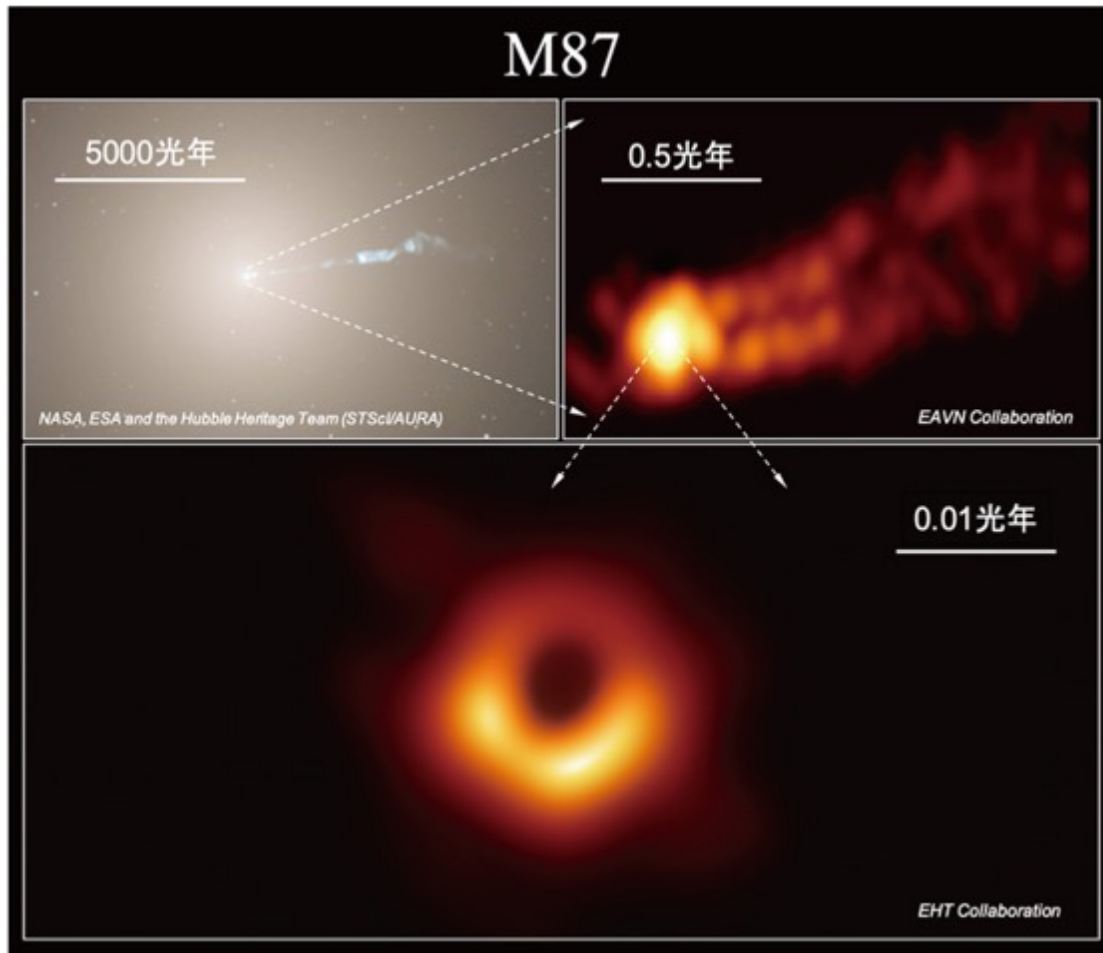


Plasma Astrophysics

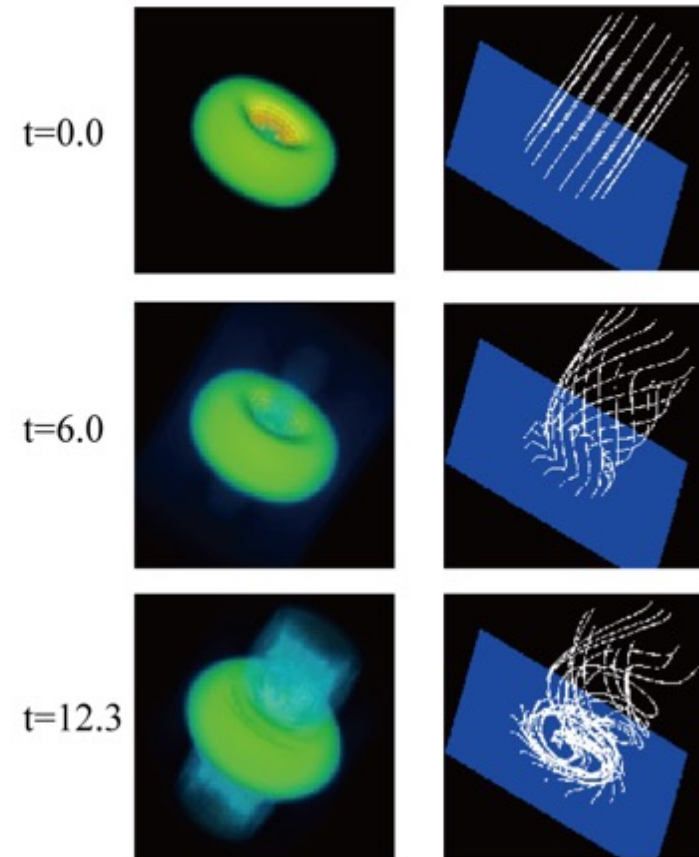
Toshiki Tajima, UCI

Class 3:PHY249 (2020Spring)



Event Horizon Telescope (2020)

3D Structure of Disk and Jet



Tajima Shibata (1997) p. 387



H. Alfvén (1981) "Cosmic Plasma"

Filamentary plasma

What are the ingredients?

gravity

plasma

B

to make a spaghetti plasma

Can you also point out other

filamentary plasma pictures

in the textbook (1997)?

Can you tell what in each picture

made them filamentary?

Fig. II.10. Contrast-enhanced photograph of the Orion nebula (*Sky and Telescope*, April, 1979).

ect. None of the observed filaments can be regarded with any certainty as resulting
m such effects, whereas in all cases there is either convincing proof, or reasonably
one evidence. Outside of the region accessible

What part of astrophysics?

- Frontiers of astrophysics only (that are not yet well understood):

highest energy particles (e.g. of cosmic rays $> 10^{20}\text{eV}$, high energy neutrinos)

highest energy photons (e.g. γ -rays up to TeV /PeV)

most violent processes (e.g. disruptive accretion; jets)

episodic and eruptive (e. g. γ -ray bursts)

young objects (e. g. AGN, Blazars, colliding galaxies (M82), microquasars (SS433),...)

neutron-star x neutron-star collision \rightarrow plasma plays essential role

.....

- I have no time to cover:

old objects (e.g. our galaxy, our Sun, our solar system), gravitational dominants

quiet, steady-state objects

objects where little plasma such as the Moon (“the older, the less plasma”)

single particle interacts with astronomical object (cf. collective interaction N^2)

.....

