frac{\lambda_B f_B(R_B)}{\lambda_A f_A(R_A)+\lambda_B f_B(R_B)} V_A(t_B) - c_A R_A$$
$$\pi_B(R_A, R_B; t_A, t_B) = \frac{\lambda_A f_A(R_A)}{\lambda_A f_A(R_A)+\lambda_B f_B(R_B)} V_B(t_A) + \frac{\lambda_B f_B(R_B)}{\lambda_A f_A(R_A)+\lambda_B f_B(R_B)} V_B(t_B) - c_B R_B$$

By suitably re-arranging these payoffs and letting $\Delta V_i \equiv V_i(t_i) - V_i(t_j)$, group i's expected payoff takes the following form $(i, j = A, B, i \neq j)$:

$$\pi_i(R_i, R_j; t_i, t_j) = \frac{\lambda_i f_i(R_i)}{\lambda_i f_i(R_i) + \lambda_j f_j(R_j)}[\Delta V_i] + V_i(t_j) - c_i R_i \qquad (3)$$

By inspecting this expected payoff, it is apparent that a necessary condition for both groups to prefer to expend positively in the contest is $\Delta V_i > 0$ for $i = A, B$ which also implies that $t_A < t_B$. When group i does not incur any expenditure in the second stage, the policy outcome is either t_j when group $j \neq i$ does invest resources or it is the majority position $\widetilde{t}$ when neither group invests any resources.

As shown in Proposition 1, the two groups will in general choose to advocate for different positions - which will also be typically different from the majority position - and the persuasion contest will be non-trivial in equilibrium. This result shows the policy non-convergence finding of Epstein and Nitzan (2004, Theorem 2) to hold even when the win-probability persuasion functions provide an incentive to both groups to choose positions closer to $\widetilde{t}$ to boost their individual chances of winning.

Proposition 1: $t_A^* = t_B^*$ *is never part of a Subgame Perfect Nash equilibrium.*[9]

How far from each other will the two lobbies choose their advocated positions, and how close will these be to their ideal positions? How will the preferences and various biases influence the choices of advocated positions and the chance each of them has to succeed? In what follows, we will be answering questions such as these.

[9] The proofs of all the propositions and corollaries in the paper are provided in the Appendix.

To facilitate closed-form solutions we will use several specific functional forms in our model.

First, we assume that the evidence production functions $f_i(R_i), i = A, B$ take the following linear form for the rest of the analysis:

$$f_i(R_i) = R_i \tag{4}$$

From (3) and (4), the first order conditions involving choice of resources along an interior optimum in the second stage are:

$$\frac{\partial \pi_A}{\partial R_A} = \frac{\lambda_A \lambda_B R_B \Delta V_A}{(\lambda_A R_A + \lambda_B R_B)^2} - c_A = 0 \tag{5}$$

$$\frac{\partial \pi_B}{\partial R_B} = \frac{\lambda_A \lambda_B R_A \Delta V_B}{(\lambda_A R_A + \lambda_B R_B)^2} - c_B = 0 \tag{6}$$

Accordingly, the equilibrium choices of resources can be shown to be the following:

$$\begin{aligned} R_A^*(t_A, t_B) = & \quad \frac{\lambda_A \lambda_B c_B \Delta V_B \Delta V_A^2}{(\lambda_A c_B \Delta V_A + \lambda_B c_A \Delta V_B)^2} \\ R_B^*(t_A, t_B) = & \quad \frac{\lambda_A \lambda_B c_A \Delta V_B^2 \Delta V_A}{(\lambda_A c_B \Delta V_A + \lambda_B c_A \Delta V_B)^2} \end{aligned} \tag{7}$$

By substituting (4) and then (7) into (1), we observe that the first stage probability of winning for group A, for any given combination of positions (t_A, t_B), such that $\Delta V_i > 0$ for $i = A, B$ is:

$$P_A(t_A, t_B) = \frac{\lambda_A c_B \Delta V_A}{\lambda_A c_B \Delta V_A + \lambda_B c_A \Delta V_B} = \frac{\lambda_A \Delta V_A}{\lambda_A \Delta V_A + \lambda_B c \Delta V_B} \text{where } c \equiv \frac{c_A}{c_B} \tag{8}$$

As seen in (8), a group's first stage winning probability depends on the bias parameters (λ_i), the cost parameters (c_i), and each group's stake from persuading the audience (ΔV_i), $i = A, B$. It turns out that in all of our subsequent analysis the ratio of marginal costs $c \equiv \frac{c_A}{c_B}$ is conveniently the sole cost parameter that matters, so it is the one we will be using from now on. By substituting (7) into the second stage expected payoffs, we obtain the first stage expected payoffs for any given combination of positions (t_A, t_B), $(t_A < t_B)$ such that $\Delta V_i > 0$ for $i = A, B$:

$$\begin{array}{l} \pi_A(t_A, t_B) = \frac{\lambda_A^2 \Delta V_A^3}{(\lambda_A \Delta V_A + \lambda_B c \Delta V_B)^2} + V_A(t_B) = P_A(t_A, t_B)^2 \Delta V_A + V_A(t_B) \\ \pi_B(t_A, t_B) = \frac{\lambda_B^2 \Delta V_B^3}{(\lambda_A \Delta V_A + \lambda_B c \Delta V_B)^2} + V_B(t_A) = P_B(t_A, t_B)^2 \Delta V_B + V_B(t_A) \end{array} \tag{9}$$

The equilibrium choices of policy positions t_A^*, t_B^* are therefore determined via simultaneous maximization of π_i as given by (9). The policy choices made by the interest groups under these specifications will naturally be influenced by how they relate to the majority position $\widetilde{t}$. For the rest of the analysis, we examine the general case of $0 < \widetilde{t} < 1$.[10]

We further assume that the policy preferences, $V_i(t)$ take the following form

$$\begin{aligned} V_A(t) = & \quad -a \mid t - 0 \mid = -at, \text{ where } a > 0 \\ V_B(t) = & \quad -b \mid t - 1 \mid = -b(1-t) \text{ where } b > 0 \end{aligned} \tag{10}$$

while the bias parameters take the form in (2) (where $\lambda_A(t_A) = \frac{\lambda}{|t_A - \tilde{t}| + 1}$ and $\lambda_B(t_B) = \frac{1}{|t_B - \tilde{t}| + 1}$). The terms $\frac{1}{|t_i - \tilde{t}| + 1}$ $(i = A, B)$ in (2) create a bias in favor of the majority position $\tilde{t}$. As already discussed, the parameter $\lambda \in (0, \infty)$ captures bias in the decision making towards the arguments presented by A (relative to B) if $\lambda > 1$. On the other hand $\lambda < 1$ indicates a bias in favor of B.

[10] We discuss, in the Supplementary Appendix, the cases in which one of the ideal points coincides with the majority position ($\widetilde{t} = 0$ or $\widetilde{t} = 1$).

3.2 Does complete polarization ever occur?

We now explore the feasibility of complete polarization occurring in the policy debate with the two lobbies choosing their ideal points as their advocated positions in equilibrium.

From (10) it follows that:

$$\Delta V_A = a(t_B - t_A) \tag{11}$$

$$\Delta V_B = b(t_B - t_A) \tag{12}$$

(11) and (12) act as "prizes" in the first stage expected payoffs of groups A and B respectively, as specified in (9), and hence naturally play an important role in influencing the first stage choice of policy positions. By themselves, these tend to induce the groups to select their respective ideal positions in order to maximize their expected payoffs, as while ΔV_A decreases in t_A, ΔV_B is increasing in t_B. Note that this differs from Epstein and Nitzan's (2002, p.206) assumption that the derivatives of the respective "prizes" are zero, which is the key to their local result (around the ideal points of lobby) of "strategic restraint."

Hence, in our setting, any tendency to move away from the ideal position on either group's part must be motivated by the impact of t_i on P_i. Now recall that (from (8)),

$$P_A = \frac{\lambda_A \Delta V_A}{\lambda_A \Delta V_A + \lambda_B c \Delta V_B} = \frac{\frac{\lambda}{\left|\widetilde{t}-t_A\right|+1}\Delta V_A}{\frac{\lambda}{\left|\widetilde{t}-t_A\right|+1}\Delta V_A + \frac{c}{\left|\widetilde{t}-t_B\right|+1}\Delta V_B}$$

Hence from (11) and (12) it follows from above that:

$$P_A = \frac{\frac{a\lambda}{\left|\widetilde{t}-t_A\right|+1}}{\frac{a\lambda}{\left|\widetilde{t}-t_A\right|+1} + \frac{bc}{\left|\widetilde{t}-t_B\right|+1}} = \frac{\frac{\frac{a}{bc}\lambda}{\left|\widetilde{t}-t_A\right|+1}}{\frac{\frac{a}{bc}\lambda}{\left|\widetilde{t}-t_A\right|+1} + \frac{1}{\left|\widetilde{t}-t_B\right|+1}} \tag{13}$$

The effect of t_A on P_A is captured by the term $\left|\widetilde{t} - t_A\right| + 1$. The farther is t_A from the majority position $\widetilde{t}$, the larger is this term and the lower is P_A. Hence this creates a countervailing "centripetal" force inducing A to choose a position towards $\widetilde{t}$. From this, it also follows that A is strictly better off with $t_A = \widetilde{t}$ relative to $t_A > \widetilde{t}$ as both ΔV_A and P_A are higher when the former condition holds relative to the latter. Hence with $0 < \widetilde{t} < 1$, in any equilibrium, $0 \leq t_A^* \leq \widetilde{t}$. Exactly analogously, ΔV_B induces B to move towards 1 while P_B provides the countervailing force drawing B towards $\widetilde{t}$ so that in any equilibrium $1 \geq t_B^* \geq \widetilde{t}$. As revealed by the second expression in (13), the level of P_A is also influenced by $\frac{a}{bc}\lambda$. The term $\frac{a}{bc}$ captures both group A's stake relative to B ($\frac{a}{b}$) as well as its marginal cost of mobilizing resources relative to B (c). A higher relative stake $\frac{a}{b}$ confers an advantage to A, whereas a higher relative cost c has the opposite effect. Overall, the term $\frac{a}{bc}$ appears multiplicatively with λ and therefore influences P_A in the same way as the bias parameter λ. Accordingly, let $\overline{\lambda}$ denote $\frac{a}{bc}\lambda$ and reflect the net effect of stake and cost asymmetries and cognitive bias that may confer an advantage to one of the groups in winning the contest which is independent of the chosen positions of the two groups. If $\overline{\lambda} > 1$, then this advantage is with A who therefore has less incentive to adjust t_A towards $\widetilde{t}$ to favorably influence P_A. If $\overline{\lambda} < 1$, then the advantage is with B.

