Age of Information in Multiple Sensing

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***Abstract*—Having timely and fresh knowledge about the current state of information sources is critical in a variety of applications. In particular, a status update may arrive at the destination much later than its generation time due to processing and communication delays. The freshness of the status update at the destination is captured by the notion of age of information. In this study, we first analyze a network with a single source, n servers, and the monitor (destination). The servers independently sense the source of information and send the status update to the monitor. We then extend our result to multiple independent sources of information in the presence of n servers. We assume that updates arrive at the servers according to Poisson random processes. Each server sends its update to the monitor through a direct link, which is modeled as a queue. The service time to transmit an update is considered to be an exponential random variable. We examine both homogeneous and heterogeneous service and arrival rates for the single-source case, and only homogeneous arrival and service rates for the multiple-source case. We derive a closed-form expression for the average age of information under a last-come-first-serve (LCFS) queue for a single source and arbitrary n homogeneous servers. For $n = 2, 3$, we derive the explicit average age of information for arbitrary sources and homogeneous servers, and for a single source and heterogeneous servers. For $n = 2$, we find the optimal arrival rates given fixed sum arrival rate and service rates.**

I. Introduction

Widespread sensor network applications such as health monitoring using wireless sensors [1] and the Internet of things (IoT) [2], as well as applications like efficient monitoring of physical environment [3], stock market trading and vehicular networks [4], require sending several status updates to their designated recipients (called monitors). Outdated information in the monitoring facility may lead to undesired situations. As a result, having the data at the monitor as fresh as possible is crucial.

In order to quantify the freshness of the received status update, the age of information (AoI) metric was introduced in [5]. For an update received by the monitor, AoI is defined as the time elapsed since the generation of the update. AoI captures the timeliness of status updates, which is different from other standard communication metrics like delay and throughput. It is affected by the inter-arrival time of updates and the delay that is caused by queuing during update processing and transmission.

In this paper, we consider AoI in a multiple-server network. We assume that a number of shared sources are sensed and then the data is transmitted to the monitor by n independent servers. For example, the sources of information can be some shared environmental parameters, and independently operated sensors in the surrounding area obtain such information. For another example, the sources of information can be the prices of several stocks which are transmitted to the user by multiple independent service providers. Throughout this paper, a sensor or a service provider is called a server, since it is responsible for serving the updates to the monitor. We assume that status updates arrive at the servers independently according to Poisson random processes, and the server is modeled as a queue whose service time for an update is exponentially distributed. We assume information sources are independent and are sensed by n independent servers.

In [5], the authors consider the single-source single-server and first-come-first-serve (FCFS) queue model and determine the arrival rate that minimizes AoI. Different cases of multiple-source single-server under FCFS and last-come-first-serve (LCFS) are considered in [6] and the region of feasible age is derived. In [7] and [8], the system is modeled as a source that submits status updates to a network of parallel and serial servers, respectively, for delivery to a monitor and AoI is evaluated. The parallel-server network is also studied in [9] when the number of servers is 2 or infinite, and the average AoI for FCFS queue model is derived.

The authors in [10] formulate a discrete-time decision problem in order to find a scheduling policy for minimizing the expected weighted sum of AoI. A multiple-source multiple-hop setting in broadcast wireless networks is investigated in [11] and a fundamental lower bound on the average AoI is derived. Different scheduling policies with throughput constraints are considered in [12] to minimize AoI. Another age-related metric of peak AoI is introduced in [13], which corresponds to the age of information at the monitor right before the receipt of the next update. The average peak AoI minimization in IoT networks and wireless systems is considered in [14], [15]. The problem of minimizing the average age in energy harvesting sources by manipulating the update generation process is studied in [16], [17].

In this paper, we study the average age of information as in [5]. We mainly consider LCFS with preemption in service (in short, LCFS) queue model, namely, upon the arrival of a new update, the server immediately starts to serve it and drops any old update being served. We derive a closed-form formula of the average AoI for LCFS and a single source. For multiple sources, AoI formula is derived for arbitrary number of sources and $n = 2, 3$ servers. In addition, the heterogeneous network with a single source is considered. To obtain the AoI, we use the stochastic hybrid system (SHS) analysis similar to [6], [7].

This paper is organized as follows. Section II formally introduces the system model of interest, and provides preliminaries on SHS. In subsection III-A, we derive the average age of information formula by applying SHS method to our model when we have a single information source and the network is homogeneous. In subsection III-B, we derive AoI for arbitrary number of information sources and $n = 2, 3$ servers. In section IV, we investigate the heterogeneous network for a single source and $n = 2$ servers, before concluding in section V.