(our textbook covers some of both kinds)

Distinction between gravity \leftrightarrow EM

- Both: range can be **infinite** \leftarrow Gauss law
- Strong and weak interactions:
 - \rightarrow range $O(\text{fermi} = \text{fm})$
- Grav: **no** negative mass; EM: + and **-**, but can be combined;
no magnetic monopole \rightarrow magnetic force range finite
- EM: if combined +/- \rightarrow atoms : range $O(\text{\AA})$
- EM: +/- \rightarrow Debye screening: range $O(v_t/\omega_p)$
 - \rightarrow collisionless skindepth: range $O(c/\omega_p)$
 - \rightarrow EM radiation infinite range

However,

- With **B** : fields screening **removed** \leftarrow **Alfven** effect
mediated @ v_A
: **texture** appears
- **Collective**/ violent proc. \rightarrow ephemeral struct. or
 \rightarrow **robust struct = wake** @ **c**

Examples of base processes

Parker instability

(ballooning instability) → Flux buoyancy

MRI → twisted magnetic amplification;
jet formation

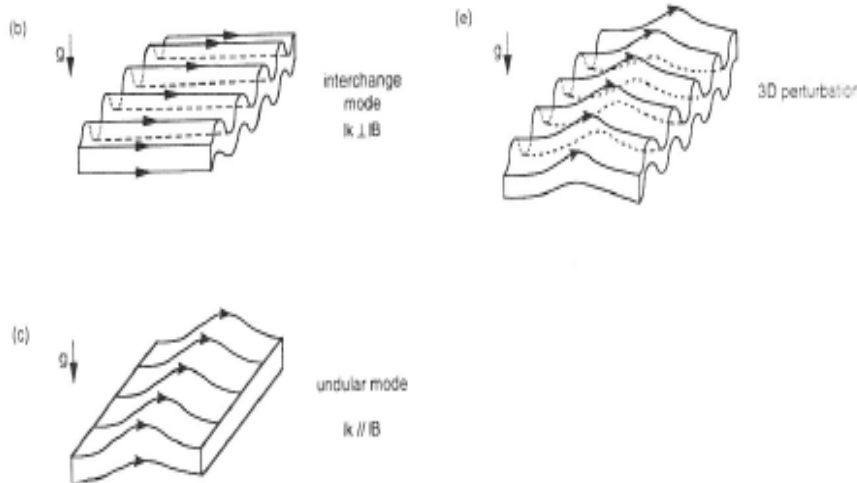


FIGURE 3.18 Interchange mode and undular (Parker) mode of magnetic buoyancy instability.

3.2.1.2 Magnetic Buoyancy Instability and Parker Instability

... to be in equilibrium. On the

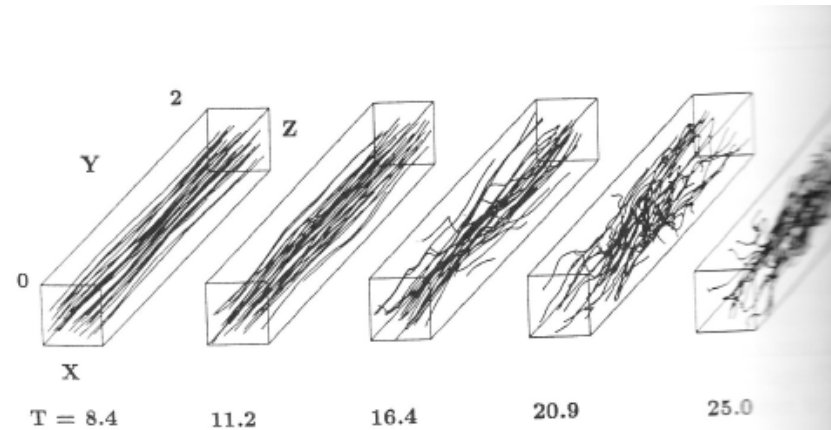


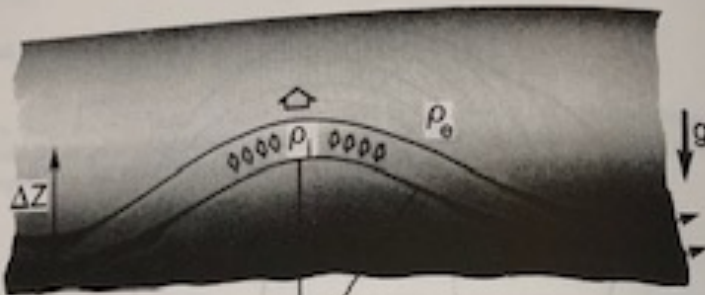
FIGURE 4.27 Magnetic field lines for model T in the eigenmode growth state [$t = (8.4 - 16.4)/\Omega$] saturation stage [$t = (20.9 - 25.0)/\Omega$] (Matsumoto and Tajima, 1995).

4.2.3.6 Effects of the Parker Instability*

When the vertical gravity is included, magnetic field escapes from the disk due to the Parker instability (the magneto-buoyancy instability; see Sec. 3.2). The growth rate of the Parker instability is $2 - 5H/v_A$, the growth rate of the Parker instability becomes comparable to that of the magnetic shearing instability as β approaches

Nonlinear evolution of Parker Instability

Collaboration of gravity and plasma + B → nonlinear evolution (can be explosive*)



Mass falls off along the flux tube
 → Stimulate further growth of balloon
 → "overshoot"

FIGURE 3.25 Undulating flux tube (or sheet).

as shown before (Eq. 3.2.4). Since the magnetic tension force is of order of $B^2/(4\pi\lambda)$, the instability condition becomes

$$\lambda > 2H. \quad (3.2.34)$$

Consequently, there is a critical wavelength below which the Parker mode is stable and the critical wavelength is of order of the local pressure scale height. As discussed earlier, above isothermal isolated flux tube is not in equilibrium, and hence these calculations are not exact (buoyancy force is too large).

Now consider a flux sheet in equilibrium and assume that both sound speed C_s and Alfvén speed V_A are constant. The unperturbed state of plasmas and magnetic field are given by the following equations,

$$p/p_0 = \rho/\rho_0 = B^2/B_0^2 = \exp(-z/\Lambda), \quad (3.2.35)$$

where

obtained from a hydrostatic balance along the flux tube (sheet) and there is no magnetic force along the flux tube, the density at the top of the raised portion of the tube becomes

$$\rho_i(\Delta z) \approx \rho_0 \exp(-\Delta z/H) \approx \rho_0(1 - \Delta z/H), \quad (3.2.37)$$

where

$$H = C_s^2/g. \quad (3.2.38)$$

On the other hand, the density outside the tube (sheet) at the same height (Δz) is

$$\rho_e(\Delta z) \approx \rho_0 \exp(-\Delta z/\Lambda) \approx \rho_0(1 - \Delta z/\Lambda). \quad (3.2.39)$$

Hence, the net density depression at the top of the loop is

$$\Delta\rho \approx \rho_e(\Delta z) - \rho_i(\Delta z) \approx \rho_0\Delta z\left(\frac{1}{H} - \frac{1}{\Lambda}\right). \quad (3.2.40)$$

The curvature radius r is rewritten using the wavelength λ as

$$r \approx \left(\frac{\lambda}{4}\right)^2 \frac{2}{\Delta z}. \quad (3.2.41)$$

Then after some manipulation, the condition for occurrence of the Parker instability $\Delta\rho/g > B^2/4\pi r$ becomes

$$\lambda^2 > \lambda_c^2 = 16\Lambda^2/(1 + 1/\beta), \quad (3.2.42)$$

where β is the ratio of gas pressure to magnetic pressure.