Keeping in mind the competing influences of ΔV_i and P_i, we examine the choice of policy positions by the groups by substituting (11), (12) and (13) in (9) to obtain the first stage payoffs as functions of policy positions as shown below:

$$\pi_A(t_A, t_B) = a\left\{\frac{\overline{\lambda}^2(t_B - \widetilde{t} + 1)^2(t_B - t_A)}{[\overline{\lambda}(t_B - \widetilde{t} + 1) + (\widetilde{t} - t_A + 1)]^2} - t_B\right\} \tag{14}$$

$$\pi_B(t_A, t_B) = b\left\{\frac{(\widetilde{t} - t_A + 1)^2(t_B - t_A)}{[\overline{\lambda}(t_B - \widetilde{t} + 1) + (\widetilde{t} - t_A + 1)]^2} - (1 - t_A)\right\} \tag{15}$$

Suppose now that $\overline{\lambda} = 1$. In this case, it is apparent from (13) that P_A is influenced purely by the relative distances of t_A and t_B from $\widetilde{t}$ and there is no a priori bias in favor or against a specific group. The first stage choice of policy positions for this case are given by Proposition 2.

Proposition 2: *Under* (2), (10)*,* $0 < \widetilde{t} < 1$ *and* $\overline{\lambda} = 1$*, both groups prefer to choose their ideal positions so that* $t_A^* = 0$ *and* $t_B^* = 1$ *in the Subgame Perfect Nash equilibrium.*

Proposition 2 illustrates a circumstance of choice of policy positions when there is cost symmetry and no inherent bias in favor of a group. It shows that in this case, the advocated positions are at the extreme ends of the policy spectrum with each group advocating for their ideal position. This implies that no group exhibits any strategic restraint and the level of the majority position does not have any effect on the choice of positions by the groups. Hence in equilibrium, one of the extreme positions is implemented.[11] Indeed, we can expect the above result to continue to hold for A as long as $\overline{\lambda} \geq 1$ as this makes A's position even stronger. B may consider moving away from 1 if the bias in favor of A is sufficiently strong. Similarly, if $\overline{\lambda} \leq 1$ we can expect B to continue to choose $t_B^* = 1$. A may move away from 0 if the bias in favor of B is sufficiently strong. We examine this more closely in the next section.

3.3 The effect of biases on choice of positions

To study the effect of bias on choice of positions, we allow for $\overline{\lambda} \neq 1$ and re-evaluate $\frac{\partial \pi_A}{\partial t_A}$ and $\frac{\partial \pi_B}{\partial t_B}$. This allows us to establish Proposition 3.

Proposition 3: *Suppose that* (2), (10), $0 < \widetilde{t} < 1$ *and* $\overline{\lambda} \neq 1$*.*

(i) *If* $\overline{\lambda} > 1$*, then it is always the case that* A *chooses its ideal position (*$t_A^* = 0$*).* B *also chooses its ideal position* $t_B^* = 1$ *as long as* $\overline{\lambda} \leq \frac{1+\widetilde{t}}{\widetilde{t}}$*. When* $\overline{\lambda} > \frac{1+\widetilde{t}}{\widetilde{t}}$ *and* $\widetilde{t} \in (0, \frac{1}{2}]$ *or when* $\overline{\lambda} \in \left(\frac{1+\widetilde{t}}{\widetilde{t}}, \frac{1+\widetilde{t}}{2\widetilde{t}-1}\right)$ *and* $\widetilde{t} \in (\frac{1}{2}, 1)$*,* B*'s choice is given by* $\widetilde{t} < t_B^* = (1 - \widetilde{t}) + \frac{(1+\widetilde{t})}{\overline{\lambda}} < 1$*. When* $\widetilde{t} \in (\frac{1}{2}, 1)$ *and* $\overline{\lambda} \geq \frac{1+\widetilde{t}}{2\widetilde{t}-1}$ *we have* $t_B^* = \widetilde{t}$*.*

(ii) *If* $\overline{\lambda} < 1$ *then it is always the case that* B *chooses its ideal position (*$t_B^* = 1$*).* A *also chooses its ideal position (*$t_A^* = 0$*) as long as* $\frac{1-\widetilde{t}}{2-\widetilde{t}} \leq \overline{\lambda} < 1$*. When* $\overline{\lambda} < \frac{1-\widetilde{t}}{2-\widetilde{t}}$

[11] As alluded to earlier, Proposition 2 also suggests that Epstein and Nitzan's (2004) finding of competing interest groups exhibiting strategic restraint (by not taking extreme positions) may not hold under plausible single-peaked preferences as in (10). This is because the marginal cost of deviating from the ideal policy position is not zero for such preferences unlike the utility functions examined by Epstein and Nitzan (2004).

and $\widetilde{t} \in [\frac{1}{2}, 1)$ *or when* $\overline{\lambda} \in \left(\frac{(1-2\widetilde{t})}{(2-\widetilde{t})}, \frac{(1-\widetilde{t})}{(2-\widetilde{t})}\right)$ and $\widetilde{t} \in (0, \frac{1}{2})$, *A's choice is given by* $0 < t_A^* = (1-\widetilde{t}) - (2-\widetilde{t})\overline{\lambda} < \widetilde{t}$. *When* $\widetilde{t} \in (0, \frac{1}{2})$ *and* $\overline{\lambda} \leq \frac{(1-2\widetilde{t})}{(2-\widetilde{t})}$ *we have* $t_A^* = \widetilde{t}$.

Proposition 3(i) allows us to make the following observations. Intuitively, starting from symmetry ($\overline{\lambda} = 1$) with $t_A^* = 0$ and $t_B^* = 1$, if there is an increase in $\overline{\lambda}$ that creates a bias in favor of A, it is only natural for A to adhere to its ideal position of 0. This suggests that the emergence of a favorable bias can only reinforce A's choice of ideal position. How B reacts to this depends both on the extent of the bias and the proximity of the majority position to its ideal point. B moves away from $t_B^* = 1$ and takes a more moderate stance when the bias in favor of A is sufficiently strong. As an illustration, consider the case when $\widetilde{t} = \frac{1}{2}$. Proposition 3(i) implies that in this case, B exhibits some strategic restraint when $\overline{\lambda} > 3$. Further, the greater is the bias, the more moderate is the position taken by B with t_B^* drawn towards $\widetilde{t}$. Indeed, for large values of $\overline{\lambda}$, $t_B^* \approx \frac{1}{2}$. Also notice that, in the range $\widetilde{t} \in (0, \frac{1}{2}]$, as $\widetilde{t}$ increases, the threshold level of $\overline{\lambda}$ that induces B to give up its ideal position falls. When the majority position is closer to B's ideal point (that is, the range $\widetilde{t} \in (\frac{1}{2}, 1)$), B is induced to select exactly the majority position when $\overline{\lambda}$ is large enough. Proposition 3 (ii) is effectively the mirror image of Proposition 3(i) and has similar implications for A's choice of policy position when there is a bias in favor of B. Hence Proposition 3 suggests that the existence of a sufficiently strong bias in favor of one of the groups can induce the other group to adjust its position closer to the majority position.[12]

A bias in favor of one of the groups can occur due to various reasons. In the context of regulatory decisions, and in the absence of any stake/cost asymmetries ($\frac{a}{bc} = 1$), this can arise due to cultural capture that creates a cognitive bias and tilts decision making within the agency sufficiently in favor of one of the groups ($\lambda > 1$ or $\lambda < 1$). In such circumstances, the disadvantaged group is induced to adopt a moderate stance when the bias against it is strong enough. Such a bias can also arise due to asymmetric stakes of the contestants as is the case when $\frac{a}{b} \neq 1$ while

[12] This tendency breaks down when one side's ideal point is also the majority position. In this case both groups pursue their most favored positions regardless of the level of bias. A proof is provided in the Supplementary Appendix.

$\lambda = c = 1$. An example of such a stake asymmetry (pursued in more detail in the next section) could be differences in sizes of the competing groups.

3.4 Choices of positions under the option of a compromise

We now show how the probabilistic interpretation of our model is essentially equivalent to a deterministic interpretation, whereby the final implemented policy position is a "compromise" one arrived at through bargaining. The chosen positions and the lobbying expenditures create disagreement (or threat) outcomes for each lobby that are used in bargaining to pull the eventual compromise position to their side.

As before, both groups choose their policy positions simultaneously in the first stage and then they choose their resource expenditures. Following these choices, both groups can choose to either agree on a compromise position (denoted by $\bar{t}$) or compete for their chosen positions. Hence, under a compromise agreement, each group's payoff is given by:

$$V_i(\bar{t}) - c_i R_i,\ i = A, B \tag{16}$$

With this, any group i, will only consider the compromise option if $V_i(\bar{t}) - c_i R_i \geq P_i[\Delta V_i] + V_i(t_j) - c_i R_i$ for $i, j = A, B$, $i \neq j$. Hence, applying the Nash Bargaining solution (or any other symmetric bargaining solution), $\bar{t}$ is given by:

$$V_A(\bar{t}) - [P_A[\Delta V_A] + V_A(t_B)] = V_B(\bar{t}) - [P_B[\Delta V_B] + V_B(t_A)] \tag{17}$$

Using (10), and substituting (11), (12) into (17), we obtain:

$$\bar{t} = t_B - P_A(t_B - t_A) = P_A t_A + P_B t_B \tag{18}$$

which is the expected value of the advocated policy positions weighted by the probabilities of winning. By substituting (18) in (16), the payoff to the two groups' under compromise is given by:

$P_A[\Delta V_A] + V_A(t_B) - c_A R_A$

$P_B[\Delta V_B] + V_B(t_A) - c_B R_B$

Notice that the above payoffs are identical to the expected payoffs in a contest due to risk neutrality. Accordingly, we assume that the competing groups opt for compromise in this stage. More generally, as long as engaging in the contest would impose additional costs to each side or if preferences were risk averse, both would strictly prefer a compromise in this static framework. Since the payoffs under compromise are the same as those in the contest, the equilibrium choices of t_i and R_i, $i = A, B$ are also the same. However, the implemented position is $\bar{t}$ as given by (18). Hence propositions 2 and 3 have direct implications for the compromise position $\bar{t}$. For example, when Proposition 2 holds and we have full symmetry with $\overline{\lambda} = 1$ and $\widetilde{t} = \frac{1}{2}$, then $\bar{t} = P_B = \frac{1}{3}(1 + \widetilde{t}) = \frac{1}{2}$. Hence, in this case, there is no divergence between the compromise position and the majority position. However, when there is a bias in favor of A as in Proposition 3 (i), with $\overline{\lambda} > \frac{1+\widetilde{t}}{\widetilde{t}}$ with $\widetilde{t} \in (0, \frac{1}{2}]$, then $\bar{t} = t_B^* * P_B = \frac{1+\widetilde{t}}{2\overline{\lambda}} < \frac{\widetilde{t}}{2}$. This implies that the compromise position is considerably skewed in favour of A relative to the majority position. Further, as $\overline{\lambda}$ increases, the compromise position drifts even more in favour of A.

4 Investing in Lobbying Infrastructure by two Income Groups

In this section we extend our model in two different directions. First, we explicitly derive the preferences and the strategies of the two lobbies from an underlying tax-and-subsidy competition between two groups, representing two income classes: the "poor" (having low income and denoted as "l") and the "rich" (having high income and denoted as "h"). The flavor of the competition can be thought of as a simplification (but also an extension in other dimensions) of the Meltzer and Richard (1981) model.