II. System Model and Preliminaries

Notation: in this paper, we use boldface for vectors, and normal font with a subscript for its elements. For example, for a vector $\mathbf{x}$, the j-th element is denoted by x_j. For non-negative integers $b \geq a$, we define $[a:b] \triangleq \{a, \ldots, b\}$, and $[a] \triangleq [1:a]$, for $a \geq 1$. If $a > b$, $[a:b] = \emptyset$.

In this section, we first present our network model, and then briefly review the stochastic hybrid system analysis from [6].

The network consists of m information sources that are sensed by n independent servers as illustrated in Figure 1. Updates after going through separate links are aggregated at the monitor side. Server j collects updates of source i following a Poisson random process with rate $\lambda_j^{(i)}$ and the service time is an exponential random variable with average $\frac{1}{\mu_j}$, independent of all other servers, $j \in [n], i \in [m]$. A network is called homogeneous if $\lambda_j^{(i)} = \lambda^{(i)}, \mu_j = \mu$, for all $j \in [n], i \in [m]$, otherwise, it is heterogeneous. In the case of a single source in a homogeneous network, we denote $\lambda^{(1)}$ simply by λ.

Consider a particular source. Suppose its freshest update at the monitor at time t is generated at time $u(t)$, the *age of information* at the monitor (in short, AoI) is defined as $\Delta(t) = t - u(t)$, which is the time elapsed since the generation of the last received update. From the definition, $\Delta(t)$ linearly increases at a unit rate with respect to t, except for the reset jump to a lower value when the monitor receives a fresher update. The goal of this paper is to study the *average AoI*, which is the limit of the average age over time, $\Delta = \lim_{T\to\infty} \int_0^T \Delta(t)dt/T$. For a stationary ergodic system, it is also the limit of the average age over the ensemble $\Delta = \lim_{t\to\infty} \mathbb{E}[\Delta(t)]$.

We note a key difference between the model in this work and most previous models. Updates come from different servers, therefore they might be out of order at the monitor and thus a newly arrived update may not have any effect on AoI because a fresher update may be already delivered.

This paper considers *LCFS with preemption in service* (in short LCFS). In this queue model, upon arrival of a new update, each server immediately drops any previous update in service and starts to serve the new update.

We view our system as a stochastic hybrid system (SHS) and apply a method first introduced in [6] in order to calculate aver-

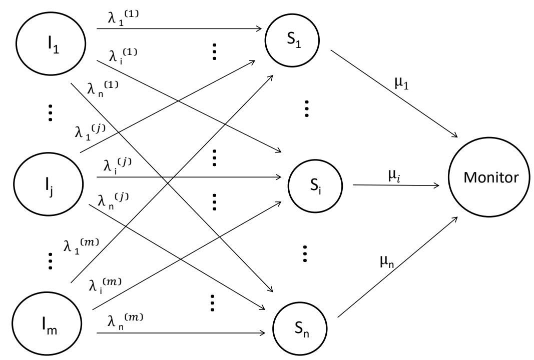

Figure 1: The n-server monitoring network with $S_1, S_2, ..., S_n$ being the independent servers and $I_1, I_2, ..., I_m$ being the independent information sources, sending the updates from the sources to the monitor.

age AoI. An SHS can be described by its states and transitions. The state is composed of a discrete state and a continuous state. The discrete state $q(t) \in \mathcal{Q}$, for a discrete set $\mathcal{Q}$, is a continuous-time discrete Markov chain (e.g., to represent the number of idle servers in the network), and the continuous-time continuous state $\mathbf{x}(t) = [x_0(t), x_1(t), \ldots, x_n(t)] \in \mathbb{R}^{n+1}$ is the stochastic process for AoI. We use $x_0(t)$ to represent the age at the monitor, and $x_j(t)$ for the age at the j-th server, $j \in [n]$. Graphically, we represent each state $q \in \mathcal{Q}$ by a node.