Finally the instability condition for wavelength becomes

$$\lambda > \lambda_c = 4\Lambda/(1 + 1/\beta)^{1/2}. \quad (3.2.43)$$

An exact treatment (Parker, 1966, 1979) shows the dispersion relation for $k_\perp\Lambda \gg 1$ as follows

$$(2/\beta + \gamma)\Omega^4 + [(4/\beta)(1/\beta + \gamma)(k_\perp^2\Lambda^2 + 1/4) + \gamma - 1]\Omega^2 + (2/\beta)k_\perp^2\Lambda^2[(2/\beta)\gamma k_\perp^2\Lambda^2 - (1 + 1/\beta)(1 + 1/\beta - \gamma)] = 0, \quad (3.2.44)$$

where

$$\Omega = \frac{\omega\Lambda}{C_s} \quad (3.2.45)$$

and y is the direction perpendicular to both vertical and magnetic fields. From this, we find the exact critical wavelength for $k_\perp\Lambda \gg 1, \gamma = 1$ as

*Explosive processes

- Cooperative process
- “vicious cycle” (or “rich gets richer process”)

- $da_1/dt = \gamma a_2 a_1,$

$$da_2/dt = \gamma a_1 a_2.$$

$$\rightarrow a_{1,2} \sim 1 / (t_0 - t)^\alpha$$

explodes in finite time (t_0) to infinite amplitude

cf. $da_1/dt = \gamma a_0 a_1 - \beta a_1.$

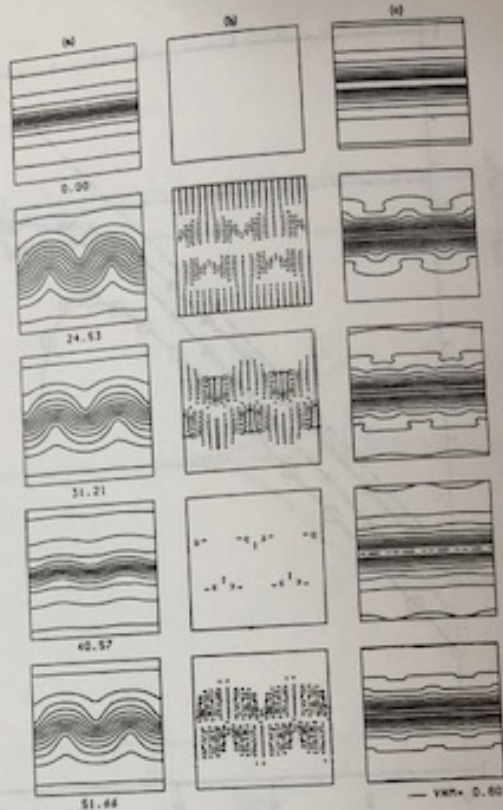
exponentiates if $a_0 > \beta/\gamma$

Beginning of structure formation

via Parker process

Consequences:

- 1. the **escape** of amplified B-field in the disk
2. pinching of plasma radially accentuated, forming **streaks** of dense regions
3. allow accentuated **magneto-rotational instability onset**
4. assist **jet formation**
5. assist **accretion of clumped matter**



3.27 Nonlinear evolution of the Parker instability in the case of weak magnetic field ($\beta = 0.88$). Note that in this case shock waves are not formed, but the nonlinear oscillation occurs

$H_m = C_s^2/g_{max}$ and $g_{max} = 0.385GM/r_0^2$. When applying this result on disks and galaxies, we can assume H_m approximately corresponds to the disk when $\beta > 1$. This result would be important to estimate the accretion rate of the accretion disk and the accretion rate of the accretion disk and in

5 Self-Similar Evolution*

et al. (1989a, 1989b) found a self-similar expansion of a magnetic loop in the case of the Parker instability. They performed 2D nonlinear simulation of an isolated magnetic flux sheet embedded in a field free gas to explain the emergence of magnetic flux tubes in the solar atmosphere. The point of their model is that the accretion rate in the accretion region

Magnetic buoyancy and twist in jets from accretion disk

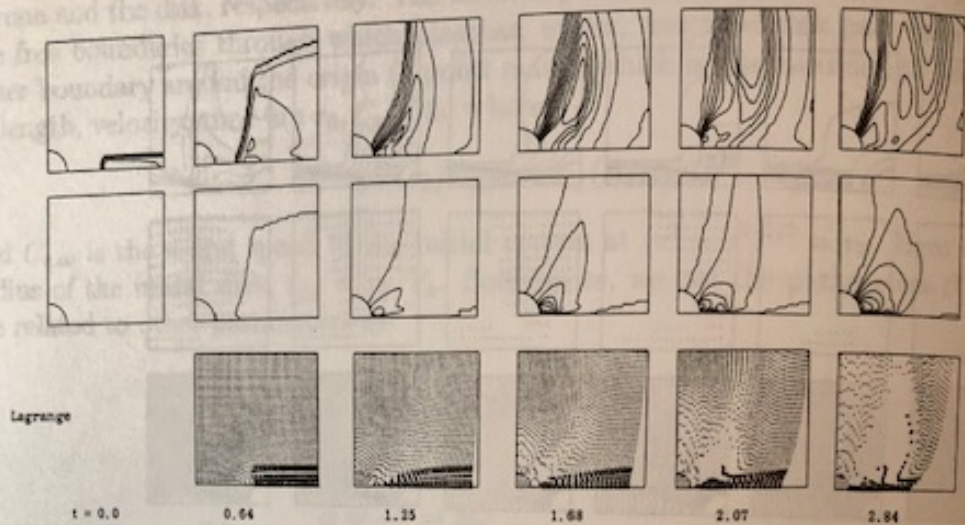


FIGURE 4.53 2.5D MHD sim. of magnetic twist jet (Shibata and Uchida, 1986a): v_ϕ , B_ϕ , Lagrange.

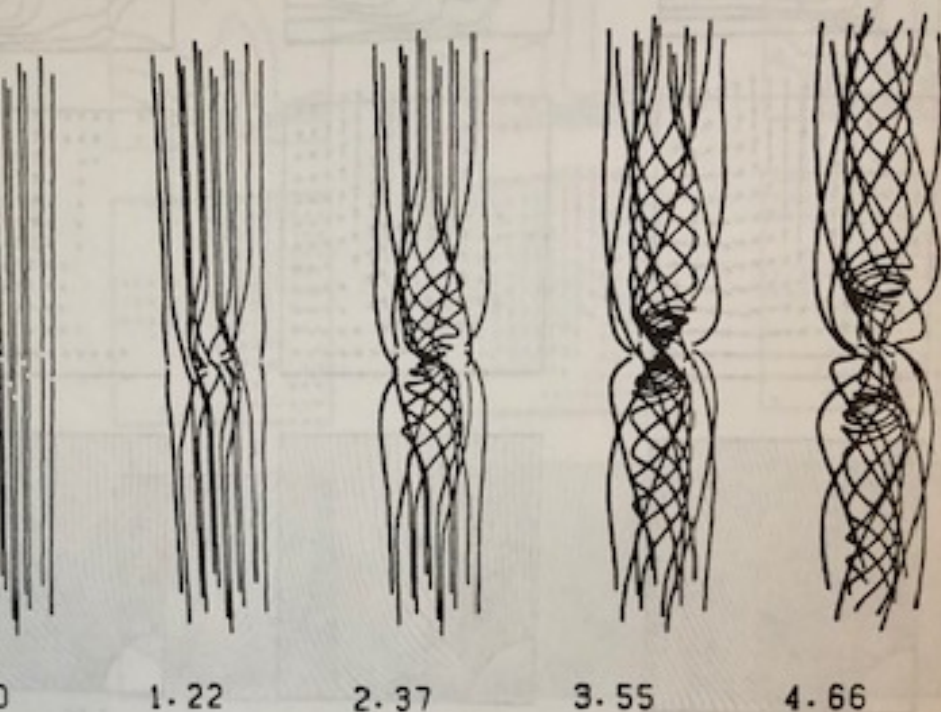
of jet.