Second, we introduce a stage of investing in a lobbying organization or infrastructure prior to the choice of the positions that each lobby will choose to advocate. We think of lobbying expenditures as having a fixed component ("capital") and a variable one ("labor"). The former can include the creation of think tanks and lobbying organizations with buildings, permanent staff, internet presence, or social connections to legislators and high-level civil servants. The variable component, which we have also examined up to this point, can include the expenditures on particular lobbying campaigns. As we will show, investments in infrastructure effectively reduce the marginal costs c_h and c_l and therefore impact their ratio c which, as we have seen already, is an important parameter in determining actual policy decisions.

Let the population share of the rich be n and that of the poor $1-n$, with corresponding gross incomes of y^h and y^l(with $y^h > y^l$). Following Meltzer and Richard (1981), the policy variable comprises of a uniform tax rate τ on income with the collected tax revenue fully given back as a subsidy to all members of the community equally. The subsidy to each individual is also the average tax collected per person: $\tau(ny^h + (1-n)y^l)$. Given this, the payoffs to the representative individuals in group h and group l for any tax rate τ are the net incomes as follows:

$$\begin{aligned} V_l(\tau) &= (1-\tau)y^l + \tau(ny^h + (1-n)y^l) = y^l + \tau n \Delta y \\ V_h(\tau) &= (1-\tau)y^h + \tau(ny^h + (1-n)y^l) = y^h - \tau(1-n)\Delta y \end{aligned}$$

where $\Delta y \equiv y^h - y^l$.

Note how the payoff of a poor individual is increasing in the tax rate whereas the payoff of a rich individual is decreasing in the tax rate. Therefore, the most preferred tax rate for the poor is 100% ($\widehat{\tau}_L = 1$) while that of the rich is 0% ($\widehat{\tau}_h = 0$). We suppose that $n < 1/2$ (i.e., there are fewer rich than poor individuals) and so the median voter's most preferred tax rate is also 100%. In Meltzer and Richard (1981), the most preferred tax rate by the median voter is less than 1 because a higher tax rate disincentivizes and reduces production. However in our case despite the absence of disincentive effects and the fact that the median voter prefers a 100% tax rate, we show that the eventual outcome is far more moderate than that due to the effect of lobbying. Obviously, allowing for disincentive effects as in Meltzer and Richard (1981) would further reduce the equilibrium tax rate.

Let τ_i, $i = l, h$ $(1 \geq \tau_l > \tau_h \geq 0)$ represent the tax positions advocated by the poor and rich respectively in the first stage. Given $V_l(\tau)$ and $V_h(\tau)$, the contestable "prize" that each side is lobbying over becomes

$$\begin{aligned} \Delta V_l &= (\tau_l - \tau_h)\, n\Delta y \\ \Delta V_h &= (\tau_l - \tau_h)\,(1-n)\Delta y \end{aligned} \tag{19}$$

The smaller is the share of the rich (that is, the smaller is n), the smaller is the contestable "prize" for the poor and the larger is the contestable "prize" for the rich. The reason for this is that the smaller the share of the rich is, the lower is the per person subsidy that the poor receive for a given difference in incomes between the two income classes (Δy). That is, the losses of the rich are more concentrated, the fewer they are whereas the gains for the poor are the more dispersed, the more of them there are; this is a key feature that drives much of the differential lobbying.[13]

Given (2), the bias parameters λ_i $(i = l, h)$ for the two sides are

$$\begin{aligned} \lambda_h(\tau_h) &= \tfrac{\lambda}{2-\tau_h} \\ \lambda_l(\tau_l) &= \tfrac{1}{2-\tau_l} \end{aligned} \tag{20}$$

Note how this formulation takes account of the fact that the median voter's position is $\tilde{\tau} = 1$. Therefore, if each side were to advocate for its most preferred position ($\widehat{\tau}_l = 1$ and $\widehat{\tau}_h = 0$), we would have $\lambda_h(0) = \frac{\lambda}{2}$ and $\lambda_l(1) = 1$, thus providing an inherent advantage to the poor (up to the point that $\lambda \leq 2$ and despite the rich having the "ear" of the decisionmaker in the range $1 < \lambda \leq 2$). This bias induces the following winning probability for the poor, P_l and the rich, P_h :

$$\begin{aligned} P_l &= \frac{\frac{1}{2-\tau_l}R_l}{\frac{1}{2-\tau_l}R_l + \frac{\lambda}{2-\tau_h}R_h} = \frac{(2-\tau_h)R_l}{(2-\tau_h)R_l + \lambda(2-\tau_l)R_h} \\ P_h &= \frac{(2-\tau_l)R_h}{(2-\tau_h)R_l + \lambda(2-\tau_l)R_h} \end{aligned} \tag{21}$$

However, one main difference with the formulation of the previous sections is that

[13] Our qualitative results would continue to hold if we were to allow for some collective group action as long as collective action is inversely related to the size of the group.

the $R_i s$ themselves are functions of two factors, of a fixed "capital" (K_i, which is invested first prior to the choice of the tax rate) and variable "labor" (L_i, expended in the end), so that $R_i = K_i^\alpha L_i$ ($i = l, h$ and $a \in (0, 1)$).[14]

The timing of moves is as follows.

1. The two lobbies, h and l, simultaneously choose lobbying infrastructure investments K_h and K_l.

2. Given the investments in stage 1, the two lobbies choose the tax policy positions they will advocate, τ_h and τ_l.

3. The variable lobbying expenditures, L_h and L_l, and the implied total lobbying efforts, R_h and R_l, are chosen. The probabilities of winning for the two lobbies are then determined.

4. The compromise tax rate is then determined by $\bar{\tau}^* = P_l^* \tau_l^* + (1 - P_l^*)\tau_h^*$, where "$*$" indicates the subgame perfect equilibrium probabilities of winning and advocated tax policy positions.

We begin our analysis with stage 3 in which the equilibrium lobbying expenditures are determined, given arbitrary choice of tax policies from stage 2 and infrastructure investments from stage 1. Note that with the infrastructure investments pre-determined from stage 1 and their costs sunk, the payoff function of the poor can be written as

$$\pi_l = \frac{(2 - \tau_h) K_l^a L_l}{(2 - \tau_h) K_l^a L_l + \lambda (2 - \tau_l) K_h^a L_h} (\tau_l - \tau_h) n \Delta y + y^l + \tau_h n \Delta y - \omega L_l$$

where $\omega > 0$ represents the price of "labor" L_l. With K_l fixed, choosing L_l automatically implies a choice of $R_l = K_l^a L_l$ and the variable cost ωL_l of stage 3 equals $\frac{\omega}{K_l^\alpha} R_l$. Note then that $\frac{\omega}{K_l^\alpha}$ effectively represents the marginal cost of the total lobbying effort R_l and a similar argument applies for R_h; note how a group's marginal

[14] See Arbatskaya and Miallon (2010) for an analysis of such "multi-activity" contests and Schaller and Skaperdas (2020) who use the same functional forms to study how bargaining and conflict affect investments.

cost is inversely related to its lobbying infrastructure investment. We shall therefore denote $c_i = \frac{\omega}{K_i^\alpha}$ (which is fixed in stage 3 by the choice of investments in stage 1).[15] Thus, we can write the payoffs functions of the two sides as of stage 3 as follows:

$$\pi_l = \frac{(2-\tau_h)R_l}{(2-\tau_h)R_l + \lambda(2-\tau_l)R_h}(\tau_l - \tau_h)\, n\Delta y + y^l + \tau_h n\Delta y - c_l R_l$$
$$\pi_h = \frac{\lambda(2-\tau_l)R_h}{(2-\tau_h)R_l + \lambda(2-\tau_l)R_h}(\tau_l - \tau_h)(1-n)\Delta y + y^h - \tau_l n\Delta y - c_h R_h$$

Our approach in the previous section remains useful in obtaining both the equilibrium lobbying expenditures and the equilibrium tax policy positions. Thus we can substitute the above stated expressions for contestable prizes and bias parameters into (7) to get the stage 3 resource expenditures of the two groups which are related in the following fashion:

$$\frac{R_h^*}{R_l^*} = \frac{1-n}{cn} \tag{22}$$

where $c \equiv \frac{c_h}{c_l}$. This relationship of the lobbying efforts indicates that the minority status of the rich ($n < \frac{1}{2}$) propels them to compete harder relative to the poor as their potential losses are more concentrated than the potential gains for the poor. If the rich also have a cost advantage (due to stronger organization or avoidance of free rider problems ($c < 1$)), which we will later show to be true under mild conditions, then this effect is further reinforced. It follows that the stage 3 win probability for the rich is:

$$P_h(\tau_h, \tau_l) = \frac{\lambda(1-n)(2-\tau_l)}{\lambda(1-n)(2-\tau_l) + cn(2-\tau_h)}$$

[15] More formally, consider the restricted cost minimization problem faced by each lobby in the second stage when "capital" is fixed at $\bar{K}_i$ and they minimize the cost of labor for a given "output" R_i of lobbying:

$\text{Min}_{L_i} \omega L_i$ subject to $R_i = \bar{K}_i^\alpha L_i$

Trivially, the optimal choice is $L_i = \frac{1}{K_i^\alpha} R_i$, and the restricted cost function is thus $\frac{\omega}{K_i^\alpha} R_i$. Thus, the marginal and average cost of lobby expenditures in the second stage is $\frac{\omega}{K_i^\alpha}$.

and expected payoffs to the two groups using (9) can be shown to be:

$$\pi_h(\tau_h, \tau_l) = y^h - \tau_l(1-n)\Delta y + \left(\lambda^2(1-n)^3(2-\tau_l)^2\Delta y\right) \frac{\tau_l - \tau_h}{(\lambda(1-n)(2-\tau_l) + cn(2-\tau_h))^2} \tag{23}$$

$$\pi_l(\tau_h, \tau_l) = y^l + \tau_h n\Delta y + \left(c^2 n^3(2-\tau_H)^2\Delta y\right) \frac{\tau_l - \tau_h}{(\lambda(1-n)(2-\tau_L) + cn(2-\tau_h))^2}$$

-

Each group chooses its tax position to maximize its stage 2 payoff as above. The equilibrium choice of positions and their implications are summarized in Proposition 4.

Proposition 4: *In the influence model between rich and poor, where the rich are in minority ($n < \frac{1}{2}$) and the majority tax position is $\widetilde{\tau} = 1$,*

(i) The rich and the poor choose in stage 2, their most preferred tax positions, so that $\tau_h^* = 0$ while $\tau_l^* = 1$ thereby resulting in complete polarization. This is regardless of the levels of λ, c and n.

(ii) The equilibrium win probability of the rich in stage 2 is $P_h^*(c) = \frac{\lambda(1-n)}{2cn+\lambda(1-n)}$ which is increasing in cognitive bias λ and decreasing in relative cost c and in the group's population size n.

(iii) The compromise tax rate as of stage 2 is $\overline{\tau}^*(c) = P_l^*(c) = \frac{2cn}{2cn+\lambda(1-n)}$ which is decreasing in cognitive bias λ, increasing in relative cost c, and increasing in the rich group's population size n.