For the discrete Markov chain $q(t)$, transitions happen from one state to another through directed transition edge l, and the time spent before the transition is exponentially distributed with rate $\lambda(l)$. Note that it is possible to transit from the same state to itself. The transition occurs when an update arrives at a server, or an update is received at the monitor. Thus the transition rate is the update arrival rate or the service rate, $\lambda(l) \in \{\lambda_1^{(1)}, ..., \lambda_n^{(m)}, \mu_1, ..., \mu_n\}$. Denote by L'_q and L_q the sets of incoming and outgoing transitions of state q, respectively. When transition l occurs, we write that the discrete state transits from q_l to q'_l. For instance, if we have 2 states and considering the transition l from state 1 to state 2, we have $q_l = 1$ and $q'_l = 2$, which shows that state 2 is an outgoing transition for state 1 and state 1 is an incoming transition for state 2. For a transition, we denote that the continuous state changes from $\mathbf{x}$ to $\mathbf{x}'$. In our problem, this transition is linear in the vector space of $\mathbb{R}^{n+1}$, i.e., $\mathbf{x}' = \mathbf{x}A_l$, for some real matrix A_l of size $(n+1) \times (n+1)$. Note that when we have no transition, the age grows at a unit rate for the monitor and relevant servers, and is kept unchanged for irrelevant servers. Hence, within the discrete state q, $\mathbf{x}(t)$ evolves as a piece-wise linear function in time, namely, $\frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{b}_q$, for some $\mathbf{b}_q \in \{0,1\}^{n+1}$.

When the discrete state $q(t)$ is ergodic, the probability of being at state q converges uniquely to the stationary probability π_q, for all $q \in \mathcal{Q}$. We can find these stationary probabilities from the following set of equations knowing that $\sum_{q\in\mathcal{Q}} \pi_q =$

1,

$$\pi_q \sum_{l \in L_q} \lambda(l) = \sum_{l \in L'_q} \lambda(l)\pi_{q_l}, \quad q \in \mathcal{Q}. \tag{1}$$

Denote by $\mathbf{v}_q = [v_{q0}, v_{q1}, \ldots, v_{qn}]$ a real vector of length $n+1$, for $q \in \mathcal{Q}$. A key lemma we use to develop AoI for our LCFS queue model is the following from [6], which was derived from the general SHS results in [18].

Lemma 1 ([6]). *If the discrete-state Markov chain $q(t)$ is ergodic with stationary distribution $\{\pi_q, q \in \mathcal{Q}\}$, and we can find a non-negative solution of $\{\mathbf{v}_q, q \in \mathcal{Q}\}$ such that*

$$\mathbf{v}_q \sum_{l \in L_q} \lambda(l) = \mathbf{b}_q \pi_q + \sum_{l \in L'_q} \lambda(l) \mathbf{v}_{q_l} A_l, \quad q \in \mathcal{Q}, \tag{2}$$

then the average age of information is given by

$$\Delta = \sum_{q \in \mathcal{Q}} v_{q0}. \tag{3}$$

III. AoI in Homogeneous Networks

A. Single Source and Multiple Servers

In this section, we derive AoI with the LCFS queue for the single-source n-server homogeneous network with arrival rate λ and service rate μ at all servers. Note that to compute the average AoI, Lemma 1 requires solving $|\mathcal{Q}|(n+1)$ linear equations of $\{\mathbf{v}_q, q \in \mathcal{Q}\}$. To obtain explicit solutions for these equations, the complexity grows with the number of discrete states. Since the discrete state typically represents the number of idle servers in the system for homogeneous servers, $|\mathcal{Q}|$ should be $n+1$. In what follows, we introduce a method inspired by [7] to reduce the number of discrete states and efficiently describe the transitions.

We define our continuous state $\mathbf{x}$ at a time as follows: the first element x_0 is AoI at the monitor, x_1 is the freshest update among all updates in the servers, and x_2 is the second freshest update in the servers, etc. With this definition, we have $x_1 \leq x_2 \leq .... \leq x_n$, for any time. Note that the index i of x_i does not represent a physical server index, but the i-th smallest age of information among the n servers. The physical server index for x_i changes with each transition. We say that the server corresponding to x_i is the i-th *virtual* server.

A transition l is triggered by the arrival of an update at a server, or the delivery of an update to the monitor. Recall that we use $\mathbf{x}$ and $\mathbf{x}'$ to denote AoI continuous state vector right before and after the transition l.

When one update arrives at the monitor and the server for that update becomes idle, we put a *fake update* to the server using the method introduced in [7]. Thus the number of discrete states is reduced to one, indicating that all servers are virtually busy. We denote this state by $q = 0$. In particular, we put the current update that is in the monitor to an idle server until the next update reaches this server. This assumption does not affect our final calculation for AoI, because even if the fake update is delivered, AoI at the monitor does not change.