Figure 4.55 shows the dependence on the plasma β . It is seen that the magnetic jet is more rigid in low β ($= 0.3$) case, while it is more undulating in high β ($= 5$) case. It is found that the velocity of jet is higher in low β case than in high β case. Empirical relation is written as

$$V_{\text{jet}} \sim \beta^{-0.3 \sim -0.4} \sim B^{0.5 \sim 0.7}$$

Interestingly, this relation is roughly in agreement with the relation of Michel's magnetic energy solution for a fast rotator (equation (4.3.35) in previous subsection). According to Michel and Shibata (1986, unpublished), the low β jet is accelerated mainly by the centrifugal force, whereas the high β jet is by the magnetic pressure gradient force. The dependence of jet velocity on initial rotational velocity is shown in Figure 4.56. From this, we find that even the initial rotational velocity is Keplerian case leads to contraction due to magnetic braking and hence eventually the jet velocity is slower than the sub-Keplerian case. More detailed comparison between the jet formation and the Balbus-Hawley mechanism (or Velikhov-Chandrasekhar

A consequence of Parker process
But we need twist in addition.



Gravity + Plasma + B → Jet formation

buoyancy (not much angular momentum)

More global – **twist (angular momentum)**
→ jet

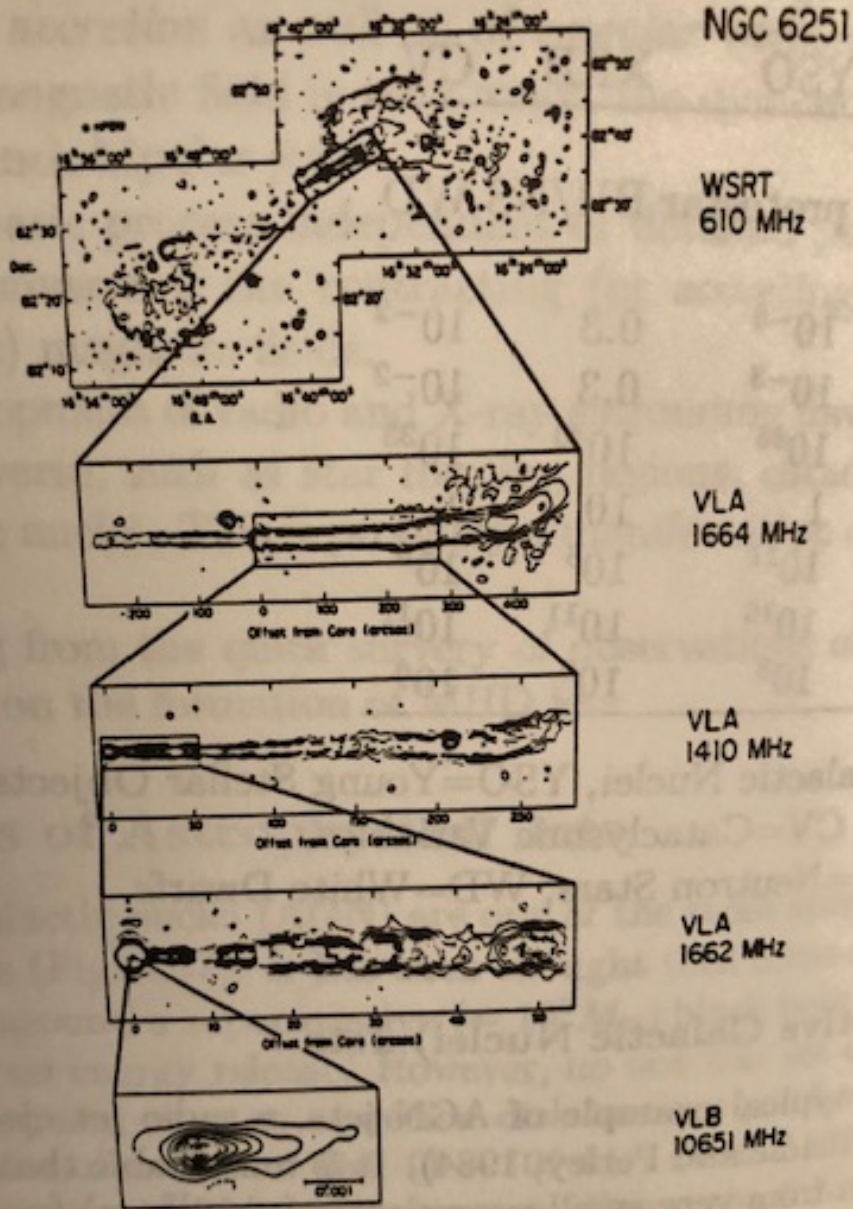
magnetic field configuration of *magnetic-twist jet* developed by Shibata and Uchiyama

Steady MHD Jets from Thick Disks*

In an AGN jet model, Matsumoto *et al.* (1996a) studied the case of a poloidal field, by performing nonsteady 2.5D MHD simulations (Uchiyama 1986a,b). (See Sec. 4.2.2.5.) Their results (see Fig. 4.60) show that (1) outflow flow occurs along the surface of thick disks. (2) Magnetic reconnection (Uchiyama and Hawley, 1991) occurs inside thick disks. (3) The velocity of the jet is comparable to the Keplerian velocity, $V_{\text{jet}} \sim 0.6 - 2.3V_k \propto B^{0.15-0.2}$. (4) The mass loss rate by the jet \dot{M} is in proportion to \dot{M}_{in} . For the three-dimensional cases see Fig. 4.60.