Thus, we see that as of stage 2 - regardless of the infrastructure investments undertaken (which impact costs), relative population size, or cognitive biases - we have a case of complete polarization in the positions advocated by the two groups: the poor advocate for a 100% tax rate whereas the rich for no taxes. The resultant compromise tax rate is between the two but the fewer are the rich, the lower is the tax rate. For example, in the absence of any cognitive biases or cost advantages so that $\lambda = c = 1$, the compromise tax rate $\overline{\tau}^* < 0.5$ provided that $n < 1/3$.

However, c is endogenous to the infrastructure investment in stage 1. With $\tau_h^* = 0$ and $\tau_l^* = 1$, the relevant payoff functions in stage 1 are $\pi_h(0,1)$ and $\pi_l(0,1)$ from

(23). However, with the choice of K_h and K_l based on $c_i = \frac{\omega}{K_i^\alpha}$ or $c = \frac{K_l^\alpha}{K_h^\alpha}$, the payoff functions can be rewritten as:

$$\begin{aligned} \pi_h(0,1;K_h,K_l) = & \quad y^h - (1-n)\Delta y + \frac{\lambda^2(1-n)^3\Delta y K_h^{2\alpha}}{(\lambda(1-n)K_h^\alpha + 2nK_l^\alpha)^2} - K_h \\ \pi_l(0,1;K_h,K_l) = & \quad y^l + \frac{4n^3\Delta y K_l^{2\alpha}}{(\lambda(1-n)K_h^\alpha + 2nK_l^\alpha)^2} - K_l \end{aligned} \tag{24}$$

-

We show that there is a pure-strategy equilibrium in stage 1 infrastructure investments under the sufficient condition that $\alpha \leq \bar{\alpha}$ for some $\bar{\alpha} \in (1/2, 1)$.[16] The implied equilibrium cost ratio turns out to equal

$$c^* = \left(\frac{2n^2}{\lambda(1-n)^2}\right)^{\frac{\alpha}{1-\alpha}}$$

Note how this cost ratio is increasing in the relative size of the rich (n) and therefore how infrastructure investments tend to favor the rich as their relative population size decreases. This suggests that the minority status of the rich confers to them an additional advantage through investments in lobbying infrastructure beyond the effects already identified in Proposition 4. For the rich to have a lower relative marginal cost than the poor (that is, for that $c^* < 1$), we need $n < \frac{\sqrt{\lambda}}{\sqrt{\lambda}+\sqrt{2}}$. To appreciate the implication of this condition, consider, for example, a case where $\lambda = 0.5$ so that cognitive bias goes significantly against the rich. Despite this, the condition implies that even a relatively high proportion of rich such as $n = 1/3$ would lead to a cost advantage for the rich $c^* < 1$.[17] We summarize our main results in Proposition 5.

[16]The condition is similar - but not identical - to those that guarantee a pure-strategy equilibrium in "Tullock" contests. When the exponent α is high enough, only mixed-strategy equilibria can exist but these are rather difficult to characterize (see Konrad, 2009, for the general problem and Alcalde and Dahm, 2010, for cases in which mixed-strategy equilibria can be characterized).

[17]In practice, we expect a cost advantage for the rich to be correlated with a cognitive bias advantage for the rich as well. The reason is that the higher infrastructure investments that imply a cost advantage would also tend to improve the position of the rich in other related decisions and thus the posterior probabilities that decision-makers would have about the relative "correctness" of the rich. The cognitive bias λ can be thought of as the posterior from previous cases of lobbying.

Proposition 5: *The influence model between rich and poor has a unique subgame perfect equilibrium, provided that $\alpha < \bar{\alpha}$ for some $\bar{\alpha} \in (1/2, 1)$, and it has the following properties:*

(i) The ratio of infrastructure investments is $\frac{K_h^*}{K_l^*} = (\frac{\lambda(1-n)^2}{2n^2})^{\frac{1}{1-\alpha}}$.

(ii) The ratio of overall efforts devoted to lobbying is $\frac{R_h^*}{R_l^*} = (\frac{\lambda^{\alpha}(1-n)^{1+\alpha}}{2^{\alpha}n^{1+\alpha}})^{\frac{1}{1-\alpha}}$.

(iii) The equilibrium win probability for the rich is $P_h^* = \frac{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}+2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}}$ *which is increasing in λ and decreasing in n.*

(iv) The equilibrium compromise tax rate is $\overline{\tau}^* = P_l^* = \frac{2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}+2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}}$ *which is decreasing in λ and increasing in n.*

Proposition 5 shows that when investments in lobbying infrastructure are taken into account, the compromise tax rate falls even more strongly, the smaller is the relative population of the rich. As implied by (i), the minority tend to invest more in lobbying infrastructure which creates a cost advantage for them. This in turn accentuates their tendency to spend more aggressively while lobbying for their ideal policy as implied by (ii). For example, with $\alpha = 0.5$ and $\lambda = 1$ (and, therefore an effective bias in favor of the poor as a result of the majority position being the one of the poor), the equilibrium compromise tax rate is about 45% when $n = 0.3$. It becomes less than 6% at $n = 0.2$, and is negligible at about 0.5% when $n = 0.1$. Further, a cognitive bias in favor of the minority complements and reinforces their advantage.

5 Conclusion

Modern representative democracy has evolved in ways that has made the state stronger and much more involved in both economic and other matters than premodern states. One key and rather special attribute is that representatives in modern democracies are not bound by mandates in what they can and cannot approve with their votes (emphasized by Stasavage, 2020).[18] A legislator or elected executive can

[18]This in contrast to other experiments in modern democracy, such those of the Dutch Republic, whereby representative had often strict mandates. As a result, centralized institutions were weak

say one thing to their voters and do something else once in office. The "no-mandates" representation allowed the development of workable consensus in policy-making and thus contributes to decisive action (that is also perceived as legitimate given that those who make decisions have been elected). However, while avoiding the "mob-rule" that Plato was warning against regarding direct democracy, modern democracy leaves room for the emergence of the "iron law of oligarchy" (Michels, 1999[1915]), the tendency of all political parties and the instruments of government to be taken over by a small group of decision-makers who are far from representing the interests of the majority.

In the first decades of the Post-World-War II period, the distribution of income in the West was relatively equitable and the electorates seemed satisfied with their lot. Since the 1970s, however, increasing dissatisfaction with both economic and political outcomes appears to have taken root and become widespread. In this paper, we have shown the political mechanisms that can create such a wedge between policies desired by the majority and the policies that are actually implemented. In addition to the direct lobbying that can be identified in particular instances of policy-making, following arguments such as those of Johnson and Kwak (2010) or Piketty (2020), we submit that many imbalances of power have their origins in how the public sphere - the media, think tanks, universities - create cultures and ideologies that favor solutions away from those favored by the majority. In turn, at least one factor in the propagation of such cultures and ideologies is the amount of resources invested in promoting them. Changing the name of inheritance taxes from "estate taxes" to "death taxes," as it was done in the US in order to reduce the appeal of those types of taxes, is only one small example of the myriad ways through which the lens used to comprehend public policy issues have been systematically changed since the 1970s thus skewing the decision making in favor of the minority. Money, however, is not the only means through which prevailing ideologies change. Collective organization could emerge to overcome the current distance between actual and majority-supported policies as it has in the past. However the conditions and circumstances that can

in the Dutch Republic and the experiment did not spread as the no-mandate form of representation did, developed more fully in England.

facilitate this in the current times are not easy to identify or immediately apparent to us.

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Appendix

Proof of Proposition 1: Suppose, contrary to the Proposition, that $t_A^* = t_B^* = t^* \in [0,1,]$. Then we would have $\Delta V_i \equiv V_i(t^*) - V_i(t^*) = 0$. By (3) it is clear that $R_i^* = 0$ for both $i = A, B$ and the implemented position would be $\widetilde{t}$ (the majority position) so that $\pi_i^* = V_i(\widetilde{t})$.

We consider the cases of $t^* = \widetilde{t}$ and $t^* \neq \widetilde{t}$.

Consider first the case of $t^* = \widetilde{t}$ and in stage 1 a deviation t_i' which is closer to the player's ideal point than $\widetilde{t}$ is. We would then have $V_i(t_i') > V_i(\tilde{t})$ and $\Delta V_i(t_i', \tilde{t}) = V_i(t_i') - V_i(\tilde{t}) > 0$. The subgame in stage 2 with payoff functions in (3) is an ordinary asymmetric contest game with conditions that satisfy those in Yamazaki (2008) (following Szidarovsky and Okuguchi, 1997, and Cornes and Hartley, 2005) and therefore has a unique equilibrium in lobbying expenditures, say (R_A', R_B'). Moreover, based on Yamazaki (2008) and Szidarovsky and Okuguchi (1997, Theorem 2), we can show that the part of the equilibrium payoff $\pi_i(R_i', R_j'; t_i', \tilde{t})$ that does not include $V_i(\widetilde{t})$ (that is, $\frac{\lambda_i' f_i(R_j')}{\lambda_i' f_i(R_i') + \tilde{\lambda}_j f_j(R_j')}[\Delta V_i] - c_i R_j'$) is positive (with both λ_i' and $\tilde{\lambda}_j$ being positive and finite) and therefore we have:

$$\pi_i(R_i', R_j'; t_i', \tilde{t}) = \frac{\lambda_i' f_i(R_j')}{\lambda_i' f_i(R_i') + \tilde{\lambda}_j f_j(R_j')}[\Delta V_i] - c_i R_j' + V_i(\tilde{t}) > V_i(\tilde{t})$$

Therefore, a deviation from $\tilde{t}$ in stage 1 by either player for a t_i' that is closer to the player's ideal point has been shown to be strictly better for i, thus contradicting the presumed equilibrium $t^*(= \tilde{t})$.

Next consider the case of $t^* \neq \widetilde{t}$. Without loss of generality, suppose $t^* < \widetilde{t}$. Then, given the monotonicity of preferences in terms of t, $V_A(t^*) > V_A(\widetilde{t})$. First, suppose $t^* > 0$ and consider any deviation $t_A' \in (\hat{t}_A, t^*)$. We would then have $V_A(t_A') > V_A(t^*)$ and $\Delta V_A(t_A', t^*) = V_A(t_A') - V_A(t^*) > 0$. Just as in the previous case, the equilibrium payoff $\pi_A(R_i', R_j'; t_i', t^*)$ that does not include $V_A(t^*)$ (that is, $\frac{\lambda'_A f_A(R'_A)}{\lambda'_A f_A(R'_A) + \lambda_A^* f_j(R_j')}[\Delta V_A] - c_A R'_A$) is positive, with both λ'_A and λ_A^* being positive and finite. Therefore, we have:

$$\pi_A(R_i', R_j'; t_i', t^*) = \frac{\lambda_A' f_A(R_A')}{\lambda_A' f_A(R_A') + \lambda_A^* f_A(R_A')}[\Delta V_A(t_A', t^*)] - c_A R_A' + V_A(t^*) > V_A(\tilde{t})$$

Thus, t^* cannot be an equilibrium for $t^* > 0$.

Finally, suppose $t^* = 0$, and consider a small deviation $t_A' = 0 + \varepsilon$ ($\varepsilon > 0$). Then, $\Delta V_i(t_A', t^*) < 0$ for both i. However, although it is optimal for B to choose $R_{BA}' = 0$, it is not optimal for A to reciprocate because doing so would yield $V_A(\tilde{t})$. Instead player A will choose a positive R_A', so that given $V_A(t^*) > V_A(\tilde{t})$, it will yield a $\pi_A(R_i', R_j'; t_i', t^*) > V_A(\tilde{t})$.