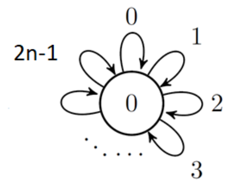

Figure 2: SHS for a single source and n homogeneous servers.

l	$\lambda(l)$	$\mathbf{x}' = \mathbf{x}A_l$
0	λ	$[x_0, 0, x_2, x_3, x_4, ..., x_n]$
1	λ	$[x_0, 0, x_1, x_3, x_4, ..., x_n]$
2	λ	$[x_0, 0, x_1, x_2, x_4, ..., x_n]$
	$\vdots$	$\vdots$
$n-1$	λ	$[x_0, 0, x_1, x_2, x_3, .., x_{n-1}]$
n	μ	$[x_1, x_1, x_1, x_1, ..., x_1]$
$n+1$	μ	$[x_2, x_1, x_2, x_2, ..., x_2]$
$n+2$	μ	$[x_3, x_1, x_2, x_3, ..., x_3]$
	$\vdots$	$\vdots$
$2n-1$	μ	$[x_n, x_1, x_2, x_3, ..., x_n]$

Table I: Table of transitions for a single source and n homogeneous servers.

When an update is delivered to the monitor from the k-th virtual server, the server becomes idle and as previously stated, receives the fake update. The age at the monitor becomes $x'_0 = x_k$, and the age at the k-th vitual server becomes $x'_k = x'_0 = x_k$. In this scenario, consider the update at the j-th virtual server, for $j > k$. Its delivery to the monitor does not affect AoI since it is older than the current update of the monitor, i.e., $x_j \geq x_k = x'_0$. Hence, we can adopt a *fake preemption* where the update for the j-th virtual server, for all $k \leq j \leq n$, is preempted and replaced with the fake current update at the monitor. Physically, these updates are not preempted and as a benefit, the servers do not need to cooperate and can work in a distributed manner.

By utilizing virtual servers, fake update, and fake preemption, we reduce SHS to a single discrete state with linear transition A_l. In Figure 2, we illustrate our SHS with discrete state space of $Q = \{0\}$. The stationary distribution π_0 is trivial and $\pi_0 = 1$. We set $\mathbf{b}_q = [1, ..., 1]$ which indicates that the age at the monitor and the age of each update in the system grows at a unit rate. The transitions are labeled $l \in [0 : 2n-1]$, and for each transition l we list the transition rate and the transition mapping in Table I. For simplicity, we drop the index $q = 0$ in the vector $\mathbf{v}_0$, and write it as $\mathbf{v} = [v_0, v_1, \ldots, v_n]$. Because we have one state, $\mathbf{x}A_l$ and $\mathbf{v}A_l$ are in correspondence. Next, we describe the transitions in Table I.

Case I. $l \in [0 : n-1]$: When a fresh update arrives at virtual server $l+1$, the age at the monitor remains the same and x_{l+1} becomes zero. This server has the smallest age, so we take this zero and reassign it to the first virtual server, namely, $x'_1 = 0$. Accordingly, virtual server $l+1$ becomes virtual server 1, and virtual server 1 becomes virtual server 2, ..., virtual server l becomes virtual server $l+1$. The transition rate is the arrival rate of the update, λ.

Case II. $l \in [n : 2n-1]$: When an update is received at the monitor from virtual server $l+1-n$, the age at the monitor changes to x_{l+1-n} and this server becomes idle. Using fake updates and fake preemption we assign $x'_j = x_{l+1-n}$, for all $l+1-n \le j \le n$. The transition rate is the service rate of a server, μ.

Below we state our main theorem on the average AoI for the single-source n-server network.

Theorem 1. *The average age of information at the monitor for homogeneous single-source n-server network where each server has a LCFS queue is:*

$$\Delta = \frac{1}{\mu}\left[\frac{1}{n\rho}\sum_{j=1}^{n-1}\prod_{i=1}^{j}\frac{\rho(n-i+1)}{i+(n-i)\rho}+\frac{1}{n\rho}+\frac{1}{n^2}\prod_{i=1}^{n-1}\frac{\rho(n-i+1)}{i+(n-i)\rho}\right], \tag{4}$$

where $\rho = \frac{\lambda}{\mu}$.