Interaction between Stellar Magnetosphere and Accretion
Field*

Extended structure of jets



Radio images of jets ejected from nucleus of radio galaxy, NGC6251 (from Bridgman et al. 1997)

Jet (magnetically-driven flow):

1D Steady Magnetically Driven (Centrifugal) Wind

One-dimensional, steady, centrifugal wind theory was developed by Weber and Davis (1967) for a solar wind on the equatorial plane. Taking spherical coordinate (r, φ, θ) , we assume steady state $\partial/\partial t = 0$, axisymmetry $\partial/\partial\varphi = 0$, no non-plane magnetic and velocity components $\mathbf{B} = (B_r, B_\varphi, 0)$, $\mathbf{v} = (v_r, v_\varphi, 0)$, ideal (adiabatic) MHD, and 1D ($\partial/\partial\theta = 0$) on the equatorial plane ($\theta = \pi/2$). Then, basic MHD equations are integrated into the following conservation equations;

$$\rho v_r r^2 = f, \quad (4.3.9)$$

$$r^2 B_r = \Phi, \quad (4.3.10)$$

$$r \left(v_\varphi - \frac{B_r B_\varphi}{4\pi \rho v_r} \right) = \Omega r_A^2, \quad (4.3.11)$$

$$r(v_r B_\varphi - v_\varphi B_r) = -\Omega r^2 B_r, \quad (4.3.12)$$

$$p = K \rho^\gamma, \quad (4.3.13)$$

$$\frac{1}{2} v_r^2 + \frac{1}{2} (v_\varphi - \Omega r)^2 + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} - \frac{GM}{r} - \frac{\Omega^2 r^2}{2} = E. \quad (4.3.14)$$

The mass flux, Φ is the magnetic flux, Ω is the angular velocity of the Sun (or star) (or rotating body) from which wind or jet comes out, r_A is the Alfvén radius (discussed later), K is a constant depending only on entropy, and E is the total energy of the wind. The parameters f , Φ , Ω , r_A^2 , K , E are integral constants. The unknown variables are v_r , B_r , v_φ , B_φ , and p . Hence, if these six constants are given, the equations are integrated to determine that six unknown physical quantities are determined at each r . Integrating v_φ in equations (4.3.11) and (4.3.12), we find

we obtain

Hence the de
Let us now
Eliminating v
using other eq

$H(r)$

where

Since the equ

Hence the poi
obtain

Here

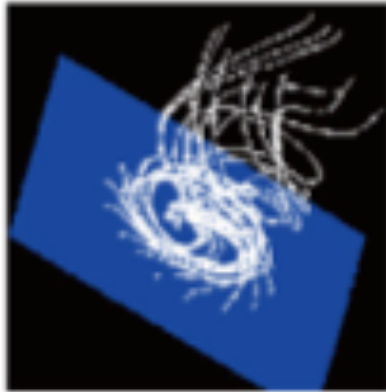
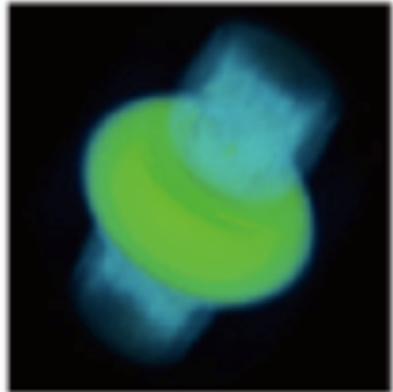
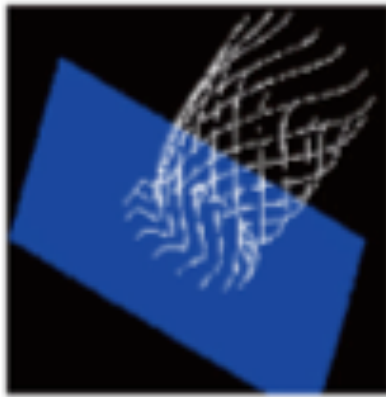
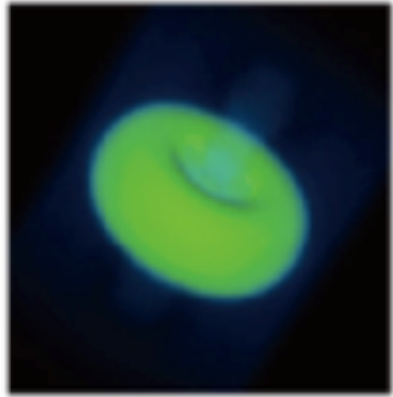
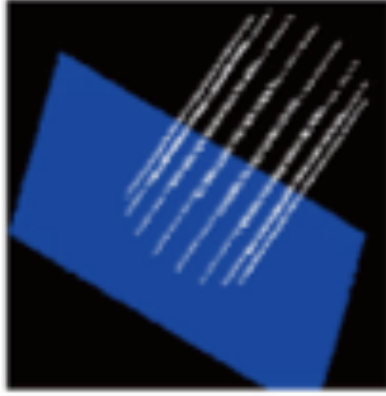
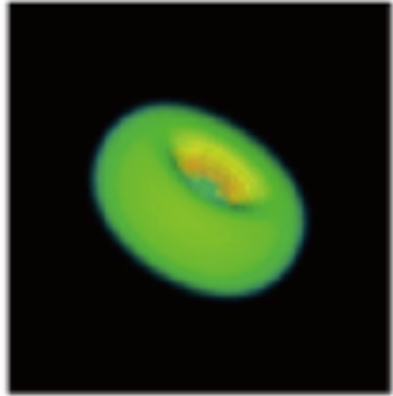
steady-state solution

Bernoulli Eq.

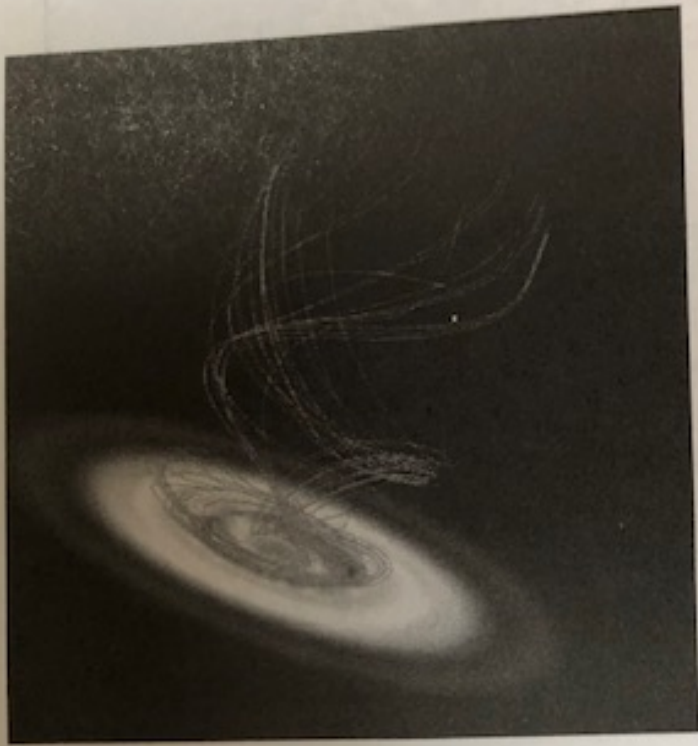
energy $E = \text{const}$
of gravity
potential ψ
angular freq Ω
(along the stream line)

3D Structure of Disk and Jet

Jets from BH and magnetic fields



Magneto-Rotational Instability (MRI)



Accretion disk
rotating plasma
B-fields

(will discuss
filamentary
singularity)

Balbus-Hawley (1991)

Matsumoto Tajima (1995)

FIGURE 4.31 (a) Magnetic field lines and equatorial density; (b) Projection of magnetic field lines (Matsumoto et al. 1995).

rotating magnetized disks (magnetic Papaloizou-Pringle instability) is observed; (iii) a helical structure is observed. Text (pp.323-353)