Therefore, in all cases the supposition of $t_A^* = t_B^* = t^*$ is contradicted.■

Proof of Proposition 2:

Suppose that $\overline{\lambda} = 1$. In this case, (14) and (15) reduce to the following respectively:

$$\pi_A(t_A, t_B) = a\left[\frac{(t_B - \widetilde{t} + 1)^2(t_B - t_A)}{(t_B - t_A + 2)^2} - t_B\right] \tag{25}$$

$$\pi_B(t_A, t_B) = b\left[\frac{(\widetilde{t} - t_A + 1)^2(t_B - t_A)}{[(t_B - t_A + 2)]^2} - (1 - t_A)\right] \tag{26}$$

Using (25) we see that:

$$\frac{\partial \pi_A}{\partial t_A} = \frac{a(t_B - \widetilde{t} + 1)^2(t_B - t_A - 2)}{(t_B - t_A + 2)^3}$$

Notice that the maximum value of $t_B - t_A = 1$. This implies that the above expression is always less than 0. Hence it follows that $t_A^* = 0$.

Using (26), we see that:

$$\frac{\partial \pi_B}{\partial t_B} = \frac{b(\widetilde{t} - t_A + 1)^2(t_A - t_B + 2)}{(t_B - t_A + 2)^3}$$

Notice that the least value of $t_A - t_B = -1$. This implies that the above expression is always greater than 0 and hence $t_B^* = 1$. ■

Proof of Proposition 3(i):

Suppose that $\overline{\lambda} > 1$. Under these conditions, using (14), we get:

$$\frac{\partial \pi_A}{\partial t_A} = \frac{a\overline{\lambda}^2(t_B - \widetilde{t} + 1)^2[(2-\overline{\lambda})t_B - t_A + (\overline{\lambda}-1)\widetilde{t} - (\overline{\lambda}+1)]}{(\overline{\lambda}t_B - t_A + \overline{\lambda}(1-\widetilde{t}) + 1 + \widetilde{t})^3} \tag{27}$$

Recall that any feasible equilibrium will involve $0 \leq t_A \leq \widetilde{t}$ and $\widetilde{t} \leq t_B \leq 1$. Since $\overline{\lambda} > 1$ and $t_B > t_A$, the denominator of (27) is always positive. It therefore follows that the sign of $\frac{\partial \pi_A}{\partial t_A}$ is determined entirely by the sign of the term $[(2-\overline{\lambda})t_B - t_A + (\overline{\lambda}-1)\widetilde{t} - (\overline{\lambda}+1)]$. Notice that when $\overline{\lambda} \leq 2$, the expression in the numerator of (27) $[(2-\overline{\lambda})t_B - t_A + (\overline{\lambda}-1)\widetilde{t} - (\overline{\lambda}+1)]$ attains its highest value when $t_B = 1$ and $t_A = 0$ which is $(1-\widetilde{t}) - (2-\widetilde{t})\overline{\lambda} < 0$. When $\overline{\lambda} > 2$, it attains its highest value when $t_B = \widetilde{t}$ and $t_A = 0$ which is $\widetilde{t} - \overline{\lambda} - 1 < 0$ as $\widetilde{t} < 1$ and $\overline{\lambda} > 2$. From this we can conclude that the numerator of (27) is strictly negative for any $t_A \in [0, \widetilde{t}]$ and $t_B \in [\widetilde{t}, 1]$. Hence we can conclude that $\frac{\partial \pi_A}{\partial t_A} < 0$ for any feasible values of t_A and t_B. Accordingly $t_A^* = 0$.

We now look at B's first stage choice, and using (15) we get:

$$\frac{\partial \pi_B}{\partial t_B} = \frac{b(\widetilde{t} - t_A + 1)^2[(2\overline{\lambda}-1)t_A - \overline{\lambda}t_B + \overline{\lambda}(1-\widetilde{t}) + (1+\widetilde{t})]}{(\overline{\lambda}t_B - t_A + \overline{\lambda}(1-\widetilde{t}) + 1 + \widetilde{t})^3} \tag{28}$$

Notice that since $(\widetilde{t} - t_A + 1)^2$ is always positive and, as argued earlier, $\overline{\lambda}t_B - t_A + \overline{\lambda}(1-\widetilde{t}) + 1 + \widetilde{t} > 0$ the term $\frac{b(\widetilde{t}-t_A+1)^2}{(\overline{\lambda}t_B - t_A + \overline{\lambda}(1-\widetilde{t})+1+\widetilde{t})^3}$ is positive and multiplicative. Hence the sign of $\frac{\partial \pi_B}{\partial t_B}$ critically depends on $[(2\overline{\lambda}-1)t_A - \overline{\lambda}t_B + \overline{\lambda}(1-\widetilde{t}) + (1+\widetilde{t})]$. Since we know that for any $\overline{\lambda} > 1, t_A^* = 0$, this expression reduces to $-\overline{\lambda}t_B + \overline{\lambda}(1-\widetilde{t}) + (1+\widetilde{t})$ which assumes its least value when $t_B = 1$ which is $1 + \widetilde{t}(1-\overline{\lambda})$. This is non-negative as long as $\overline{\lambda} \leq \frac{(1+\widetilde{t})}{\widetilde{t}}$. Hence, given $t_A^* = 0$, as long as $\overline{\lambda} \leq \frac{(1+\widetilde{t})}{\widetilde{t}}$, then $\frac{\partial \pi_B}{\partial t_B} > 0$ for any $t_B \in [\widetilde{t}, 1)$ and $\frac{\partial \pi_B}{\partial t_B} \geq 0$ at $t_B = 1$. Hence $t_B^* = 1$.

When $\overline{\lambda} > \frac{(1+\widetilde{t})}{\widetilde{t}}$, $\frac{\partial \pi_B}{\partial t_B} < 0$ when $t_B = 1$. When $t_B = \widetilde{t}$, the sign of $\frac{\partial \pi_B}{\partial t_B}$ depends on the sign of the value $\overline{\lambda}(1-2\widetilde{t}) + (1+\widetilde{t})$. This is strictly positive whenever $\widetilde{t} \leq \frac{1}{2}$. When $\widetilde{t} > \frac{1}{2}$, it is strictly positive when $\overline{\lambda} < \frac{(1+\widetilde{t})}{(2\widetilde{t}-1)}$.

Let us first examine the case of $\overline{\lambda} > \frac{(1+\widetilde{t})}{\widetilde{t}}$ and $\widetilde{t} \leq \frac{1}{2}$. Given that $\frac{\partial \pi_B}{\partial t_B} > 0$ at $t_B = \widetilde{t}$, $\frac{\partial \pi_B}{\partial t_B} < 0$ at $t_B = 1$, and $\frac{\partial \pi_B}{\partial t_B}$ is continuous in t_B, it follows that $t_B^* = (1 - \widetilde{t}) + \frac{(1+\widetilde{t})}{\overline{\lambda}}$ which uniquely solves $\frac{\partial \pi_B}{\partial t_B} = 0$. From this it also follows that when $\widetilde{t} \in (\frac{1}{2}, 1)$ and $\frac{(1+\widetilde{t})}{\widetilde{t}} < \overline{\lambda} < \frac{(1+\widetilde{t})}{(2\widetilde{t}-1)}$, $t_B^* = (1-\widetilde{t}) + \frac{(1+\widetilde{t})}{\overline{\lambda}}$.[19] When $\overline{\lambda} \geq \frac{(1+\widetilde{t})}{(2\widetilde{t}-1)}$, $\frac{\partial \pi_B}{\partial t_B} \leq 0$ when $t_B = \widetilde{t}$. Recall that the sign of $\frac{\partial \pi_B}{\partial t_B}$ depends on the sign of $-\overline{\lambda} t_B + \overline{\lambda}(1 - \widetilde{t}) + (1 + \widetilde{t})$ which assumes its highest value over the interval $t_B \in [\widetilde{t}, 1]$ when $t_B = \widetilde{t}$ which is non-positive. This implies that $\frac{\partial \pi_B}{\partial t_B} \leq 0$ over the entire range $t_B \in [\widetilde{t}, 1]$. Hence group B's optimal choice is $t_B^* = \widetilde{t}$. ■

Proof of Proposition 3(ii):

Let us examine $\frac{\partial \pi_B}{\partial t_B}$ as given by (28) under the assumption $\overline{\lambda} < 1$.

Notice that $b(\widetilde{t} - t_A + 1)^2$ is always positive as by assumption $b > 0$. Further, the lowest value of the denominator term $\overline{\lambda} t_B - t_A + \overline{\lambda}(1 - \widetilde{t}) + 1 + \widetilde{t}$ is $1 + \overline{\lambda} > 0$ which is attained when $t_A = t_B = \widetilde{t}$. From this we can infer that the term $\frac{b(\widetilde{t} - t_A + 1)^2}{(\overline{\lambda} t_B - t_A + \overline{\lambda}(1-\widetilde{t}) + 1 + \widetilde{t})^3}$ is positive and multiplicative and therefore does not play a role in determining the sign of $\frac{\partial \pi_B}{\partial t_B}$. The critical term is $[(2\overline{\lambda} - 1)t_A - \overline{\lambda} t_B + \overline{\lambda}(1 - \widetilde{t}) + (1 + \widetilde{t})]$. When $\overline{\lambda} \geq \frac{1}{2}$, this term attains its lowest value when $t_A = 0$ and $t_B = 1$ as given by $1 + \widetilde{t}(1 - \overline{\lambda}) > 0$ as $\overline{\lambda} < 1$. When $\overline{\lambda} < \frac{1}{2}$, this term attains its lowest value when $t_A = \widetilde{t}$ and $t_B = 1$ as given by $1 + \overline{\lambda}\widetilde{t} > 0$. Hence it follows that as long as $\overline{\lambda} < 1$, then $\frac{\partial \pi_B}{\partial t_B} > 0$ for any $t_A \in [0, \widetilde{t}]$ and $t_B \in [\widetilde{t}, 1]$. Hence it follows that $t_B^* = 1$.