Proof: Recall that $\mathbf{v}$ denotes the vector $\mathbf{v}_0$ for the single state $q=0$. By Lemma 1 and the fact that there is only one state, we need to calculate the vector $\mathbf{v}$ as a solution to (2), and the 0-th coordinate v_0 is AoI at the monitor. As mentioned before, $\mathbf{v}A_l$ is in correspondence with $\mathbf{x}A_l$, so we have:

$$\begin{aligned}(n\lambda+n\mu)\mathbf{v} = \quad & [1,1,1,1,1,1,1,...,1] \\ & +\lambda[v_0,0,v_2,v_3,v_4,...,v_n] \\ & +\lambda[v_0,0,v_1,v_3,v_4,...,v_n] \\ & +\lambda[v_0,0,v_1,v_2,v_4,...,v_n] \\ & \qquad\vdots \qquad\qquad \vdots \\ & +\lambda[v_0,0,v_1,v_2,v_3,...,v_{n-1}] \\ & +\mu[v_1,v_1,v_1,v_1,v_1,...,v_1] \\ & +\mu[v_2,v_1,v_2,v_2,v_2,...,v_2] \\ & +\mu[v_3,v_1,v_2,v_3,v_3,...,v_3] \\ & \qquad\vdots \qquad\qquad \vdots \\ & +\mu[v_n,v_1,v_2,v_3,...,v_{n-1},v_n]. \end{aligned} \tag{5}$$

From the 0th coordinate of (5), we have $(n\lambda+n\mu)v_0 = 1 + n\lambda v_0 + \mu\sum_{j=1}^{n} v_j$, implying

$$v_0 = \frac{1}{n\mu} + \frac{\sum_{j=1}^{n} v_j}{n}. \tag{6}$$

From the 1st coordinate of (5), it follows that $v_1 = \frac{1}{n\lambda}$. Then, to calculate v_0, we have to calculate v_i for $i \in [2:n]$. From the i-th coordinate of (5),

$$((n-i+1)\lambda+(i-1)\mu)v_i = 1+\mu\sum_{j=1}^{i-1} v_j + \lambda(n-i+1)v_{i-1}. \tag{7}$$

For $i \in [2:n-1]$, from (7), we obtain

$$(i\mu+(n-i)\lambda)(v_{i+1}-v_i) = \lambda(n-i+1)(v_i - v_{i-1}).$$

Hence, $w_{i+1} \triangleq v_{i+1}-v_i = \frac{\lambda(n-i+1)}{(i\mu+(n-i)\lambda)}w_i$. Setting $i=2$ in (7), we have

$$((n-1)\lambda+\mu)v_2 = 1+\mu v_1 + \lambda(n-1)v_1. \tag{8}$$

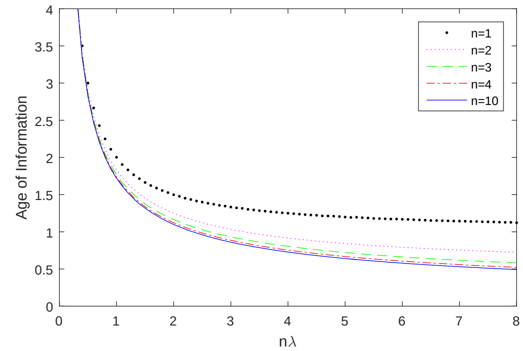

Figure 3: AoI versus the number of servers, for fixed total arrival rate. For each server, the service rate $\mu = 1$ and the total arrival rate $n\lambda$ is shown in the x-axis.

Simplifying (8), we obtain $w_2 = v_2 - v_1 = \frac{1}{(n-1)\lambda+\mu}$. Therefore, we write

$$w_j = \frac{1}{n\lambda}\prod_{i=1}^{j-1}\frac{\lambda(n-i+1)}{i\mu+(n-i)\lambda}, 2 \le j \le n. \tag{9}$$

Finally, setting $i=n$ in (7),

$$(\lambda+(n-1)\mu)v_n = 1+\mu\sum_{j=1}^{n-1} v_j + \lambda v_{n-1},$$

implying $\mu\sum_{i=1}^{n} v_i = \mu\sum_{j=1}^{n-1} v_j + \mu v_n = (\lambda+(n-1)\mu)v_n + \mu v_n - 1 - \lambda v_{n-1}$. Hence,

$$\frac{1}{n}\sum_{i=1}^{n} v_i = \frac{\lambda}{n\mu}w_n + v_n - \frac{1}{n\mu}. \tag{10}$$