Growth and patterns of MRI

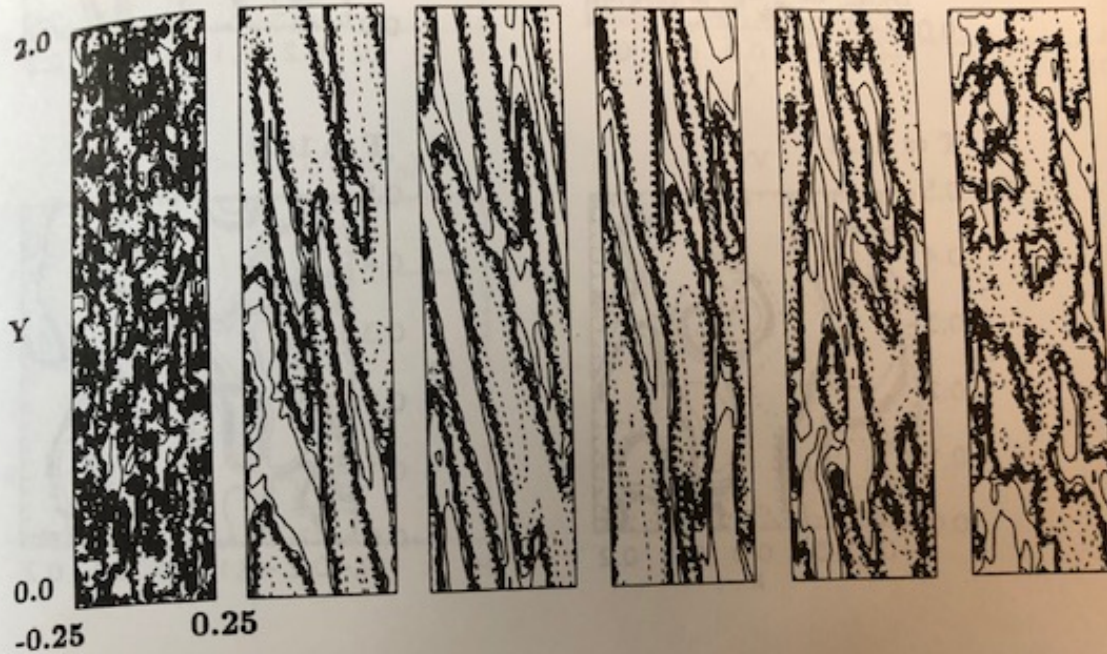


FIGURE 4.28 Result of three-dimensional MHD simulation of the disk with the Keplerian velocity shear (model T). The unperturbed magnetic field is uniform and purely toroidal. The initial plasma β is 100. The isocontours of the azimuthal velocity perturbation δv_θ in the $z = 0.25H$ plane are shown. The contour step width is 0.5 in logarithmic scale. Solid curves and dashed curves represent the positive velocity and the negative velocity, respectively (from Matsumoto and Tajima, 1985).

Accretion plasma flow patterns:

← θ - r plane : δv_θ -pattern



Fluctuations: Magnetic (δB) + plasma velocity (δv)



Enhanced transport = anomalous viscosity η

Next page: coupled equations for δv , $\delta B \rightarrow 2^{\text{nd}}$ order ordinary diff. eq. in radial dir.

\rightarrow analytical and physical solutions (even though there is an Alfvén singularity exists) found

4.2.3.2 Alfvén Singularities and Eigenmodes*

In this subsection we derive the wave equation in differentially rotating magnetized disks and solve the eigenvalue problem. In the unperturbed state, the density, pressure, and magnetic fields are assumed to be uniform. By assuming that $v_x = v_z = 0$ in the unperturbed state, the unperturbed momentum equation is

$$\mathbf{g} + 2\mathbf{v}_0 \times \boldsymbol{\Omega} + (\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega} = 0. \quad (4.2.49)$$

We further assume that $B_z = 0$ in the unperturbed state. We linearize the basic equations and look for eigenmode solutions of the form $\bar{\phi}(x, t) \exp[i(k_y y + k_z z)]$ according to the above discussion. The Laplace transform of the momentum equation and the induction equation are

$$-i\omega_0 \bar{\mathbf{v}} - 2A\bar{v}_y \hat{y} - 2\bar{\mathbf{v}} \times \boldsymbol{\Omega} - \frac{(\mathbf{B}\nabla)\bar{\mathbf{b}}}{4\pi\rho} + \nabla \left(\frac{\mathbf{B}\bar{\mathbf{b}}}{4\pi\rho} + \frac{\delta p}{\rho} \right) - \nu \nabla^2 \bar{\mathbf{v}} = \bar{\mathbf{v}}(x, 0), \quad (4.2.50)$$

$$-i\omega_0 \bar{\mathbf{b}} + 2A\bar{b}_y \hat{y} - (\mathbf{B}\nabla)\bar{\mathbf{v}} - \eta \nabla^2 \bar{\mathbf{b}} = \bar{\mathbf{b}}(x, 0), \quad (4.2.51)$$

$$\omega_0 = \omega + 2Ak_y x \quad (4.2.52)$$

is the Doppler shifted frequency, $\bar{\mathbf{b}}$, $\bar{\mathbf{v}}$, and δp are Laplace transform of the magnetic field, velocity, and the pressure perturbations, respectively.

Substituting these results into the Laplace transform of the continuity equation $\nabla \cdot \mathbf{v} = 0$ yields the initial value equation

$$\omega_0^2 \omega_{\text{ep}}^4 \frac{d^2 v_x}{dx^2} + 4A\omega_0^2 k_y \omega_{\text{ep}} \omega_{\text{ep}}^2 \frac{dv_x}{dx} + \left[-(k_y^2 + k_z^2) \omega_0^2 \omega_{\text{ep}}^4 - 8A^2 k_y^2 \omega_0^2 \omega_{\text{ep}}^2 + \kappa^2 k_z^2 \omega_0^2 (\omega_0^2 + A\omega_0^2 / \Omega_B) \right] v_x = \Gamma(x, \omega) \quad (4.2.53)$$

$$\omega_0 = \omega_0 + i\eta(k_y^2 + k_z^2 - \frac{d^2}{dx^2}), \quad (4.2.54)$$

$$\omega_0 = \omega_0 + i\nu(k_y^2 + k_z^2 - \frac{d^2}{dx^2}), \quad (4.2.55)$$

$$\omega_{\text{ep}}^2 = \omega_0 \omega_{\text{ep}} - \omega_A^2. \quad (4.2.56)$$

Here $\Omega_B = \Omega - A$, $\kappa = 2(\Omega\Omega_B)^{1/2}$ is the epicyclic frequency, and ω_A is the Alfvén frequency

$$\omega_A^2 = \frac{(\mathbf{k} \cdot \mathbf{B})^2}{4\pi\rho} = k_z^2 v_A^2. \quad (4.2.57)$$

Alfvén singularity

The initial condition enters through the right-hand side of Eq. (4.2.53).