We now examine group A's choice. Recall that $\frac{\partial \pi_A}{\partial t_A}$ is given by (27). Since $\frac{a\overline{\lambda}^2 (t_B - \widetilde{t} + 1)^2}{(\overline{\lambda} t_B - t_A + \overline{\lambda}(1-\widetilde{t}) + 1 + \widetilde{t})^3} > 0$ and appears multiplicatively, the sign of $\frac{\partial \pi_A}{\partial t_A}$ depends on the sign of the expression $[(2 - \overline{\lambda})t_B - t_A + (\overline{\lambda} - 1)\widetilde{t} - (\overline{\lambda} + 1)]$ which attains its highest value when $t_B = 1$ and $t_A = 0$ which is $(1 - \widetilde{t}) - (2 - \widetilde{t})\overline{\lambda}$. Notice that this is non-positive as long as $\overline{\lambda} \geq \frac{1-\widetilde{t}}{2-\widetilde{t}}$. Hence in this range of $\overline{\lambda}$, $t_A^* = 0$. We now look at the case of $\overline{\lambda} < \frac{1-\widetilde{t}}{2-\widetilde{t}}$ with the knowledge that $t_B^* = 1$. It therefore follows that $\frac{\partial \pi_A}{\partial t_A} > 0$ when $t_A = 0$. Given this, we now explore the sign of $\frac{\partial \pi_A}{\partial t_A}$ when $t_A = \widetilde{t}$. In this case the value of the expression $[(2 - \overline{\lambda})t_B - t_A + (\overline{\lambda} - 1)\widetilde{t} - (\overline{\lambda} + 1)]$ equals $(1 - 2\widetilde{t}) - (2 - \widetilde{t})\overline{\lambda}$. Notice that for $\widetilde{t} \geq \frac{1}{2}$, it is always the case that $(1 - 2\widetilde{t}) - (2 - \widetilde{t})\overline{\lambda} < 0$, implying that $\frac{\partial \pi_A}{\partial t_A} < 0$ at

[19] Note that when $1 > \widetilde{t} > \frac{1}{2}$, $\frac{(1+\widetilde{t})}{(2\widetilde{t}-1)} > \frac{(1+\widetilde{t})}{\widetilde{t}}$.

$t_A = \widetilde{t}$. Hence in this range of $\widetilde{t}$, since $\frac{\partial \pi_A}{\partial t_A}$ is continuous in t_A and given that $\frac{\partial \pi_A}{\partial t_A} > 0$ at $t_A = 0$ and $\frac{\partial \pi_A}{\partial t_A} < 0$ at $t_A = \widetilde{t}$, it follows that $0 < t_A^* = (1-\widetilde{t}) - (2-\widetilde{t})\overline{\lambda} < \widetilde{t}$. When $\widetilde{t} < \frac{1}{2}$,$(1-2\widetilde{t}) - (2-\widetilde{t})\overline{\lambda} < 0$ as long as $\overline{\lambda} > \frac{1-2\widetilde{t}}{2-\widetilde{t}}$ so that $\frac{\partial \pi_A}{\partial t_A} < 0$ at $t_A = \widetilde{t}$. Hence when $\widetilde{t} < \frac{1}{2}$ and $\overline{\lambda} \in (\frac{1-2\widetilde{t}}{2-\widetilde{t}}, \frac{1-\widetilde{t}}{2-\widetilde{t}})$ for exactly the same kind of reasoning we will have $0 < t_A^* = (1-\widetilde{t}) - (2-\widetilde{t})\overline{\lambda} < \widetilde{t}$. When $\overline{\lambda} \leq \frac{1-2\widetilde{t}}{2-\widetilde{t}}, \frac{\partial \pi_A}{\partial t_A} \geq 0$ at $t_A = \widetilde{t}$. Further, since the expression $[(2-\overline{\lambda}) - t_A + (\overline{\lambda}-1)\widetilde{t} - (\overline{\lambda}+1)]$ attains its least value at $t_A = \widetilde{t}$, it follows that $\frac{\partial \pi_A}{\partial t_A} > 0$ for any $t_A < \widetilde{t}$ given $t_B^* = 1$. Hence it follows that when $\widetilde{t} < \frac{1}{2}$ and $\overline{\lambda} \leq \frac{1-2\widetilde{t}}{2-\widetilde{t}}, t_A^* = \widetilde{t}$. ■

Proof of Proposition 4: We'll examine the rich group's choice of tax rate first from (23). Observe that

$$\frac{\partial \pi_h(\tau_h, \tau_l)}{\partial \tau_h} = \left(\lambda^2(1-n)^3(2-\tau_l)^2 \Delta y\right) \frac{\partial}{\partial \tau_h}\left(\frac{\tau_l - \tau_h}{(\lambda(1-n)(2-\tau_l) + cn(2-\tau_h))^2}\right)$$

Since $(\lambda^2(1-n)^3(2-\tau_l)^2\Delta y)$ is always positive, the sign of $\frac{\partial \pi_h(\tau_h,\tau_l)}{\partial \tau_h}$ is determined by the sign of $\frac{\partial}{\partial \tau_h}\left(\frac{\tau_l-\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}\right)$.

$\frac{\partial}{\partial \tau_h}\left(\frac{\tau_l-\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}\right) = -\frac{\lambda(1-n)(2-\tau_l)+2cn(1-\tau_l)+cn\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^3} < 0$. Hence it follows that $\frac{\partial \pi_h(\tau_h,\tau_l)}{\partial \tau_h} < 0$ for any (τ_h, τ_l). Further,

$\frac{\partial^2 \pi_h(\tau_h,\tau_l)}{\partial \tau_h^2} = (\lambda^2(1-n)^3(2-\tau_l)^2\Delta y)\, \frac{\partial^2}{\partial \tau_h^2}\left(\frac{\tau_l-\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}\right)$

$\frac{\partial^2}{\partial \tau_h^2}\left(\frac{\tau_l-\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}\right) = \frac{cn(-4\lambda(1-n)(2-\tau_l)-2cn-6cn(1-\tau_l)-2cn\tau_h)}{(\lambda(1-n)(2-\tau_l)+(cn(2-\tau_h))^4} < 0$ so that $\frac{\partial^2 \pi_h(\tau_h,\tau_l)}{\partial \tau_h^2} < 0$.

Given this, it follows that the rich group always sets $\tau_h^* = 0$.

We now examine the poor group's choice of tax rate, also from (23). Observe that:

$$\frac{\partial \pi_l(\tau_h, \tau_l)}{\partial \tau_l} = (c^2 n^3 (2-\tau_h)^2 \Delta y) \frac{\partial}{\partial \tau_l}\left(\frac{\tau_l - \tau_h}{(\lambda(1-n)(2-\tau_l) + cn(2-\tau_h))^2}\right)$$

Since $(c^2n^3(2-\tau_h)^2\Delta y)$ is always positive, the sign of $\frac{\partial \pi_l(\tau_h,\tau_l)}{\partial \tau_l}$ is determined by the sign of $\frac{\partial}{\partial \tau_l}\left(\frac{\tau_l-\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}\right)$.

$\frac{\partial}{\partial \tau_l}\left(\frac{\tau_l-\tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}\right) = \frac{2\lambda(1-n)(1-\tau_h)+cn(2-\tau_h)+\lambda(1-n)\tau_l}{(\lambda(1-n)(2-\tau_l)(+cn(2-\tau_h))^3} > 0$. Hence it follows that $\frac{\partial \pi_l(\tau_h,\tau_l)}{\partial \tau_l} > 0$ for any (τ_h, τ_l). Further,

$\frac{\partial^2 \pi_l(\tau_h,\tau_l)}{\partial \tau_l^2} = (c^2 n^3 (2-\tau_h)^2 \Delta y) \frac{\partial^2}{\partial \tau_l^2} (\frac{\tau_l - \tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2})$

$\frac{\partial^2}{\partial \tau_l^2} (\frac{\tau_l - \tau_h}{(\lambda(1-n)(2-\tau_l)+cn(2-\tau_h))^2}) = \frac{\lambda(1-n)(\lambda(1-n)(6(1-\tau_h)+(2-\tau_l))+4cn(2-\tau_h)+3\lambda(1-n)\tau_l)}{(\lambda(1-n)(2(-\tau_l)+cn(2-\tau_h))^4} > 0$ so that $\frac{\partial^2 \pi_l(\tau_h,\tau_l)}{\partial \tau_l^2} > 0$.

Given this, it follows that the poor group always sets $\tau_l^* = 1$. This proves part (i).

Straightforwardly, $P_h^* = P_h(0,1) = \frac{\lambda(1-n)}{2cn+\lambda(1-n)}$

Hence, $\frac{\partial P_h^*}{\partial \lambda} = \frac{cn(1-n)(2-\tau_l)(2-\tau_h)}{(\frac{\lambda(1-n)}{cn}+2)^2} > 0$.Further, $\frac{\partial P_h^*}{\partial c} = \frac{-\lambda n(1-n)(2-\tau_l)(2-\tau_h)}{(\frac{\lambda(1-n)}{cn}+2)^2} < 0$ and $\frac{\partial P_h^*}{\partial n} = \frac{-\lambda c(2-\tau_l)(2-\tau_h)}{(\frac{\lambda(1-n)}{cn}+2)^2} < 0$. This proves part (ii).

Part (iii) follows from $\bar{\tau}^*(c) = P_l^* \tau_l^* + (1-P_l^*)\tau_h^* = P_l^* 1 + (1-P_l^*)0 = P_l^*$ and the fact that $P_l^* = 1 - P_h^*$, which, by definition, must have properties that are exactly opposite to those of P_h^* which are mentioned in part (ii). ■

Proof of Proposition 5: We first derive the equilibrium levels of K_h^* and K_l^* and show the properties (i)-(iv). In the end, we show the sufficient conditions under which this equlibrium exists.

By taking the derivatives in (*24*) with respect to K_h and K_l we obtain

$$\frac{\partial \pi_h(0,1;K_h,K_l)}{\partial K_h} = \frac{4\alpha K_h^{2\alpha-1} K_l^{\alpha} \lambda^2 (1-n)^3 n \Delta y}{(\lambda(1-n)K_h^{\alpha} + 2nK_l^{\alpha})^3} - 1$$

$$\frac{\partial \pi_l(0,1;K_h,K_l)}{\partial K_l} = \frac{8\alpha K_l^{2\alpha-1} K_h^{\alpha} \lambda (1-n) n^3 \Delta y}{(\lambda(1-n)K_h^{\alpha} + 2nK_l^{\alpha})^3} - 1$$

Setting these derivatives equal to 0 and dividing them yields:

$$\frac{{K_h^*}^{1-\alpha}}{{K_l^*}^{1-\alpha}} = \frac{\lambda(1-n)^2}{2n^2}$$

From this, we readily obtain property (i) in Proposition 5 as well as the fact that $c^* \equiv \frac{\frac{\omega}{K_h^{*\alpha}}}{\frac{\omega}{K_l^{*\alpha}}} = \frac{K_l^{*\alpha}}{K_h^{*\alpha}} = (\frac{2n^2}{\lambda(1-n)^2})^{\frac{\alpha}{1-\alpha}}$.

By substitution into the first-order condition of each player, we can find the analytical solutions for K_h^* and K_l^* :

$$\begin{aligned} K_h^* &= \frac{2\alpha[\lambda^2(1-n)^{3+\alpha}2n^{1+\alpha}]^{\frac{1}{1-\alpha}}\Delta y}{[\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}+2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}]^3} \\ K_l^* &= \frac{2\alpha[\lambda(1-n)^{1+\alpha}4n^{3+\alpha}]^{\frac{1}{1-\alpha}}\Delta y}{[\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}+2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}]^3} \end{aligned}$$

To prove part (ii) of Proposition 5, we use c^* in (*22*) to obtain:

$$\frac{R_h^*}{R_l^*} = \frac{1-n}{n\left(\frac{2n^2}{\lambda(1-n)^2}\right)^{\frac{\alpha}{1-\alpha}}} = \left(\frac{\lambda^\alpha(1-n)^{1+\alpha}}{2^\alpha n^{1+\alpha}}\right)^{\frac{1}{1-\alpha}}$$

To prove part (iii) of Proposition 5, we recall that from Proposition 4 part (ii) it follows that:

$$\begin{aligned} P_h^* = P_h^*(c^*) = \frac{\lambda(1-n)}{2c^*n+\lambda(1-n)} &= \frac{\lambda(1-n)}{2n\frac{2n^2}{\lambda(1-n)^2}^{\frac{\alpha}{1-\alpha}}+\lambda(1-n)} \\ &= \frac{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}+2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}} \end{aligned}$$

This is straightforwardly increasing in λ and decreasing in n.