Combining (6) and (10), we obtain the average AoI as

$$\Delta = v_0 = v_n + \frac{\lambda}{n\mu}w_n = \sum_{j=2}^{n} w_j + \frac{1}{n\lambda} + \frac{\lambda}{n\mu}w_n,$$

which is simplified to (4) using (9). ■

Figure 3 shows AoI when the total arrival rate $n\lambda$ is fixed and $n = 1,2,3,4,10$. We observe that for up to 4 servers, a significant decrease in AoI occurs with the increase of n. However, increasing the number of servers beyond 4 provides only a negligible decrease in AoI. In Figure 4, LCFS (with preemption in service), LCFS with preemption in waiting, and FCFS queue models are compared numerically. As can be seen from the figure, LCFS outperforms the other two queue models, which coincides with the intuition that exponential service time is memoryless and older updates in service should be preempted. Moreover, we observe that the optimal arrival rate for FCFS queue is approximately 0.5 for all $n \le 50$.

B. Multiple Sources and Multiple Servers

In this subsection, we present AoI calculation with the LCFS queue for the m-source n-server homogeneous network. Due to the limited space, proofs of theorems are provided in the online version of the paper [19]. The arrival rate of source i at any server is $\lambda_j^{(i)} = \lambda^{(i)}$, for all

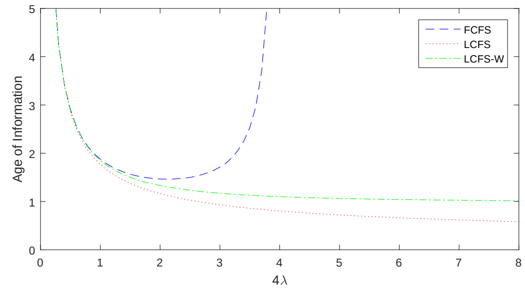

Figure 4: Comparison of LCFS, FCFS, and LCFS with preemption in waiting (LCFS-W). The number of servers is $n = 4$ and $\mu = 1$ for each server.

$i \in [m], j \in [n]$. The arrival rate of the sources other than source i is $\overline{\lambda^{(i)}} \triangleq \sum_{i' \neq i} \lambda^{(i')}, i \in [m]$. The service rate at any server is μ. Let Δ_i denote the average AoI at the monitor for source $i \in [m]$. Without loss of generality, we calculate Δ_1 for Source 1 under LCFS.

The continuous state $\mathbf{x}$ represents the age for Source 1, and similar to the single-source case, it is defined as follows: x_0 is AoI of source 1 at the monitor, x_i is the age of the i-th freshest update among all updates of source 1 in the servers. Therefore $x_1 \leq x_2 \leq .... \leq x_n$, for any time. Using fake updates and fake preemption as explained in Section III-A, we obtain an SHS with a single discrete state and $3n$ transitions described below:

Case I. $l \in [0 : n-1]$: A fresh update arrives at virtual server l from source 1. This update is the freshest update, so $x_1' = 0$. Now, the previous freshest update becomes the second freshest update, that is $x_2' = x_1$, and so on. Then $\mathbf{x}' = [x_0, 0, x_1, \ldots, x_l, x_{l+2}, \ldots, x_n]$. The transition rate is $\lambda^{(1)}$.

Case II. $l \in [n : 2n-1]$: A fresh update arrives at virtual server $l' \triangleq l + 1 - n$ from source $i \neq 1$. The age at the monitor does not change, namely, $x_0' = x_0$. The l'-th freshest update is preempted. Moreover, if the virtual server l' does complete service, it does not reduce the age of the source of interest. Thus, the l'-th virtual server becomes the n-th virtual server with age x_0. Therefore, we have $\mathbf{x}' = [x_0, x_1, \ldots, x_{l'-1}, x_{l'+1} \ldots, x_n, x_0]$. The transition rate is $\overline{\lambda^{(1)}}$.

Case III. $l \in [2n : 3n-1]$: the update of source 1 in virtual server $h \triangleq l + 1 - 2n$ is delivered. The age x_0 is reset to x_h and the virtual server h becomes idle. Using fake update and fake preemption, we reset $x_l' = x_h, h \leq j \leq n$. The transition rate is μ.