First, we consider the case when there is no dissipation ($\eta = \nu = 0$). The homogeneous part of Eq. (4.2.53) reduces to the second order differential equation as

$$\frac{d^2 v_x}{dx^2} + \frac{4A\omega_0^2 k_y}{\omega_0(\omega_0^2 - \omega_A^2)} \frac{dv_x}{dx} + \left[-(k_y^2 + k_z^2) - \frac{8A^2 k_y^2 \omega_0^2}{\omega_0^2(\omega_0^2 - \omega_A^2)} + \kappa^2 k_z^2 \frac{\omega_0^2 + A\omega_0^2 / \Omega_B}{(\omega_0^2 - \omega_A^2)^2} \right] v_x = 0.$$

It is noted that in the presence of shear flows the eigenmode equation such as Eq. (4.2.58) does not take the self-adjoint form anymore. This is in contrast to the standard MHD problem without shear flows, which has the self-adjoint form (Bernstein et al, 1958). Note that although this can be formally changed to self-adjoint form (4.2.58) must not be converted into the self-adjoint form since it includes singular term (Arfken and Weber, 1995, p. 539). Thus the eigenvalues ω are not guaranteed to be real or pure imaginary. We express Eq. (4.2.58) in terms of

$$\xi = \frac{2Ak_y x}{\omega_A} \quad (4.2.59)$$

$$\frac{d^2 v_x}{d\xi^2} + \frac{2\omega_A^2}{\omega_0(\omega_0^2 - \omega_A^2)} \frac{dv_x}{d\xi} + \left[-\left(1 + \frac{1}{q}\right) \left(\frac{\omega_A}{2A}\right)^2 - \frac{2\omega_A^4}{\omega_0^2(\omega_0^2 - \omega_A^2)} + \left(\frac{\kappa}{2A}\right)^2 \left(\frac{\omega_A^2}{q}\right) \frac{\omega_0^2 + A\omega_0^2 / \Omega_B}{(\omega_0^2 - \omega_A^2)^2} \right] v_x = 0, \quad (4.2.60)$$

$$\text{where } q = \frac{k_z^2}{k_y^2}. \quad (4.2.61)$$

This differential Eq. (4.2.60) has two singularities at $\omega_0 = \pm\omega_A$. These are the shear Alfvén singularities where the absorption and mode conversion of Alfvén waves take place (e.g. Russ et al, 1982). The locations of Alfvén singularities in the complex plane are

$$\xi_A = \pm 1 - \frac{\omega}{\omega_A}. \quad (4.2.62)$$

By applying the Frobenius method around $\omega_0 = \pm\omega_A$ (or $\xi = \xi_A$), and solving the indicial equation, we find that the exponent s in the series expansion

$$v_x(\xi) = \sum_{n=0}^{\infty} a_n(\xi - \xi_A)^{n+s}$$

becomes complex. Thus the solutions which pass through these points are singular (called regular singular points). The corotation point $\omega_0 = 0$ (or $\xi = -\omega/\omega_A$) appears to be also singular in Eq. (4.2.60). However, by solving the indicial equation, we find that the solution is regular at the corotation point.

The solutions of Eq. (4.2.60) which vanish as $\xi \rightarrow \pm\infty$ have an asymptotic form

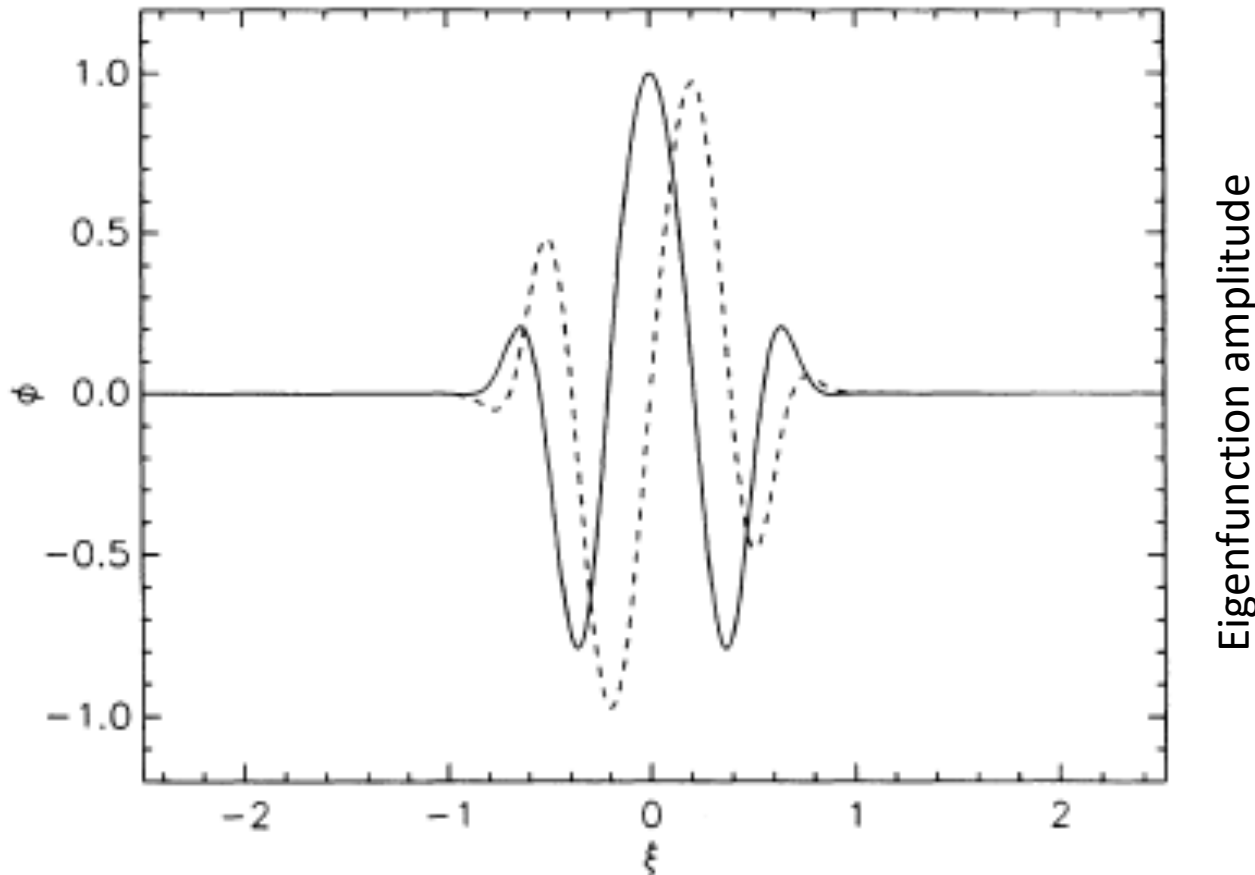
$$v_x \propto \exp\left[\mp(1 + 1/q)^{1/2} \omega_A \xi / (2A)\right]. \quad (4.2.63)$$

Eigenfunction w/ singularity

Frobenius' method index s degree of singularity → Nature of filamentary singularity

For those aficionados of math physics, enjoy the math of regular singular point eigen functions (see: Matsumoto-Tajima, Ap. J. 1995)

Solution by Frobenius Method to the MRI eigenfunction



Shear flow in the azimuthal θ
direction \rightarrow
amplitude oscillations in r
(or ξ)

FIG. 1b Normalized radial position $\xi=0$ is Alfvén singularity

Anomalous viscosity

in accretion disk

$$\text{Anomalous viscosity } \eta \sim \delta B^2 \sim O(\alpha_B)$$

$$\delta v^2 \sim O(\alpha_v)$$

$$\text{where } \alpha_B \sim (\delta v_A / C_s)^2 \\ \alpha_v \sim (\delta v / C_s)^2$$

General structure of anomalous transport

Here we set the classical viscosity $\nu=0$ and resistivity $\eta=0$

Typical astronomical *observed* and our *simulational* values of

$$\alpha \sim 0.1$$

$$\eta = \left(\frac{\pi}{2}\right)^{1/2} \frac{1}{4\pi n_e m C_s} \left(\frac{k_y^2}{k^3}\right)_{\max} \langle \delta B^2 \rangle, \quad (4.2.71)$$

where $(k_y^2/k^3)_{\max}$ is evaluated at $k = k_{\max}$.