Similarly, to prove part (iv) of Proposition 5, we have from part (iii) of Proposition 4:

$$\begin{aligned} \overline{\tau}^* &= \overline{\tau}^*(c^*) = \frac{2cn}{2c^*n+\lambda(1-n)} = \frac{2cn}{2n\frac{2n^2}{\lambda(1-n)^2}^{\frac{\alpha}{1-\alpha}}+\lambda(1-n)} \\ &= \frac{2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}+2^{\frac{1}{1-\alpha}}n^{\frac{1+\alpha}{1-\alpha}}} \end{aligned}$$

It is apparent from the above expression that $\overline{\tau}^*$ is straightforwardly decreasing in λ

and increasing in n.

We next examine the conditions under which the pure-strategy equilibrium we have just derived exists. First, we show that each payoff function is strictly concave in a player's own strategy. Second, we show that the equilibrium payoffs are positive (and, therefore, investing in lobbying infrastructure is better than non-participation).

With respect to the first task, by taking the second derivatives with respect to each player's own strategy we eventually obtain:

$$\frac{\partial \pi_h^2(0,1;K_h,K_l)}{\partial K_h^2} = \frac{4\alpha K_h^{2\alpha-2} K_l^{\alpha} \lambda^2 (1-n)^3 n \Delta y}{(\lambda(1-n)K_h^{\alpha} + 2nK_l^{\alpha})^4}[-(\alpha+1)\lambda(1-n)K_h^{\alpha} + (2\alpha-1)2nK_l^{\alpha}]$$
$$\frac{\partial \pi_l^2(0,1;K_h,K_l)}{\partial K_l^2} = \frac{8\alpha K_l^{2\alpha-2} K_h^{\alpha} \lambda (1-n) n^3 \Delta y}{(\lambda(1-n)K_h^{\alpha} + 2nK_l^{\alpha})^4}[(2\alpha-1))\lambda(1-n)K_h^{\alpha} - (1+\alpha)2nK_l^{\alpha}]$$

The first term of each derivative is positive. Therefore, the derivative is negative if and only if the second term inside the square brackets is negative. Note that for any $\alpha \in (0, 1/2]$ both terms inside the square brackets are negative since the constituent terms multiplied by $(2\alpha - 1)$ are non-positive and the other constituent term that are multiplied by $-(1 + \alpha)$ would be negative. Therefore, for any $\alpha \in (0, 1/2]$ each player's payoff is strictly concave in the player's own strategy.

To show that the equilibrium exists for some values of α greater than $1/2$, we can consider $\frac{\partial \pi_h^2(0,1;K_h^*,K_l^*)}{\partial K_h^2}$ and $\frac{\partial \pi_l^2(0,1;K_h^*,K_l^*)}{\partial K_l^2}$ and, in particular, evaluate the two critical terms in square brackets (as shown above) at K_h^* and K_l^*. Using the fact that $K_l^{*\alpha} = \frac{2n^2}{\lambda(1-n)^2}^{\frac{\alpha}{1-\alpha}} K_h^{*\alpha}$, we have:

$$[-(\alpha+1)\lambda(1-n)K_h^{*\alpha} + (2\alpha-1)2nK_l^{*\alpha}]$$
$$= \lambda(1-n)K_h^{*\alpha}[-(\alpha+1) + (2\alpha-1)\frac{2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}}] \quad (29)$$
$$[(2\alpha-1))\lambda(1-n)K_h^{*\alpha} - (1+\alpha)2nK_l^{*\alpha}]$$
$$= \lambda(1-n)K_h^{*\alpha}[(2\alpha-1) - (1+\alpha)\frac{2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}}(1-n)^{\frac{1+\alpha}{1-\alpha}}}] \quad (30)$$

It is clear that the negativity of the two second derivatives depends on the expression encapsulated by square brackets on the right-hand-side of (29) and (30) each being negative. Let $l(\alpha) \equiv \frac{2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}}} \in (0, \infty)$. Then (29) and (30) being negative is equivalent to:

$$-(\alpha + 1) + (2\alpha - 1)l(\alpha) < 0 \tag{31}$$

-

$$(2\alpha - 1) - (\alpha + 1)l(\alpha) < 0 \tag{32}$$

Both inequalities hold for all $\alpha \leq 1/2$ regardless of the value of $l(\alpha)$. Lemma A (proved below) shows the derivative properties of $l(\alpha)$ depend on whether $l(\alpha)$ is less or greater than $\frac{1-n}{n}$. We will use those two cases as well as $l(\alpha) = \frac{1-n}{n}$ to prove the existence of a $\bar{\alpha} \in (1/2, 1)$ such that our equilibrium exists for all $\alpha < \bar{\alpha}$.

Case I: $l(\alpha) < \frac{1-n}{n}$

Proving (*31*) also holds for a range of $\alpha > 1/2$ is equivalent to showing:

$$\alpha < \frac{1 + l(\alpha)}{2l(\alpha) - 1} \tag{33}$$

As α increases beyond 1/2, the right-hand-side (RHS) of the above inequality increases with α since $\frac{\partial}{\partial \alpha} \frac{1+l(\alpha)}{2l(\alpha)-1} = \frac{-3l'}{(2l(\alpha)-1)^2} > 0$ as in Case I. However the left-hand-side (LHS) of this inequality also increases with α. Given this, there are two possibilities to consider. One possibility is that rate of increase of RHS always weakly exceeds that of LHS so that the inequality continues to hold for any $\alpha \in (1/2, 1)$. The other possibility is that the rate of increase of LHS may sufficiently exceed that of RHS. In the latter case, since both RHS and LHS are continuous functions of α, it follows that there exists an $\alpha_1 \in (1/2, 1)$, such that $\alpha_1 = \frac{1+l(\alpha_1)}{2(l(\alpha_1))-1}$. Hence in this case (*31*) will be satisfied for any $\alpha < \alpha_1$.

Now, proving (*32*) also holds for a range of $\alpha > 1/2$ is equivalent to showing:

$$\alpha < \frac{1 + l(\alpha)}{2 - l(\alpha)}$$

As α increases beyond 1/2, note that the RHS of this inequality is decreasing in α given that $l'(\alpha) < 0$ when $l(\alpha) < \frac{1-n}{n}$ while the LHS is increasing in α. Hence as with the case of (*31*), either the inequality will continue to be satisfied for any $\alpha \in (1/2, 1)$ or since both LHS and RHS expressions are continuous in α, an $\alpha_2 \in (1/2, 1)$ will exist such that $\alpha_2 = \frac{1+l(\alpha_2)}{2-l(\alpha_2)}$. In this case (*32*) is satisfied for all $\alpha < \alpha_2$.

Hence we can conclude that for $l(\alpha) < \frac{1-n}{n}$, (*31*) and (*32*) are satisfied either for any $\alpha < 1$ or at least for $\alpha < \widetilde{\alpha} = Min(\alpha_1, \alpha_2) \in (1/2, 1)$.

Case II: $l(\alpha) > \frac{1-n}{n}$

Proving (*31*) also holds for a range of $\alpha > 1/2$ is equivalent to showing[20]:

$$\alpha < \frac{l(\alpha)+1}{2l(\alpha)-1}$$

As α increases beyond 1/2, $\frac{d\frac{l(\alpha)+1}{2l(\alpha)-1}}{d\alpha} = -\frac{3l'(\alpha)}{(2l(\alpha)-1)^2} < 0$, given $l'(\alpha) > 0$ for $l(\alpha) > \frac{1-n}{n}$. Hence the RHS ($\frac{l(\alpha)+1}{2l(\alpha)-1}$) is decreasing in α while the LHS is increasing in it. Depending on the rate of decrease and the curvature of RHS function, we can either have a situation where the inequality continues to be satisfied for any $\alpha \in (1/2, 1)$ or there exists an $\widetilde{\alpha} \in (1/2, 1)$ such that $\widetilde{\alpha} = \frac{l(\widetilde{\alpha})+1}{2l(\widetilde{\alpha})-1}$. Hence, for all $\alpha < \widetilde{\alpha}$, the inequality $\alpha < \frac{l(\alpha)+1}{2l(\alpha)-1}$ holds and (*31*) is satisfied.

Observe that (*32*) is always satisfied in this case, given that $(2\alpha-1)-(\alpha+1)l(\alpha) < (2\alpha-1)-(\alpha+1)\frac{1-n}{n} < (2\alpha-1)-(\alpha+1) = \alpha-2 < 0$.

Hence, for $l(\alpha) > \frac{1-n}{n}$ we can conclude that both (*31*) and (*32*) are satisfied either for any $0 < \alpha < 1$ or at least for $\alpha < \widetilde{\alpha} \in (1/2, 1)$.

Case III: $l(\alpha) = \frac{1-n}{n}$

Satisfying (*31*) for a range of $\alpha > 1/2$ is equivalent to $-(\alpha+1)+(2\alpha-1)\frac{1-n}{n} < 0 \Leftrightarrow \alpha < \frac{1}{2-3n}$. We let $\widetilde{\alpha} = \frac{1}{2-3n}$. Since $0 < n < 1/2$, $\widetilde{\alpha} > 1/2$. Further when $n \in [1/3, 1/2)$, $\widetilde{\alpha} \geq 1$. In this case, (*31*) holds for any $\alpha \in (0, 1)$. For $n \in (0, 1/3), \widetilde{\alpha} \in (1/2, 1)$ and (*31*) holds for $\alpha < \widetilde{\alpha}$. (*32*) is always satisfied in this case for $\alpha > 1/2$ by the same argument as presented for Case II.

[20] Note that $2l(\alpha) > 1$, given that $2l(\alpha) > 2\frac{1-n}{n} > 2$ with the last inequality following $n < 1/2$.

Hence, for $l(\alpha) = \frac{1-n}{n}$, (*31*) and (*32*) are satisfied for either for $0 < \alpha < 1$ or at least for $\alpha < \widetilde{\alpha} = \frac{1}{2-3n} \in (1/2, 1)$.