Dropping the index $q = 0$ and denoting $\mathbf{v}_0 = \mathbf{v} = [v_0, v_1, \ldots, v_n]$, the system of equations for the model is

$$\begin{aligned} n\mu v_0 &= 1 + \mu \sum_{i=1}^{n} v_i, \\ v_1(\overline{\lambda^{(1)}} + n\lambda^{(1)}) &= 1 + \overline{\lambda^{(1)}} v_2, \\ n(\lambda+\mu)v_i &= 1 + (i-1)\lambda^{(1)} v_i + (n-i+1)\lambda^{(1)} v_{i-1} \\ &\quad + i\overline{\lambda^{(1)}} v_{i+1} + (n-i)\overline{\lambda^{(1)}} v_i \\ &\quad + \mu \sum_{j=1}^{i-1} v_j + (n-i+1)\mu v_i, 2 \leq i \leq n, \end{aligned} \tag{11}$$

where $v_{n+1} \triangleq v_0$ and $\lambda = \overline{\lambda^{(1)}} + \lambda^{(1)} = \sum_{i=1}^{n} \lambda^{(i)}$.

The theorems below state the average AoI for $n = 2, 3$ servers, and determine the optimal arrival rate given the sum arrival rate.

Theorem 2. *For m information sources and $n = 2$ homogeneous servers, the average AoI at the monitor for source i, $1 \leq i \leq m$, is*

$$\Delta_i = \frac{1}{2(\lambda+\mu)} + \frac{\lambda+\mu}{2\mu\lambda^{(i)}}. \tag{12}$$

Theorem 3. *For homogeneous m sources and $n = 3$ servers,*

$$\Delta_i = \frac{1}{3\mu} \frac{(5\rho^{(1)} + 2(\rho+1)^2)(\rho+1)}{2\rho^3 + 5\rho^{(1)}\rho + 2\rho^{(1)}}, \quad 1 \leq i \leq m,$$

where $\rho = \frac{\lambda}{\mu}$ and $\rho^{(i)} = \frac{\lambda^{(i)}}{\mu}$.

Theorem 4. *Consider homogenous m sources and 2 servers. The optimal arrival rate ${\lambda^{(i)}}^*$ minimizing the weighted sum of AoIs, i.e., $w_1\Delta_1 + w_2\Delta_2 + ... + w_n\Delta_n$ for $w_i \geq 0$, subject to the constraint $\lambda^{(1)} + \lambda^{(2)} + ... + \lambda^{(m)} = \lambda$, is given by*

$${\lambda^{(i)}}^* = \frac{\lambda\sqrt{w_i}}{\sum_{i=1}^{m} \sqrt{w_i}}, i \in [m].$$

IV. Heterogeneous Networks for a Single Source

In this section, we consider a single source and assume that the arrival and service rates of the servers are arbitrary. We denote by $\lambda_j^{(1)} \triangleq \lambda_j$ the arrival rate of the single source at server j, and μ_j the service rate of server $j \in [n]$. For this setting, we can no longer use the same technique used in the homogeneous case to reduce the state space and derive AoI. In particular, we need to keep track of the age of updates at the physical servers as well as their ordering, resulting in $n!$ number of states. In the following, we illustrate the steps for deriving AoI in the case of $n = 2$ servers.

Theorem 5. *Consider $m = 1$ source and $n = 2$ heterogeneous servers. The average AoI is given by*

$$\Delta = \frac{1}{\mu_1+\mu_2} + \frac{1}{\lambda_1+\lambda_2} + \frac{1}{\mu_1+\mu_2}\frac{1}{\lambda_1+\lambda_2}\left(\frac{\mu_1\lambda_2}{\lambda_1+\mu_2} + \frac{\mu_2\lambda_1}{\lambda_2+\mu_1}\right). \tag{13}$$

Proof: We define State 1 as the state that Server 1 contains a fresher update compared to Server 2 and State 2 as the state that Server 2 has the fresher update. Table II summarizes the transition rates and the mappings in the system.

Based on Table II and solving (1) and (2), we can derive the exact expression of AoI. Explicit derivations can be found online at [19]. ∎

For $n = 2$ servers, we find the optimal arrival rates. The derivation can be found in [19].

l	$\lambda(l)$	Transition	$\mathbf{x}' = \mathbf{x}A_l$	$v_{q_l}A_l$
1	λ_1	$1 \to 1$	$[x_0, 0, x_2]$	$[v_{10}, 0, v_{12}]$
2	λ_1	$2 \to 1$	$[x_0, 0, x_2]$	$[v_{20}, 0, v_{22}]$
3	λ_2	$1 \to 2$	$[x_0, x_1, 0]$	$[v_{10}, v_{11}, 0]$
4	λ_2	$2 \to 2$	$[x_0, x_1, 0]$	$[v_{20}, v_{21}, 0]$
5	μ_1	$1 \to 1$	$[x_1, x_1, x_1]$	$[v_{11}, v_{11}, v_{11}]$
6	μ_1	$2 \to 2$	$[x_1, x_1, x_2]$	$[v_{21}, v_{21}, v_{22}]$
7	μ_2	$1 \to 1$	$[x_2, x_1, x_2]$	$[v_{12}, v_{11}, v_{12}]$
8	μ_2	$2 \to 2$	$[x_2, x_2, x_2]$	$[v_{22}, v_{22}, v_{22}]$

Table II: Table of transitions for $n = 2$ heterogeneous servers.