Since magnetic fields induced by the magnetorotational instability eventually dominate over seed magnetic fields, we equate B^2 to $\langle \delta B^2 \rangle$ to evaluate the saturation level. The saturation level of magnetic fluctuations can be determined by equating the growth rate γ of the instability with the anomalous resistivity damping ηk^2 . By using Eq. (4.2.64) for γ and Eq. (4.2.71) for η , we obtain

$$\frac{\langle \delta B^2 \rangle}{4\pi n_e m C_s^2} = \chi_e \left(\frac{2}{\pi}\right) \left(\frac{k^2 k_z^2}{k_y^4}\right) \left(\frac{k_{\parallel}^2}{k_z^2}\right) f(q), \quad (4.2.72)$$

where $\chi_e = n_e/n$ is the ionization rate. The factor k_{\parallel}^2/k_z^2 is unity for purely poloidal field. For cases with a purely toroidal field $k_{\parallel}^2/k_z^2 = q$. In the marginally stable state k_{\parallel}^2/k_z^2 is between q and 1 because the magnetic fields are already perturbed.

As the disk plasma is close to the marginality, the magnetic viscosity parameter $\alpha_B = -\langle \delta B_x \delta B_y \rangle / (4\pi \rho C_s^2)$ may be approximately written (Matsumoto and Tajima, 1995) by using the linearized momentum equation [Eq. (4.2.50)] and linearized induction equation [Eq. (4.2.51)] with $\eta = \nu = 0$ as

$$\alpha_B = \chi_e \frac{\langle \delta B^2 \rangle}{4\pi n_e m C_s^2} \frac{-\langle \frac{\delta B_y}{\delta B_x} \rangle}{\langle \left(\frac{\delta B_x}{\delta B_x}\right)^2 \rangle + \langle \left(\frac{\delta B_y}{\delta B_x}\right)^2 \rangle + 1}, \quad (4.2.73)$$

where

$$\left\langle \frac{\delta B_y}{\delta B_x} \right\rangle = \frac{-2\Omega\gamma + (\gamma^2 + \omega_A^2 - 4A\Omega)(k_y/k_x)}{\gamma^2 + \omega_A^2 + 2\Omega\gamma(k_y/k_x)}, \quad (4.2.74)$$

and

$$\left\langle \frac{\delta B_x}{\delta B_x} \right\rangle = -q^{1/2} \frac{(\gamma^2 + \omega_A^2)(k_x/k_y + k_y/k_x + 2A/\gamma)}{\gamma^2 + \omega_A^2 + 2\Omega\gamma(k_y/k_x)}. \quad (4.2.75)$$

When deriving these equations, we replaced d/dx by ik_x . The notations $\langle \delta B_x \delta B_y \rangle$ etc. denote the spatial average.

The instability-induced velocity fields also contribute to the radial angular momentum transport. The viscosity parameter corresponding to the Reynolds stress $\rho \langle v_x v_y \rangle$ due to this instability is expressed (Matsumoto and Tajima, 1995) as

$$\alpha_v = \frac{\langle v_x v_y \rangle}{C_s^2} = -\alpha_B \left(\frac{\gamma^2}{\omega_A^2}\right) \frac{\langle v_y/v_x \rangle}{\langle \delta B_y/\delta B_x \rangle}, \quad (4.2.76)$$

where

$$\left\langle \frac{v_y}{v_x} \right\rangle = \left\langle \frac{\delta B_y}{\delta B_x} \right\rangle + \frac{2A}{\gamma}. \quad (4.2.77)$$

Evolution of accretion disks

- Anomalous viscosity → “growth hormone”
of the AGN (young galaxies), $0 < \alpha < 1$
stars, Universe in general
 - Feeder of matter / energy to AGN and jets
brightening and growth of AGN, jets
- Fundamentals for evolution of the Universe

Episodic transitions of accretion disks with B

(Quasi-)steady state accretion disks

cf.

Episodic accretion disks

“breathing” disks

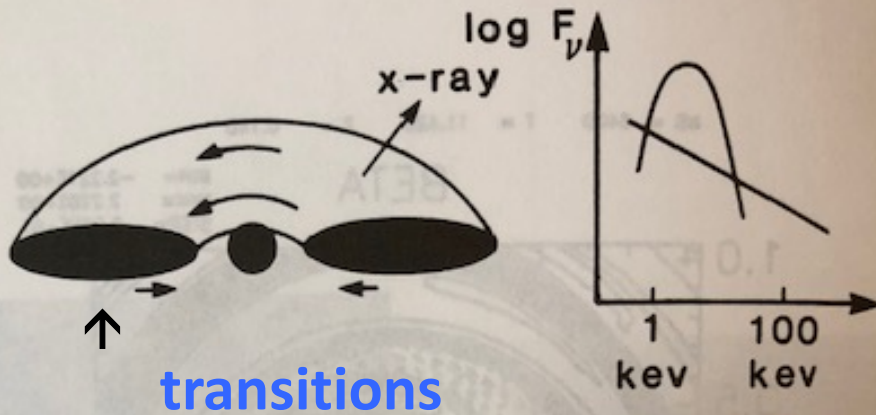
transitions between the **states**

soft state ($\beta \gg 1$)

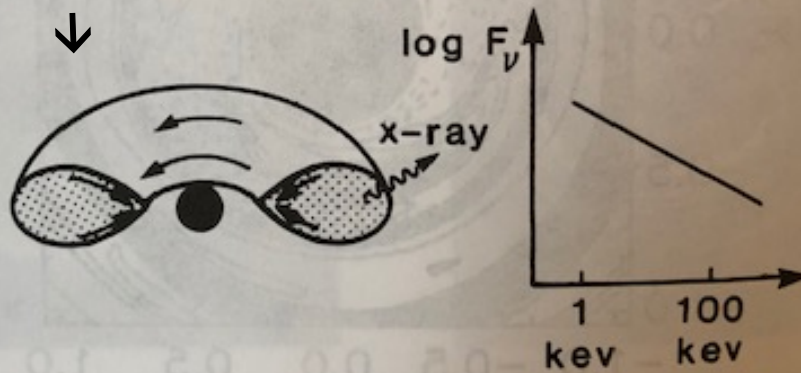


hard state ($\beta \sim 1$)

● High State (Soft State)



● Low State (Hard State)



3 Two states of accretion disks: “High state” (soft state) vs. “Low state” (hard state); the spectra

known that accretion disks in black hole candidates have two spectral states (Miyamoto et al., 1984). One is the high state and the other is the low state. In the high state, the spectra has blackbody component which can be explained by emission from optically thick accretion disks. On the other hand, in the low (or hard) state, the spectra has power-law component which may come from optically thin accretion disk (Fig. 4.33). Other