To show the positivity of equilibrium payoffs, these payoffs can be calculated by replacing K_h^* and K_l^* (see above) in the payoff functions:

$$\pi_h(0,1; K_h^*, K_l^*) = y^h - (1-n)\Delta y + \frac{\lambda^2(1-n)^3 \Delta y K_h^{*2\alpha}}{(\lambda(1-n)K_h^{*\alpha} + 2nK_l^{*\alpha})^2} - K_h^*$$

$$= y^h - (1-n)\Delta y + \frac{\Delta y \lambda^{\frac{2}{1-\alpha}} (1-n)^{\frac{3+\alpha}{1-\alpha}} [\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}} + (1-2\alpha) 2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}]}{[\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}} + 2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}]^3} \quad (34)$$

$$\pi_l(0,1; K_h^*, K_l^*) = y^l + \frac{4n^3 \Delta y K_l^{*2\alpha}}{(\lambda(1-n)K_h^{*\alpha} + 2nK_l^{*\alpha})^2} - K_l^*$$

$$= y^l + \frac{\Delta y 2^{\frac{2}{1-\alpha}} n^{\frac{3+\alpha}{1-\alpha}} [(1-2\alpha)\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}} + 2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}]}{[\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}} + 2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}]^3} \quad (35)$$

Note that both equilibrium payoffs are guaranteed to be positive when $\alpha \leq 1/2$. More generally, as a sufficient and far from necessary condition and derived from the last two terms of the equilibrium payoffs above, we need:

$\alpha < \frac{1}{2} + \min\{\frac{\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}}}{2^{\frac{\alpha}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}, \frac{2^{\frac{\alpha}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}}}\} = \widehat{\alpha}$

Let $\overline{\alpha} = \min(\widetilde{\alpha}, \widehat{\alpha})$ whenever either or both $\widetilde{\alpha}, \widehat{\alpha} \in (1/2, 1)$ exist (as defined in each of the three alternative cases above).

By drawing from our proofs of cases I, II and III, we can then conclude that the first stage expected payoff of each group is strictly concave and positive at the equilibrium either for any $\alpha \in (0, 1)$ or at least for all $\alpha < \bar{\alpha} \in (1/2, 1)$. This ensures the existence of equilibrium for this range of α. ■

Lemma A: $l'(\alpha) < 0$ if $l(\alpha) < \frac{1-n}{n}$ and $l'(\alpha) > 0$ if $l(\alpha) > \frac{1-n}{n}$.

Proof: $l'(\alpha) = \frac{\partial \frac{2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}}}}{\partial \alpha} = \frac{2^{\frac{1}{1-\alpha}} n^{\frac{1+\alpha}{1-\alpha}}}{(1-\alpha)^2 \lambda^{\frac{1}{1-\alpha}} (1-n)^{\frac{1+\alpha}{1-\alpha}}} [\ln(2n^2) - \ln(\lambda(1-n)^2)]$ which is negative or positive as $2n^2$ is less or greater than $\lambda(1-n)^2$. The latter is, in turn, equivalent to $\frac{2n^2}{\lambda(1-n)^2}$ and $\frac{2n^2}{\lambda(1-n)^2}^{\frac{1}{1-\alpha}}$ being less or greater than 1. By re-arranging the terms, $l(\alpha)$ can be expressed as $l(\alpha) = \left(\frac{2n^2}{\lambda(1-n)^2}\right)^{\frac{1}{1-\alpha}} \frac{1-n}{n}$. Hence $l(\alpha) < \frac{1-n}{n}$ when $\left(\frac{2n^2}{\lambda(1-n)^2}\right)^{\frac{1}{1-\alpha}} < 1$ thus implying that $l'(\alpha) < 0$. Similarly, when $l(\alpha) > \frac{1-n}{n}$, it follows that $\left(\frac{2n^2}{\lambda(1-n)^2}\right)^{\frac{1}{1-\alpha}} > 1$ thus implying that $l'(\alpha) > 0$. ■

Supplementary Appendix

In this supplementary appendix, we explore equilibrium choice of policy positions when the majority position coincides with one of group's ideal policy position.

The case of $\widetilde{\mathbf{t}} = 1$.

Suppose that Assumption 1, (2) and (10) hold and $\widetilde{t} = 1$ so that the majority position is identical to group B's ideal position. In this case, using (13), we observe that:

$$P_A = \frac{\frac{a\lambda}{\left|\widetilde{t}-t_A\right|+1}}{\frac{a\lambda}{\left|\widetilde{t}-t_A\right|+1} + \frac{bc}{\left|\widetilde{t}-t_B\right|+1}} = \frac{\frac{\overline{\lambda}}{2-t_A}}{\frac{\overline{\lambda}}{2-t_A} + \frac{1}{2-t_B}} = \frac{\overline{\lambda}(2-t_B)}{\overline{\lambda}(2-t_B)+(2-t_A)}$$

Substituting for P_A as given by the above equation, we get:

$$\pi_A(t_A, t_B) = a\left\{\overline{\lambda}^2(2-t_B)^2\frac{(t_B-t_A)}{(\overline{\lambda}(2-t_B)+(2-t_A))^2} - t_B\right\}$$

From the above expression it follows that the sign of $\frac{\partial \pi_A}{\partial t_A}$ is completely determined by the sign of $\frac{\partial}{\partial t_A}\frac{(t_B-t_A)}{(\overline{\lambda}(2-t_B)+(2-t_A))^2} = \frac{1}{(\overline{\lambda}(2-t_B)+(2-t_A))^3}\left(\left(2+\overline{\lambda}\right)t_B - t_A - 2(\overline{\lambda}+1)\right)$.

Since $0 \leq t_A, t_B \leq 1, (\overline{\lambda}(2-t_B)+(2-t_A))^3 > 0$. The term $\left(\left(2+\overline{\lambda}\right)t_B - t_A - 2(\overline{\lambda}+1)\right)$ assumes its highest value when $t_B = 1$ and $t_A = 0$ which is $-\overline{\lambda}$. Hence it follows that $\frac{\partial}{\partial t_A}\frac{(t_B-t_A)}{(\overline{\lambda}(2-t_B)+(2-t_A))^2} < 0$ for any $0 \leq t_A, t_B \leq 1$. Hence $t_A^* = 0$ regardless of the level of $\overline{\lambda}$.

Since $P_B = 1 - P_A$, it follows that:

$$\pi_B(t_A, t_B) = b\left\{(2-t_A)^2\frac{(t_B-t_A)}{(\overline{\lambda}(2-t_B)+(2-t_A))^2} - (1-t_A)\right\}$$

From the above expression it follows that the sign of $\frac{\partial \pi_B}{\partial t_B}$ is completely determined by the sign of $\frac{\partial}{\partial t_B}\frac{(t_B-t_A)}{(\overline{\lambda}(2-t_B)+(2-t_A))^2} = \frac{1}{(\overline{\lambda}(2-t_B)+(2-t_A))^3}\left(\left(2(\overline{\lambda}+1\right) + \overline{\lambda}t_B - (2\overline{\lambda}+1)t_A\right)$.

Since $0 \leq t_A, t_B \leq 1, (\overline{\lambda}(2-t_B)+(2-t_A))^3 > 0$. The term $\left(2(\overline{\lambda}+1\right) + \overline{\lambda}t_B - (2\overline{\lambda}+1)t_A)$ assumes its lowest value when $t_B = 0$ and $t_A = 1$ which is $1 > 0$. Hence from this it follows that $\frac{\partial \pi_B}{\partial t_B} > 0$ for the entire policy space $0 \leq t_A, t_B \leq 1$. Hence

$t_B^* = 1$ regarless of the level of $\overline{\lambda}$.[21]

The case of $\widetilde{\mathbf{t}} = 0$.

Suppose that Assumption 1, (2) and (10) hold and $\widetilde{t} = 0$ so that the majority position is identical to the ideal policy position of group A. In this case, using (13), we observe that:

$$P_A = \frac{\overline{\lambda}(1+t_B)}{\overline{\lambda}(1+t_B)+(1+t_A)}$$

Substituting for P_A as given by the above equation, we get:

$$\pi_A(t_A,t_B) = a\left\{\overline{\lambda}^2(1+t_B)^2\frac{(t_B-t_A)}{(\overline{\lambda}(1+t_B)+(1+t_A))^2} - t_B\right\}$$

From the above expression it follows that the sign of $\frac{\partial \pi_A}{\partial t_A}$ is completely determined by the sign of $\frac{\partial}{\partial t_A}\frac{(t_B-t_A)}{(\overline{\lambda}(1+t_B)+(1+t_A))^2} = \frac{1}{(\overline{\lambda}(1+t_B)+(1+t_A))^3}\left(-(\overline{\lambda}+1)-(2+\overline{\lambda}\right)t_B+t_A)$. Since $0 \le t_A, t_B \le 1$, $(\overline{\lambda}(1+t_B)+(1+t_A))^3 > 0$. The term $\left(-(\overline{\lambda}+1)-(2+\overline{\lambda}\right)t_B+t_A)$ assumes its highest value when $t_B = 0$ and $t_A = 1$ which is $-\overline{\lambda}$. Hence it follows that $\frac{\partial}{\partial t_A}\frac{(t_B-t_A)}{(\overline{\lambda}(1+t_B)+(1+t_A))^2} < 0$ for any $0 \le t_A, t_B \le 1$ Hence $t_A^* = 0$ regardless of the level of $\overline{\lambda}$.

Since $P_B = 1 - P_A$, it follows that:

$$\pi_B(t_A,t_B) = b\left\{(1+t_A)^2\frac{(t_B-t_A)}{(\overline{\lambda}(1+t_B)+(1+t_A))^2} - (1-t_A)\right\}$$

From the above expression it follows that the sign of $\frac{\partial \pi_B}{\partial t_B}$ is completely determined by the sign of $\frac{\partial}{\partial t_B}\frac{(t_B-t_A)}{(\overline{\lambda}(1+t_B)+(1+t_A))^2} = \frac{1}{(\overline{\lambda}(1+t_B)+(1+t_A))^3}\left((\overline{\lambda}+1\right)-\overline{\lambda}t_B+(2\overline{\lambda}+1)t_A))$. Since $0 \le t_A, t_B \le 1$, $(\overline{\lambda}(1+t_B)+(1+t_A))^3 > 0$. The term $(\overline{\lambda}+1)-\overline{\lambda}t_B+(2\overline{\lambda}+1)t_A$ assumes its lowest value when $t_B = 1$ and $t_A = 0$ which is $1 > 0$. Hence from this it follows that $\frac{\partial \pi_B}{\partial t_B} > 0$ for the entire policy space $0 \le t_A, t_B \le 1$. Hence $t_B^* = 1$

[21] This result is a natural extension of Proposition 3 (i) where it can be seen that when $\widetilde{t} \approx 1$, the interval $\left(\frac{1+\widetilde{t}}{\widetilde{t}}, \frac{1+\widetilde{t}}{2\widetilde{t}-1}\right)$ is almost empty as both $\frac{1+\widetilde{t}}{\widetilde{t}}$ and $\frac{1+\widetilde{t}}{2\widetilde{t}-1}$ are approximately equal to 2. This implies that B effectively always chooses its ideal position regardless of the level of bias in favor of A.

regardless of the level of $\overline{\lambda}$.[22]

The above results imply that Proposition 2 holds for any value of $\overline{\lambda}$ when the majority position coincides with one of the group's ideal points.

[22]This result is a natural extension of Proposition 3 (ii) where it can be seen that when when $\widetilde{t} \approx 0$, and therefore the majority position is almost aligned with A's ideal point, $\frac{1+\widetilde{t}}{\widetilde{t}} \approx \infty$, which implies that B prefers to stick to its preferred position irrespective of the degree of bias against it.