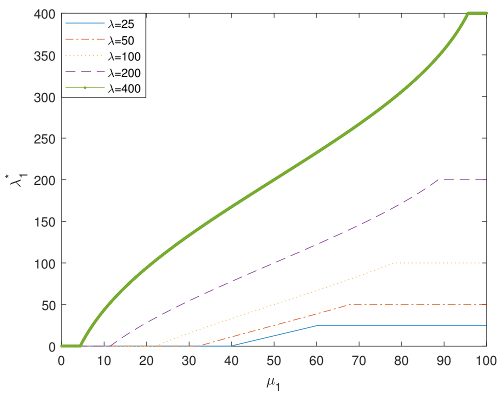

Figure 5: Optimal value of λ_1 as a function of μ_1. $\lambda_1 + \lambda_2 = \lambda, \mu_1 + \mu_2 = 100$.

Theorem 6. *For $m = 1$ and $n = 2$ heterogeneous servers, given μ_1, μ_2 and fixed $\lambda_1 + \lambda_2 = \lambda$, the optimal ${\lambda_1}^*$ satisfies*

- *if $\mu_1 < \mu_2$ and $\mu_2^2 - \frac{\mu_1(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_2} < 0$:*

$$
{\lambda_1}^* = \frac{-(\mu_2 + c(\lambda+\mu_1)) + \sqrt{\mu_1(\lambda+\mu_2)(2 + \frac{\mu_2}{\lambda+\mu_1} + \frac{\lambda+\mu_1}{\mu_2})}}{1 - \frac{\mu_1(\lambda+\mu_2)}{\mu_2(\lambda+\mu_1)}},
$$

- *if $\mu_1 < \mu_2$ and $\mu_2^2 \geq \frac{\mu_1(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_2}$: ${\lambda_1}^* = 0, {\lambda_2}^* = \lambda$,*
- *if $\mu_1 > \mu_2$ and $\mu_1^2 \geq \frac{\mu_2(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_1}$: ${\lambda_1}^* = \lambda, {\lambda_2}^* = 0$,*
- *if $\mu_1 > \mu_2$ and $\mu_1^2 < \frac{\mu_2(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_1}$:*

$$
{\lambda_1}^* = \lambda - \frac{-(\mu_1 + \frac{(\lambda+\mu_2)}{c}) + \sqrt{\mu_2(\lambda+\mu_1)(2 + \frac{\mu_1}{\lambda+\mu_2} + \frac{\lambda+\mu_2}{\mu_1})}}{1 - \frac{\mu_2(\lambda+\mu_1)}{\mu_1(\lambda+\mu_2)}},
$$

where $c = \frac{\mu_1(\lambda+\mu_2)}{\mu_2(\lambda+\mu_1)}$.

The optimal ${\lambda_1}^*$ is illustrated in Figure 5. When $\mu_1 = \mu_2$ the optimal rates that minimize AoI are ${\lambda_1}^* = {\lambda_2}^* = \frac{\lambda}{2}$. As Figure 5 illustrates, for $\mu_1 = \mu_2 = 50$, optimal rates are ${\lambda_1}^* = \frac{\lambda}{2}$ and in the regimes that one of the service rates is much greater than the other one, AoI minimizes when all the updates are sent to the server with the greater service rate.

V. Conclusion

In this paper, we studied the age of information in the presence of multiple independent servers monitoring several information sources. We derived AoI for the LCFS queue model using SHS analysis when we had a homogeneous network and a single source. We also provided the AoI formula for m sources and $n = 2, 3$ servers in a homogeneous network. For a single-source heterogeneous network, the case of $n = 2$ servers were investigated. Moreover, optimal arrival rates are obtained when the sum arrival rate and the service rates are given. Future directions include deriving explicit formula of AoI for multiple sources in a homogeneous and heterogeneous sensing networks where the update arrival rate and/or the service rate are different among the servers for any number of sources and servers.

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